Control Engineering
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Module - 11
Lecture - 01
Part – 02
Navigation – Dead Reckoning and Reference Frames

Hello and welcome to module eleven lecture 1 part 2 of navigation. It is a warm day in Chennai. So, I hope I do not sweat too much when talking now alright. So, in the previous lecture we saw some basics of navigation. So, we saw some we discussed some really interesting story of Alexander's march, how dessert ants search for food how to do celestial navigation and so on.

We briefly discussed dead reckoning celestial navigation, coastal navigation and something about GPS and other things although that was very brief. What we are going to do today is to dig a little bit deeper into the basic the first cut equations of dead reckoning, and we will see why reference frames are very important. In the next lecture I will actually show to you the kind of errors that are induced because of dead reckoning alright and those errors primarily because of sensor characteristics very good.

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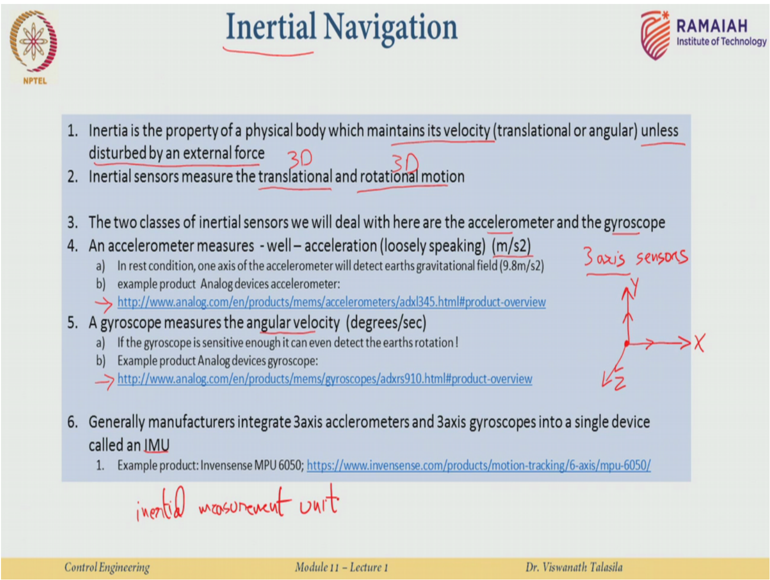

So, let us start inertial navigation. So, the reason why it is called inertial navigation is because we use sensors which respond to Newton's laws. So, you consider an accelerometer or gyroscope and these objects have certain inertia like all physical objects, and these objects they maintain their velocity either translational velocity for acceleration or angular velocity for the gyroscope. They maintain their acceleration and velocity unless they are disturbed by an external force that is an Newton's inertial law right.

So, this sensor when they make the measurements, when they disturbed by an external force this is why they are inertial sensors alright. So, what do they actually measure well they measure two things. One is the translational motion which is say the motion of this object as it is going in a particular line right. So, it could rather go straight, it could go left right up down alright. It could also measure the rotational motion and the rotational motion would basically be I have this aircraft, and I have an angular motion with respect to an axis, I have this rotation and I have the third rotation we will see little bit more about this. So, since we live in a 3 D world we are always going to look at 3D translator motion and 3D rotational motion alright. Now there are various types of inertial sensors the two sensors we will deal within in this and the next series of lectures are the accelerometer and the gyroscope. We could also be doing the magnetometer, but due to lack of time we will focus on just these two.

So, what does an accelerometer measure? Well loosely speaking it measures the acceleration, and the units of that is meters per second square alright and accelerometer is a very interesting object. So, whenever we talk of accelerometer or gyroscopes from this time onwards, we will be referring to what are called as three axis sensors right. And three axis basically means that because we live in a three dimensional world, we have motion along an axis we can call it x axis, we have a motion along perpendicular to the x axis called the y axis. So, that is in a plain, and then perpendicular to the plain we have the z axis. In the literature they commonly do not call it as x y and z, they are specific notations for this we will see this a little bit later, but for now we really need to know that motion takes place in three dimensions, and each of these three axis sensors they measure the motion along each of these axis alright. It could be a gyroscope it could be an accelerometer as well.

So, I have also included some links for an accelerometer and for a gyroscope both are analogue devices, but there are plenty of other options as well there is inventions and the Honeywell products and so on. This is just an example for you to see what these products are about. A gyroscope measures the angular velocity we will see this a little bit later what I specifically mean by this. So, the gyroscope is used to measure the rotational behaviour of an object the accelerometer is used to used to measure the translational behaviour of an object alright.

Historically accelerometers and gyroscopes would come as two independent products, for may be the last decade or two decades manufacturers have integrated these accelerometers and gyroscopes into a single device called the IMU IMU is basically an inertial measurement unit. So, you these small chips which we shall show you may be in a couple of lectures with some experiments which one of my research associates Vinay Sridhar will be presenting, we will show you the basic IMU which are the sensors mounted in that and examples of these ok.

So, we are going to be working predominantly with IMUs which have an integrated accelerometer and a gyroscope on board good.

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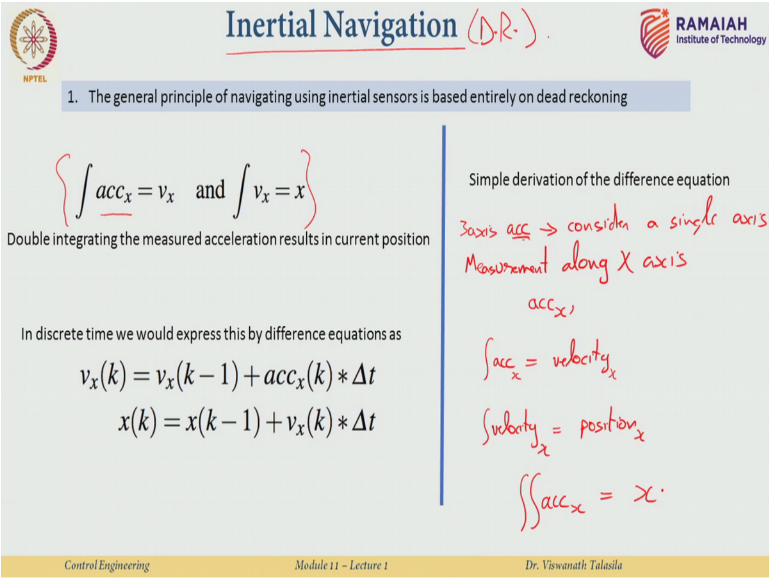

So, let us look at little bit of dead reckoning in the very basic sense. So, this slide is basically on dead reckoning alright. So, as I have said the general principle of navigation using inertial sensors, is based entirely on dead reckoning. Now let us look at the example of accelerometers in this slide, in the next slide we will go to gyroscope. So, what does an accelerometer measure? We saw that it measures the acceleration. So, here I have written.

So, let us say you have a three axis accelerometer, we consider a single axis let us say the x axis, you could also consider y or z and the measurement along that x axis will simply we can denote it as we can denote the measurement along the x axis as simply a c c underscore x, and along the y which I am not mentioning here would be a c c underscore y along the z axis a c c underscore z alright. So, let us say we have made a measurement along the x axis, and now we want to see how to compute position based on this. By the basic laws of physics we know that the integral of acceleration is nothing, but velocity right, inverse of that is derivative velocity is nothing, but acceleration. So obviously, if you have if you are integrating over the x axis, the velocity which you compute will be also along the x axis.

Again from basic physics we know that the integration of velocity is nothing, but position alright the position of the object. And as before if you are integration along x the position computed will also be along x. So, that is basically what this expression over here says right, which is basically the same as saying if you double integrate the acceleration what you are going to end up with is a position x right we will do this . So, now, that we seen the basic say calculus of how we compute position from acceleration, all of you have done this in high school. Now remember that our sensors are usually connected to micro controllers right or to computers or to any electronic device being computers and micro controllers, they can take only indiscrete values of the actual data. So, if your real let us say this is time axis, this is the acceleration along let us say they y axis right.

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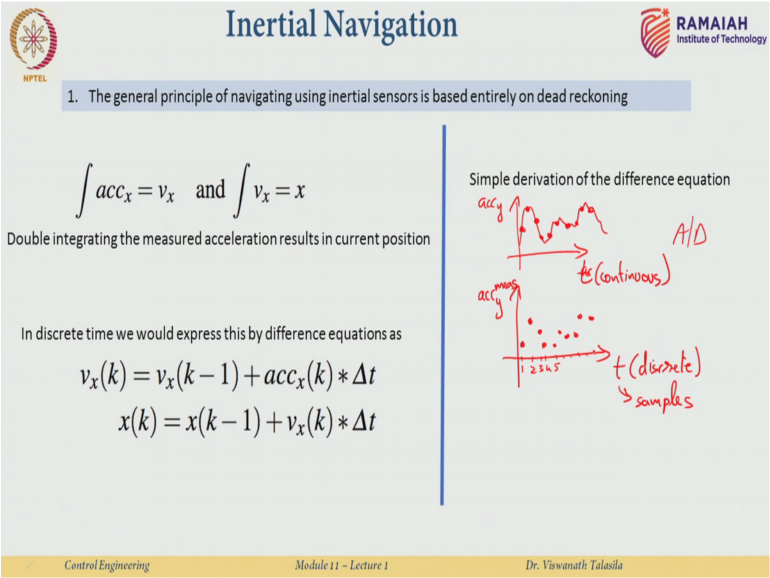

So, let us say the vehicle was accelerating in the particular manner, when you actually try to convert this or connect this sensor to a micro controller, you need to pass it through a analogue to digital converter right, and depending on the resolution of the analogue to digital converter, you actually get a 16 bit 10 bit 8 bits so on and so forth right.

So, the accuracy increases with the increase in resolution. So, with whatever accuracy with whatever resolution we have taken, we will be able to recover some parts of this data right. So, the complete curve is a true acceleration, what your sensor is able to measure are these points it cannot measure anything in between, unless you increase the sampling rate of the sensor and unless you increase the resolution analogue to the digital converter alright. So, now, in the discrete world or in the discrete times, this is the continuous time. So, in the discrete time which as all of you know nothing, but samples how will this look like the measurements. So, I call it a c c y actually measured or measured and then discretized and sampled it will basically look like this right that is going to be your measurement.

Now this is the actual data which is going to come into your micro controller. So, I would call this as a sample number 1, sample number 2, 3, 4, 5 and so on. Now based on these measurements which we are taking the acceleration measurements our objective is to compute our position right that is a whole point of dead reckoning, based on whatever you know in the past you want to predict your future. So, based on the current acceleration, I would like to know what happens next or a different way of putting based on the acceleration which I have measured now I want to know where exactly I am. So, you have to double integrate the acieration.

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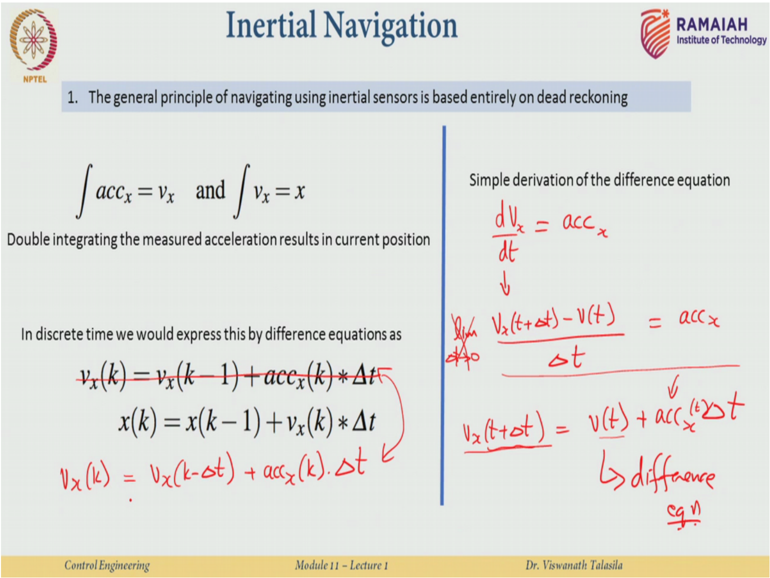

How did we do this in the discrete world? Because you are now inside a computer your data has come inside your micro controller or you computer. So, how we actually compute this? Well let us look at the basic equations first right. So, we see that d V by d t where V is a velocity is nothing, but your acceleration. So, let us just take it along the x axis this is nothing, but the limit V x of t plus delta t minus V of t whole divided by double delta t is equal to acceleration x we know this from basic calculus that the is derivative expressed as a limit delta t tends to 0 blah blah blah. So, on now of course, in the discrete world we do not have the have the delta t really tending to 0, it it is really small you know it could be 0.01, may be 0.0001 and so on.

But it is never perfectly 0. So, let us fix the delta t to a certain value we will just call it as delta t, it could be any number depending on how fast you are able to sample your signal alright. So, we can remove the limit for now because it never really goes to 0 and you are left with this basic expression in the discrete time. How do I if I expand this expression? I will basically get V x of t plus delta t is equal to V of t plus a c c x times delta t. This is your difference equation which does the following; it uses the current information of the measured acceleration note that this is a function of time, it uses the current velocity which is known from the previous calculation which we have done and it calculates the next velocity. So, it predicts what the next position the next velocity is going to be. Now I have expressed this slightly differently in this equation over here. So, instead of t I have taken k and ok.

So, let us see how this looks like. So, if I write this equation over here this will basically look as follows. So, V x of k note that in my case I took t plus delta t, in this case I am taking k because I am doing a k minus delta t in the other place. So, V x of k is nothing, but V x of k minus delta t alright plus acceleration x times k times delta t alright. This is a same as this equation x accept that I have used a slightly different notation over here and here.

So, we will not be using this notation so much this is the kind of notation which you will see when you write your code, and I will show you this a little bit later. For our discussion now we will use this notation we will come back to this when I explain the code to you. So, this is how you compute the acceleration the velocity from the given acceleration alright. Now how do we compute the position in exactly the same way that we have looked at the computation of the velocity equation? So, now, that we know what is a current velocity which we have computed by using the acceleration measurement, we are just gaping to repeat the same process for computing the position and that is again follows the same rule that d x by d t is nothing.

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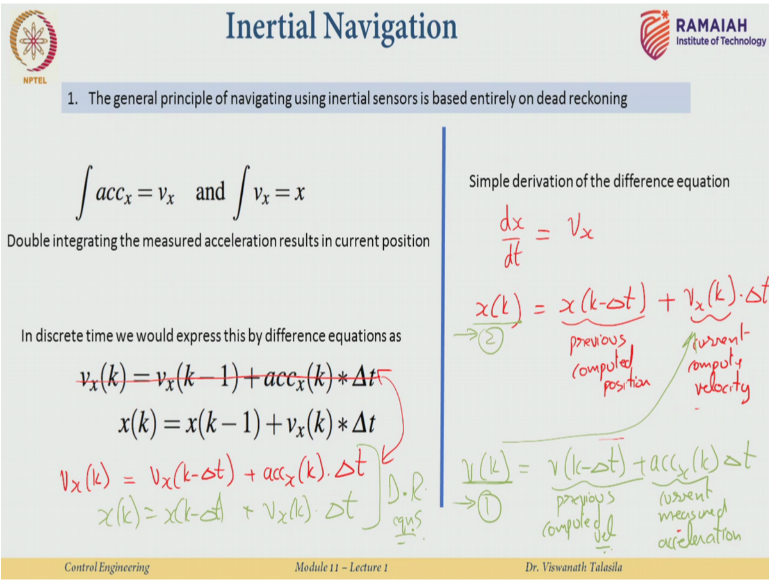

But V x the velocity, this basically would say if I write this in the notation of k, it basically says x of k plus or let us do it. So, x of k is nothing, but x of k minus delta t the previous computed position plus the current velocity which I have computed from integrating the acceleration before times the delta t ok.

So, this is the previous computed position, this is the current computed velocity; let me recall the previous equation of how we computed the velocity. So, there we had V of k was equal to V of k minus delta t plus a c c x of k times delta t. So, in that case again as we have seen now say. So, this was the previous computed velocity right this is the current measured acceleration by the accelerometer. So, this would be step one this would be step two. So, you first compute the velocity based on the previous known velocity and the current acceleration then having the value of this velocity now you plug this into this equation for computing the position.

So, the current position is now equal to the previous computed position plus the current computed velocity times delta t and that is basically this expression over here. So, this is just be x of k equal to k of k minus delta t plus that is wrong. We are going to take the velocity plus velocity which is computed at time k times delta t right. So, these are my dead reckoning equations, will basically follows from the mathematical principle that if I double integrate the acceleration I get position. To do that in a computer you need to first integrate it once to get velocity integrate that the second time to get position. So, these are the equations we are going to be using in our micro controller programming which I will show you tomorrow to compute the actual position.

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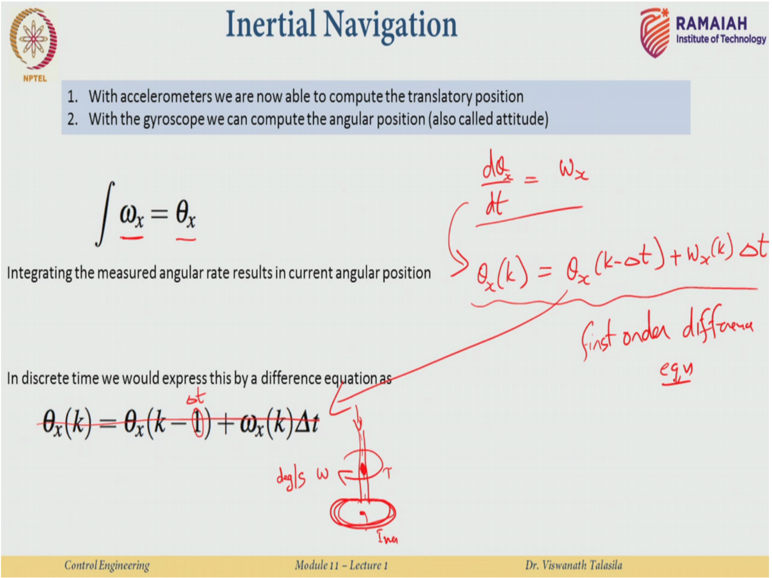

So, this is the translation position because we using the accelerometer data, what happens when we use the gyroscope data if you want to visualise this just think of this as let us say there is a disk over here, and it is spinning along this axis. So, this disk will have some inertia, it will have some mass and so on and you apply a torque on this disk and you rotate it right.

Say for example, I have connected a motor over here and I am I am continuously rotating the disk, and here if I place a gyroscope at this location it is going to measure the angular rotation rate in the units of degrees per second. Now that is what the angular rotation rate is. Now what I want to compute is the angular position. At every instant of time I want to know where this disk is. I want to know whether I have rotated 5 degrees or 10 degrees and so on. Given the measurement is angular velocity.

So, of course, by basic physics we know that integration of angular velocity is nothing, but angular position, and we use exactly the same story as before since d theta by d t along the x axis of course, is omega x. I will rewrite this in time in the discrete time domain which we get after sampling the angular velocity data from the gyroscope. So, this will basically become theta x at the sample number k is the previous computed angular displacement. So, k minus delta t plus the current measured angular velocity, which comes in from my gyroscope times delta t and that is exactly what we have over here ok.

So, this is actually should be delta t it is more a computer notation which I have used over there. So, this will come to later when we looking at the code, we will consider this equation for now. So, we see again this is a first order difference equation because we only had a first order difference equation over here alright. So, in both the cases for the accelerometer and the gyroscope we compute the first order difference equations in order to get the angular position and the angular and the translatory position alright. So, that is done and now of course, we have a slightly different issue, which is basically that each sensor as I have said before is a three axis sensor.

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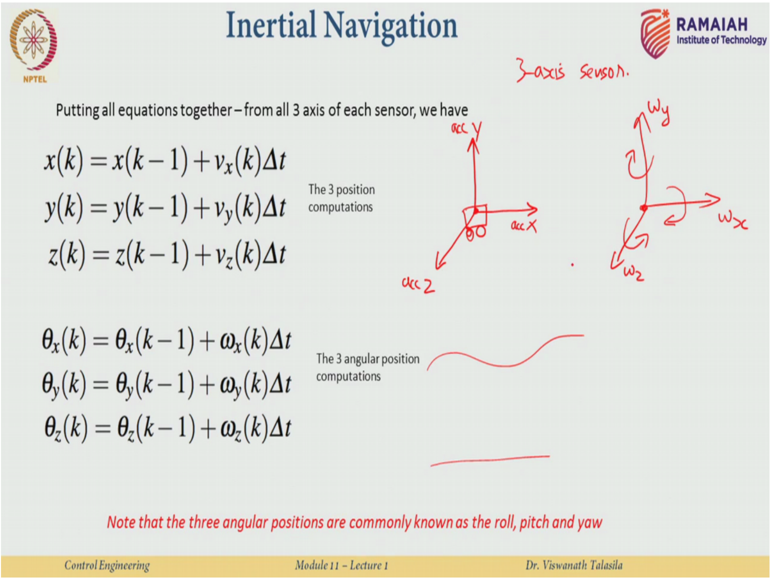

So, this specifically means is that let us consider an accelerometer. So, again I will explain this very very briefly before and let us say that you have a vehicle it is a car or something like that it is a pretty ugly car, but is good enough for our story. So, let us say you have a car and it is moving, it is its moving on a terrain right it is not a street road it is moving on a terrain like an off road vehicle and in the Himalayas or somewhere, and let us say we have these axis which we denote with x y and z. Now if it is moving on only one in only one direction in only axis say it is a perfectly flat road.

So, let us say it is moving only along the x direction, the three axis accelerometer which can be visualised like this. So, this will be one axis the other axis and the third one each one is orthogonal or perpendicular to each other. So, these three axis accelerometer will measure the acceleration of the car only in the direction in which it is moving right. So, it will not measure this direction or the acceleration measurement will be 0 in this direction it will also be 0 in this direction alright.

It measures only in the forward in which it is moving similarly if it is just moving left and right say a plane or helicopter moving left and right, along this axis over here right. So, along this axis it actually gives me a non zero acieration, but along the forward axis it will show 0 along the along the vertical axis it will show 0 right. If you move in a combination of two different directions, both the x and the y axis measurement of the acceleration will show non zero values. If it moves in 3D all three axis will show non zero values. So, that is what an accelerometer measures, a gyroscope in a similar way again have.

So, I can call this as a c c along x, a c c along y a c c along z or gyroscope will measure the angular rotation rate along with respect to an axis I will give a brief of that now. So, let us say that I have my three axis sensor, we will call this the one pointing say pointing towards you as a forward axis, the one pointing to the right as one axis and the vertical axis completes the orthogonal system or the perpendicular axis system. So, I have three axis all perpendicular to each other. Now let us say I take the system and I rotate it about this axis alright how would I rotate it about the forward axis? I keep the forward axis fixed and I rotate it right. So, you can see the motion over here.

So, this motion is a angular rotation about the forward axis, what about the rotation about say the right side axis the x the y axis. So, I keep this one fixed and I rotate it this way alright. So, this is our angular rotation about this axis finally, I keep this fixed and I rotate it over here, this is another rotation about this axis. So, we have three rotations and the rotations are measured with respect to each of the axis. So, it would look like this. So, I have a rotation about the x axis or I can have a rotation about the y axis or rotation about the z axis or you can have a combination of rotations.

So, it need not just be a rotation like this or rotations like this, it can be a rotation like this right and that is a combination of all three rotations. When you take all of these then basically what you will get are these three equations in the computer in the discrete time case right. So, this is the computed position along the x direction assuming you have computed the velocity along x, you have the position along y position along z and similarly we have the angular position along x, angular position y, angular position along z after we made the measurements along x y and z angular velocity measurements. For those of you who either watch lots of movies or play some nice video games like flat simulators and those or read some crazy novels, you may immediately recognize that these three angular positions are very commonly known as the roll the pitch and the yaw alright we will see this in the next slide.

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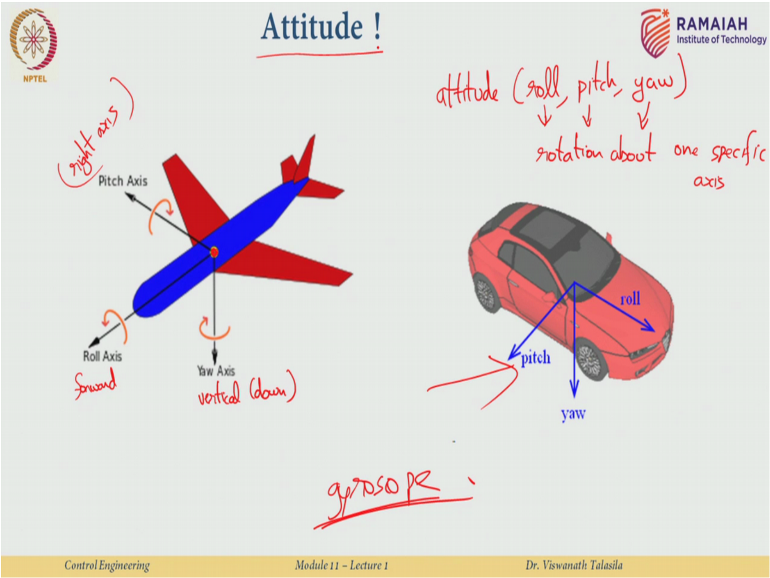

So, just small terminology the word attitude in navigation it does not refer a good attitude or bad attitude, it refers to these three angles we are talking about. The roll angles the pitch angle and the yaw, and note that each of these angles is a rotation about one specific axis. So, for example, the roll angle is usually called as a rotation about the forward axis. So, I keep the forward axis fixed and I roll my vehicle right. If I keep the side axis fixed and I pitch up and down you know.

So, the pilot who is sitting in the plane they pull on the you can call it a joystick they pull on the joystick and the plane pitches up you push it down it pitches down alright the final one is a rotation about the z axis, this is actually the heading or the yaw. So, you keep it over here and you ask the plane to move to change the direction to the right. So, it rotates to the right left it rotates to the left about this axis. So, that is a roll pitch and yaw and you can see this small illustration over here. So, you mount your sensor at this place the gyroscope and this is the forward axis, this is the vertical down axis as it is called, and this is the right axis which completes the orthogonal axis system. So, whenever you talk of roll you typically talk of the aircraft rolling with respect to the forward axis, as you can see my forward axis is not moving. When you talk of pitch this axis does not move. So, it is just a rotation like this and the same for the yaw. This does not apply only to aircraft it applies to cars as you can see over because cars especially off road cars when they move on uneven terrains you have a roll pitch and a yaw.

As many of you would have experienced if you take a car ride through let us say Bangalore's amazing roads filled with pot holes, you often you know you go into these kind of roll pitch and yaw. And of course, this applies to all movement you can talk of human movement right. So, I am standing here with respect to you I may be doing a yaw I may be I may be doing a pitch up my hand may be doing a combination of angular rotations and so on. Every one of this can be measured by the use of a gyroscope fine now attitude ok.

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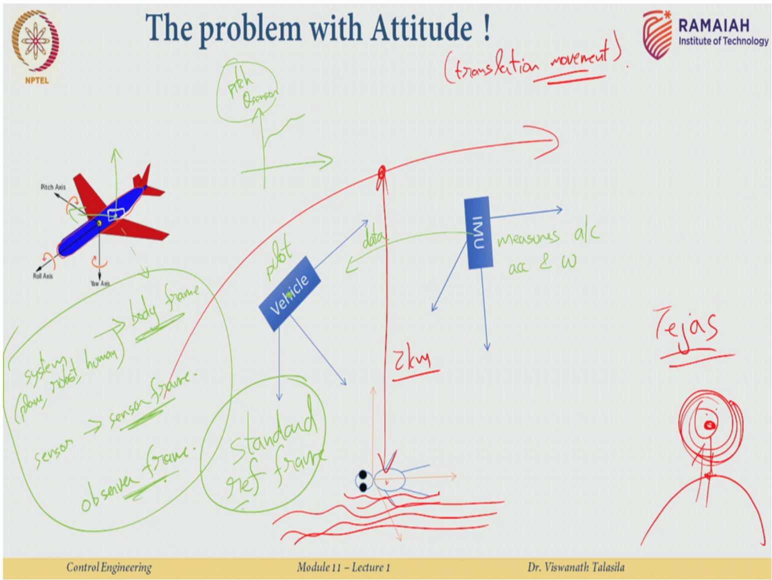

So, the roll pitch yaw attitude is good, but sometimes attitude is not good and it is always problem with attitude well. Let us see what is a problem with attitude over here as it is not just attitude, I need to be more precise attitude as well as a translational position or translation movement you can call it. So, let us say there one of you cool people are lying on a beach you know wearing some dark shades and they are just looking up at the sky, and you see a nice aeroplane flying across over here alright. And you know there is a friend of yours who is sitting in another part of the of the town or the city, and you want to call up this guy and say hey look my nice or this amazing the LCA Tejas aircraft you know Indias own fighter combat plane brilliant stuff.

Now, this guy is going to ask. So, the guy who is sitting in the town he is going to ask his friend dude, where did you actually see this plane. Now this guy is going to say I saw the plane let us say approximately two kilometres above this guy has no clue what it means by 2 kilometres above him. He does not know where he is right. So, the guy in the town does not know where the guy on the beach is does not know what 2 kilometres above him means especially if this guy is sitting on top of a mountain, and this guy is on the beach at almost at the sea level.

So, he has no clear idea of where the plane actually is with respect to him. The person on the beach knows where the aircraft is, but the person here based on the information which the beach dude has given him he is unable to configure or figure out where exactly the aircraft is. Is a further problem, you have the aircraft shown over here now of course, we need the aircraft flight control system needs to be able to compute it is own position. So, it places this gyroscopes accelerometers and all these things in the aircraft. Now where does we place it let us say for example, it has placed at let us use a different colour white let us say it has placed at over here. So, this box is actually a very small box is placed over here, and this box is actually going to the measurements of angular rotations as well as the accelerations the IMU.

Now, what is a problem with this as you can see the aircraft axis is shown over here with a roll the pitch and the yaw, and aircraft axis is typically defined with respect to the center of mass of the aircraft or the center of gravity of the aircraft. The IMU in this case is placed at a slightly different location and it is not aligned with the aircraft axis. So, if the aircraft axis is like this, the IMU axis could be like this right because it depends how I placed my aircrafts. So, if this is a aircraft very bad example I agree, if this is a aircraft let us say this is a IMU. if I place it perfectly aligned with a axis of the aircraft that is good, but because of various reasons, I have placed it like this and the axis are completely miss aligned. So, in that case the axis of the IMU will be like this that would be the forward axis this would be the pitch axis if you want to call it, and that would be the vertical axis. And you can see that the two axis are not aligned at all, which means the IMU which measures that aircraft acceleration and angular velocity, it is measuring in it is own frame of reference right the three axis of the accelerometer. With respect to these three axis it is telling the aircraft is rotating about a certain degrees per second, but the aircraft is in a different axis representation.

So, the aircraft is in this axis representation the IMU is in a different axis representation alright. So, you essentially have now a bigger problem; a guy the pilot in the aircraft if she or he looks at the data coming from the I MU. So, let us say the data coming from the IMU is displaced on the screen of the pilot and it is saying that look man let us say your pitch angle, is currently going like this.

And the pilot is not really sure if this is correct or not because this pitch angle is measured by the sensor, and it is not the same as the pitch I actual pitch angle of the aircraft because it is in the slightly different position. For all you know if this is on my left hand this is aircraft axis the right hand is the sensor axis, the way the designers mount the sensor on the plane it could be completely 90 degree shifted. So, the pitch axis becomes the roll axis roll axis becomes the pitch axis. So, when the pilot is looking at the data from the pitch over here, it is actually the roll axis of the aircraft and the pilot has to make a decision based on that it can be catastrophic.

So, this is a problem with reference frames the problem in latitude, problem in transitional movement, every object which measures movement, measures it in it is own frame of reference, it is own axis or coordinate system. The actual system the plane or a robot or a human walking is experiencing motion in it is own reference system. So, typically you have the system which could be a plane or a fancy robot a human walking and so on, this has it is own frame of reference it is own coordinate system this is called the body reference frame.

A sensor which is mounted on the body to measure the motion of the body or you can actually say there is a camera over there and it is capturing my movement, the camera is recording my movement or the sensor on my body is recording my movement in it is own reference frame called the sensor frame, and then you have an observer like you or me standing outside and just you know watching this cool plane flying above, then you have a observer frame. Now with three different frames how are you going to say the aircraft is exactly there or the robot is exactly over here, you know autonomous robot which are wandering all over the place. Like this terminator movies, how would you say that the terminator is right over here. You can only do that if there is a standard reference frame and this is actually a very important problem in robotics in ship and aircraft navigation, human movement everything. Every movement must be expressed with respect to what standard reference frame it cannot be expressed with respect to his reference frame or her reference frame, it should be with respect to a standard reference frame which everyone understands.

However, measurements are done in the sensor frame, the body is moving in it is own frame, the observer has his own frame. So, what we will need to do is whatever measurements are done in the sensor frame we need to translate it or we need to transform it into the standard reference frame. Whatever the body is experiencing transform that into a standard reference frame, then you know exactly what is happening to the motion of the object alright.

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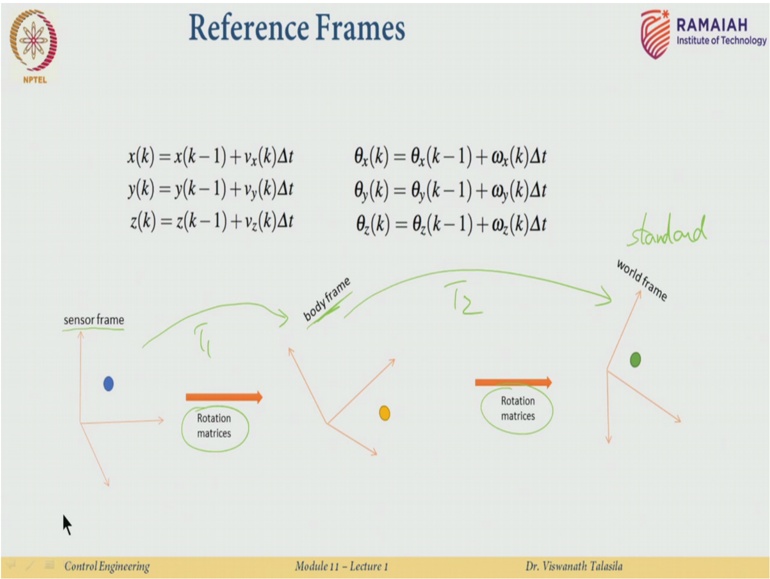

So, this is the problem which I have been talking for the last two three minutes, and the way that people address this problem in navigation is, when a sensor measures you know the acceleration and angular velocity and all three axis. We transform the measurements usually first into the body frame into the aircraft or to the robot. So, we know exactly what is a measurement of the aircraft movement, we do not care what this sensor is experiencing really we care what the aircraft is experiencing specifically the movements right. So, we transform the sensor measurements into the coordinate system of the aircraft or the robot or the human being who is walking, once you are in the body frame of course, you are body frame is different from his body frame right it is the body coordinate system is different from his body coordinate system. So, you need to now transform it into the standard reference frame. So, for each of these transformations you have something called a rotation matrix. So, apply the rotation matrix and you actually go from one reference frame to another reference frame. We will see now one simple example of how to actually do the rough the rotation matrix then we will go to the complex 3D case. So, let us see how we can do this reference frame transformation, we will start with a very single simple example.

Let us frame the question properly. So, the question would actually be given a motion which is measured in one reference frame, let us say it is the sensor reference frame ok.

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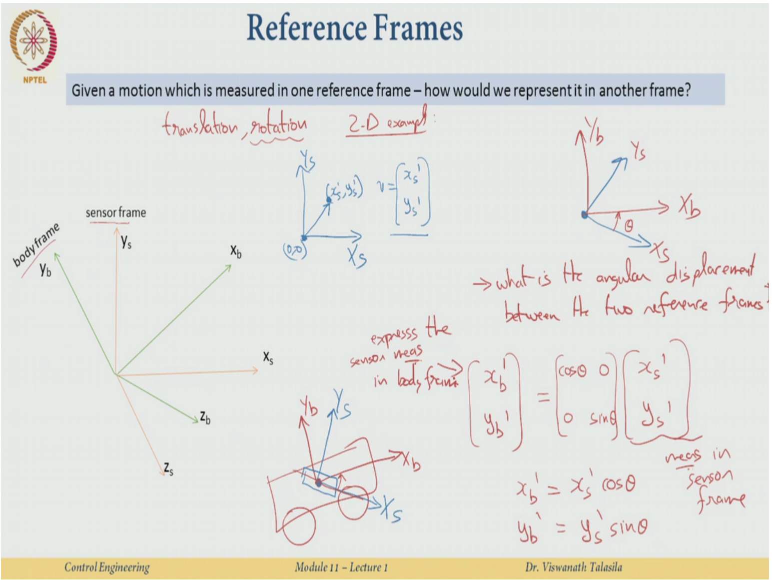

How would you represent this motion in another reference frame, let us say the body reference frame alright. So, that is the basic question which we are asking, we will actually pose the question as follows. So, given a motion which is measured in one reference frame, how would we represent it in another reference frame? We always have to keep track of two quantities one is the translation the other is the rotation. We will focus primarily on the rotation over here, I will briefly explain how to take of the translation afterwards. So, we have two reference frames as you can see here the sensor reference frame and the body reference frame, you need to represent one in terms of the other, let us take a very simple 2D example, and in this 2D example I will assume I have got a vehicle and the center of gravity of the vehicle is over here, with a axis as follows. So, I will call this x body axis and y body axis.

Now what I have done is to mount IMU at exactly at the center of gravity of the vehicle, now usually this will not be the case and if it is not the case you need to take care of the translation part as well. If you mount at exactly at the center of gravity of the vehicle, you only have a rotational problem which is what we will deal with here. So, let us say we mount the sensor like this and the sensor origin coincides with the origin of the body coordinate system. Except that the sensor is misaligned with respect to the body coordinate system. So, the sensor coordinate system will be like this.

I will call this as or let me rewrite that a little bit I will call this as x sensor and this is y sensor. So, I can see that the two origins are perfectly aligned, there is a difference in the rotation between the two sensors; and it is in intuitively very clear already by now to most of you all you need to do is to merely rotate the sensor reference frame by certain angle to match with the body reference frame alright, and that is exactly what the 2D example is all about. So, let us say that in the sensor reference frame the X s and Y s, I have said with respect to the origin of course, we can call it 0 comma 0, it does not really matter it can be any x comma y because two origins are perfectly aligned.

Now with respect to the origin let us say I have measured a certain motion which is over here and this could be acceleration, it could be angular velocity, whatever it is depending on whether I am using an accelerometer or a gyroscope over here any one of the sensors you can assume. And you see that any point in this is a measurement with coordinates and it can be called as x s 1 and y s 1. What is x s 1 and y s 1 with respect to the origin it is simply a vector right basic high school physics. So, this is a measurement which is done, I need to now take this vector which is the sensor measurement x s 1 and y s 1, and I need to transform this into the body reference frame. Because I want to know in my body reference frame what is the measurement, which I am seeing I do not care what is in the sensor reference frame I want to know in my body reference frame.

So, given this how do I transform it into the body reference frame, very simple. So, we have the two this is the sensor frame, and then you have the body reference frame right X b and Y b this is X s Y s. Now you need to know only one important quantity in the 2D example only one quantity, which is what is the angular displacement between the two reference frames that is all I mean it is intuitively clear to all of you by now what it is that that we need. So, if we know what is this angular difference let us just call it theta. If you are able to measure this theta we know exactly what we need to do well; let us say our gyroscope is actually able to measure this theta or we know from a third sensor or we have manually measured it ourselves, that the two axis are slightly deviated with respect to theta and you can actually measure this right.

So, you can actually put some sensors over here and you can say that there is a angular displacement between these two reference frames. Once you know that you just going to do the following. So, given x sensor 1 and y sensor 1 I want to express this in my body coordinates right. I want to know what is x body 1, x y body 1. And the way you will do it is very simple you would actually do it as follows. So, x body 1 would be nothing, but x sensor one times cos theta; y body 1 would be nothing, but y sensor one sin theta and how do you express this as a matrix very simple. So, I would just put cos theta over here 0 0 sin theta that is it.

So, when you do this, now I have a measurement in sensor frame, now I am able to express the measurement the sensor measurement in the body frame brilliant, simple straight forward in the 2D case. The 3D case becomes a little bit tricky, but it is almost the same logic, how we do that well as follows, in a 3D case note that you can have a rotation about all three axes.

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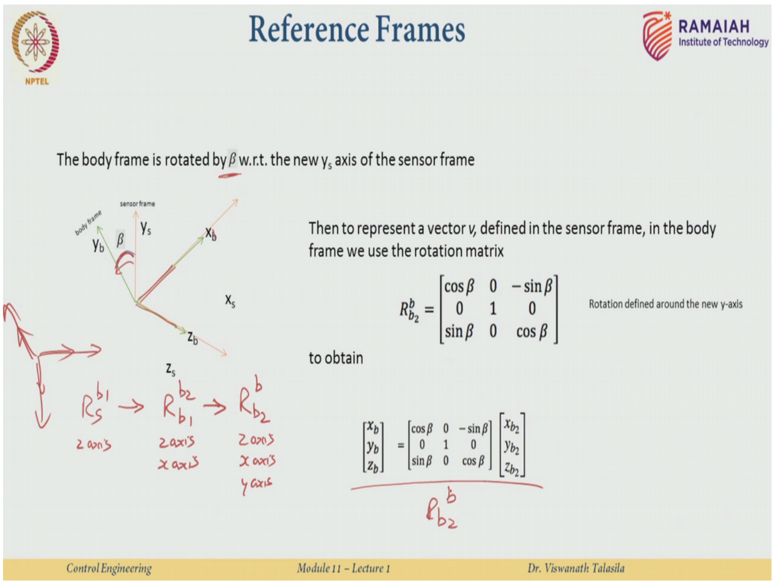

So, if you look at the previous slide, we basically said that there was a rotation of theta degrees about the x axis, which means even the y axis will also be theta degrees rotated with respect to the other y axis alright. So, what I am basically saying is if this is theta degrees over here by the property of orthogonality or perpendicularity, this will also be theta degrees exactly the same because both axis are orthogonal for both the reference frames. Not true for 3 axis system, it is little bit more involved and the reason is little bit more involved is as follows, 2 reference frames let us take the forward axis this one over here, and let us say that you have a slight yaw of the second reference frame, yaw is a rotation about the vertical axis right.

So, the above the vertical axis the second reference frame is slightly yawed by some angle gamma. Now because it is 3D this is not the only case which we can have about this axis over here, which I am showing with my left finger, about this axis I have a slight pitch with respect to this guy over here. And now I still have a third axis left the forward axis, the roll axis I have a slight displacement with that. So, I have to now consider three angles, because my axis can be misaligned with respect to this axis in three different ways, I can have a yaw displacement I can have a pitch displacement and a roll displacement. All three conditions have to be considered it is not difficult it is fairly straight forward, you just need to use the correct math.

Again we take the same example of the sensor frame and the body frame, but please remember you can use this for any two frames. So, the assumptions I am making here is that the sensor frame and the body frame have the same origins. So, in that case we have this origin over here the two reference frames, because we have three possible angular displacements between the two reference frames, we are going to consider each angular displacement independently.

So, first in this particular example I am taking we will assume that the sensor frame is rotated by an angle gamma right over here, with respect to the body frame with respect to the z axis or the body frame. Turns out if you do the math properly, because when we are doing a rotation about the z axis notice that the bottom one is fixed. So, when you rotate about the z axis, the third component which is the z axis component will all be zeros except the z axis itself because the z axis is fixed, it is not moving it is the other two axis which are moving right.

The other two axis by standard trigonometry relationships will follow this relationship, you do not need to worry about the derivations of this may be those of you who going for higher studies, you can study this later not too relevant for now it is important to know, that if I want to go from the sensor frame to a frame called b 1 which is body frame 1 where the z axis are now aligned. So, remember I had 3 misalignments, a misalignment about the z axis, misalignment about the y axis that is a pitch axis and about the roll let me correct these one by one. So, first let me correct the z axis. So, I do a rotation about the z axis this one over here. So, that rotation I do first the second rotation which I will show in the next slide will be about the pitch axis. So, I will do like this, and the rotation will be about the roll axis. So, I am going to dot this step by step.

So, about the z axis, if you blindly go ahead and use this expression this is called the rotation matrix, and this rotation matrix helps you in this example to align sorry about that, it helps you to align the two z axis of the two reference frames that is all what it does. So, which means that if I have a vector my measurement X s Y s Z s in the sensor frame, this I can use this rotation matrix and bring it to the body frame where the z axis of the two reference frames are aligned we still have to take care of the other two reference frames. So, this is step 1; step 2 let us say that your body frame was now rotated by some theta about the x axis, in the previous case it was gamma about the y axis now theta about the x axis. In this case again since it is about the x axis, x axis will always remain all terms will be 0 and x itself will be fixed. So, it will always be one, and we look at the rotations in the other two components and it has a very similar structure as the previous one. So, this is again my rotation matrix and notice what it is doing in the previous example we had this rotation matrix with the notation R s b 1 which means it is a rotation or a transformation from the sensor frame s to the sensor frame b 1 with the z axis are aligned.

In this slide we Rb1 to Rb 2 this is a rotation from b 1 reference frame to b 2 reference frame where z axis is aligned as before and now we are aligning the x axis as well . So, now, two axis are aligned and you can see that over here right both the axis z and the x are aligned after you apply the rotation and again this is a standard expression. So, the measurement in body frame 1 where the z axis were aligned, is now transformed to a measurement in body frame 2 where x and z are both aligned, that you can see in this slide after the previous two transformations the z and the x are now aligned with respect to reference frames; and what is left out is the one axis.

So, let us say you have the angular displacement by beta with respect to the new y axis, do exactly the same thing as we done before. So, we have had from the sensor frame to body frame 1, after this we have done body frame 1 to body frame 2 in this case we align the z axis. In this case we have aligned the z axis and the x axis and now finally, we look at the rotation between the 2 y axis of say body frame to the final correct body frame b 2 to b, and now all three axis will get aligned x axis z axis and the y axis, and you can see that over in the next slide.

So, when you rotate this by the appropriate angle all three axis will be perfectly aligned with each other right. So, like. So, and this is a rotation to go from b 2 to b. The second body axis to the final body axis, and interestingly all you need to do in order to do this very complex sounding mathematical description is very is one very simple operation.

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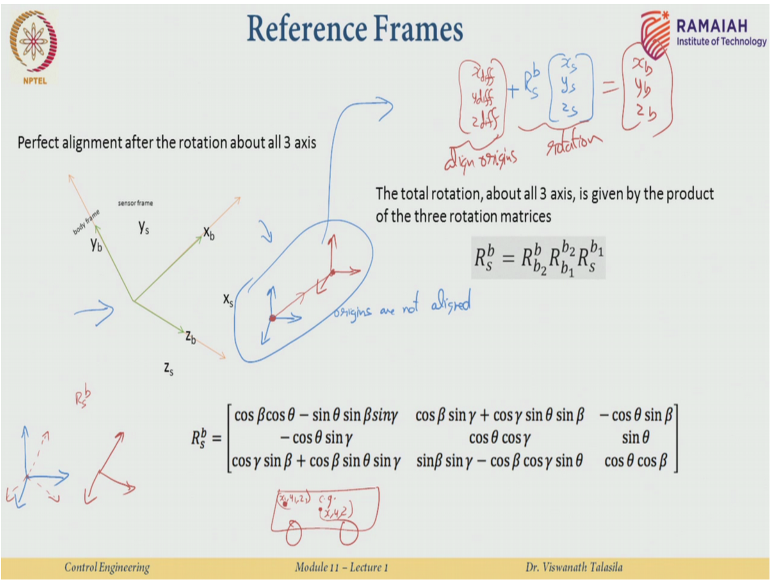

Take the three rotation matrices and multiply them in this specific order; on the right hand side from the sensor frame to body frame 1 that is one rotation matrix, multiply that with body frame 1 to body frame 2 the second rotation matrix and finally, multiply that result with the body frame 2 to the actual body frame the third rotation matrix. You actually do the multiplication and you get this horrible looking expression, which you do not need to worry about considering you are an under graduate student, you can actually use this as it is. And this follows from very basic trigonometry rules and really nothing more than that.

So, the interested student can contact me or on the forum n p t e l forum or you can look up in Google to find out how you actually derive these expressions. Now this means. So, this formula what it actually means is this, it means that any measurement I do in the sensor frame. So, I can call it X s Y s Z s any measurement done in the sensor frame, if I multiply it with this rotation matrix, I will exactly get what the actual measurement is in the body frame. Now the body and the sensor frame are aligned, they are now exactly what they are both talking about like where the vehicle actually is then you just multiply with this which is this matrix alright.

Now, note that I have talked only about rotation, I have not talked about translation. Well translation is slightly different it is even simpler. So, let us say that you have two rotation frames like this, and a second rotation frame over here origins are not aligned, that is a basic difference between the example considered here and the example I am now considering well how to handle this really simple. So, this example we will write down over here and it is really really straight forward how we are going to do this is as follows. So, again we have the measurement in the sense of range Z s, the first thing we are going to do is to align the rotation axis. So, by multiplying with our matrix which we have already computed, after doing that we shift, I will actually draw this over here. This is one sensor axis and this was the other sensor axis the body axis let us say. Now what we are going to do when we apply R s to b, this axis over here it gets shifted displaced rather angularly displaced to this axis.

So, now, the angles at which the two reference frames are being considered in are exactly the same, what is missing is I need a translation from here to here, I need to go I need to shift the origin from this place to that place. Now let us not worry how that is actually calculated, let us say someone actually tells you by measurement. So, you have your car and this is the center of gravity and you have placed your. So, it is in some x y z body position origin, and you have placed your IMU over here, you can physically just measure it right.

So, you know that this is some x 1, y 1, z 1 where you have placed your sensor and you know the difference between x and x 1, you know the difference between y and y 1, z and z 1 subtract all those differences and all you need to do is to add that small difference. So, I will call it x difference in the translation position, y difference in the translation position, z difference in the translation position. If you add this to the rotated values you are going to get perfectly x b, y b, z b.

So, first take care of the rotation between the two axes, then align the origins done essentially done three dimensional inertial navigation. So, let me try to summarise this entire story which we have been talking about which may seem a little bit complex when you encounter it at the first time, if you go through this two or three times it actually fairly simple all we are doing is just trigonometry in 3D. So, let me just try to summarise it as neatly as possible. So, the problem was as follows.

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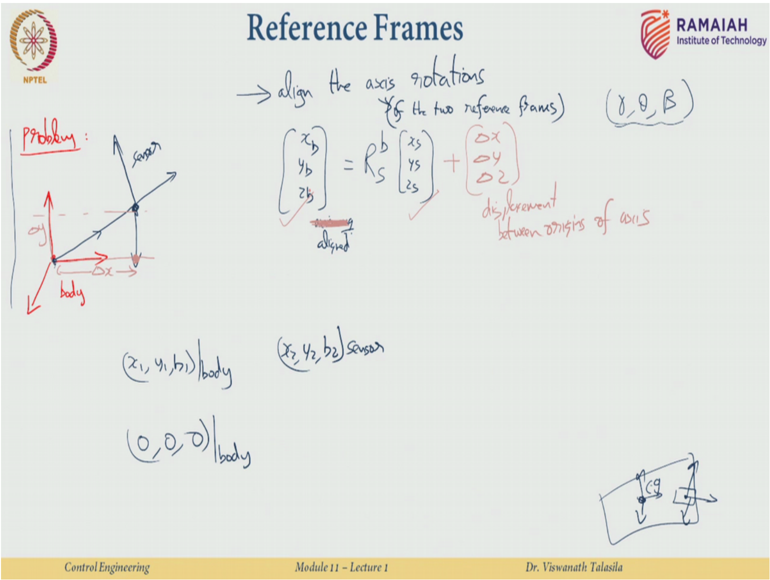

So, given two reference frames let us call this as a body frame, and another reference frame is the sensor reference frame. Now we know that a measurement is always done in the senor reference frame, the sensor measures values with it is own three axis reference frames, which needs to be converted to the body measurement frame. Then of course, we need to convert the body measurement frame to the standard reference frame, but will come to that later.

So, how do we actually do this well as we have seen there are two steeps, the first is align the axis rotations align the axis of the two reference frames that is. So, if you align the axis of the two reference frames even though they are widely separated from each other, as you can see this is angularly different from this one let us first align them together alright.

Now to align them you only need that one R s to be matrix you know that big three cross three matrix which you had you. Only need three angles as you saw there was a gamma theta and what was the last one a beta. So, if you have three angles, you can actually align the reference frames together angularly align them. How do you compute these gamma theta and beta there are various techniques, for our case let us actually supposedly we do it manually. So, as I said before you have a vehicle with the center of gravity over here, and the sensor mounted over here I mean that is a axis like this all we need to do is to find out what is angular displacement between these two things alright it is a bunch of ways of doing it, but we can just literally do it manually take I mean if you are really lazy just take a protractor and try to align them and that is that will be reasonably accurate alright. If you want very high degree of accuracy there are really fancy techniques, but let us not worry about that.

So, all you had to do was to compute the R s b matrix, and then multiply the vector the sensor measurement that you do with a R s to be matrix, then you get the measurement in the body coordinates with axis aligned. So, this is the axis angularly aligned body coordinates it will still not align in the translation way for that all you need to do is to just compute this guy. So, if you know the x 1 y 1 b 1 of the body and you know the x 2 y 2 b 2 of the sensor.

So, again very easy to do the easiest way to do it without actually breaking your head too much is to assume the body axis origin is 0 0 0, just assume it 0 0 0 no worries. You compute the displacement along the x direction of the body to see whether sensor is. So, if the body is over here let us say this is the origin alright origin of the sensor. You basically just go along the x direction and you measure and you get the displacement delta x with respect to the sensor the x displacement, same thing do with y.

So, keep going until you see what is that y and the same thing with z then all you need to do is to add the delta x, delta y, delta z. This is the displacement the translation displacement between the origins of the axis. And once you do all that you get a completely aligned reference frames and whatever you measure in the sensor axis now you know it is exactly the same as what you actually have in the body axis that is a basic principle you are free to contact me if you want specific code for this if you want more explanation literature about all this you are free to contact us on the forum.

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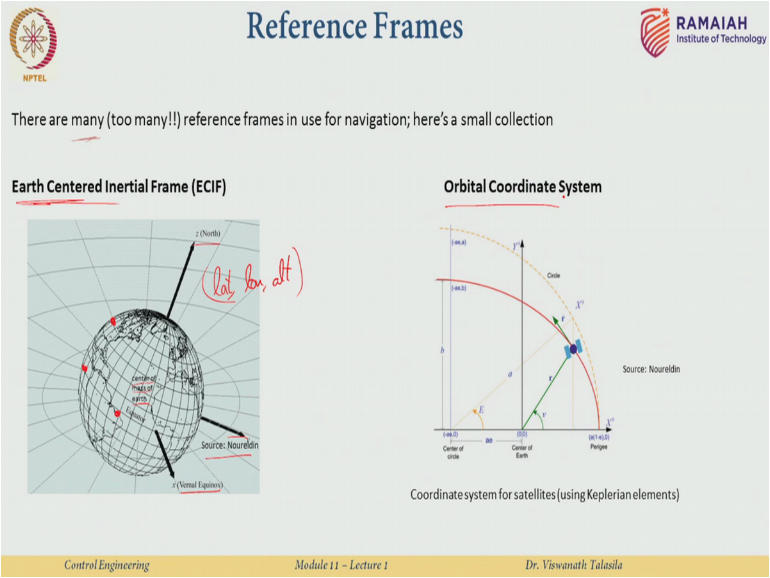

Well this is fairly simple there are fortunately or unfortunately, there are many too many reference frames with which are actually in use. So, while we like to say that the there is one standard reference frame it really depends on what your purpose is. If you are on earth one of the standard reference frame is called the earth centered inertial reference frame, where the origin of the reference frame is the center of mass of the earth and one axis points towards north other axis point towards the vernal equinox, and the third one just completes the right angle triangle along the equator.

So, this is one reference frame. So, anywhere you are on earth right can I see India here no I cannot see in the map, but anywhere you are on earth you can define your position with respect to these three axis, and actually this is what will give you the standard latitude longitude and altitude measurements. But this is if a vehicle is on earth, but for a satellite, you have a different reference frame altogether and one of them is called the orbital coordinate system. If you are inside a room like the one that I have it really makes no point to worry about orbital coordinate system and for very specific applications, you may not even care about earth centered reference earth centered inertial frame, you may just literally worry about this room axis.

How do you define the room axis? I can just take the corner of the room, I go down where all the walls are meeting at one corner and I can define that to be the origin. So, reference frames are very important and it is up to you to decide what reference frame you want, you can choose the reference frame of this room. Assume your robot is navigating within this room it is only job is to worry about what it is doing in this particular room. A person outside who wants to monitor the robot is also interested in what the robot is doing in the room we do not really care if this room is in Chennai or Bangalore or Mumbai or wherever right. It is enough for us to know that in this room the robot is doing it is job properly, for that you just need a room reference frame I can take the axis over here and then define it.

If you are going from actually I am planning a trip from Chennai to Mahabalipuram tomorrow, because you know I am visiting Chennai from Bangalore then my room reference frame is of is of no use, my car reference frame is of no use I really need a earth centered inertial reference frame. I need a GPS kind of reference frame the latitude longitude altitude that is more relevant for me. So, what you use how you use is upto your task, in all cases the way that you would compute the way that you would express a motion in one reference frame in another reference frame is exactly by the story which was described. This was the basic story of say navigation where we discussed different navigation techniques.

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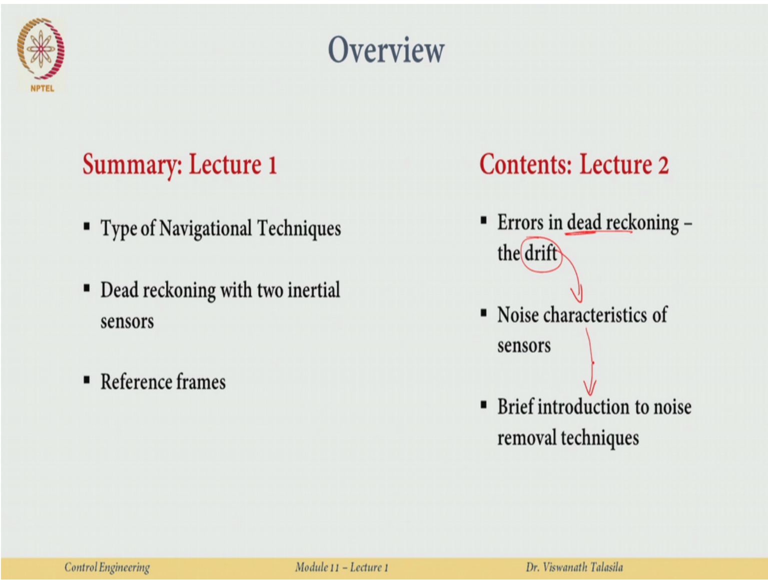

We looked at very briefly what is dead reckoning which was basically the integration process, and we represented the integration process with a difference equation which is going to be implemented in a micro controller and finally, we concluded with reference frames.

In the next lecture what I will be talking about is one of the most important or the most important problem in dead reckoning it is a problem called drift and we will see that drift arises because of certain noise characteristics of sensors, and very briefly I will introduce one or two techniques of how we can remove this noise in order to get a better computation or estimation of position. And we will finally, conclude with a very nice detailed experimental example right here my research associate Vinay Sridhar will actually be taking care of that.

Thank you very much.