Control Engineering
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Module - 01
Lecture - 04
Example of Modelling

In this lecture we will learn few more examples about modelling. Again using all the steps of modelling that was listed earlier; to see if you could still identify basic elements, again we go with conservation laws or any of those physical laws that govern the system. So, the first example that we will deal is very simple model of a cruise control of a car.

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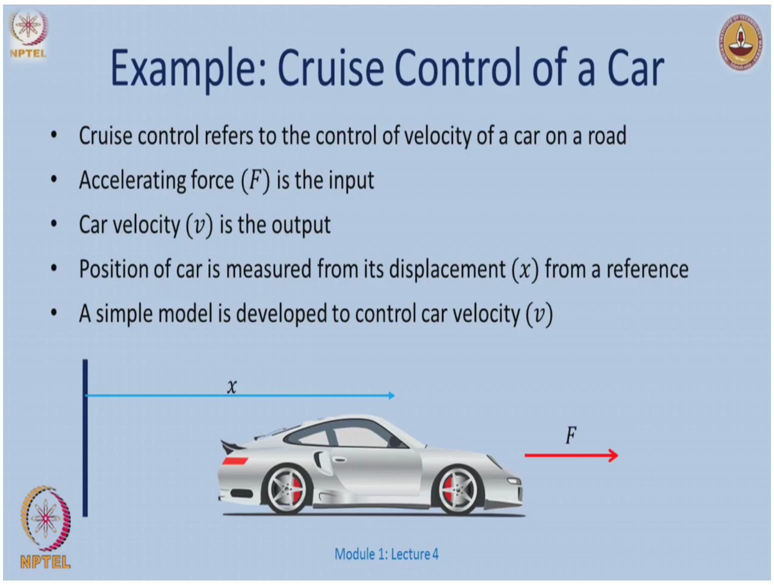

So, cruise control refers to the control of a velocity of a car on a road, where the input comes in the form of some kind of an accelerating force, the measure output is usually the car velocity. And then I usually or we usually measure the position of the car how much distance it has travelled; typically from reference points.

So, how do we capture this dynamics at there is a certain force which generates a displacement and intern a velocity.

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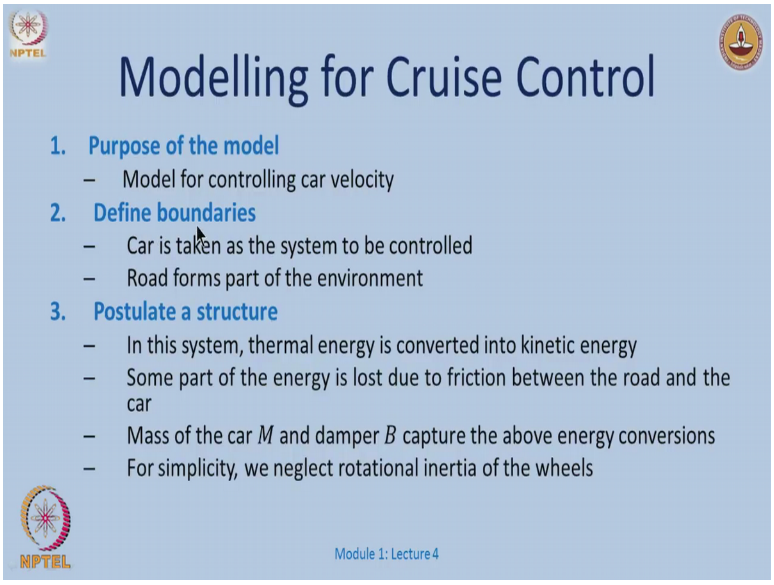

So, if we go back to the steps of modelling the first step would say well what is the purpose of the model; to purpose of the model is to understand the velocity behaviour of a car, and therefore to control the velocity. How do we define the boundaries? Well, we look at the car as a system to be control and then it travels on a highway or road that parts that forms part of the environment.

So, what could be a typical structure here? So, there is again a force displacement and velocity. So, we could say that in this system the thermal energy which comes from the engine structure is converted into kinetic energy or the velocity. Some part of the energy is lost due to friction between the road and car. We will not take or take into consideration for simplicity the thermal losses in this case.

So, the car would have certain mass and then since we already said that there is some energy loss, there should be some kind of a damping element introduced in the model, and before we write down the relevant energy conservation of the force conservation loss. And also to be little simple to begin with we will neglect the rotational energy of the ways.

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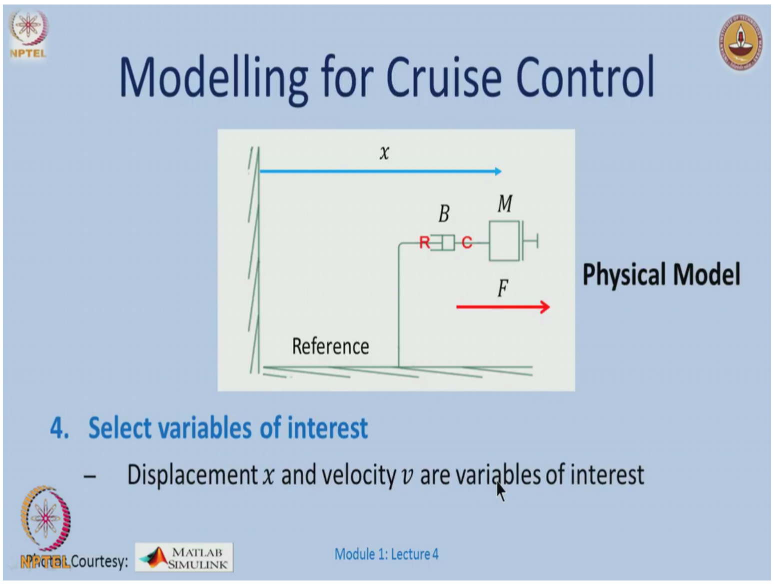

So, once I do this I would like to select some basic variables of interest. So, the variables of interest are the displacement x which again gets the velocity all where this force F. So, have this car modelled as a mass M and the friction between the car and the surface on which it which it is moving the road as the modelled as a damper with the B.

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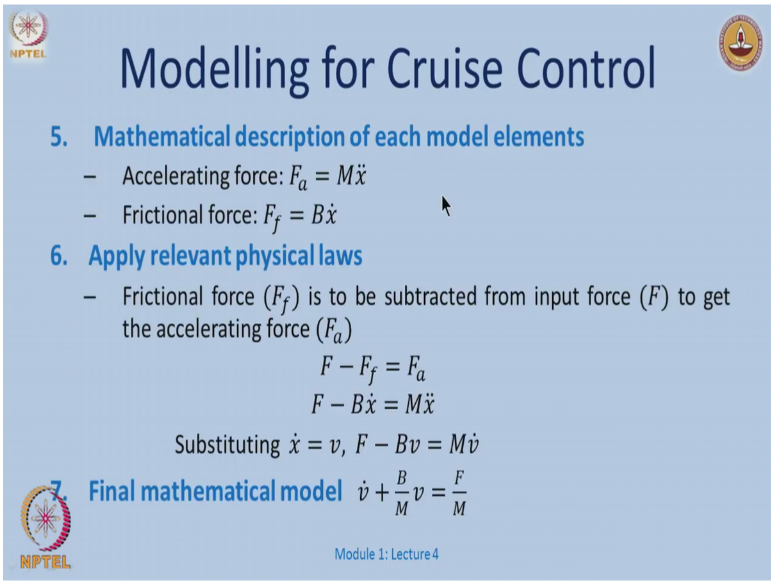

So, there are two elements here: one is the inertia element which is modelled as F equal to M x double dot, as we did in the previous lecture. Frictional force would be related to the velocity via damping coefficient B. Are there any physical laws? Shall once we identify this structure and the structure here I can directly write that the summation of all forces is 0, which means something like this is an accelerating force; the frictional force and the accelerating force.

So, what is the frictional force? Well, this is B x dot and the accelerating force is M x double dot and substituting that dx by dt is the velocity. I can write my equation as F minus B times v minus M times v dot; sorry F minus B times v would be equal to M times v dot or the final mathematical model would look like v dot plus B over M times v is F over M; a very simple model which captures the cruise control in a car. This is very simplified model, but still gives some basic understanding of the dynamics of the system.

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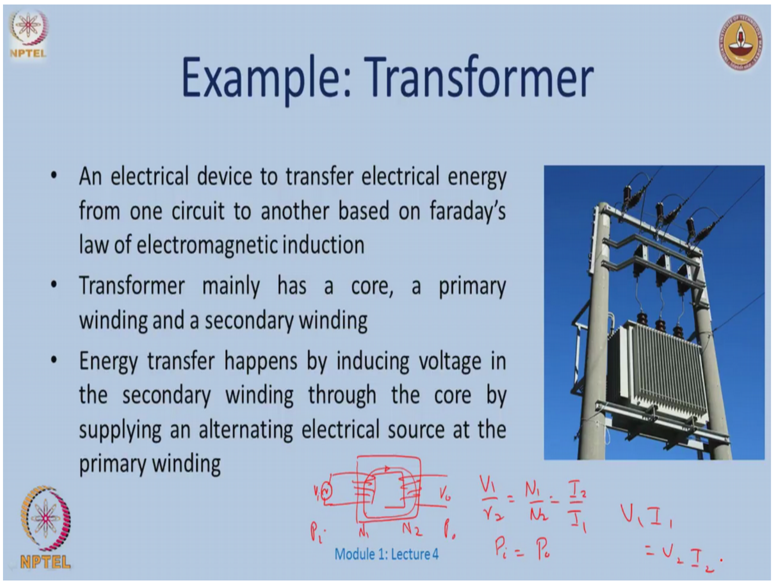

The next example I can we come back to the electrical domain is that of the transformer. So, what is again transformer? Just to recollect itself an electrical device which transfers electrical energy from one circuit to the other and it works based on the laws of electromagnetic induction introduced by faraday. How does it look like where it has core something called a primary winding and a secondary winding? And an energy transfers happens by inducing voltage in the secondary winding via the core. And where does the core get energy? The core gets its energy through some electrical source in the primary winding.

So, if I just quickly write a very small diagram of this could say that this is my core, so I have this all this as my primary winding, this is my secondary winding, I have voltage source and then I will call this v i which will generates some output voltage v over here. And then this is the magnetic core there will be some flux, and with all those electromagnetic induction laws, there would be voltage induced in the secondary and then you will have certain N 1 N 2. So, is that V 1 over V 2 is again N 1 over N 2 equal to I 2 over I 1. And then what is a region between the power here and the power here? Is that it is very simply the input power should be equal to the output power or V 1 I 1 is V 2 I 2.

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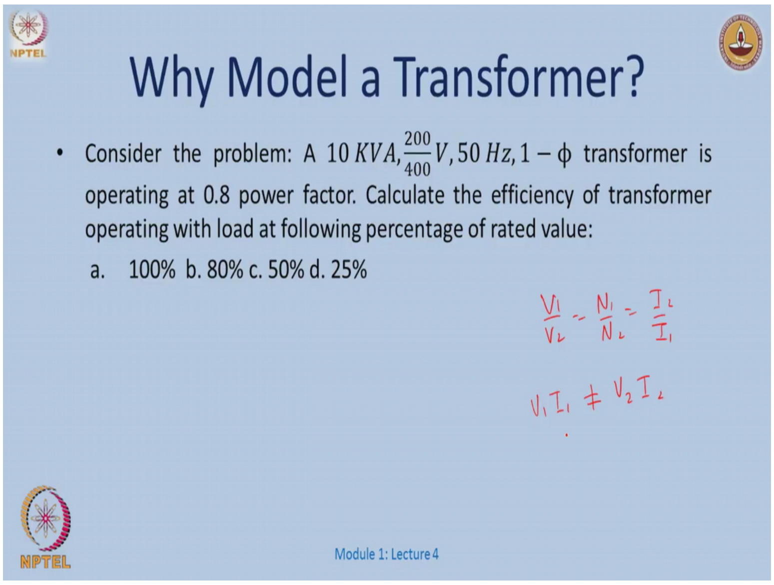

Now, this is something what we do in our course. All text books have some problems like this. Consider a problem where I given a rating of a transformer goes from 200 to 400, a step of transformer frequency is given to be 50 hertz single phase transformer some power factor. And if you calculate the efficiency of the transformer operating with the load with some load at 100 percent load, at 80 percent load, 50 percent load and 20 percent, 25 percent load.

So, how does solve this problem? Though I take this transformer and say well, let we get us voltage supplied let me get a load which is 100 percent and then do my experiment each time for 100 percent 80 percent 50 percent and 25 percent and get the answers. Or there is any other easier way. The risk when I do this experiment is that is when I may not be very careful or I could burn other transformer and so on.

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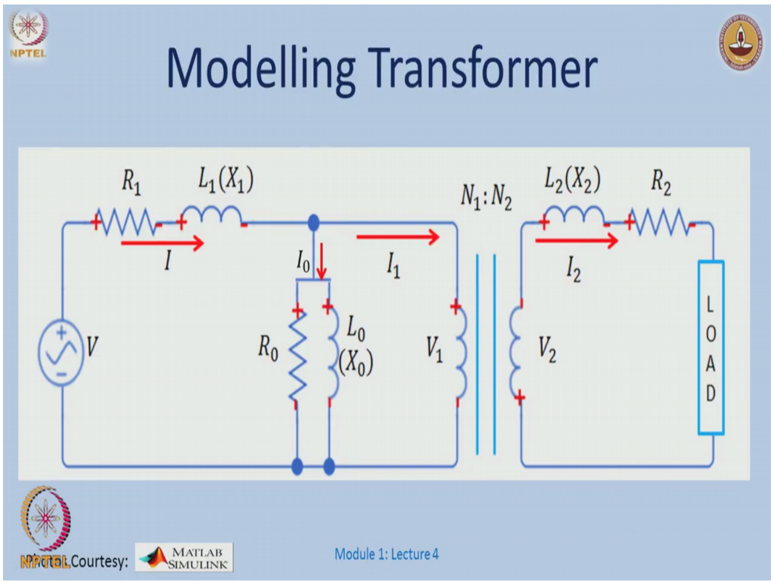

And therefore, what we equivalently see this problem is a problem translated to something like this which is the equivalent circuit model of a transformer. Now, once I read this question; solving it through this picture seems very tricky right that I really have to get this transformer down open all the wires, we know get a big Crain because this seems a very big transformer. But if I look at this transformer as something like this then things get much easier.

Now what is this process called? This process is where we call what we call as the model of the transformer for solving problems of this kind. And why to problems of this kind arise? When we say that V 1 by V 2 is N 1 by N 2 or I 2 by I 1; this was true all the time then we would never position equivalent circuit, because this thing usually does not; it is not always true that V 1 V 2 is I 1 I 2 there is something going wrong here and that something going wrong is what we will try to observe.

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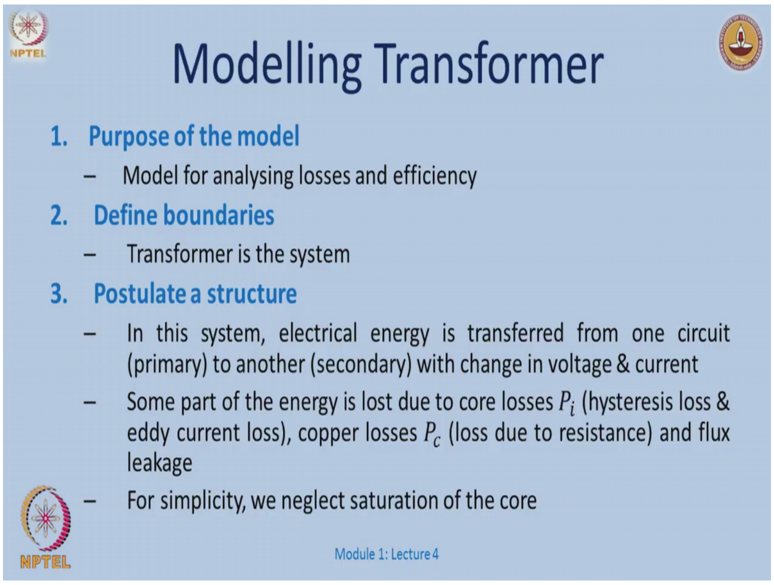

So, we will propose a model for analysing losses, and therefore in turn we could compute what is efficiency. So, the first step we identified what is the purpose, second step is define the boundaries which is the transformer, possibly connected to load. And then the structure when the structure of the transformer; so we have the primary winding, we have a secondary winding, we have voltages currents. Or since I have winding would have its own resistance its own inductance the core would come with its own properties which might lose some energy and so on.

And just for a movement it will assume that we will neglect the saturation of the core. If I take into consideration all these other things of what in the text books would be called as the core losses and the copper losses.

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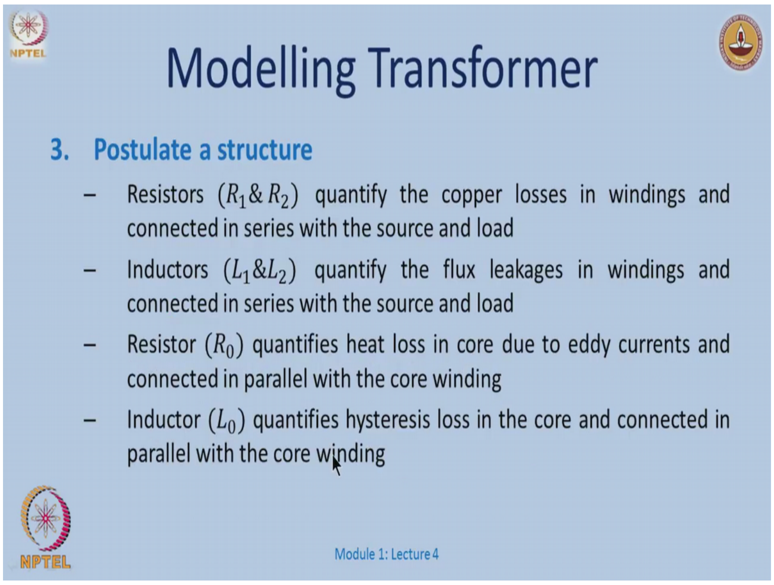

So, I then give a structure to that. So, I have a primary winding and a secondary winding, then I call this with each of this guides would have some resistance let me call this R 1 and R 2. There would be some flux leakages in both the windings of primary and secondary I will call them as L 1 and L 2. There are certain losses which happen in addition to the losses in the winding also in the core, they are I will quantify them as R 0 the resistors which called quantifies the heat loss in the core and similarly the inductance of the core.

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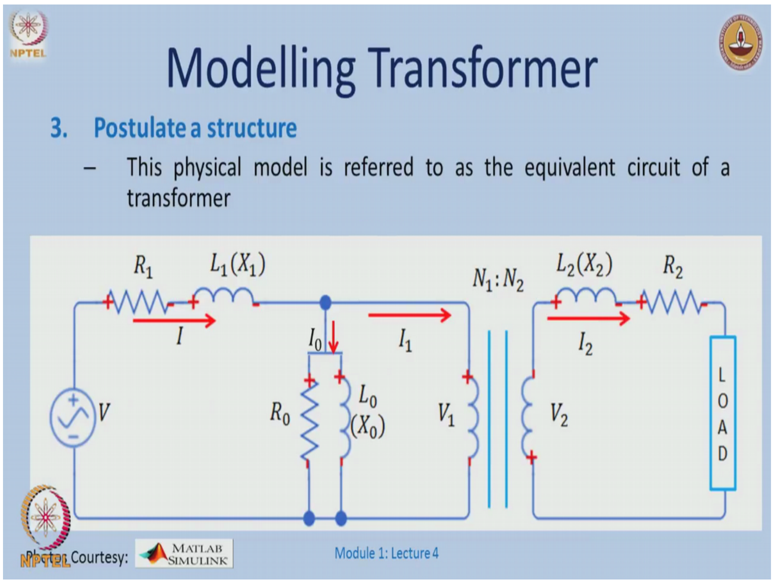

So, once I have this, I have this beautiful looking circuit. I have a voltage source, resistance, L 1, the inductance R 0, I have L 0, I given there is a turns ratio L 2 R 2 and so on. So, this is the physical model of the transformer. Now given a transformer how test we what is R 1, L 1, what is R not, what is L not what is L 2 and so on. The load I would know because I am just connecting this externally to the transformer.

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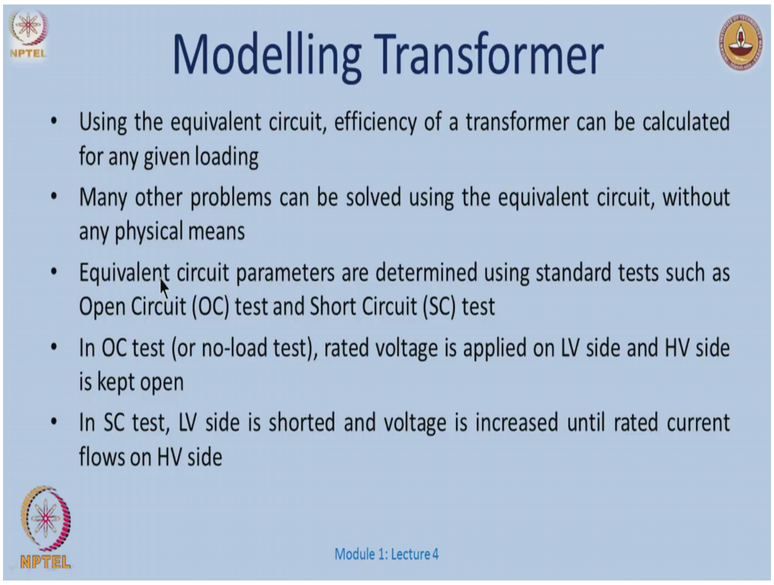

Once I have is equivalent circuit I would know that I could calculate the efficiency of the transformer. And who gives me this values of the R several resistance and inductances? Well, we know that we could do something called an open circuit test or a short circuit test. So, when I do the open circuit test what do I do? The open circuit there is no load that this guy just be is just open and that is why we call this. The open circuit test I apply a voltage here which is equal to the rated voltage. Then most of the current consumed would just be here. And based on the measurements of the power and the current and so on I could estimate what are these guys. What is R, what is L; just by the relation between the voltage the current and power. That we call in our lab experiment as the no load test.

Similarly, to identify the resistance and the inductance of the winding I do something called a short circuit test, where the low voltage side is the short circuit. So, this guy is simply a short circuit. And I keep on increasing the voltage from 0 to a value where the current goes to the rated value. That is usually typically given in our problem statement or even if you look at a physical transformer I will have it as the rated current; you know the supplier would he would give me some.

So, once I have the rated current going through I this I 1 and I 2 then this becomes a little negligible and all the parameter which I measure or all the losses would mainly be accounted in I square R losses in the primary winding and I square R losses in the secondary winding. And doing this these two experiments would give me all this values of R 1, L 1, R 2, L 2, R naught and L naught.

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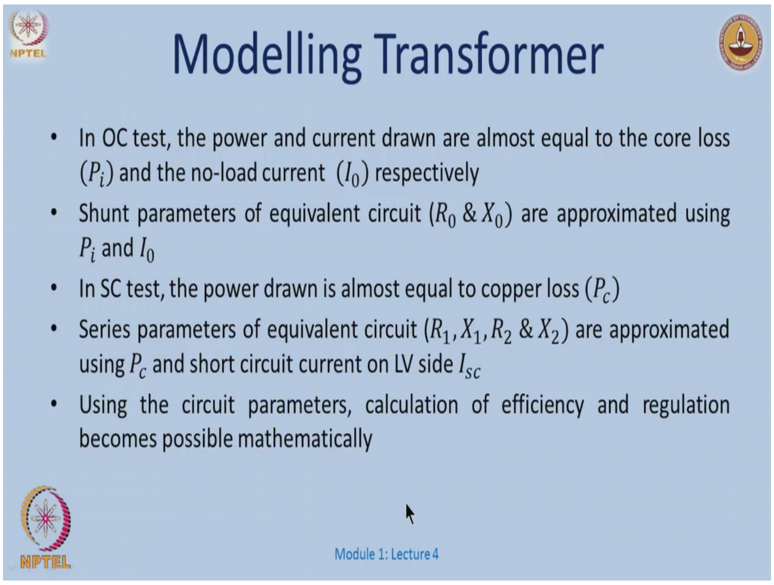

So, this process: the open circuit test, the short circuit test, identified parameters, so once I identify all this parameters the calculation just becomes now basic problems in circuits. So, I can just look at the circuit and all these problems translate to; so this is given may be possibly the certain power factor or this is could be a resistor. So, I am just solving problems using a pen and a paper.

So, what I have a done? I have I have looked at this guy the big transformer which is sitting at a substation, I want do experiments on this which will give me answers on how this transformer would behave to different changes in load; no loads, from full load different power factor, resistive load, inductive load, all these experiments I have translated to simple experiments which I could just use pen and a paper and possibly a calculator. And that is the beauty of the model I can answer all the questions without having to touch the transformer.

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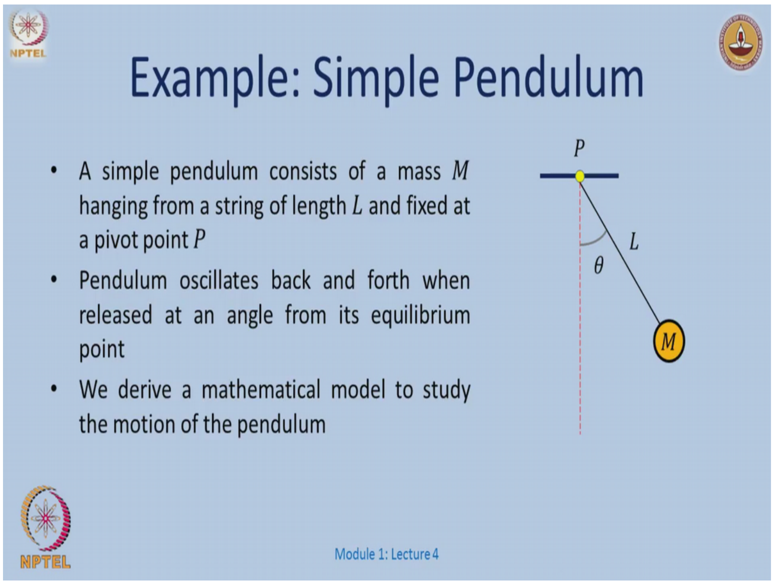

So, next example is something which is very familiar to us something which we learn in high school. Now simple pendulum; well its by construction it has mass M, a little string L hills to a pivot point P and we all know that if I start from this position given theta just let it go it will just keep on oscillating back and forth around its equilibrium point. Now how do we explain this dynamics or explain what this pendulum does as a mathematical model.

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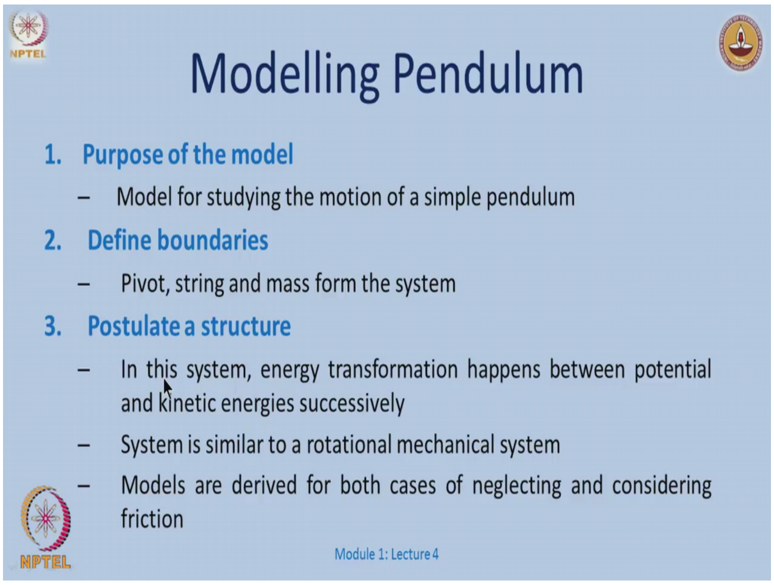

Let us go back to high school physics for a while. The purpose of the model again is I want to study the motion of the simple pendulum, how would it behave if I release it from theta of 5 degrees or theta of 10 degrees or 90 degrees and so on, and edifying the relevant boundaries of the system. And what is the structure of the system? In this system I know that there is some kind of energy transformation happening in the system.

So, if I see here that it goes all the way here till minus theta and comes back. And at this point where it stops and goes back that is a point where we have complete change from the kinetic to the potential energy: kinetic energy goes to 0, all the energy is the potential energy. This is the point where all the energy is into the kinetic energy and back and forth. So, system oscillates between points of maximum potential energy to maximum kinetic energy and so on.

We will see how looks mathematically. As the structure is similar to a rotational mechanical system right you have rotational motion, possibly at torque external torque or the systems own torque because of its inertia and so on.

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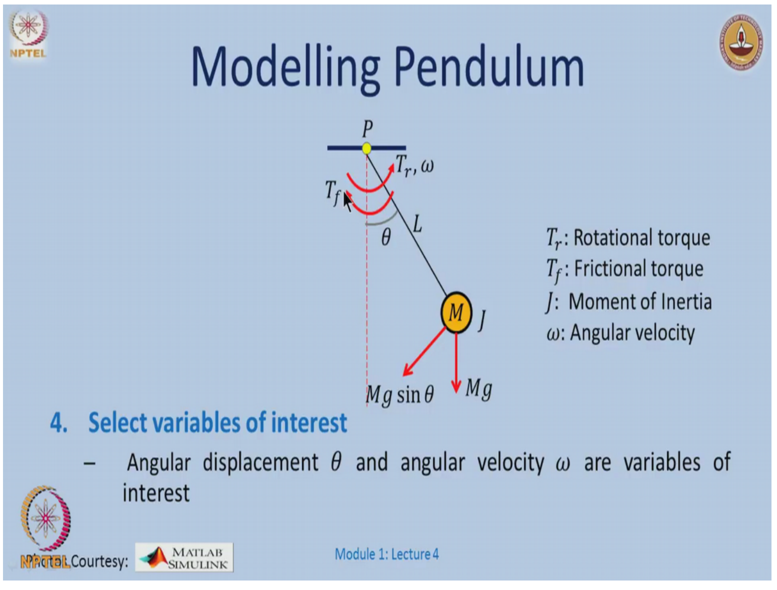

Next is friction I just draw something like the free body diagram which I am use to in my twelfth standard physics. I have the rotational torque, there is some there some could possibly some frictional torque, a moment of inertia j and some angular velocity omega.

So, the variables of interest here are I would as in the car I was interested in how the position and the velocity evolves. Here I would be interested in how the theta the angular displacement and the angular velocity evolve a time based on these parameters of system the M the L g is natural the gravity.

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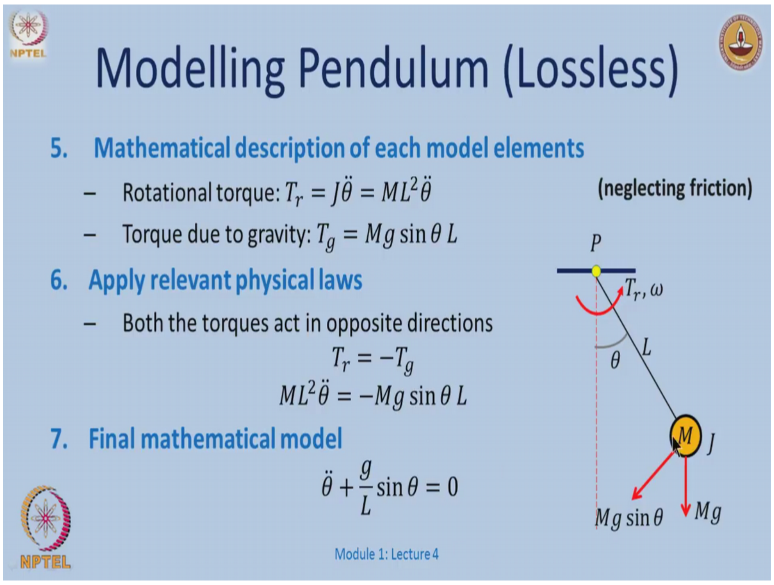

Again just if you recollect those basic building blocks in a mechanical system I had the inertia element modelled as torque is related to the angle acceleration via j. And again if I just recollect some of my high school physics I would know that the moment of inertia is M L square. Now I also in additional have gravity right in my system and the torque into gravity would be in this direction and I can be easily computed to be M g sin theta multiplied by l.

So, this is my gravitational force M g the force in this direction would be M g sin theta and the torque would be given by this form. And both this torques are attain in opposite direction. So, at the moment I do not consider friction over here, we just a lossless system. So, M L square theta double dot the torque this guy is equal to is compensated by this guy minus T g. And therefore, I have a dynamics written as this one: theta double dot g over L sin theta is 0.So, this is just lossless a pendulum. Once you leave it from this position theta it will keep on oscillating forever.

So, as a simple exercise can be try and find out and equivalent of these kind of behaviour in an electrical circuit, where I just heard from an initial condition and my systems are such or the system component are such that the total energy is of course conserved and it also shows an oscillatory behaviour. Think about it try to do it may be in the next class to see if there is an equivalent of simple pendulum behaviour in an electric circuit.

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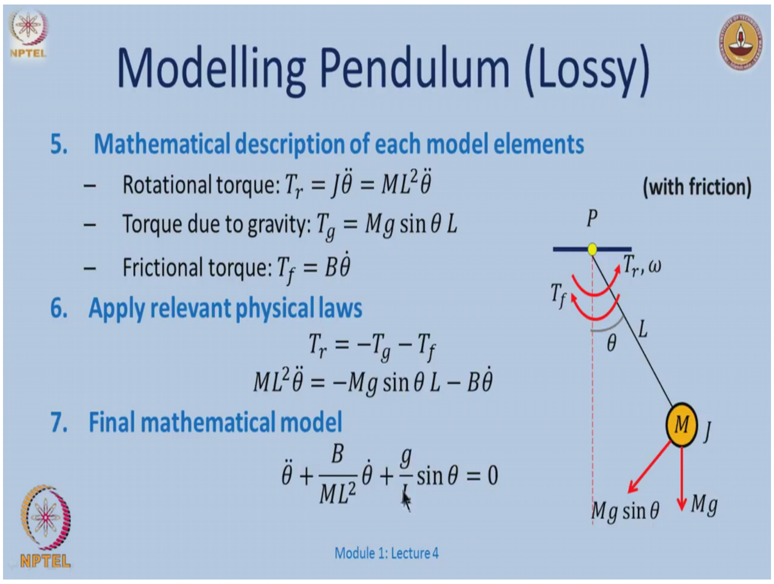

Next we move on to modelling friction. In addition now I have the frictional torque T f again if you recollect the previous lecture it is modelled as relation between the torque and theta dot by a resistive element beta. So, I will have one more equation in my or one more term in my equation that T r would be the negative of T g and the negative of T f both acting in the opposite direction, and I just write down the relevant substitute this T r T g and T f over here to get final mathematical model something in theta double dot theta dot and sin theta.

So again, this is my kinetic energy element, this is my potential energy element, this is my frictional element if you really want it relate it to mass spring and the damper kind of system. Think of another example of an electrical example of which exhibits behaviour like this. Like a lossless behaviour combined with the dissipate element; think of it.

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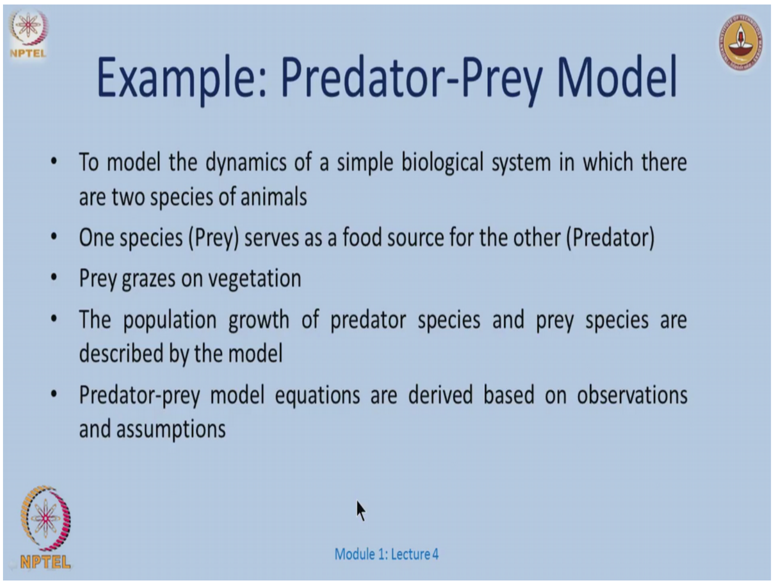

So, something which we will do now is, something outside what we have learned so far in text books. So, I just briefly explain you what we are trying to do here.

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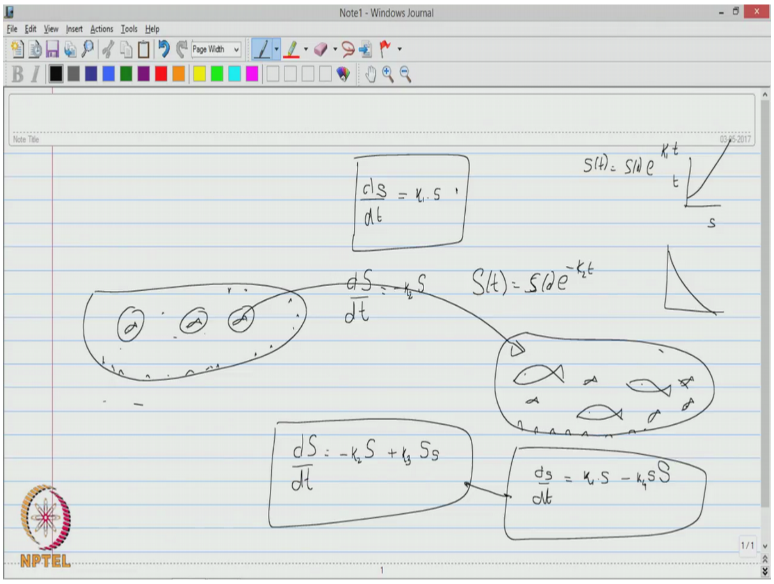

So, let me start with some scenario here say- here I have like a little bowl here lots of water, little little leads here, little little vegetation inside and I have some you know small fish and this small fish nice nice it will guys the feed on this little vegetation inside the pond. So, this is good looking scenario where these guys, bunch of those small fish, the small fish here some hiding inside here and so on. So, this small fish feed on all the vegetation and then they are they leave happily they multiply the growing population and so on. So, how can I model this population?

So, let me call this small fish as some small s and I want to see how these guys involve with time. And if there are a bunch of them with some initial condition a nonzero initial condition they have plenty of food to eat you will imagine that they will keep on growing forever. So, this could be modelled as some constants say I will call this k 1 times s; they are keep on growing forever. This is a very simple model which captures growing on forever phenomena.

Now, let us see there is another pond here where the bigger fish many of them, but this fishes such that they do not feed on vegetation but they feed on some smaller fish. And if there is no small fish here for them to feed on it is natural that all of them will die out. There is no food the fish go on dying. And I will model this as minus k 2 times big S. So, the first equation tells the how the small fish will grow when they; of course, I assume that they do not naturally die and they have lots of food to eat. So, if I write down this just be like an exponential growth like so the S at any time t will be S at 0 e power k 1 times t; this is a solution to this equation and then if I just draw a graph between small s and t it might just grow like very fast.

So, this guys what will happen, they do not have food to eat so they will die down eventually and start the initial condition they have nothing to feed the solution will not look very good, we have s the big S at 0 e power minus k 2 t and with time they go smaller and smaller asymptotically it go to 0.

Now interest in thing happens when I take some of these guys small fish and put them here. So, there is the small fish nice time it will once; it is say one of them could be with no one. So, what will happen now? So, this big fish have some small fish to eat. So, what we would expect to happen in the second equation. So, d of big S over dt, these guys had nothing to eat so they all were dying now have something to eat.

So, naturally we would expect that their population will grow. So, I will right that this big fish they come in contact with the small fish at some rates I will call this say from k 3. And you will see now the population increases that because they have food to eat they can multiply and so on.

What happens to this guy? Will this guy still grow exponentially? What happens to d small s by dt? Here we assume that they never die, they have lot of food to eat and therefore they keep on multiplying. Now these d s by dt which was earlier k 1 times small s will now see some decrease, and the decrees in based on how many big fish come in contact to the small fish and at what rate and we call this k 1. So, this equation together with this equation; this is small s and big S I hope here able to distinguish between those let me just this is my d of small s. So, this two equations fill together in some way explain me the behaviour of how small fish and big fish their population will change with time based on may be some initial conditions availability of food and so on.

So, we are just written down this vaguely in terms of some mathematical equations. Just made some assumptions and we assume that they grow exponentially or sometimes they die when they have food and or when they small fish die when they come in contact with big fish and so on. So, let us do this little systematically and see how it goes.

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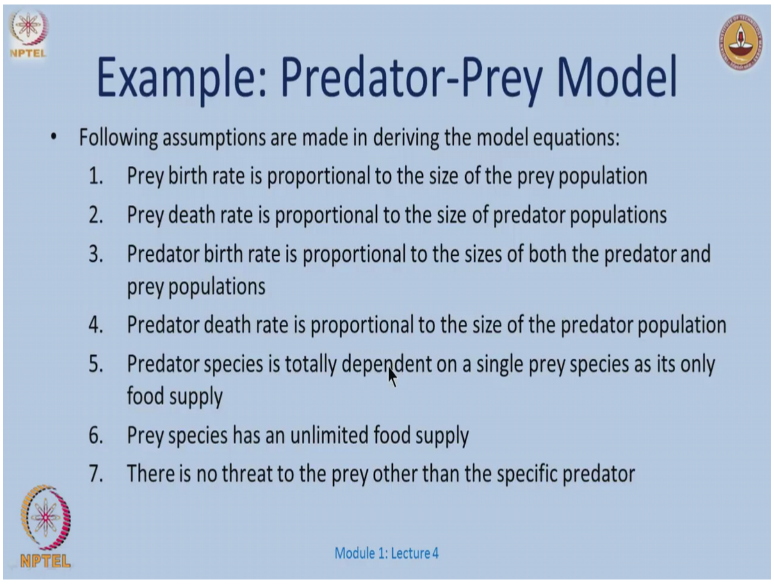

So, these models are typically referred to as the predator prey models. So, what is this? So, there are two spices: one of them serves as a source of food for the other. So, the bigger fish are the predators and the smaller fish are called the prey. And this prey they feed on vegetation. So, how do we go about describing a model for this? So, earlier these are based on certain assumptions and observations.

So, we make the following assumptions when we derive this model. At the prey birth rate is propositional to the size of its own population. And the way the prey die is only when they come in contact with the predator, and therefore their death rate would be proportional to in some way to the size of the predator population. The third assumption we make is that the predator birth rate is proportional again to the size of both predator and the prey population because, when they are in contact with each other they have food to eat and then they could multiply. And the predator death rate is proportional again to be its own size.

Further assumption are that: the predators are have nothing else to eat accept the small fish the big fish nothing to eat expect the small fish; or other words it means at the predator spices is totally depended only on single source the small fish for food supply. And the preys spices are such that they have unlimited food supply and nobody else eats them expect the big fish; there is no other thread to the prey apart from this specific predator which is the big fish.

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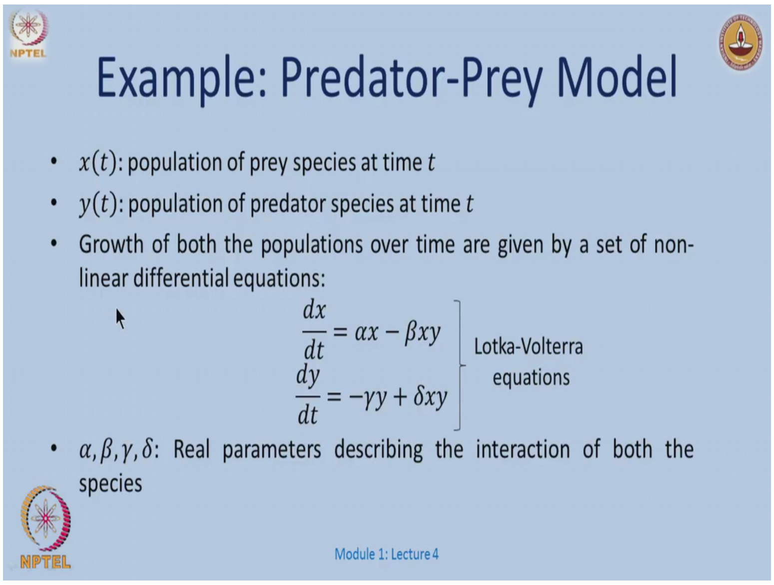

So, let us say I want to define; these are my system variables like in a way the x t in the population of the preys spices at some instant of time t; y t the population predator at time t. So, based on how we derived the model previously dx by dt would be some constant alpha greater than 0 which will this first term will denote how the predator population increases, then they are their own and they have lot of food to eat. The second term minus beta x y captures how their population decreases when they come in contact with the bigger fish or the predator.

The next expression dy by dt is minus of gamma y which means the first term means that they will die in the absence of food. The second term captures the increase in their population when they come in contact with the prey or the smaller fish then how their population increase. And this all this parameters alpha, beta, gamma, and delta depend on how fast they die or you know how frequently the predators and the prey they come in contact with each other; all these are like greater than 0.

So, these equations are usually referred to in literature as the Lotka-Volterra equation.

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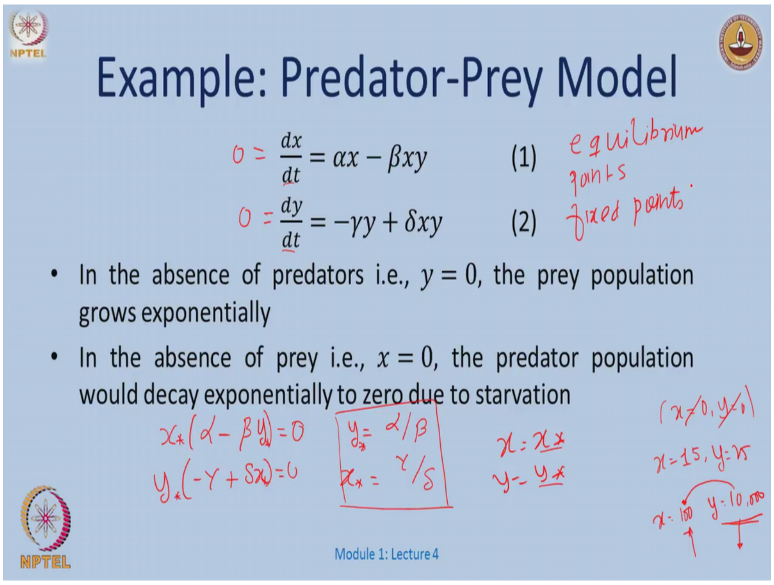

So, again we just re writing what we just said earlier: in the absence of predators what happens to the prey, in absence of prey what happens to the predators and so on.

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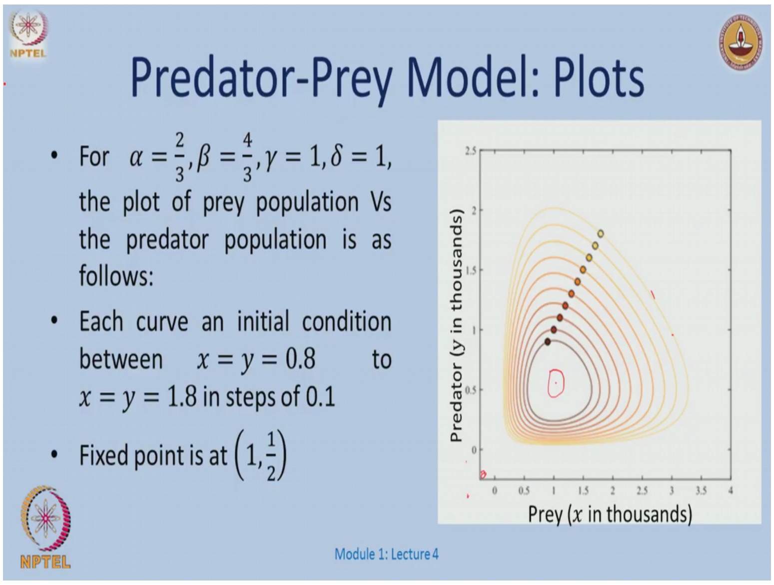

If I want to just look at look at the graph of what happens right. So, let me say that if I want to just right down this things- if I say well what if the initial condition says that x equal to 0 y equal to 0 then means nothing. Wt what if say some x is 1 say 15 and y is some 25, who will increase first? Who will decrease first? Let us say for example, if I start with say some something x is 100 and y is say 10000. So, what we would see initially is that there are lots of big fish to eat the small fish, so you will see a drastic reduction first in the size of x.

And then now these guys will have less food to eat then y will go down for a while. And y goes down these guys will increase and then they keep on increasing and decreasing forever, unless there is a point where if there is some number x star and y equal to y star they start from these numbers and remain that forever; which means that if x start is constant for all times, y star is constant by all time which means dx by dt will go to 0 and also dy by dt will go to 0. Let us say this is this is possible or not.

So, I have two equations here: alpha x plus beta alpha x minus beta x y going to 0 and minus of gamma y plus delta x y going to 0. So, the first equation would mean x alpha minus beta y is 0, the second equation would mean y minus gamma plus delta times x is 0. We are not interested in the solution x equal to 0 and y equal to 0. Therefore, the solution would be y is alpha over beta and x let me call this is star because this is looking like solution when dx by dt goes to 0 and dy by dt was goes to 0. So, this x star would be if I rearrange gamma over delta and y star would be alpha over beta. So, if a initial conditions are size these two things hold then may population will just be the same throughout; that is why this model takes place.

What happens for any other values? So, this point here the centre point is a x star y star somewhere here. For every other thing I just see that the population of x increases, then because x increases y will also increase, but we will y increases x will see some decrease and so on. Let us be some oscillatory kind of behaviour depending on where it starts from here then I will just follow this curve here, if I start close here these curves which look like here. If I start here I will just remain here. There is no other thing because I cannot go here because x and y can never be equal to 0 and I do not even would not start over here. So, these are fairly good conditions where I start at some point and then I just keep on re visiting the same point over and over again.

This is just simulations for some certain values of alpha beta gamma and delta. So, this is what we call as a fix point or an equilibrium point when dx by dt goes to 0 and dy by dt goes 0, this is like the equilibrium points and we will define this eventually more formally; equilibrium points their also called fix points. So, do not worry about this now, but we will define this formally little later right.

So, this point is 1 comma 1 by 2; L l may be we loss a share code of how to draw this kind of pictures; just for your own understanding and all this could be drawn for easily in meta lab. So, is there any drawback of the model?

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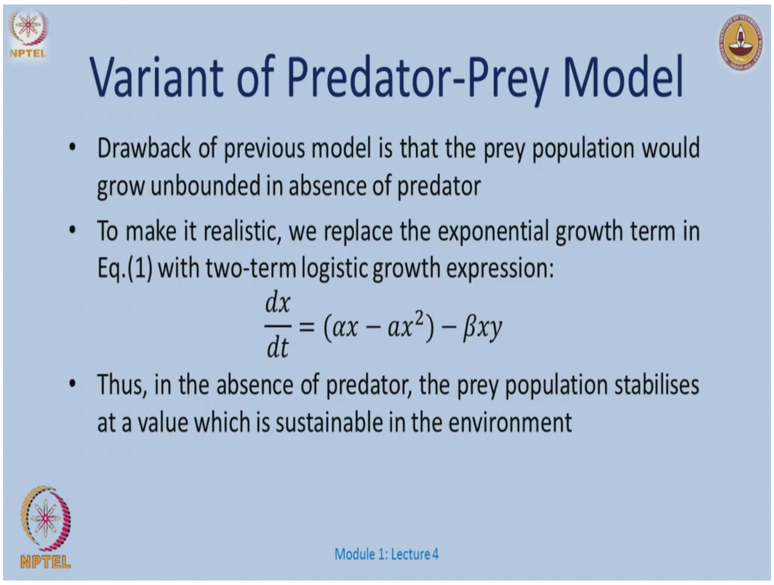

When we e look at the steps of the model we say no step five step six step seven says come to the mathematical model. And the when the step says validate your model; if I say validate my model will I you know if I just go to upon I just by some you know I just own upon for a while there is vegetation, I buy small fish big fish and I say I will you know I do this experiments will this hold true? Well, they may not because the assumptions we made are really strong.

Now let us see if we need to make some more assumptions or relax assumptions or may be there should be some stronger assumption to arrive at more realistic models. So, the drawback of the previous model was that the prey population would grow on bounded in absence of predator, which means they would never die by themselves which is contradicting to nature. If you know there will be people, there will be small fish who will die naturally.

And therefore, to make it realistic we replace the exponential growth term. So, earlier we had dx by dt was alpha x which represented an exponential growth in population, now we will also add some term which will take care of the death rate. Say some constant a multiplied by x square. Therefore, if you look at this equation if there is no big fish y goes to 0, and in the absence of y the prey population well it puts a stabilize at some value it will not be that really a big a large number of fish, there is not enough to eat, but will. So, the assumptions are little little unrealistic.

This is an assumption which will be closer to reality or which is sustainable in the environment.

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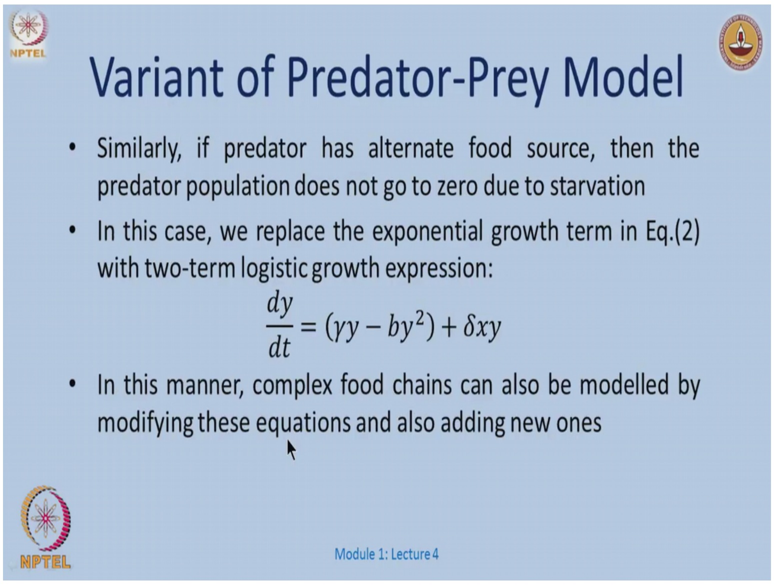

Similarly, if we say that the predator, we assumed earlier that it will of course die naturally, but if what we also assume that if there are no smaller fish they will just die because they have nothing to eat. That is also a very strong assumption, but if I say that the predator has some other alternate source of food then the predator population then they will not go to 0 to starvation. So, mess not the only source of the food for hostel guys so they could go out they can leave you know much longer.

So, that I would account for say dy by dt this takes into consideration by increasing population, because of the small fish, I modify this little bit to take care of the account of the natural death minus b y square and this is the increase in population when I come into contact with some other restaurant food not the mess food. So, in this manner we could models complex food chains, you can just look at some more assumptions something's could be based on observations, we could also model what there are multiple spices interacting with each other. That is how the ecosystem of the of the nature is build right that one species is depending on the other species survival. That tigers are depending on deer's, deer's are depending on trees and so on.

So, all these nice kind of natural balance equations can be captured just by observations and then writing down in terms of some of simple differential equations.

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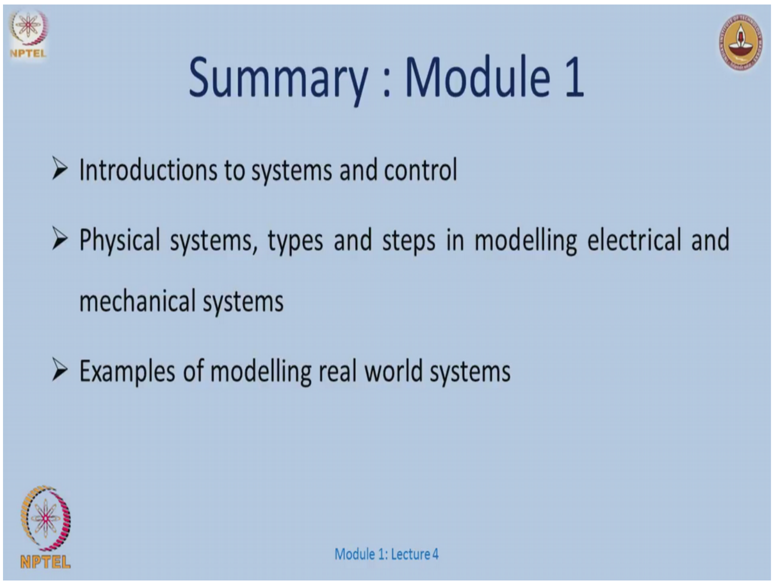

So, what we are learns so for is very general lotions of systems; what could we do with systems in terms of control? Open loop control, we had close loop control, we have classified system into two types of physical system which we tell extensively, electrical and mechanical systems. And we also see well of you know how to build models or approximate all these real world phenomena in terms of some basic elements from electrical and mechanical systems.

And even if I do not have this basic electric and mechanical systems I could still model systems in terms of some kind of differential equations just by observation and looking at some facts which I know that living being will die eventually they eat food to survive and so on.

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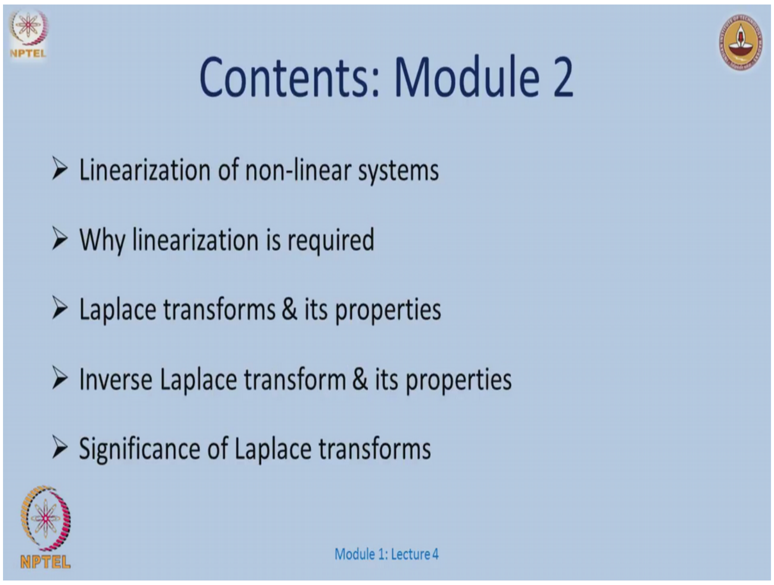

So, next lecture what we will talk is: before that we will do some problems and after we finish with those problems we will do something called as linearization of non-linear systems, more systems we counter RSS non-linear, how do we linearize them and then how does a linearization helps us in analysis. We will just look at why is linearization required. And then we will recollect some tools which we learned in maths or even single system course; the Laplace transforms its properties inverse Laplace transforms and why do we need Laplace transforms and what is relevance through the rest of the course.

Thank you.