Control Engineering
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Module - 10
Lecture - 01
Experimental Determination of Transfer Function

In this module we will learn couple of things. So, we will start with experimental determination of a transfer function. So, what we had learnt earlier in terms of modelling was, you have a system you first build a physical model of the system physical models would mean that you realize it with some basic components, if it is an electrical domain you will have resistors capacitors inductors from mechanical domain, you will have equivalent elements in terms of mass spring dampers, you could also model electromagnetic coupling and so on.

And once you have the physical model, what you would do is write down the physical laws like the KCL KVL the Newton's laws, and arrive at a mathematical model. So, another way to do is what if I do not have any information on the system. I may not be able to realize it with physical components and therefore, I may not be able to directly write down the transfer function in terms of offers by writing down the KCL and KVL kind of things.

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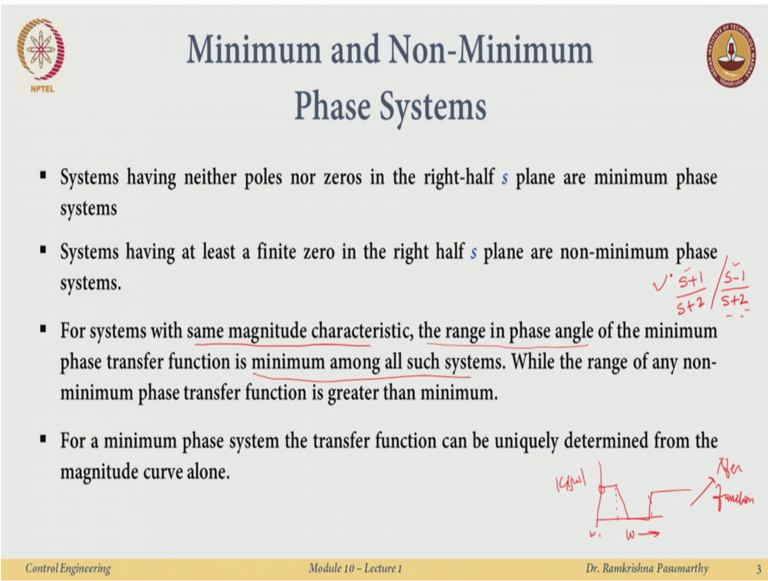

So, can I do something else, can I start with determining the transfer function with the help of experiments. So, we need to be careful while we do that. So, and at least first to define the concept of minimum and non-minimum phase systems. So, I will just briefly run you through these things before I tell you why we need these things. So, systems which have neither poles nor zeroes in the right half s plane are minimum phase systems. So, far we characterized poles being on the right hand side and identified them with essentially unstable systems, we never talked about zeroes either on the left or on the right all we knew while we were drawing the root locus was that the poles go to the zeroes and so on.

So, now what are these non-minimum phase systems? So, these are systems which have some amount some number of zeroes finite number of zeroes in the right half s plane. So, how do I identify when is a system a minimum phase or a non-minimum phase. So, there could be several systems with the same magnitude characteristic for example, if I and we also revisit this example. So, if I have a transfer function which is like s plus 1, s plus 2 and I compare it with a transfer function s minus 1 and s plus 2. If I plot down the magnitude characteristics both of these systems will have the same magnitude characteristic ok.

Now, how do I identify which is. So, by just by looking at the magnitude plot, I may not know if it is a minimum phase or a non-minimum phase. So, what happens in the ins in case of non-minimum phase systems is that, the range in the phase angle of the minimum phase transfer function is minimum along among all such combinations right all such systems which have the same magnitude characteristics. So, you have same magnitude characteristic here and here, how do I identify which one is the minimum phase, you just look at the range in the phase angle of this guy, and the range in the phase angle of this guy. And you take that one which has the minimum range right range in the phase angle of the minimum phase transfer function is the minimum along all such systems right.

So, I will show you some plots also shortly. So, while the range of any other thing would be. So, if this one is the minimum any other thing would be the non-minimum characteristic. And now for a minimum phase system the transfer function can be uniquely determined by the magnitude curve alone. So, once I know this I can do something nice, let us say if I just give you magnitude characteristics, if this is my frequency, this is my magnitude; magnitude G of j omega it looks something like this.

Then I know that this essentially what I do is a bode a plot, and with the corner frequencies the gains at say omega equal to 1, I can actually write down the transfer function here . So, for that I need to first make sure that I know that my phase would be of minimum phase. So, for a minimum phase system the transfer function can be uniquely determined from the magnitude curve alone. So, I will assume that whenever I am doing this experimental determination that my system would be minimum phase. So, more on this we will keep coming.

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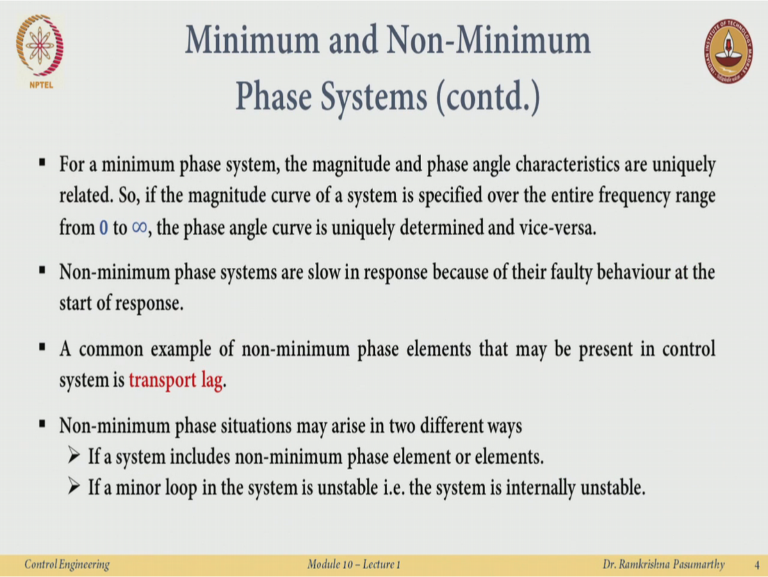

So, what are the other ways how of distinguishing between minimum and non-minimum phase. So, for a minimum phase system the magnitude and phase characteristics are uniquely related. So, just take any minimum phase like this. So, given the magnitude plot there will be a direct one to one relation with the phase plot. So, if the magnitude curve is specified over entire frequency range, then I could also find the phase angle curve. So, non-minimum phase systems what do they or why are they not really good for us, and I will spend an entire lecture on analyzing these three bullet points which are coming up.

So, non-minimum phase systems are slow in response because of the faulty behaviour at the start of response. So, if we remember while we were in in one of the earlier modules I think in number 7, we were looking at the role of zeroes in speeding up the response. There were also one plot where we talked about a 0 being on the right hand side, where we experienced and under shoot. So, I will spend time on that in the next lecture. For the moment we will concentrate on determining experimentally the transfer function of a system.

So, a common example we will again come to this examples is that of a transport like systems. So, when can non-minimum phase situations arise practically? Well if the system includes non-minimum phase elements right. So, I think we will do this revise this examples or revisit this examples in detail, or when internally there is a minor loop that is unstable.

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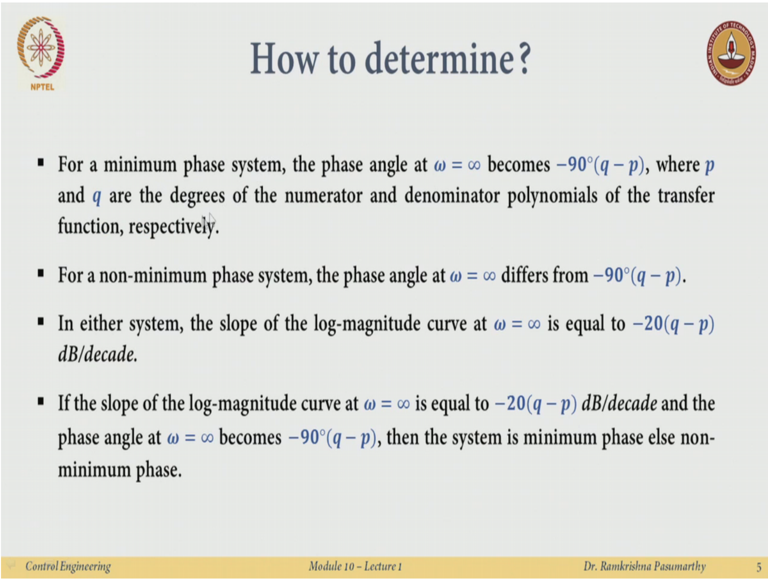

Now what are further techniques just look at the transfer function or the phase characteristic and the magnitude characteristics, is there any way can we can identify between minimum and non-minimum phase systems. So, for a minimum phase system as the frequency tends to infinity right the phase angle becomes minus 90, q minus p. This is just a difference in the number of the poles and the zeroes right with the p and q are the degrees of the numerator and the denominator, what essentially number of poles minus number of zeroes.

For a non-minimum phase system this will not be to be true, that the phase angle as omega tends to infinity will be different from this number. However, in either case at it could be a minimum phase or a non-minimum phase; the slope would be equal to this guy. So, the slope at omega equal to infinity or as omega tends to infinity would be equal to minus 20 or some multiple of minus 20, sorry would be equal to minus 20 q minus p.

So, multiple will you come over and if the slope of the log magnitude curve is equal to minus 20 decibel per decade, and the phase angle is ten minus 90 q minus p, then the phase system is minimum phase as it is non-minimum phase.

So, what you need to determine is if it is minimum phase or non-minimum phase is this number or the phase angle minus 90 q minus p ok.

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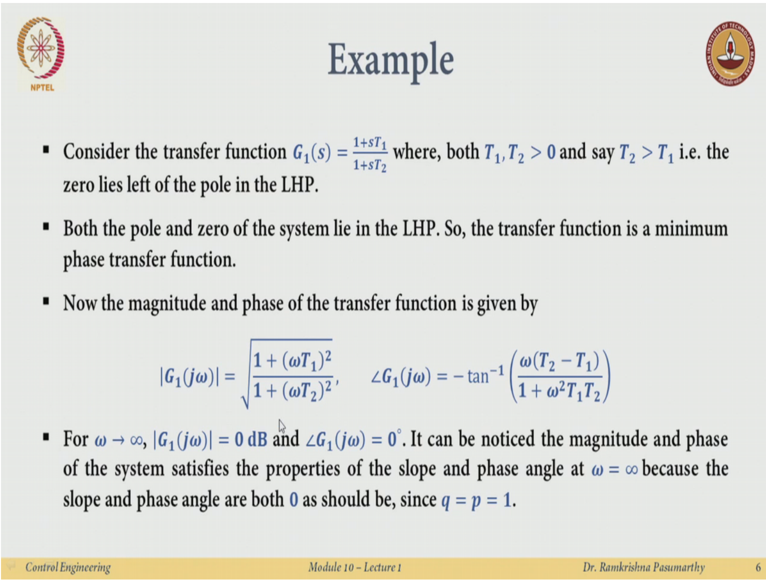

So, let us start with an example let us so, I has consider a transfer function 1 plus sT 1, 1 plus sT 2 both of them are greater than 0, and let us assume for simplicity that T 2 is greater than T 1 or in other words this 0 lies to the left of the pole in the now left of plane. So, it is a stable system and also minimum phase as per the definition which we just had. So, well both poles and the zeroes lie in the left of plane. So, the transfer function is a minimum phase transfer function.

Now, look at the magnitude and let us compare with what we had said over here in this 4 bullet points. So, the magnitude and the phase can be determined by this way. So, G 1 of G omega would be the magnitude of this guy, and the (Refer Time: 09:47). So, say something like this similarly with the angle right the angle contribution of this minus the angle contribution of this one. So, as omega tends to infinity in this case, we see that the magnitude goes to the 0 dB line or in absolute terms it will just be 1. And the angle goes to 0. So, what should be from here?

So, the magnitude for a minimum phase system as omega tends to infinity should be minus 20 q minus p, what is q in this example what is p both are equal to 1. So, what is minus 20 q minus p? That would just be 0 q minus p is 0 therefore, you see as omega goes to infinity the magnitude goes to 0. What happens to the angle? Well as the angle as omega increases and goes to infinity, we can see that the angle goes to 0. Now is this consistent with the definition here where the angle should become minus 90 q minus p, q minus p is 0. Therefore, the angle contribution should be 0. So, based on these observations here or this remarks, we can notice that the magnitude and the phase satisfies the properties of a minimum phase system. Both the magnitude criterion and the phase criterion both are 0 because q and p both are equal to 1.

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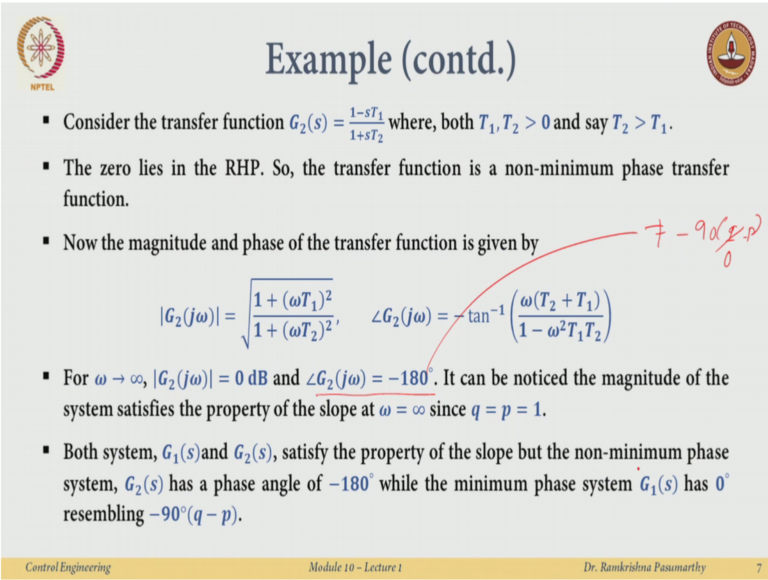

Now, as a contrast let us take another example, where I have a 0 in the right half plane the system is still stable by the way. So, G 2 s is 1 minus sT1, 1 plus sT 2, T 1 T 2 again greater than 0, with T 2 being greater than T 1, the 0 lies in the right half plane. So, the transfer function as per our definition is a non-minimum phase transfer function. So, let us do all the comparison of the magnitude and the phase again, so G 2 of j omega. So, this magnitude is like this, and it is exactly the same as in the previous case. So, the magnitude just remains the same and therefore, even as omega goes to infinity the magnitude will go to 0 dB nothing will change here right.

So, this does not give me any information if my system is a minimum phase or a non-minimum phase. Let us go to the angle right if I do this one and my angle now is negative of tan inverse, omega T 2 plus T 1 and so on. So, at little different that is why we have a T 2 plus T 1 here instead of a T 2 minus T 1. So, now, if I compute the angle as omega goes to infinity, I can easily check that the angle goes to minus 180 and this is not equal to. So, this angle is not equal to minus 90 q minus p. Q minus p in our case is 0 therefore, this angle if it is a minimum phase system should also be 0, and since this does not hold we say that the system is a non-minimum phase from these two conditions.

So, sorry this condition this is satisfied trivially, but this condition is not satisfied. So, to conclude our observations both G 1 s and G 2 s have the same magnitude criterion, but they have different phase angle as omega goes to infinity. G 2 has minus 180, G 1 has 0 degree. G 1 satisfies this criterion minus 90 into q minus p whereas, G 2 does not and therefore, G 2 is the non-minimum phase and because of this definition of the 0 being on the right hand side, it satisfies these four conditions.

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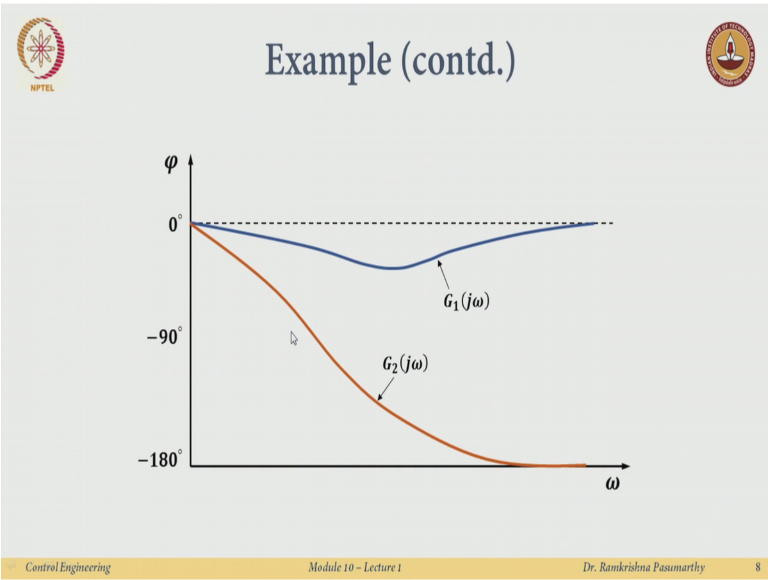

So if I were to just plot this right for some values of T 1 and T 2, you see that this is G 1 and this starts from 0 goes back to 0, G 2 will start here and then go to minus 180 again. So, given these two how do I identify which one is minimum phase? Well I go here because if I say I just do not know what is the number or what are the number of poles and zeroes. So, I take the phase plots and I say the range in phase angle of the minimum phase transfer function is the minimum among all subsystems. So, the range in the phase angles here it goes 0 say minus 45, and here it keeps on going down. So, this is the minimum phase system and this is the non-minimum phase system, because of the range also of the phase angles.

So, I need not always uniquely in if I do not know; what is the number of q s and p s, I can make use of the other definition.

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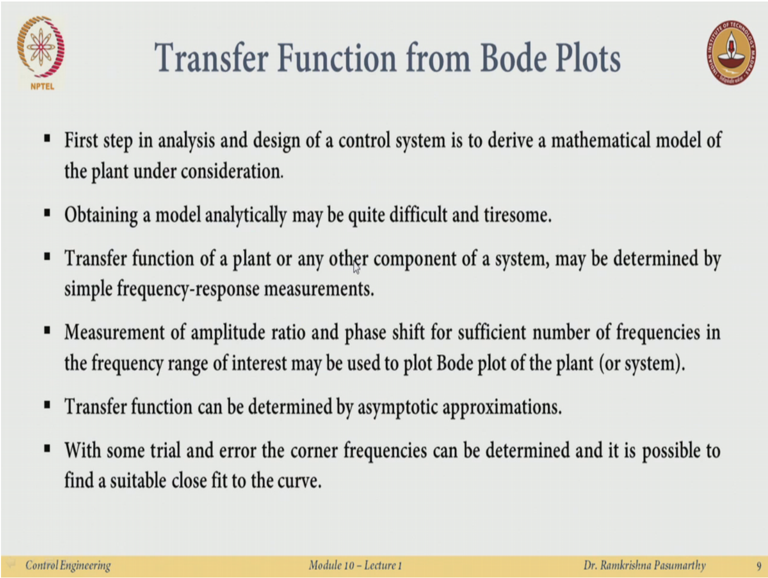

Ok. So, now, this brings us to the question which we started with can we experimentally determine the transfer function, and by experimentally determining I say well can I subject my system to different frequencies and look at the magnitude plot ok.

So, as we saw earlier the first step in analysis and design was to derive a mathematical model. The mathematical model which we dealt or which we used so far with extensively was the transfer function. Sometimes it may not be possible analytically or even I may not even have any information of what are the physical components sitting in the system. Therefore, I employ a technique where I can just plot the frequency response of the system, and see if I could get to the transfer function.

So, I measure the amplitude over a large range of frequencies and this and because of this I can easily plot the bode. So, bode is essentially was that I am looking at how my magnitude changes, which changes in frequency. And the transfer function can be determined by asymptotic approximations, and I will show you a couple of examples how we do this and of course, I can just keep on adjusting the corner frequencies until I get as close to the real model as possible; and we also know how to uniquely determine what were the errors associated with the corner frequencies for each of the cases I have a 0 of pole complex conjugate poles and zeroes and so on.

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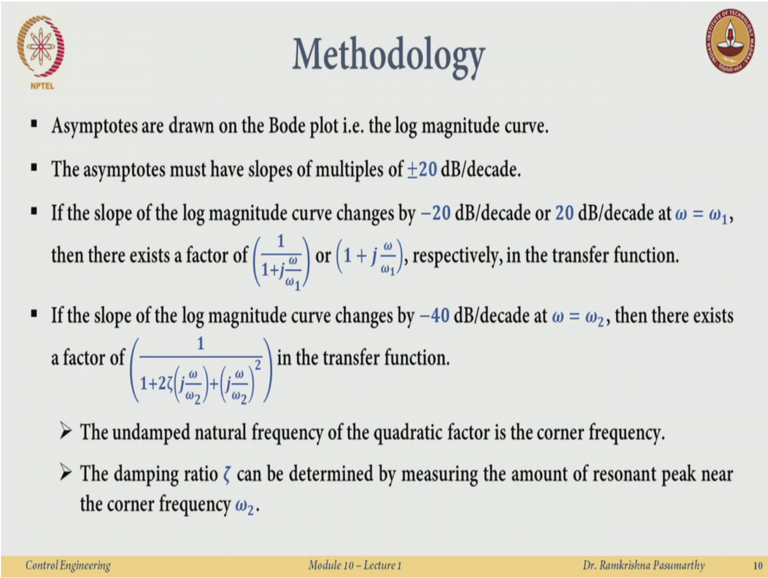

So first we will just recollect bode plot. So, this is just recollecting bode plots in in a reverse way right. So, first I will just draw a asymptotes of the of all the bode plots and well all this asymptotes must be of multiples of plus minus 20 dB per decade depending on if it is a pole if it is a 0 if there are multiple poles multiple zeroes and so on. So, if the slope changes by minus 20 dB per decade, then I know that there exists a factor resembling a pole like this. Just remember what we did recall, what we did in while we were constructing the body parts now I am just doing the reverse of that.

And similarly if there is a slope of plus 20 decibels per decade, then I am actually looking at a 0. So, this is again I assume that everything is minimum phase and therefore, I could do this, and similarly if the magnitude changes by minus 40 decibels per decade then there exists a factor something like this, these are complex conjugate poles and similarly for plus 48 it would be complex conjugate zeroes and then the un damped natural frequency. Here is the corner frequency and the damping ratio can be found out by measuring the amount of resonant peak near omega 2. So, we are plotted the bode for different values of zeta at the corner frequencies.

So, based on those observations which I had earlier I could get an estimate of what is the damping ratio.

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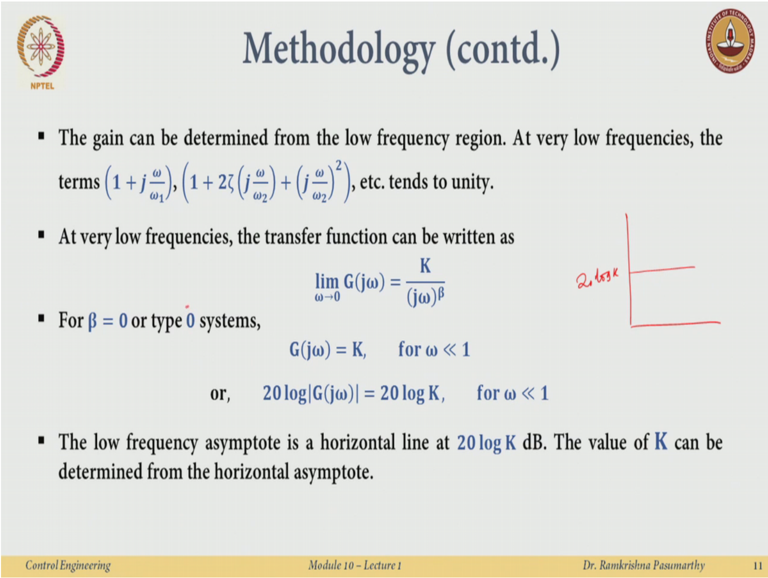

And again it can be determined from the low frequency region right and we also saw how the low frequency region also helps me; what is low frequency region helps me estimate the error constants K p K v K a and these are directly related to the low frequency gain right. So, the low frequency behaviour I can find out what is the gain of the system. And well these are all we know so, we know these from earlier right at low frequencies.

So, this derivations we did when we were looking at computing the errors constants from the bode plots. So, at low frequencies I am just interested in this kind of things, that could where beta is like the type of the system if you just type system G of j omega equal to K for a very low frequencies or in the log scale 20 log G of j omega would just be 20 log K right.

And then in this case the low frequency is behaviour is just a horizontal line at minus 20 K dB per decade (Refer Time: 19:10). So, let it just be a straight line of minus 20 log K and just be a constant line across all frequencies if I just keep one drawing this. So, this is a low frequency behaviour when I have a type 0 system.

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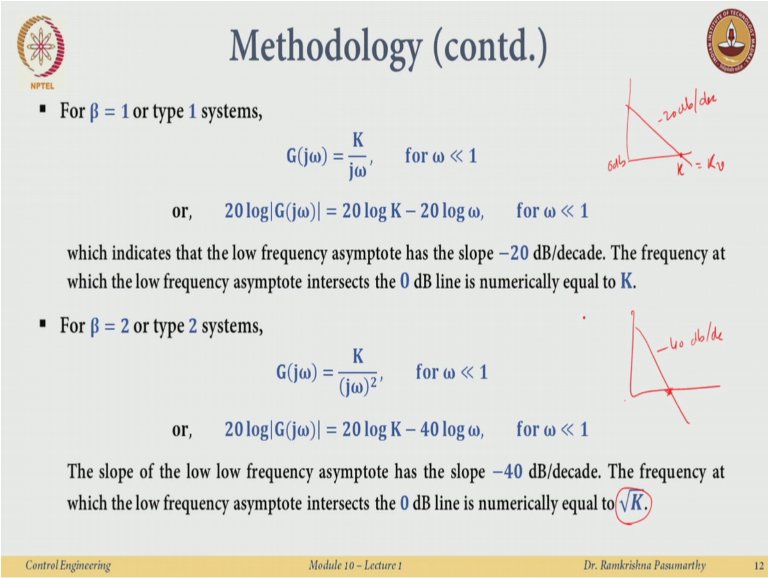

Similarly, for a type one system: soo, G of g omega can be approximated for very low frequencies as K over G of g omega and I see that the behaviour is something like this, which indicates that the low frequency has a slope of minus 20 dB per decade and where it intersects the 0 dB line this was my constant or this constant was also equal to the velocity error constant.

Similarly, for type two systems I will have a slope of minus 40 decibels per decade, and the intersection with the 0 dB line will tell me the gain right. With this we have derived this formulas right when we were looking again at how to determine kv and ka directly from the bode plots ok.

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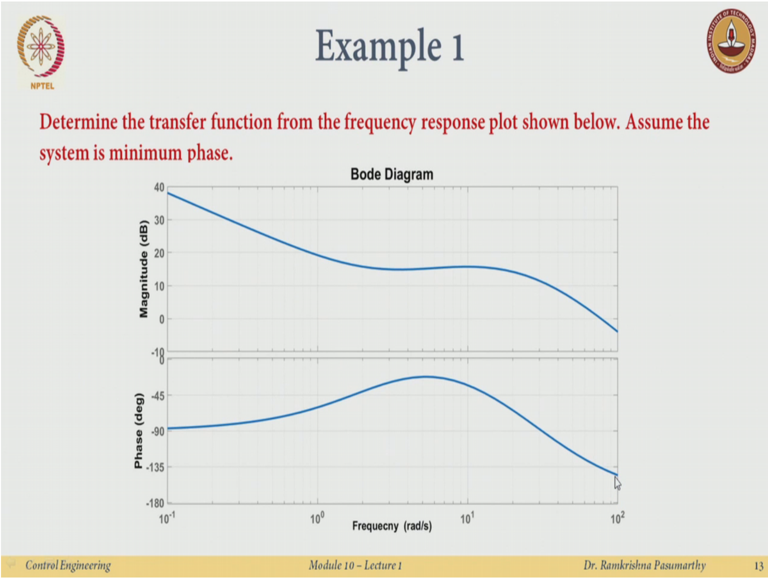

So, just as here is an example. So, how do we do this? So, just I just do a frequency testing of a signal I subject frequency testing of a (Refer Time: 20:39) system, I subject my system to several frequencies and I see a plot like this, ok.

So, and I say now if I say that I do not know anything of the system, I just give you this plot just the magnitude plot, forget even the phase for the moment. If I just have the magnitude plot, can I find out what is the transfer function?

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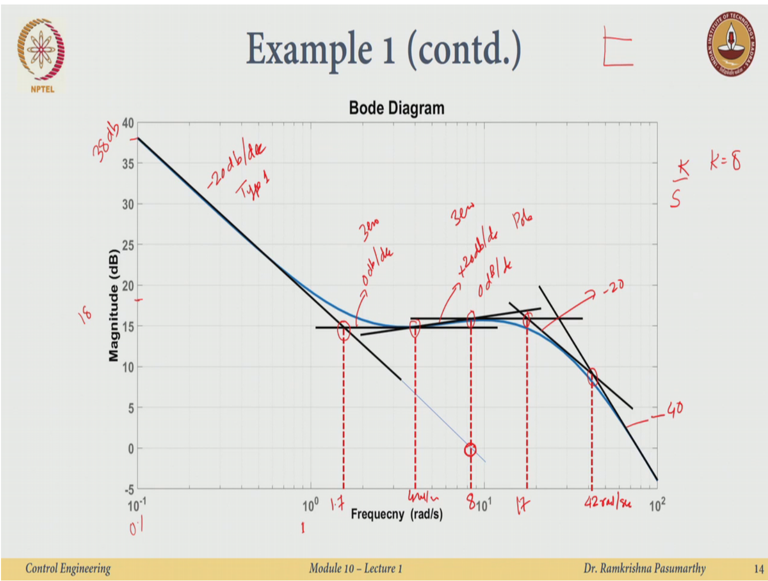

The way we do this the way we identify, now the transfer function is to first construct an asymptotic plot of the magnitude plot of the body. As said earlier I am not too worried about what happens in the phase plot, because I am essentially dealing with minimum phase system in which case I can purely determine my transfer function based on the asymptotic plot ok.

So, what happens first in the low frequency region? Well if I know that if it were just a straight line then the system would be of type 0, now there is some initial slope. So, that a system is no longer of types either it could be type one type two type three and so on. So, let us find out what is the slope of this one. So, I set here at a frequency of 0.1, I reach a decade here at ten power 0 this is 1. So, let us measure the magnitude. So, here I am roughly about 38 dB, and I go down and then where is a point where I am at a 10 power. So, so I am somewhere here I go here and say and roughly about 18.

So, in a decade from 0.1 to 1, my slope has decreased by 20 decibels. So, this is 20 decibels per decade right which means I have a system which is now of type one. Now second thing what we need to identify are the corner frequency. So, I start as a slope at 20 dB per decade and I see that my slope changes from minus 20 to 0 dB here right; so at this corner frequency. So, we will identify what this is so, I start with this is minus 20 this is 0 decibel per decade and I keep going and I see that again at this corner frequency, I am at plus 20 decibel per decade and I go here again at somewhere around this frequency again it becomes 0 dB per decade then at this frequency I am at minus 20 and here I am at minus 40 ok.

So, let us see. So, here this is a type one system. So, I will definitely have a 1 over s kind of thing or a pole at the origin. So, I also need to identify what is again k. So, we will come back to that. So, so start from here to here what is this corner frequency if I go here this occurs roughly at a corner frequency of one point seven now next I go further. So, this is this is now I sort at minus 20 I go I increase a slope by I encounter a quantity which increases a slope by plus 20. So, there is a 0 here, similarly at this frequency which is about 4 radians per second I can just measure it through this this max here.

Four radians per second I encounter another 0, because slope increases from 0 to 20, at this frequency which is roughly about 8, the slope decreases from 20 to 0. So, I have a pole, again at this frequency I go which is roughly about 17, again there is a pole and so on until there is the slope decreases from 0 to minus 20 minus to minus 20 to minus 40 here. So, also here there is a pole at a frequency of about 42 radians per second. Now the next thing to find out is what is the gain k. So, the way we find out the gain is it is a value at which the initial slope intersects the 0 dB line. So, here that is your, we also saw this when we learnt how to compute the velocity error constant through the bode plot.

So, this is a. So, my constant K which sits in the numerator, where the initial low frequency region is of this form, so, the K is simply found to be 8 by the intersection of the initial line with the 0 dB line. Now things are straight forward right.

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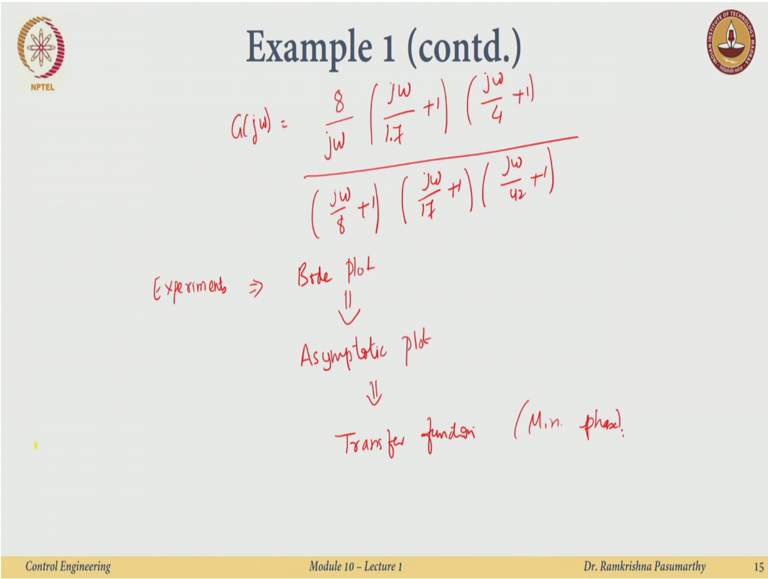

So, I just can write down my transfer function of the form G of j omega is now I have the gain, this is the low frequency behaviour I am just writing down this sinusoidal transfer function, then what happens after this. I encounter a 0 at 1.7. So, this would show up in this way j omega by 17 plus 1 after that there is another 0 at a frequency of 4 radians per second.

So, this will be 1.7 sorry j omega by 4 plus 1, and then after 4 I am at a corner frequency of 8 right. So, that the slope now decreases from plus 20 to 0 therefore, I encounter a pole here j omega over 8 plus 1, similarly I have another pool at 17 and another pole at j omega by 42 at 42 where the slope now becomes minus 40 decibels per decade. So, what is the simple process that we follow here right? So, we start with first by experiments, these experiments lead to a frequency response which is the bode plot. From this bode plot I get I can construct the asymptotic plot and finally, from the asymptotic plot I can find out what is the transfer function of the system.

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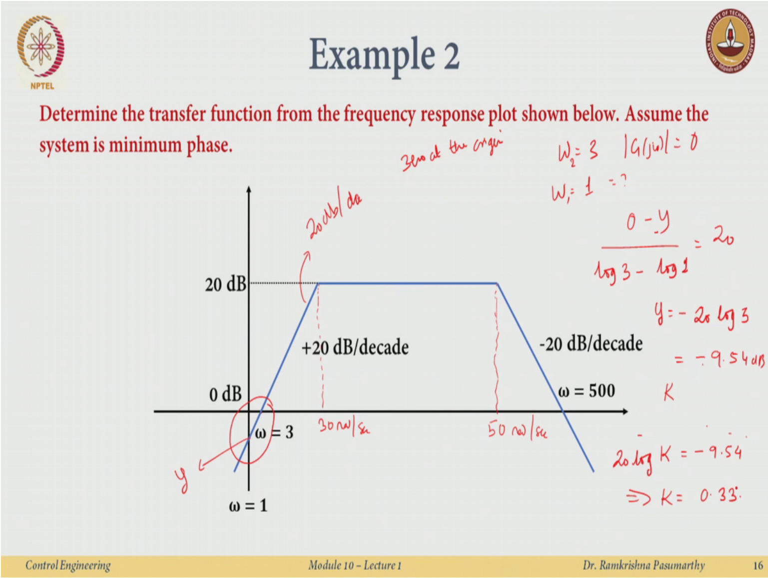

Again the key is the assumption of minimum phase. So, let us do another example where I am directly given the asymptotic bode plot. So, things are much easier for me here. So, how do I go about doing this? So, first is well I have this low frequency behavior, yes I can say that we my numerator or this is like 20 dB per decade and therefore, there is a some like a 0 at the origin and then there is. So, what is this corner frequency? So, at omega equal to 3 I am at 0 and I pass one decade to be 20. So, this is what is the decade? You multiple by 30 this is 30 radians per second.

Now, similarly now I go here and there is another corner frequency here. So, at this point I want to find what is corner frequency I am at 20, I go to 0 at five hundred and this all happens in a decade and again what is the decade. So, here I divide by 10 say at 50 radians per second. So, now, I have something like a 0 at the origin, then one pole here and another pole here. Now something strange is happening here at omega equal to 3 my magnitude is 0, at omega equal to 1, but I do not really know what this is ok.

So, now these are just straight lines at which go at 20 decibels per decade if I look at in in in the log scale. So, what I would know if I just called this as a omega 2, omega 1. So, what I know is that log of omega 2 I will just write it as (Refer Time: 29:32) now compare with the y axis. So, what is on the y axis? So, I have a 0 let me call this some number y, this magnitude here 0. So, y two minus y 1 by s, 2 minus y 1 is the slope right that is what you have given in the corner geometry. So, here on this axis I have log of 3 minus log of 1 is 20.

Ok. So, now, from this I have y is minus 20 log of 3, this means y is minus 9.454 decibels. Now I should find; what is the gap value of gain k. So, I compute it in the following way. So, at omega equal to one I have 20 log of k is minus 9.54 and this gives me a value of k as 0.33. So, now, I just want to write down my transfer function ok.

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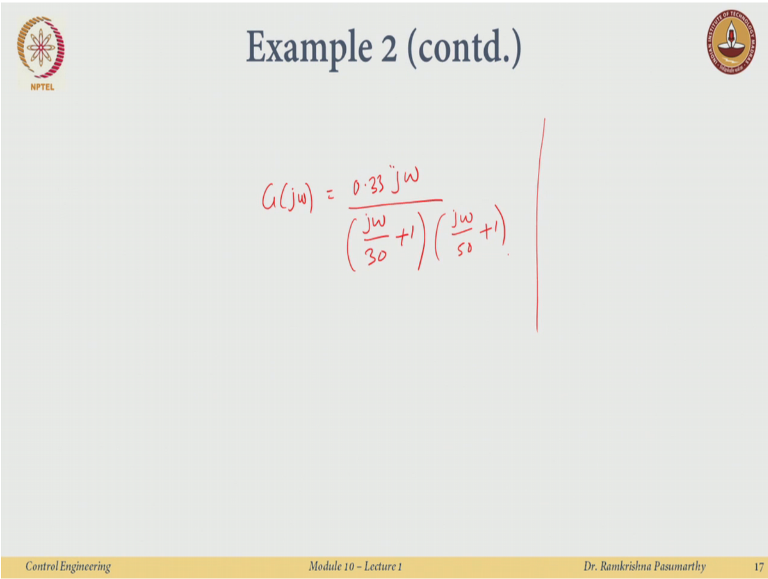

So, G of j omega I know where it has a 0 starting from the origin if I gain and at 0.33, what happens after this. So, from here till here I have a pole now right.

By slope decrease from 20 dB per decade to 0 it is like a minus 20 from here right. So, there will be a pole what is the corner frequency corresponding to that pole? That will be 30 radians per second. I will write this down here sorry this this has to be j omega this will be 0.33 j omega, j omega over 30 plus 1 and here this is a corner frequency of 50. So, this would be j omega over 50 plus 1. So, this is the overall transfer function of my system and we can re check this by just plotting the bode of this. So, we get to this asymptotic plot right. So, so what would we have seen at?

So, we have seen two examples where we start with a pole at the origin, we start with the 0 at the origin and so on right. So, this is now a very general procedure you could apply this to several other systems, it is just a matter of manual exercise picking the asymptotes carefully and accurately, and you just arrived at the appropriate transfer function ok.

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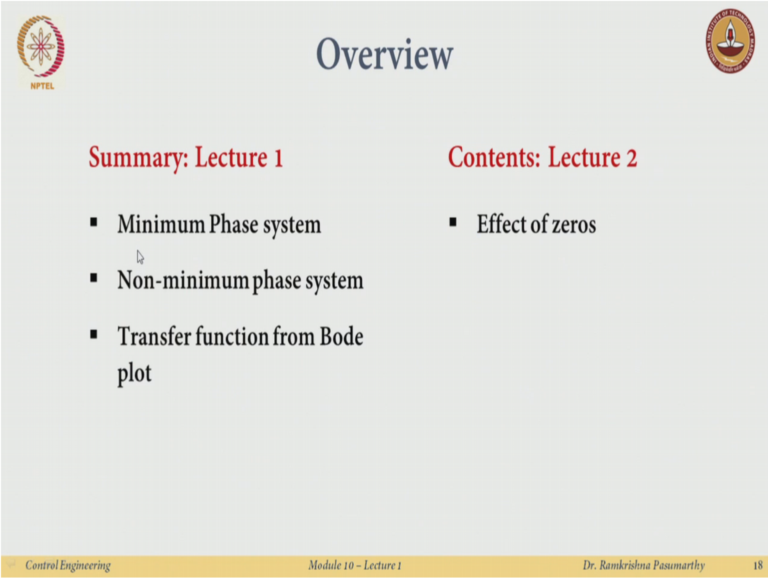

So, what we have learnt here is well we started with the aim of experimentally determining the transfer function, but the catch was that we had to distinguish between minimum and non-minimum phase, and we said that well for the minimum phase there is a unique relation between the magnitude plot and the transfer function and also the phase right. So, we learnt about these things, and then we plotted or we given a minimum phase transfer function, we learnt how to get the transfer function from a bode plot. So, I know that the system is minimum phase.

So, we will next class we will see a little more on this effect of zeroes. Zeroes the effect of zeroes to the left half plane we required extensively in module 7, even in module 8 while we were designing what we called as the lead compensators here. Here we will see some strange behaviours of zeroes being on the right half plane or zee a non-minimum phase zeroes. So, that will be coming up shortly.

Thank you.