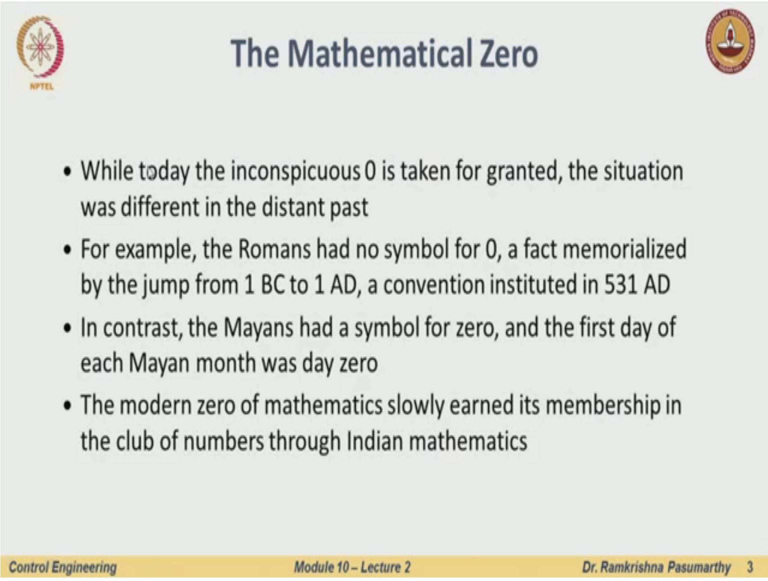
Control Engineering
Dr. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 10
Lecture - 02
Effect of Zeros on System Response

Hello everybody. In this particular lecture we will consider effect of zeros on the system response. And I will not really worry now about well let me with a abuse of terminology call it and call it a stable 0, we would be mostly interested in zeros on the right half plane. What about the zeros on the left half plane? We have done several analysis on that in module 7 and even through other design process procedure in module 8 and 9. So, we know; what is the effect now of 0 on the left half plane how it helps improving the transient response and so on.

So, So far we have not really discussed about in detail about zeros. We talked about zeros only when we were facing some problems in the design or in the performance specifications. All we were interested even while drawing the root locus was how my poles change as again varies from 0 to infinity. Similarly even while we were doing the bode plot, you never really worried much about what is the role of zeros and what are the implications of having different kinds of zeros.

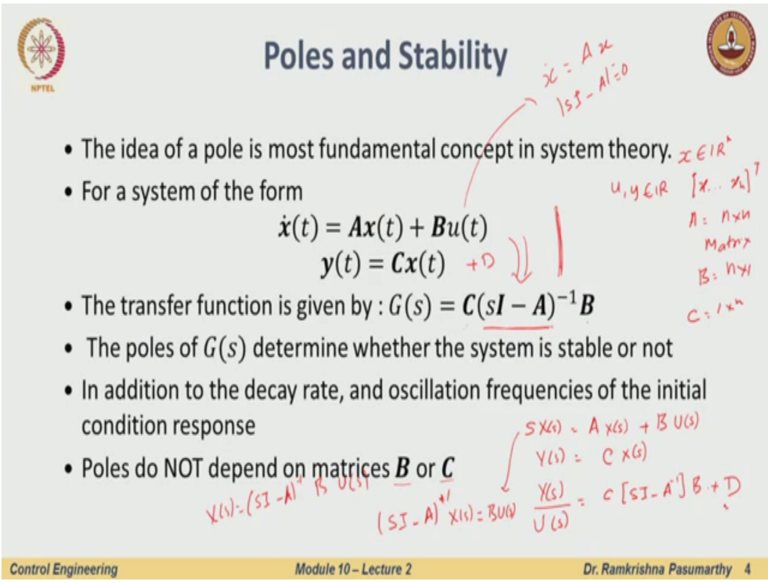
(Refer Slide Time: 01:36)



So, in this can we start with some basic things right. So, what is a mathematical 0 well, so if you back to history a little bit well there was no symbol for 0, right? If you remember what we learn in roman numerals in school there was no concept of zeros, there you had x v then l and the c, I do not even know, why we why we are forced to learn those things even though we had a much better number system.

So, and there was few other things other civilizations where they had some kind of symbol for 0 and first day of each Mayan month was zero. But significant advances happened in through Indian mathematics. So, the 0 invented here went through the Arabic world and then to the west through Europe and so on. So, if you typically ask European who invented 0, they would actually think it is actually an Arabic number system because that is where they imported it from. But it is this is the biggest contribution of Indian mathematics ok.

(Refer Slide Time: 02:43)



So, we will not discuss about the Indian mathematics here, what we will look at is the role of zeros in our context; in the context of control systems. So, so far in our analysis even when we started with poles or when we started with the analyzing stability everything was depending on poles zeta omega and everything and the characteristic equation everything can be in the denominator. And for a system of the form now this is these are what as referred to as state space equations.

This will come up in detail in module 12, but for the moment assume that we have our system in a differential equation form of this way, where x dot A are my states. A is the system matrix, B is the input matrix y is an output vector and C is an is the output matrix. So, typically what this would be mean is well I have A x as an n dimensional state vector, it could be x 1 till x n x transpose, a is an n cross n matrix. Similarly, B if there is just one input would just be an n cross 1 matrix. C if there is only one output. So, one input one output means u and y are simply scalars. And C would then be 1 cross n matrix in some cases there could be D also here and ok.

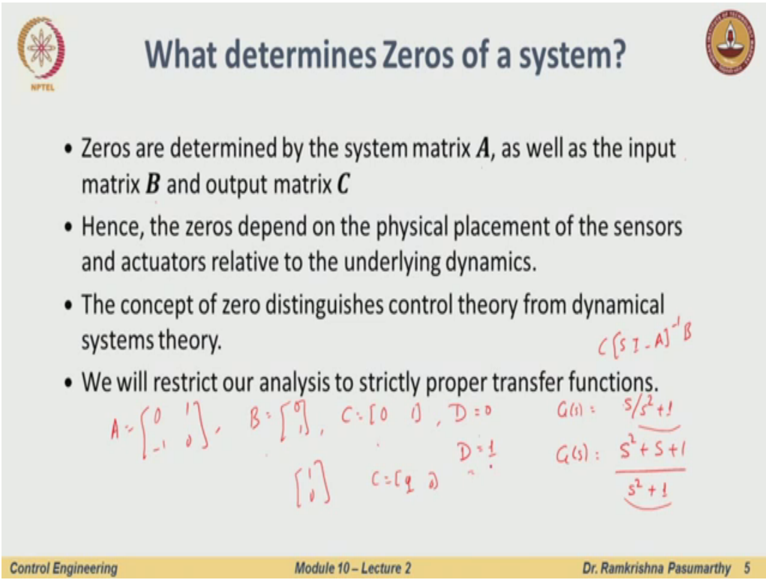
So, how do I go from here to here? The way I go from here till here is, So I just take the Laplace of this as x of S is A x of S plus B u of S. And then y of S is C times x of S. And then I keep eliminating things and what I find is I will skip the steps here, but there should be very straight forward to find out. So, this would be C x of S comes from here this would be SI minus a inverse times b. So, so you can just eliminate x of S from those. So, this will be SI minus a inverse x of S is B times u of S and this is equal to the manipulations. So, this is what we call as the system in the state space form and we had discussed this very briefly in one of our first lectures of course, we even not analyzed much on what is the nature of a B and C that will come in detail later.

But from here to here is this is the very straight forward procedure, you just take the Laplace transforms and take the ratio of y over u. And if you have a D here the D will very D will set here. So, you are eliminating x from this equation and just substituting for here sorry, this is just some arrow here. So, this would be this would not be an inverse this would be plus here and when I do it on the other side x of S would be SI minus a inverse times B u of S. And then it is a little straight forward. So, this is also I just made a mistake here this should be a plus 1. Now, this is also what is this is our transfer function. And you if you if you look at it carefully what goes in the denominator is this SI minus a inverse right.

So, you have this the way you compute the inverse of a matrix you just take the determinant and you take the adjoint and so on. So, the determinant or the denominator here will only depend on what is SI minus a inverse I am not talking anything about C B or event B at the moment. The poles determine whether a system is stable or not, in addition to the decay rate and oscillation frequencies of the initial condition response.

So, it is it talks about stability it also talks about the transients the decay rate the frequency of the oscillation in terms of omega D decay rate is determined by zeta. And what is important to note here is while I do this analysis is that the poles do not depend on B or C. Therefore, instead of this having this if I just say I want to analyze x dot equal to A x the autonomous system, the characteristic equation will just give me the poles SI minus a equal to 0 it is same thing right. So, I do not really worry about what is B C and even D for that matter.

(Refer Slide Time: 07:42)



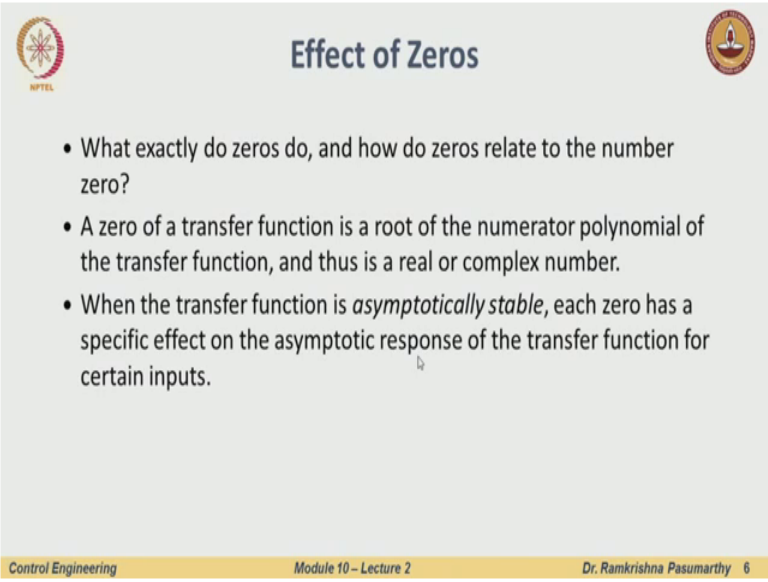
So, the message from this slide is that the poles only depend on the a matrix, just because of this guy. And they have nothing to do with B C or even D. Now zeros are determined by the system matrix A and as well as the input B and the output C. Now what is this these inputs suggestion? What is output suggestion? The input matrix or the way the input enters to the system is via placement of a certain actuators actuating elements. In similarly why are the measurements, which we will or the why voice are the quantities which we will measure physically. And therefore, why comes through some sensors and voice will determine or the C will determine where my sensors are placed.

So, these zeros are determined by the system matrix A as well as the input matrix B and output matrix C. Practically what do they mean practically this zeros depend on the physical placements of the sensors and actuators relative to the underlying dynamics. Now this is what distinguishes a control system from a standard dynamical systems emanating from physics. So, the concept of 0 is what distinguishes control theory from dynamical systems theory and of course, throughout this.

So, most of the analysis we will do is for strictly proper transfer functions. Let us see this with some very, very simple example. So, I have a as 0 1 minus 1 0 say, B is 0 1 C is also 0 1. And say D equal to 0, and then I do the maths C SI minus a inverse B D 0. So, what I get as a transfer function G of S is S over S square plus 1. Now I keep all this, So a does not changes, B does not change, C does not change, D does not change, D does and say I put D equal to 1 then what I get as a transfer function G of S is S square plus S plus 1 over S square plus 1. You see nothing changes in the denominator the poles are the same.

What changes from here till here is the location of the zeros, there is one more 0 added here and you can similarly check you know may be B as 1 0 C as 0 C has 1 0 and you will get different numerators for each of them. So, the zeros are determined by not only a, but also B C and then also many cases D ok.

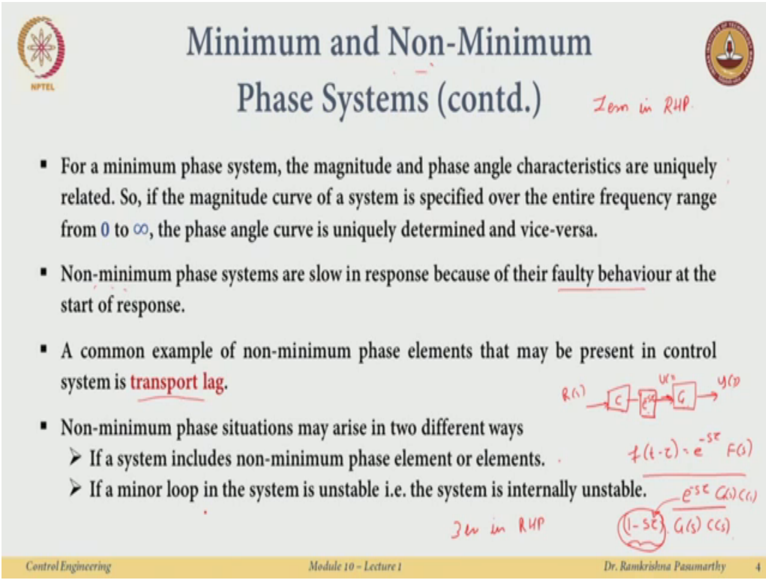
(Refer Slide Time: 10:20)



So, what do I exactly these zeros do, right? So, so 0 of a transfer function is the root of the numerator polynomial of the transfer function and it is can be a real or a complex number or a just be with a number 0 also.

Then when the transfer function is asymptotically stable; so in this analysis we will only be interested in transfer functions which are asymptotically stable which means all the poles are to the left half plane and each 0 then has the specific effect on the asymptotic response of the transfer function for certain inputs. And this is what we will do throughout this lecture. So, let us just re visit in the slides which we had last time ok.

(Refer Slide Time: 11:14)



So, what we will in this lecture typically be interested is in non minimum zeros, it is zeros in the right half plane. So, what we discussed last time about all these things how do I distinguish between minimum phase and non minimum phase via the bode plot. We also concluded that if the system is minimum phase the magnitude response will uniquely determine the transfer function. And well, so there is things like we said in we will actually detail this a little more that non minimum phase systems are slow in response, because of the faulty behavior at this at the start of the response.

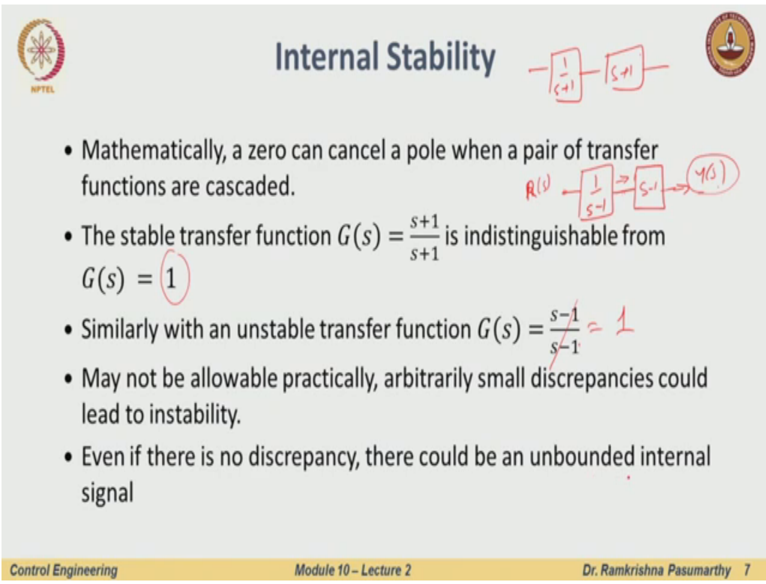
Common example is transport lag. So, let us see I have very, very simple example that I have a plant here G, I have a controller and there may be some reference signal then, as usual some output here there is some input to the plant I say well there is a little delay between the time the control signal is sent and so, then the control sends the signal to the actuator and the actuator generates a control signal.

And this is modeled in transfer function as e power minus S tau, assuming tau is the delay and this actually comes from the Laplace transform of a delay signal f t minus tau is e power minus S tau f of S, is this is a straight forward derivation we will not go in to the details of that. But why do I say that this system has is has or it will be faulty behavior at the start of the response?

Now, what is the contribution of this delay here? That in the numerator it adds up e power minus S tau and then you have a C G of S and C of S. And if I take a small approximation of this would just be e power S tau can be approximated at 1 over S tau and then you have G of S C of S. Now what this and this is not a very elaborate proof? But just a very small evidence of what happens when I when I do this. So, when I take a when I have a transportation delay or transportation line. So, this is a system. So, what is the nature of this, So now, this introduces a 0 in the right half plane. And this will result in a system which is now non minimum phase, ok.

So, we will see what is the role of this 0 on the right half plane, how is it related to a transported lag well it is related via this relation. And what is the faulty behavior here that we are talk talking of right. So, we will talk of these 2 points also right. So, non minimum phase situation arises when the system included non minimum system has a non minimum phase element or a minor loop inside the system which is internally unstable, ok.

(Refer Slide Time: 14:31)



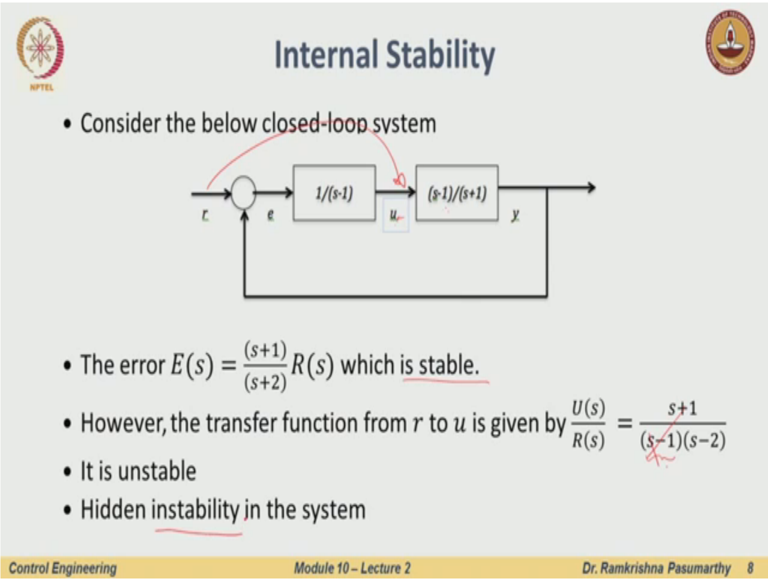
So, let us first talk of internal stability and what does it mean? Mathematically a 0 can cancel a pole then there are 2 transfer functions in cascade. Let us a loosely speaking say

I have 1 over S plus 1 and there is a cascade S plus 1. So, mathematically they will cancel each other and this is 1. Now we will not worry about how can I realize this and so on. Not just the worry at the moment.

Similarly, when I have unstable things this is S minus 1 this is S minus 1, they will also cancel each other and I will have 1 right. So, first is, is this a stable realization? So, if I say I have a unstable plant, I want to track a reference say some r of S, so we can put capital R and the input here. So, this is S minus 1 and then this is the output. Then each time I could say that well I have a unstable plant here, unstable pole cancel it with unstable 0 and I will get the output which is exactly the reference. I want to take this is straight forward, right. We just looking at it mathematically, but is it practically feasible well it may not be allowed practically because if the changes by even a very small number say, by 1 millions for example.

Then well there is some instability there is no cancellation. More over what is the initial conditions are not 0? Then it never do this because the signal from here till here is unstable, right. And we will we will talk of this then we will talk of unbounded internal signals. So, even if there is no discrepancy between these numbers there could be an unbounded internal signal ok.

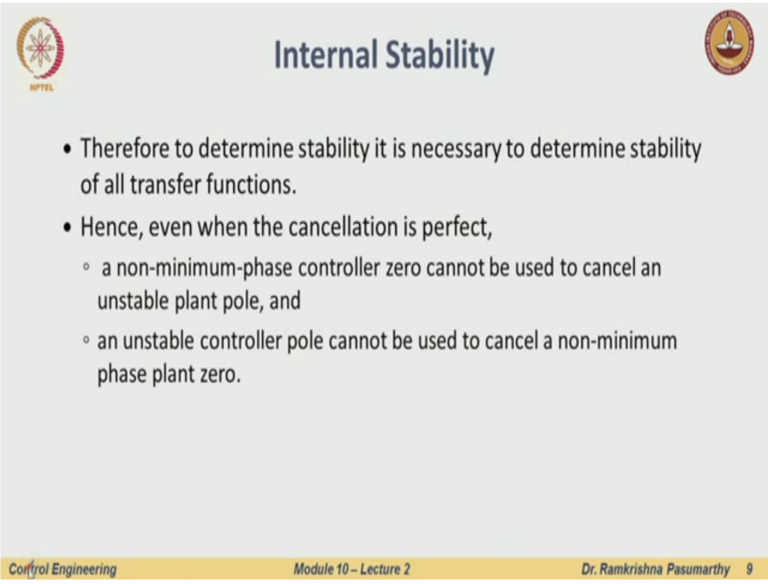
(Refer Slide Time: 16:23)



What does that mean? So, if I take a configuration like this, right. Say I have the standard reference I have the output the error unstable pole here and I have a 0 on the, right. And I have a S plus 1 only; so overall if I just look at the cancellation. So, the system might look stable, not only that how does the error goes? So, what we are worried about in the system is usually the error and all the analysis in the transients or the studies sorry, all the study state analysis we were doing in terms of this error signal ok.

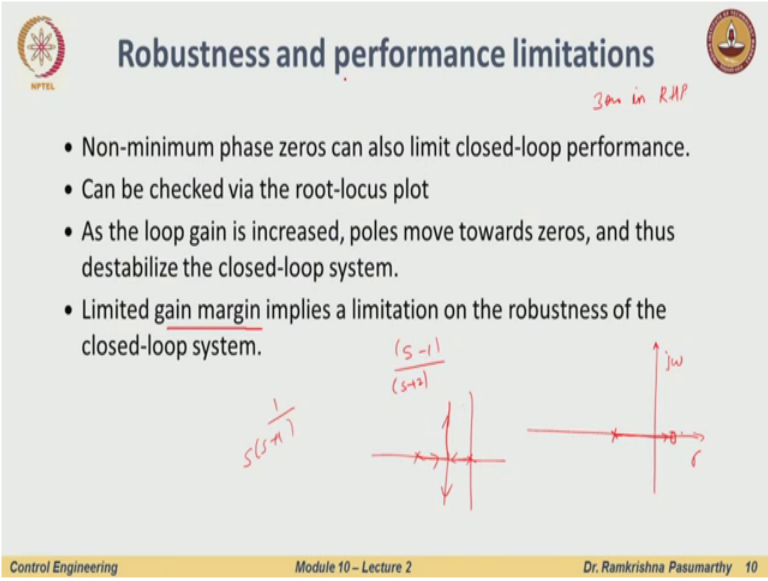
Now, this error signal or the transfer functions from r to e looks something like this looks this looks nice, right. This was this is actually a stable; however, if I look at the transfer function from r to u well, I have something such I have S minus 1 here. Now this is unstable and this is not allowed right. So, there is some amount of heat and instability in the system, so the signal from here to here. So, the u which goes to this plant if I may say o, is actually unstable right. So, this u would be unbounded and therefore, I cannot do much about this system is system, which has a instability hidden within the system right

(Refer Slide Time: 17:43)



So, now what do we do, right? In this cases well, So therefore to determine stability is necessary to determine the stability of all the transfer functions within the system. So, even when there is a perfect cancellation we should not just blindly, so the unstable pole is cancelled by a 0 on the right half plane. So, system is this is stable, no. So, even with the arguments in the previous slide we just cannot do this. A non minimum phase controller 0 cannot be used to come to cancel an unstable plant pole. And in the same way an unstable controller pole cannot be used to cancel a non minimum phase 0, either ways.

(Refer Slide Time: 18:36)



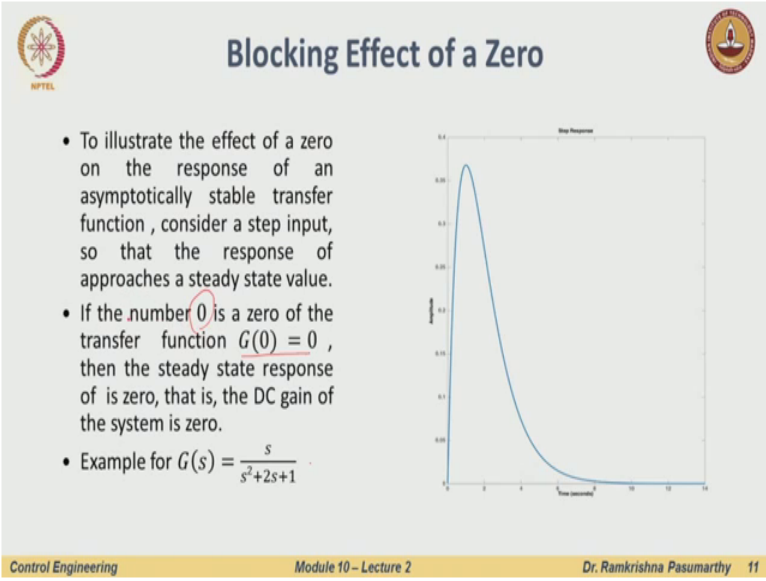
So it is, this is we have to be very careful of this of this cancellations.

Second thing what I have what happens with this zeros? Again we are just talking of zeros in the RH plane r right half plane. Now say I just look at say a very simple transfer function of the form say S minus 1 S plus 2, and I just want to plot the root locus of this. So, I have pole here and at minus 2 and 0 here at plus 1. So, what happens as the gain k increases well I go here and has k goes k gain k goes to infinity I am in the unstable region in the a plane of the sigma and j omega axis.

So, this non minimum phase zeros also limit the closed loop performance, well how do I do this? Well, my root locus plot tells me that has the loop gain is increased the poles move towards the zeros, right. And then once I cross this line here I am already in the unstable region. And therefore, whenever I have a 0 in the right half plane my gain margin would be limited. And if the gain margin is limited it would imply that the system has less robust stability properties.

Contrast this with this case, right. Where we had a transfer function of S over S plus 1 a pole here a pole here it is a stable for all case no matter what you do. So, this is, this is the effect also of a right half 0 where it in where it introduces some additional performance limitations. Now is that all that is that that right half 0 does there are, some more interesting things which right half 0 does ok.

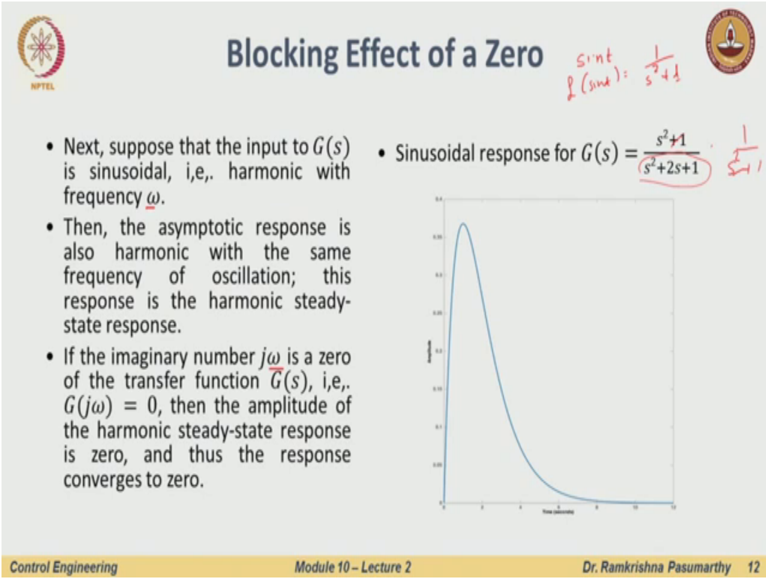
(Refer Slide Time: 20:21)



So, while doing the rest of the analysis we will not do proofs, some of the proofs are still open problems, but we will just try to understand the several phenomenon associated with it. And try to relate with what we had already studied in the earlier lectures ok.

So, the first effect is what is called as a blocking effect of a 0. So, let us take a simple example here where the number 0 this is the 0 the transfer function which means G of 0 is equal to 0. Then the steady state response of it is also 0. And which means the dc gain is also 0. So, the study state is a D C gain. So, for example, if I take this plant here and I say well what is this response well the response usually goes up and it just goes back to 0. This is when I have G of 0 equal to zero; however, what am I tracking I am tracking is this is a step input right.

(Refer Slide Time: 21:42)

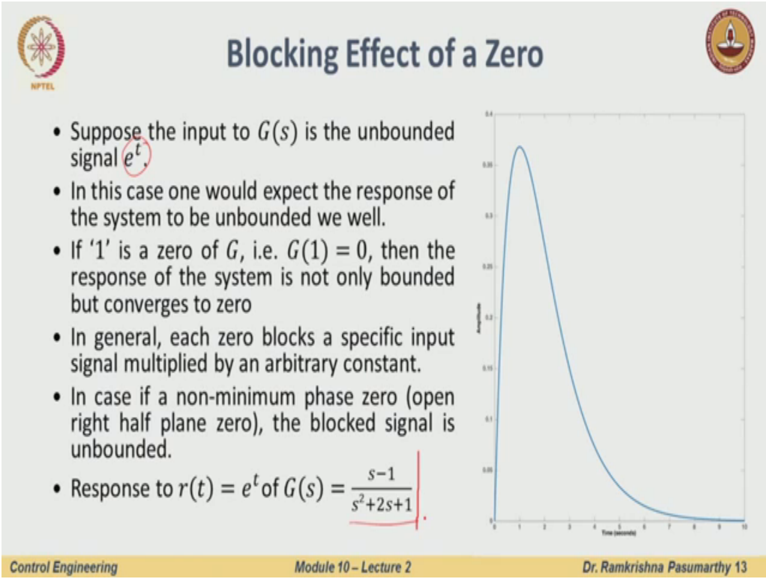


So, this is a step response of a system which has a 0 at this number 0 ok.

Next what happens when I have a sinusoidal input? Say input of the form input in such a way that the frequencies of the input is sinusoidal with some frequency omega. And then if the imaginary number J omega is a 0 of course, minus j omega will be a also a 0, if these 2 omegas are the same, so in this right. So, I am I have roots at plus minus j omega now, this plus minus j omega. So, if I have an input of the form sin t this is the Laplace transform of 1 over S square plus 1 then, typically what we would assume or what we had learnt earlier that if the input is a sinusoidal signal the output for a linear time invariant system will also be a sinusoidal signal.

What will change? Well, the magnitude will change. Frequency will remain the same and there might be a bit of phase shift depending on what are the components sitting in my plant or in my process. But that is not always true now the sum something strange will happen here now, if I subject this signal this plant to a sinusoidal thing which means I am especially multiplying with 1 over S square plus 1. So, this guys goes away and what remains is just the impulse response of this signal which is stable, right. I just recover back or I just start from 0 it is a peak and then go back to 0. So, the output is not a sinusoid for this case at least and the condition is what that this omega must be equal to this omega. If this is S square plus 2 and something else might happen, you have got a actually see a sinusoidal signal at the output.

(Refer Slide Time: 23:47)

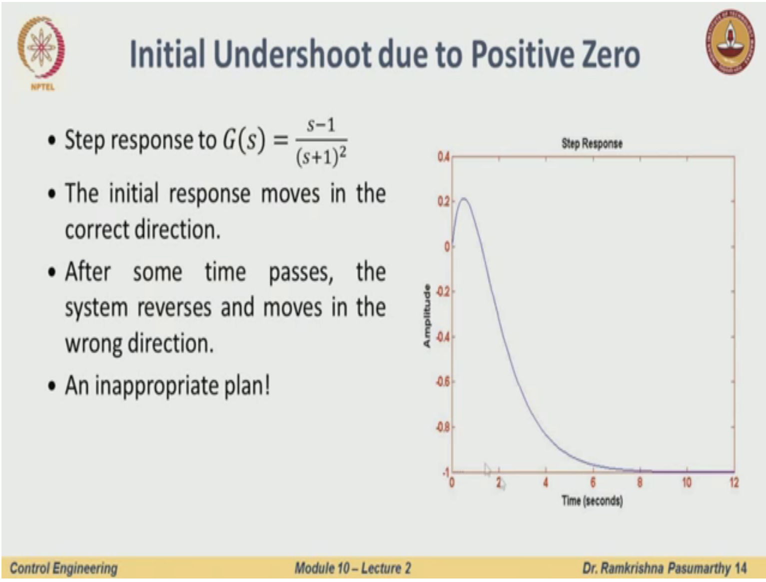


So, this is also. So, these 2 were called blocking effect of a zeros. Now something even stranger, when we talked about stability, So one of the first ways. So, how was stability defined stability? Was defined in a way that if my plant is or if my sub cell system is subject to an initial condition will it come back to it is rest state. From physics it will be will it come back to it is point of minimum potential energy right. So, that is how we defined a stability we defined stability.

Another notion of stability was bounded inputs leading to bounded outputs. And this analysis of bounded inputs leading to bounded outputs helps us to characterize stability in terms of the location of poles of the system. But we never talked about what happens when the input is unbounded if it gets mere unbounded output is a system stable. Well, I do not know the answer what I know is if the input is bounded and if that results in an unbounded output the system is unstable. Now what if the input is unbounded, right? So, something strange happens here. So, say I apply as I take a system with transfer function of this form, right. It has again a right half 0 and I subject it to an unbounded signal e power t unbounded input signal ok.

So, what would you we typically expect that we would expect that the response of the system would also be you unbounded. Now the number 1 is the 0 of the system which means G of 1 is equal to 0. Then the response of the system surprisingly is not only bounded, but it also converges to 0, right. You can just see with the plot. So, it is this is again start from 0 e power minus j is the input and I just come back to 0. So, this is one such thing. So, so what we had left out while discussing about stabilities what about unbounded inputs. Well unbounded inputs could also result in bounded outputs in under certain conditions here. Where the system is naturally stable because the poles of the system are always on the left half plane ok

(Refer Slide Time: 26:09)

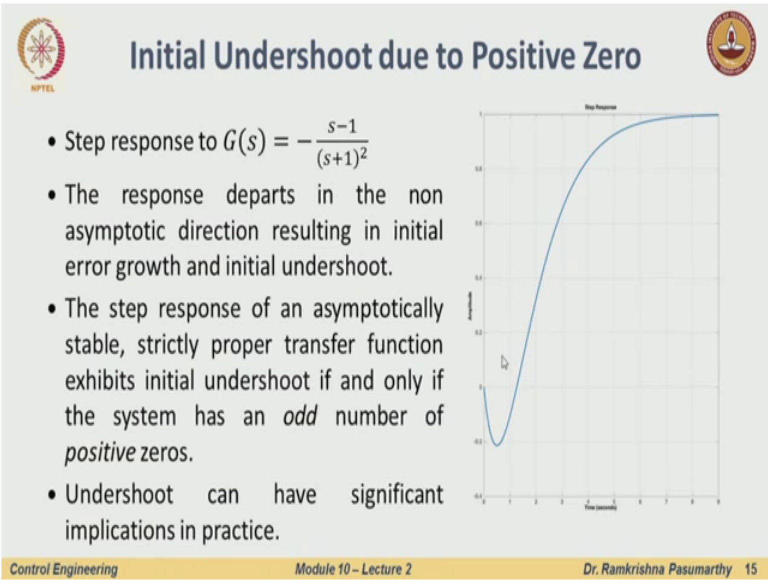


Now, some more strange phenomena: so after we discussed about what we called as the blocking effects of zeros, that we see something strange happenings to what we had learnt in the earlier 9 modules, right. If a input is a sinusoid output is also a sinusoid well no there is something strange happening here. Input is unbounded output might also be unbounded, no, something strange is happening here that output actually is bounded, right. Ok.

Now, there is something called initial undershoot right. So, we will we will analyze these some of these examples here. So, let me take as a step response to a signal which looks like this and again it has an unstable 0, if I may call it is a or plus 0 in the right half plane. So, what you see here is a starting from 0, if I were to track a step I am actually moving in the correct direction. I move in the correct direction, but here something happens and I just go back to 0. So, after sometime passes the system reverses and moves in the wrong direction right. So, if this was a plans say, I may be I hire a new CEO in my company, which was not doing very good until yesterday. There is a big positive sentiment in the market the stock prices go up after few months, I realize that this is not a good decision to have this new CEO, nothing changes in my in my in my profit or even my turnover and I see well that the response actually goes down right.

So, this could be encountered in several situations and inappropriate plan, right. Even sometimes you a elect a new government, right. Which promises a lot during the election campaign, the market sentiment is high everybody is very happy stock prices go up, but after few months it is all the same story again and again it is ok.

(Refer Slide Time: 28:12)



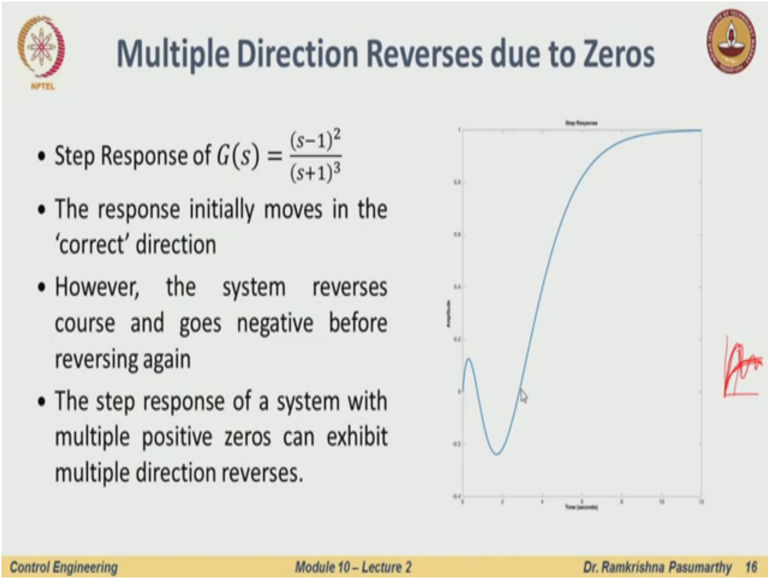
Now some other things could also happen here. What is happening I take a decision and the initial trends look very nice, but then after a while I have made I realized that I made a mistake by hiring this new CEO or you know even hiring or even voting for new government because of this is the trend that follows later on. Now here something else happens right. So, if I say I again I take the same transfer function with a negative sign here. And what I observe here is if I just start here and I just wait for a while and see what is happening here I am actually tracking this step properly there is no error nothing. But what is happening initially is I am actually going down. I go down and then I start rising after a sometime. For example, if you look at the demonetization, the initial trends suggested that the productivity in the market was going down, some analysis analogist suggest that maybe the government would say it oh it actually it went from 0 to 1 in instantly.

But we are not we are not looking at what the politicians say, but just say initial trends would suggest that because of demonetization there was a cash crunch and we have somewhere here now, right. At this at this stage as well as we are recording this lecture, but what we hope or the government hopes is that we will go somewhere here eventually we will start rising possibly rise to higher levels, but; however, the opposition things that we just go down and down under the desolation when the opposition gets selected and then the upward trend will start moving. So, we will not do the analysis of what the politics or the economics say, but decision sometimes leads to the movement in the wrong direction and eventually it might just pick up.

So, a response of this departs in the non asymptotic direction what is the asymptotic direction that I want to track this number 1 and these results in a initial error growth. So, the error well if I just started t equal to 0 plus it will just be about zero, but the error is actually growing, but after while you see that it is it is it is decreasing. So, the step responses of an asymptotically stable strictly transfer functions. Strictly proper at this is a little important, for proper transfer functions some something else might happen and we will not worry about that at the moment. So, the observation is that the step response of an items asymptotically stable strictly proper transfer function exhibit is and initial undershoot, if and only if it has the odd number of positive zeros.

Again we will not really do the proof of this one, but if there are odd numbers of positive zeros we expect a trend like this, now what happens?

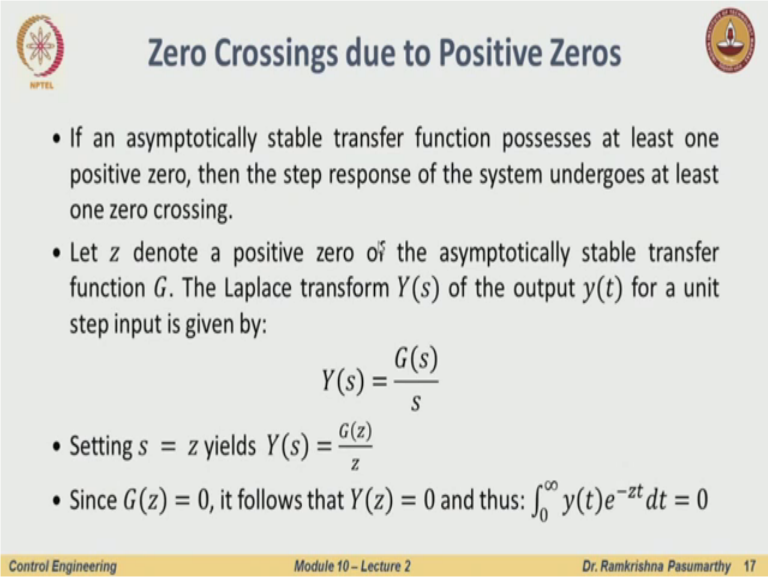
(Refer Slide Time: 31:00)



When I have an even number of zeros? So, I have S minus 1 and S minus 1. The response initially moves into the correct direction. And So, this is the after a while and then I actually goes down, and again I am actually picking up, right. So; however, the system reverses here it goes negative and again reverses here. So, this the step response of a system with multiple positive zeros can exhibit multiple reverses in the direction.

Again I will not do the proof of this, right. It just a we will just observe few things what are happening and these are things which would not be done in standard control syllabus now the gate or even under undergrad techs, but these are actually little bit of fun things to know. What are the things nice things or even the bad things that could happen with non minimum phase zeros ok.

(Refer Slide Time: 31:59)



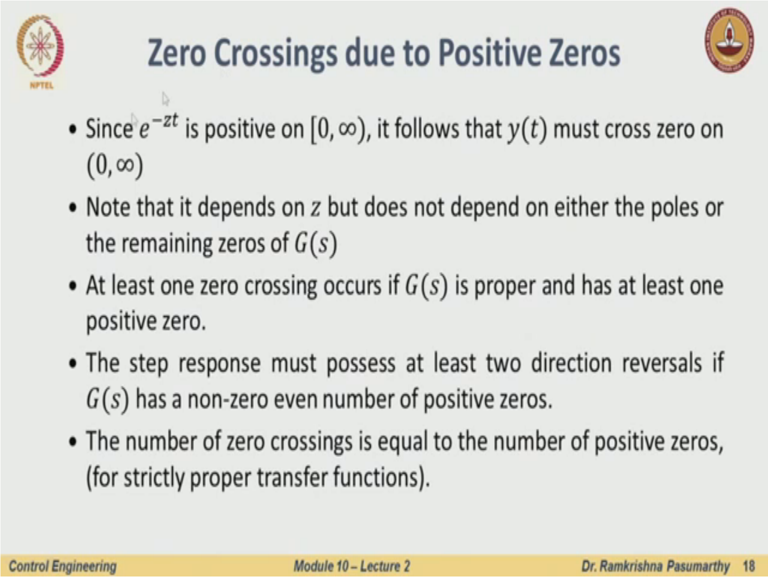
So, what we see in these plots is that well there is a 0 crossing, ideally what you would expect? Well, if it is if it is a say under damped or critically damped system, but the response is like this if it is a under damped is, So if it is over damped or critically damped this is this is the case, if it is under damped then I get expect some oscillatory behavior overshoot and so on. Here I experience something else that I start here and I go here I again cross the 0 once, I come here I again cross the 0 once more here I am crossing the 0 once I go down I cross this 0 here ok.

So, when do these 0 crossings happening happen and how many of them happened. And is why what is the evidence that is actually happens? So, if an asymptotically stable transfer function possesses at least one positive 0 then the step response undergoes at least one 0 crossing. So, if there is at least one 0 sitting on the right half plane there will be at least one time where the plot will cross 0 or when the response will cross 0. So, how do we how do we verify this?

Let us say the number z denotes a positive 0 of the asymptotically stable transfer function. This is important, right unstable systems will be there is nothing we could analyze. The Laplace transforms of the output for and if I am just looking at a step is y of S is G of S over S. And what happens at a S equal to z? Which is the positive 0 well I have y of S is G of z over z, G of z equal to 0 it follows that y of z equal to 0, and what is y of z?

Now, this y of S is the Laplace transform coming from this formula. This is simple formula e power minus st ft dt, right. I am just substituting z for S and what I see here is this value of the exponent is positive all the time, right. This guy e power minus zt and what could only go to 0 is y of t and therefore, we say that there is actually a 0 crossing, when this happens that y of a G of z going to 0 and actually y of z going to 0 actually means something like this.

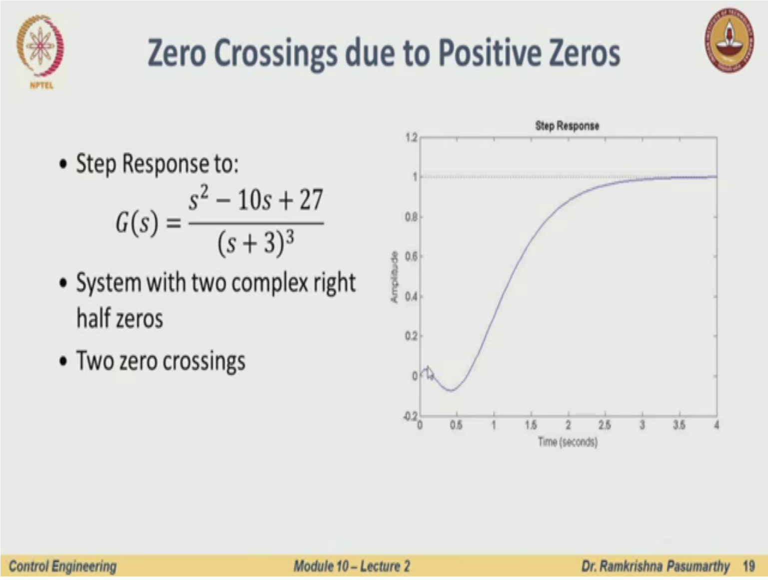
(Refer Slide Time: 34:28)



And because e of z is always positive y all z should go to see you a sometime. That is what this argument says since e of z is positive for all times it follows that y of t my must cross 0 somewhere in this interval, right. Now this is important here, right. Note that this only depends on the positive number z, it does not depend on either the poles or the remaining of zeros just this guy is what is crossing a to cross 0.

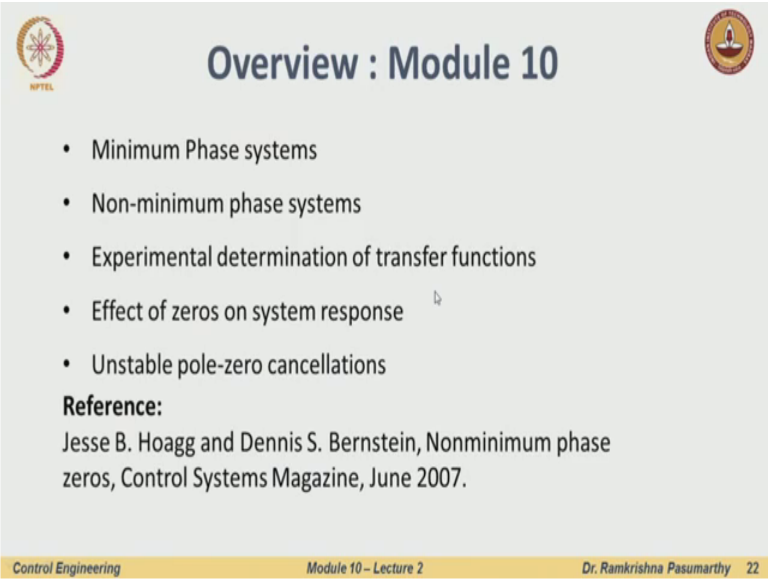
So, at least 1 0 crossing occurs if the transfer function is proper and at least and as at least 1 0 on the right half plane. Now if there are more then you will have multiple reversal of directions in such a way that the step response must possess at least 2 direction reversals if it has a non 0 even number of positive zeros. So, I am just writing down this statement which I observed here. I have 2 0 2 direction reversals and 2 times I cross 0 1 is here and one is here. And the number of 0 crossings is equal to the number of positive zeros. This is only for a strictly proper transfer functions. And these are also most many of these are just by observations and I think this statement is still not been proved mathematically ok.

(Refer Slide Time: 35:49)



Now, 0 crossings are they only with positive zeros one may ask what happens when my roots are imaginary or the zeros on the right half plane are in or complex conjugate. Well, it still has 2 0 crossings right. So, things also extend to the case when we have poles which are not only positive, but all sorry, right half zeros which are not only positive, but also complex conjugate again these are just a little examples ok.

(Refer Slide Time: 36:23)



So what we, what we have done. So, far is in this module. So, which is possibly shortest of all modules is to analyze the role of zeros from a different perspective. Not just looking at a lead compensator or a derivative control action. We are seeing what happens when the zeros are on the, right. There is there are several other facets of this particular topic.

But which is restrict to just what we need just the basic information of what we need and why we are be interested in this? Because we were interested in doing the experimental determination of transfer functions why are the bode plots why are the magnitude of the bode plot magnitude plot of the bode thing. So, in that case there was a unique correspondence between the transfer function and the bode plot if and only if the system was minimum phase.

So, what we saw here was a minimum phase systems and then the definition of non minimum phase and how the concept of minimum phase systems was crucial in the experimental determination of transfer functions. And we saw the effect of right half zeros on the system response things like the unstable pole 0 cancellations blocking effects and so on. So, all this material from this from I just referred directly to this paper. It is a very beautiful paper to read if you if you have time we can there are several other examples of the inverted pendulum some examples related to the to even server systems and so on. And also good examples of you know of day to day life examples where when you say I am riding a bicycle and I want to turn to the left.

A typical answer if you ask anybody how would you turns your bicycle to the left he would think you know it is it is stupid question to ask. That you just going, going and then you just do turn left of what is happening, but that is not true right. So, what you do is you first turn a bit initially to the right create an angle So that then you turn to the left. Just observed this when you are next time on a bicycle right. So, these are several phenomena which are not very obvious to us, but there if you go a little bit detailed into detailed observation you observe the effects of what we are actually saw in this lecture in terms of plots ok.

So, in the next module what we would do is I mean most people are interested in learning some practical applications or applications could be several, right. From process industry a very popular application is a robotics, and people say where all robotics is just guy buy a robot you know that a microcontroller program it and do. So, we will spend an entire module on some, some applications. So, a colleague of mine Dr. Vishwanath will be handling that that module. He is very experienced in, he has a rich experience in industry he was experienced he was involved in the development of the LCA which is now called the tejas. He worked with honey well for a very long time. So, he has good experience in a in practical aspects of control and he will teach you few things about inertial measurements and sensors and so on.

So, I hope that will be a little more interesting again, this will not be a part of the standard control curriculum for either gate or even while creating a normal course in the in the in the university. But never the less we thought it will be little more interesting to add a bit of practical aspects. We cannot add all the possible application by at least one application where you can directly see what you are learning and how to apply that in real life.

And I see you back again during the last lecture of in the in the module 12 on state space analysis.

Thank you.