Control Engineering
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Module - 09
Design using Bode plots
Lecture - 04
Design of Lead-Lag compensators using Bode plots

In this last lecture of design with using frequency domain methods or in particular the bode plot we will look at a designing both lead and lag compensators.

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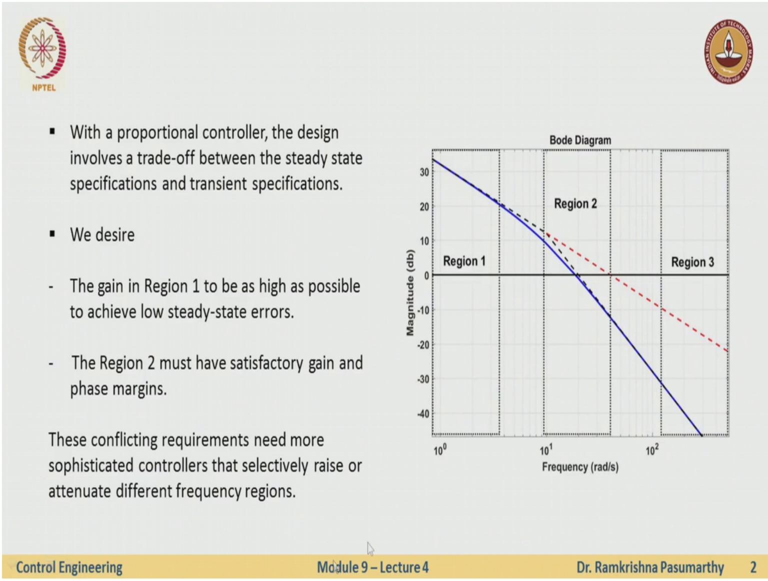

So, just too again recap what we did earlier was you have this region one which was a low frequency region where we want the gain to be I as high as possible so that we have low steady state errors in the region two it should have some satisfactory gain and phase margin. So, this corresponds to the transient region the steady state region and the high frequency region.

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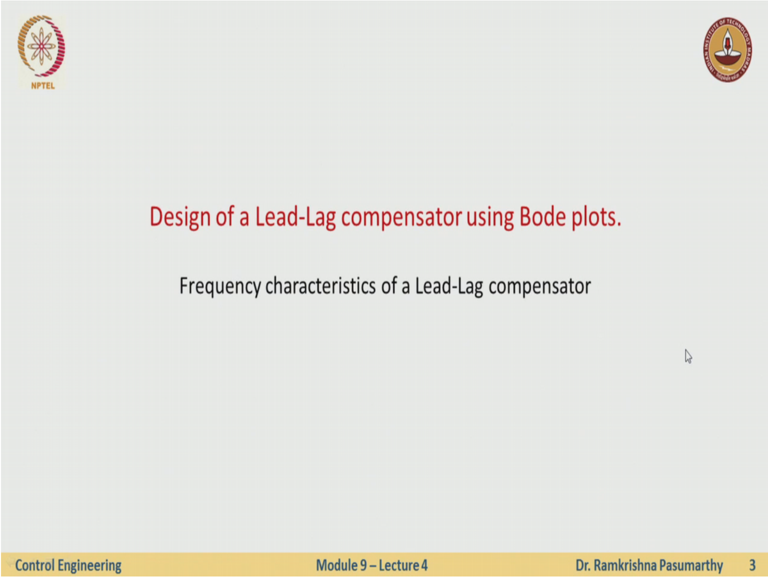

So, what is this lead lag compensator all about?

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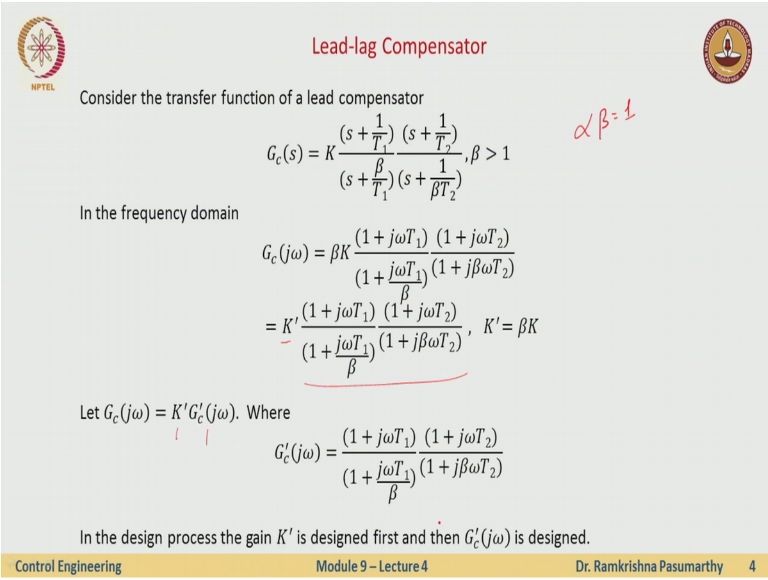

So, if we remember what we had what we had done while we were doing the design with root locus methods. We had a component which was essentially a lead component and the lag component and of course, taking care of all the lead and lag components and beta should be one for something greater than 1 and an alpha and beta here are 1. If we directly map to what we had done in terms of that once alpha times beta is 1. So, I have a lead and the lag components and essentially I am now looking at a designing this compensator with a gain K prime and G c is again split into the gain component and then the dynamic component. So, something like this. So, as usual in the design process, we design the gain K prime first and then go about designing this one.

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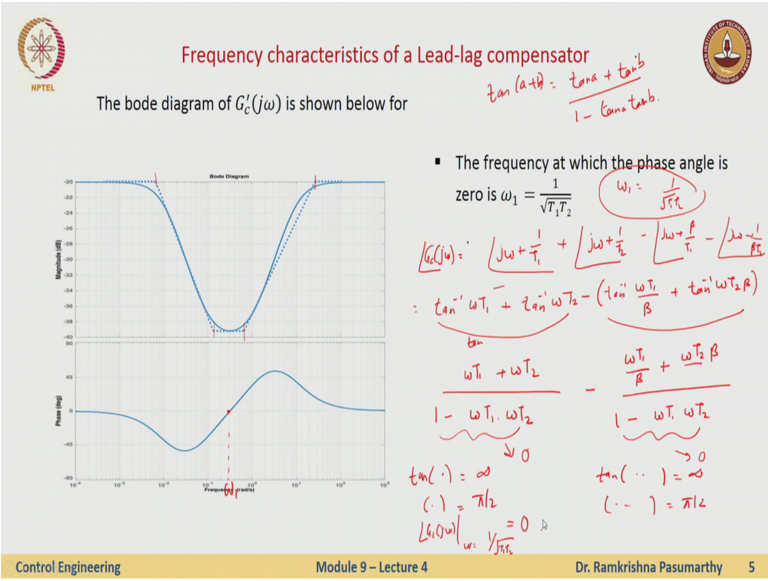

So, let say how the frequency response or the frequency characteristic of this lead lag compensator looks like. So, if I just draw the bode, so what I see is well I have the lag component here and then the lead component across see it with the magnitude plot. So, this response to my lag where you attenuation at higher rather this frequencies for example, and you also have this to be the lead component.

So, what we observe here is of course, these are my corner frequencies 1, 2, 3 and 4, and this is this will this four corner frequencies will decide how my design of the compensated look like or how the poles and zeros of my compensator are placed, and I if look at the phase plot, there is a certain frequency over here at which the angle goes to zero. Now, what is this angle this angle the frequency at which the phase angle is 0, if I call this as omega 1, the frequency at which the phase angle is 0 is this is one.

Now, how do we compute this, now what are this T 1 and T 2? This t 1 and T 2 essentially come from here the t one corresponds to this part of the compensator T 2 corresponds to this part of the compensator. So, how do we do this? So, as I just quickly run you through the proof. So, you have the angle of G c j omega is so you have the angle of j omega plus 1 over T 1. Similarly, you have the angle j omega T 2, now the angle of the pole. So, the negative sign j omega beta over T 1 j omega plus beta T 2. This could also be written as the tan inverse of make a T 1 tan inverse of omega T 2 minus tan inverse of omega T 1 over beta plus tan inverse of omega T 2 times say beta. So, there I will just split these two quantities this guy and this one and I just use the simple formula which we learn in trigonometry. So, tan of a plus b is tan a plus tan b over 1 minus tan a than the tangent of b.

So, if I take the tangent of this entire guy, what will I will be left with is I have omega T 1 omega T 2 over 1 minus omega T 1 omega T 2. Similarly, on this side I will have omega T 1 over beta plus omega T 2 times beta over 1 minus omega T 1 and omega T 2. Now, what happens when this is true, when omega one or at a frequency over omega 1 is 1 over square root T 1 times T 2 I substitute it over here and I see that the denominator tends to zero denominator tends to zero here also. So, what I will be left with is ok, so the tan of this entire first term is infinity if I take the inverse tan what I get is that whatever is remaining inside has an angle of pi over 2 similarly here. So, the tangent of these two terms added up would be infinity.

Therefore, whatever terms in the bracket is pi by 2. So, I have a pi by 2. And this happens only when at this frequencies. I am looking at the situation where omega is 1 over square root of T 1 times T 2. And at this frequency, I get if I substitute it here, I get a pi by 2 minus pi by 2 and therefore, the angle contribution the angle of G c of j omega at this frequency omega is 1 over square root T 1, T 2 is actually 0.

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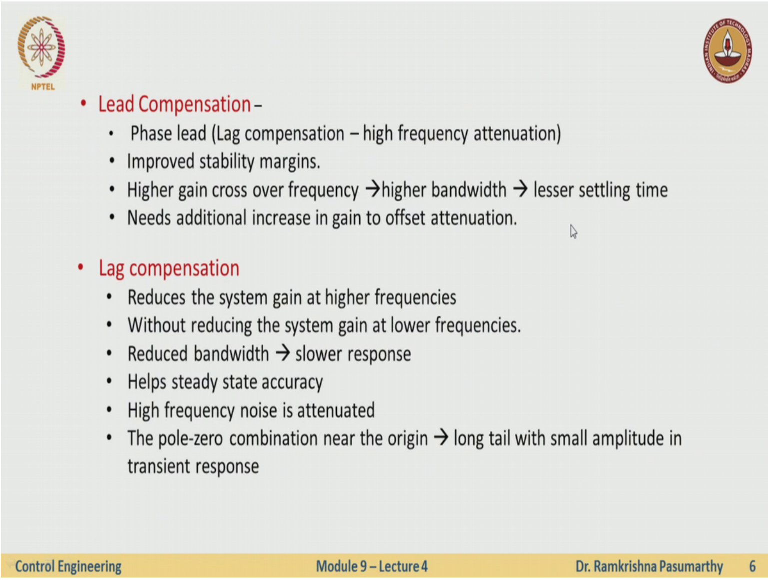

So, just a little summary of what we learnt so far. So, if I look at lead compensation well as the name such as it provides a phase lead which means it improves the stability margins. It also has a higher gain cross crossover frequency which means it has a higher bandwidth. And we know what is the relation now between bandwidth and the damping ratio that suggests that higher the bandwidth which would mean lesser setting time. And of course, it needs an additional gain increasing gain to offset the attenuation, the alpha was less than one therefore, we have to put a some kind of for gain there it so as to offset this say attenuation.

So, the lag compensator it reduces the system gain at higher frequencies in such a way that it does not affect the system gain at lower frequencies. So, only the higher frequencies are attenuated. Reduce bandwidth obviously mean slower response and it helps in steady state accuracy right as we see in also in the time domain. Because of this property of reducing the system gain at higher frequencies, it helps attenuating the higher frequency noise and another drawback was well the pole zero combination near the origin. So, the compensator poles and zeros in the lag case are placed very close to the origin and this will lead small some kind of variation in the transient response in terms of the settling tangent it will have a long tail with small amplitude in the transient response.

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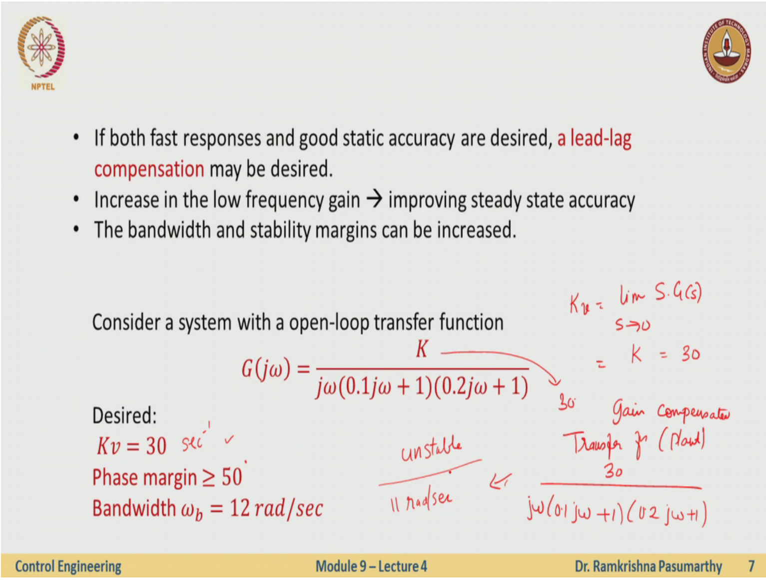

So, now if both fast responses and good static accuracy are desired then you go for a lead lag compensation when increase in the low frequency gain would main improving steady state accuracy. And of course, the bandwidth and stability margins can also be increased when we do the lead and the lag design together. So, let us start with an example and you see what I am trying to say here that I am given a transfer function, which looks like this and this it is desired that K v is 30. If I write down the units, it is third second phase margin is greater than 50 degrees and the bandwidth of certain number of 12 radians per second.

So, what I will do here is I will try to solve this in MATLAB; I will just plotted up for everything for you. And we will see how the design process goes. Many times while designing I had mentioned that well the design is not a one step process; you may have to do it in over and over until you reach closer to the solution. So, in the first iteration, you may be close to the desired results, but not then in the may be in the secondary iteration you may chose in your parameters which essentially in the beta, may be you can choose a different alpha and beta as we did in the time domain analysis to just tweak it a little bit. So, that you get the desired response. So, I will just quickly run you through that process of going about that.

So, before that right so we are given this K v, now K v should be 30 second. So, what is the K v in my case K v in my case is the standard formula limit as tends to 0 S times G of s that becomes K here. So, for K v to be 30, this guy K should also be 30. So, I am looking at a gain compensated transfer function or the plant also right the gain compensated plant which now looks like say 30 and there is sort in the same; now, j omega 0.1 j omega plus 1 and 0.2 j omega plus 1. Now, let us go to MATLAB and try to see what this gain means in terms of stability margins.

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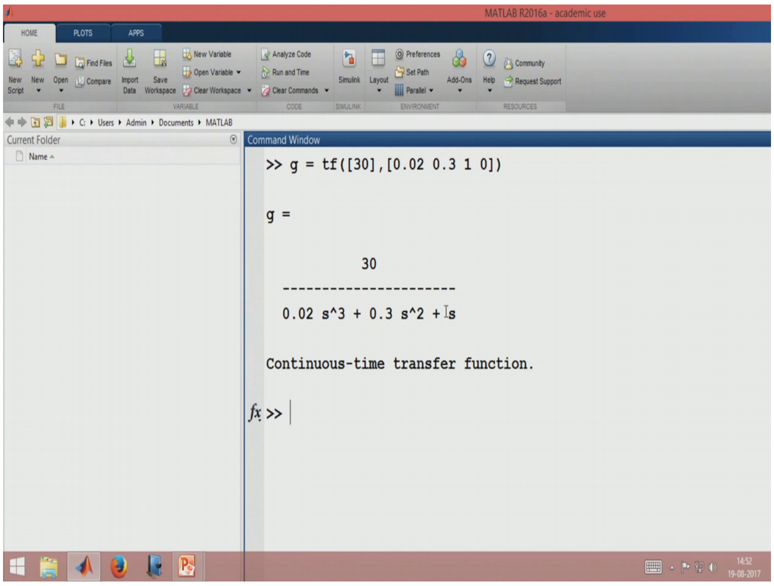

So, I have G as 30 and here I will have 0.02, 0.3, 1 and 0 like a transfer function 30, 0.02 as cube, and 0.03 as square and s, this is the transfer function for this guy.

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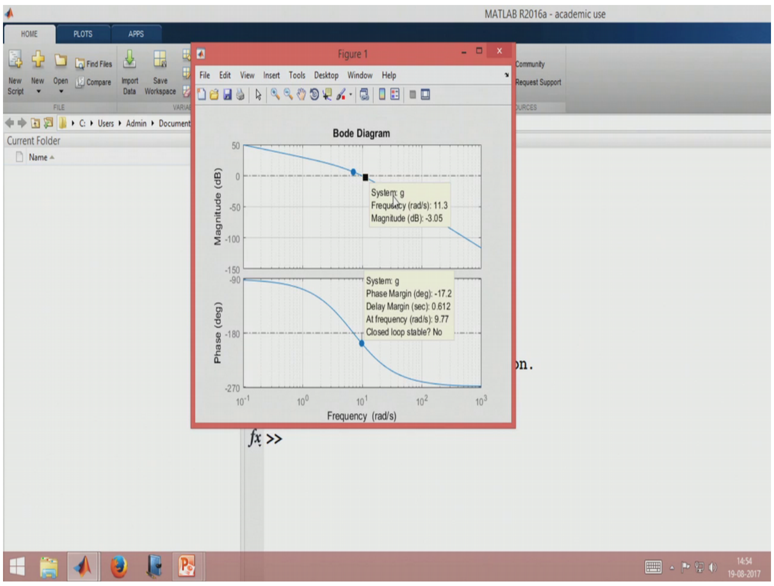

So, if I do the bode plot of this to see what does the gain compensation do to me. So, I have plot here. So, I will see which why the great the characteristic all stability margins. So, I have phase margin of minus 17 and negative gain margin and it shows that the closed loop system is unstable something not very good has happened here. So, if I look at what is the benefit, at what time am I reaching minus 3. So, look at this go here. So, roughly about eleven some something about eleven point what you say which we says about 11.3 radiance per second. So, let us just note that down.

So, the gain compensated system has when this is first this is unstable has a negative phase margin and negative gain margin, and also it has a bandwidth of and a roughly about 11 radians per second. And this is not very useful to me, because the system is unstable. Now, let us just write what happens we just using a lag compensator. I will just write down the steps here. Let me just take arbitrary and any let us start with say a lead compensator a lead compensator.

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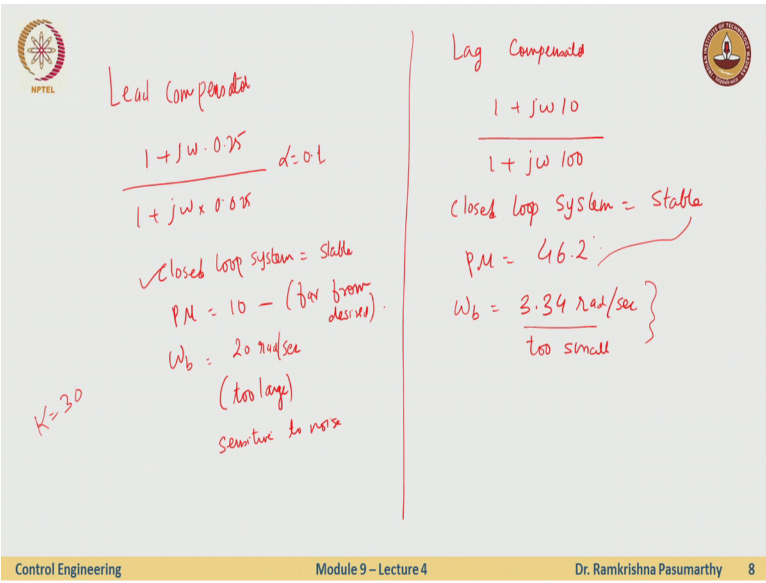

So, let me say it looks just for me j omega into 0.25 over 1 plus j omega multiplied by 0.025 I take a very liberal alpha which gives a very big phase lead of say 0.1. So, now let us see what happens with this lead compensation. So, there is no basis for me to choose this, but just to see a basic analysis what it is.

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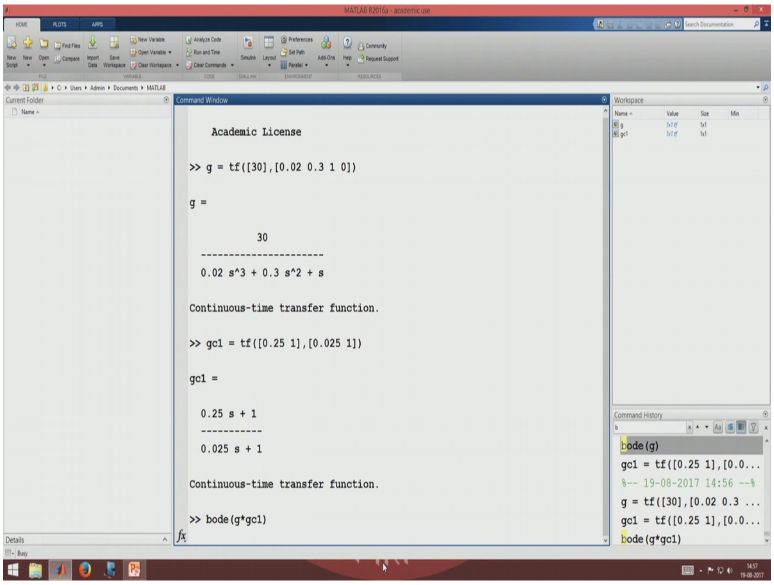

So, this is my G c 1 I will call it is as a transfer function. So, it has one so 0.25, 1, 0.025 and 1. So, this is how my compensator looks like. So, now, if I do the bode plot of G with G c 1 which is my lead compensator when I get something like this.

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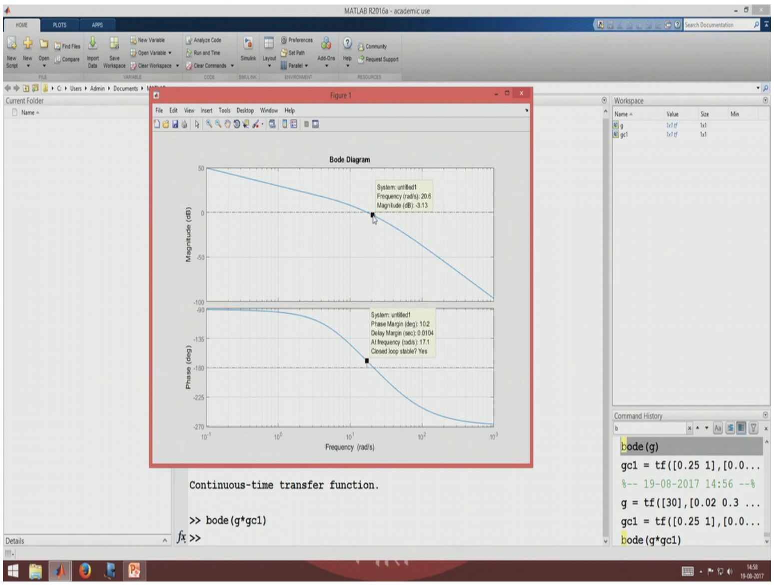

See, I have some improvement in the phase margin good thing is at the closed loop system is stable it has a phase margin of 10. And gain margin of about 3.5 db, and not bad because at least I could get the system close to stability and the bandwidth if I go back bandwidth of close to 20 roughly about 20. So, let us just note this down. So, just by using a lead compensator what I achieve is well the closed loop system is stable.

Now what is the phase margin, phase margin is about what to say phase margin was about 10 degrees and bandwidth was fairly large 20 radiance per second. So, what is good, well this is good, this is a good thing, but it is far from desired, desired was more than 50. And this is too large if I look at attenuating higher frequency said this was also decide to be 12, so this just about designing with the lead compensator. Let me just do with the help of a lag compensator. And just take some standard, this one some standard compensator. Let us say it could be lag 1 plus j omega 10 and 1 plus j omega 100. So, let us see what MATLAB says because of this.

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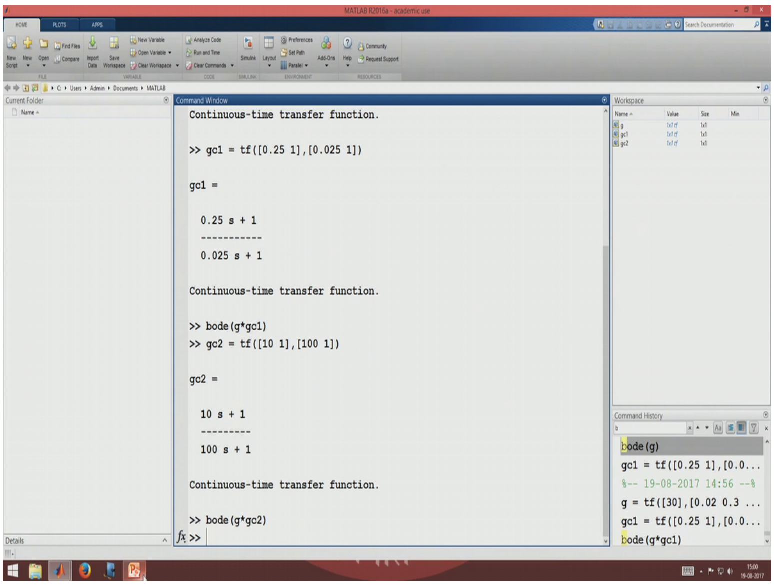

So, let me call this g c 2 enough 10 j omega plus 101. Now, if I do the bode plot of this plant with the lag compensator.

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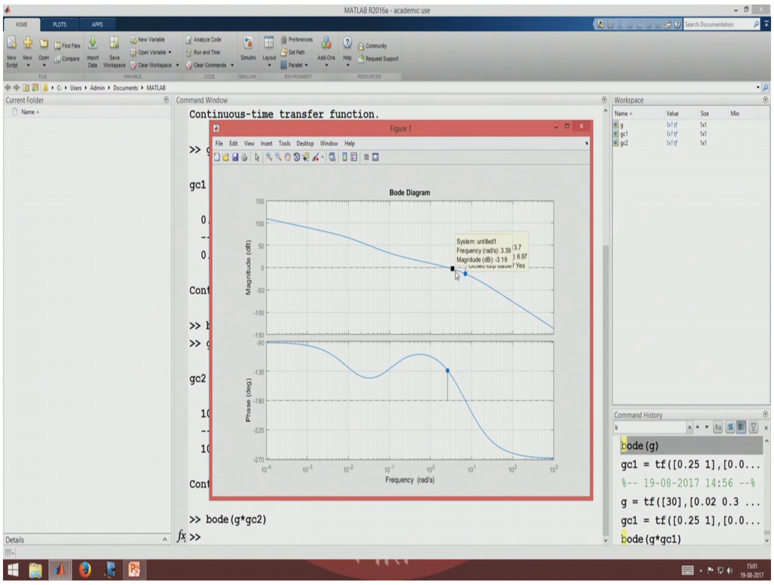

And it looks something like this, there it is. And I am looking at the stability margins. So, I see that my phase margin is quite good now, it is for (Refer Time: 18:27) may good enough gain margin. Now, let us say what is happening to the bandwidth when I am at minus 3 db is roughly about 3.4 radiance per second. So, with this my phase margin has shown some improvement. Well the closed loop system is stable which is good. My phase margin has become almost like more than 46 degrees then I have the bandwidth is 3.3 something like this because to 3.4 radiance per second.

So, if I look here the closed loop system was stable bandwidth increase. So, this will be sensitive to noise. And then now I see here well the bandwidth is too small. If I can play around a bit with beta to make this 50, but you know this is now where near beings satisfactory. So, there some contradiction here right even though one the individual compensators achieve something better than what were what we had when K was 30 after taking care of the steady state requirements. Well this is good, but something here is this is bad well here well things are better, but something here is not very desirable. So, in situations like this we may need to go for both compensations.

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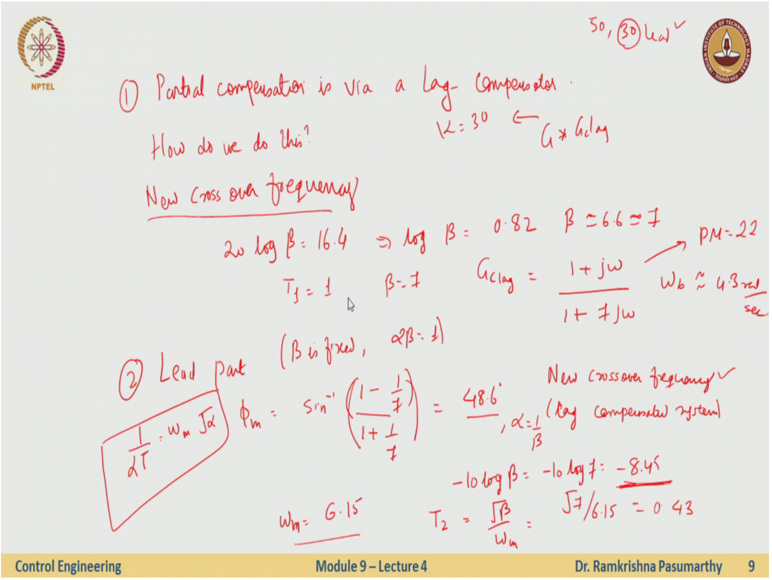

So, as a rule of thumb with may not be very proper justifications, we first design the lag compensator. So, partially or partial compensation is via a lag compensator. And if you get the system to behaves slightly better than we can use the lead to overcome the deficiencies where the lag compensator; say here, the bandwidth increases in phase margins are kind of thing then if it is like say the desired is 50, this guy gives me 30. I can nano compensate this with a lead and also the bandwidth can in can be increased because the lead naturally has this tendency to increase the bandwidth of the system. So, how do we do this? So, we will so sorry first thing we need to choose is; what is the new crossover frequency. So, I will again go back to my plots.

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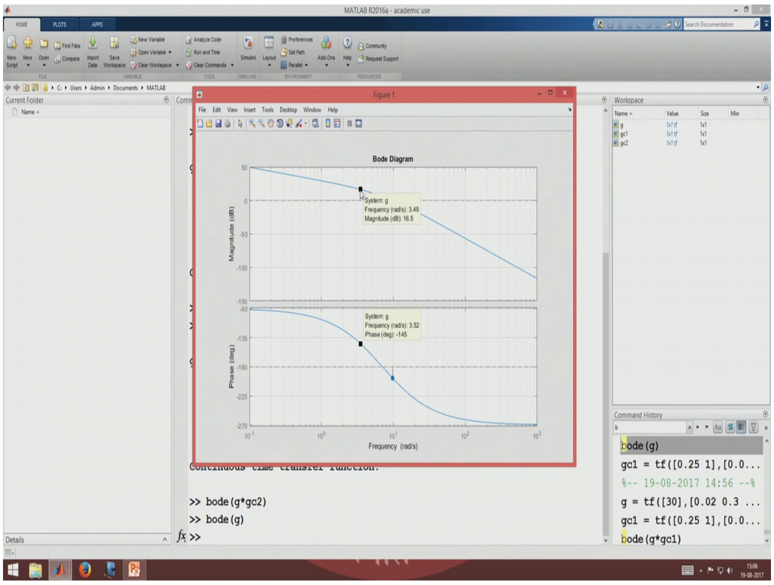

So, I go back to my plot here, which is just the gain compensated system with K equal to 30 and I choose just (Refer Time: 21:48) some partial compensation. Let me say I choose this frequency here where the phase is minus 145 which means the phase margin is about 35 degrees. So, then we will this become the phase margin when this frequency 3.52 is actually the gain crossover frequency. So, I go here. So, roughly about this point I want the magnitude to be 0. What is the magnitude here it is about 16 db.

So, this is the lag compensator should provide a negative gain here of minus 16.5 such that this is the new gain crossover frequency. I am just assuming some partial thing (Refer Time: 22:31) also do go to 150 and check you can also go to 1 minus 140 and check, so no hard and fast rule here. So, what I need to do is compensate for this magnitude via the lag compensator or in other words I need to first decide the beta.

So, how is the beta designed well I am looking at now 20 log beta is about 16.4 in decibels this means that log of beta is 0.82 and which essentially means beta is about 6.6 and let us assume that it is 7. Now, I have to assume or find out what or assume something for the corner frequencies. Let me just start by assuming T 1 equal to 1 and with beta equal to 7; T 1 equal to 1, my lag compensator if I call it G c lag would look something like this. And I have sorry 1 plus j omega 1 plus 7 j omega. Now, let us see what happens when I just plug in this compensator into my plant, I will call this as my G c lag.

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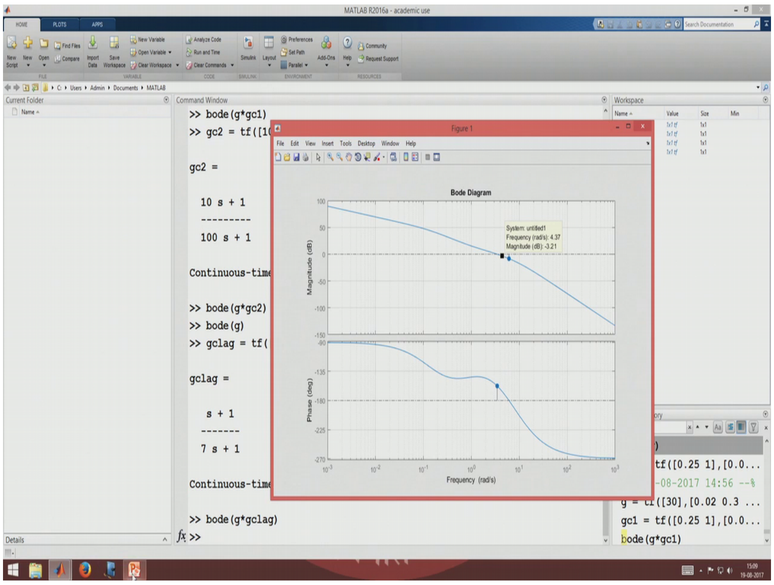

So, something like this. And then I look at the bode plot of the lag compensated system. So, G is the gain adjusted and then I have G c lag. So, what is happening with this now. So, I have now phase margin of a 220 degrees, it is fine and I have a gain margin of 8.2 and well as check what is the bandwidth, the bandwidth is let us like pretty small say about say close to 4.3. Now, I have this lag compensated system which now has a phase margin of 22.5 and a bandwidth of about four and half.

Now, what I know now is that I could use the lead compensator to provide an additional lead here; and with the lead compensator, my bode plot also shifts to the right. So, I could improve the phase margin and also improve the bandwidth. So, first what I did is to partially compensate with the help of a lack compensator and I will just do the lead part now. So, how we will go about in the lead part? The step two would be in design the lead part, because this compensator had given me a phase margin of 22 together with a bandwidth of foremost about 4.3 radians per second.

So, now the lead part; so what is at beta is fixed and alpha times beta is 1. So, I know what is alpha now. So, the phi of m if I write it in terms of this beta would be sin inverse of 1 minus what is my alpha is 1 over 7 by it is big bigger bracket here 1 plus 1 by 7. This is comes out to be 48.6 degrees. Now, what we will change is there should be a another new crossover frequency. So, this new crossover frequency now we have to adjust this into the lag compensated system lag compensate of this one. So, these G c where g times G c lag where G was again at this system with gain K be in equal to 30.

So, once I know phi m, I also know alpha I can compute alpha or in this case I know beta where alpha is 1 over beta, I can compute the new crossover frequency where the magnitude of the lag compensated system take this value minus 10 log beta. So, this is minus 10 log of beta is 7 sorry log of the beta is 7, this is minus 8.45 and this 8.45 occurs at omega m, I will show you this one in the in the bode plot is about 6.15.

Now, once I get this omega m, so I know alpha or beta I know omega m, I know phi m I can compute T 2 as square root of beta by omega m. And this turns out to be square root of 7 over 6.15, this is 0.43. So, if you remember the formula, it was like 1 over alpha times T was is equal to omega m square root of alpha, let us just go back to the slides of the lead compensator I will get to know this.

So, I am looking at this magnitude this where the lag compensated system has a magnitude of minus 8.45 that will be the new crossover frequency. So, that is turns out to be 6.15 as we will see from the bode plot bode plot of the lag compensated system.

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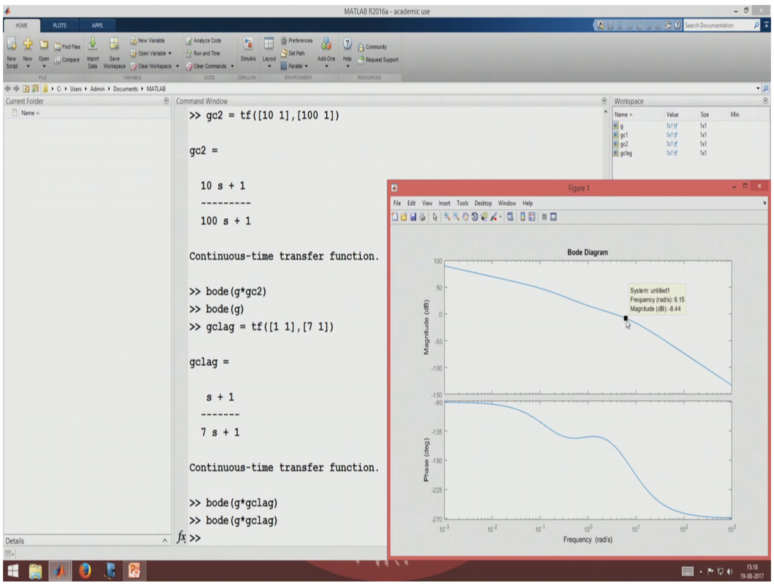

So, that is it minus 8.45 is roughly around here right 6.15, this is the frequency here right. So, now, based on these things I can write down how my lead part of the compensator would look like.

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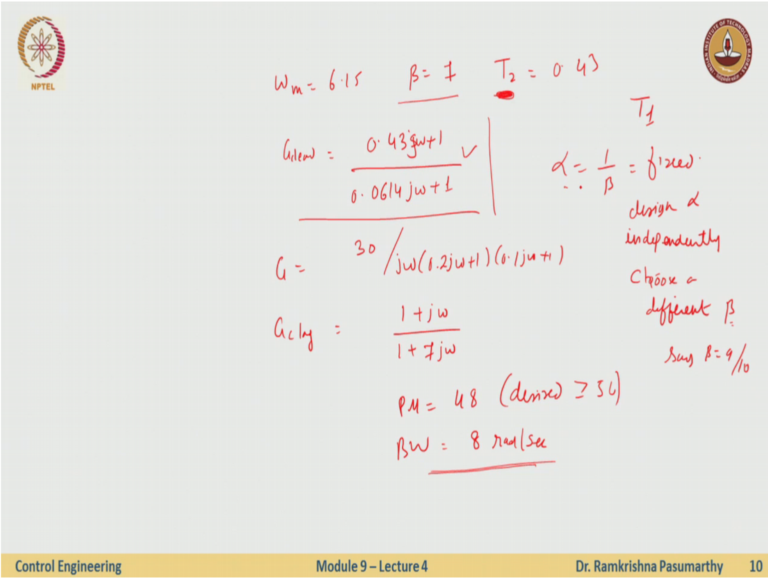

So, the lead compensator has now the form based on omega m is 6.15, beta beings 7, T 2 being 0.43. This T 2 is essentially that the time the T 2 the time constant of or or the numerator part of the lead compensator that which we had here. So, the unknowns in my design were T 1 beta and T 2. So, now I am actually going to find out what is T 2 and by doing all this process I get T 2 to be 0.43, which gives my g lead if I call it g c lead to be of the form 0.43 as plus sorry j omega plus 1 over 0.0614 j omega plus 1. Let us see how the overall compensation now looks like. So, I have G which was 30 over j omega 0.2 j omega plus 1.1 j omega plus 1 I had the G c lag like which looked like 1 plus j omega 1 plus 7 j omega and I have the lead.

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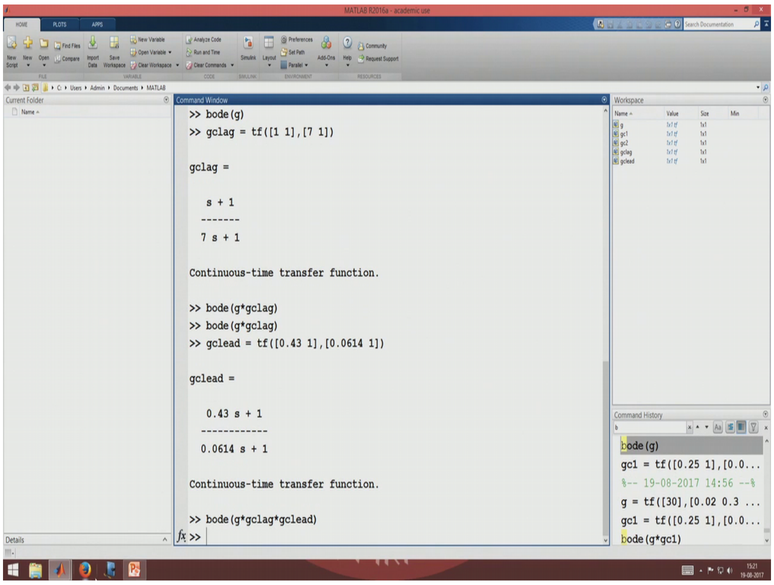

So, let us see how my bode plot has change when I plug in this lead compensator. So, my G c lead is 0.43 and the 1, 0.0614 and 1, something like this. So, my overall compensated system with the lead and the lag compensator I have g lag and I have a g c lead.

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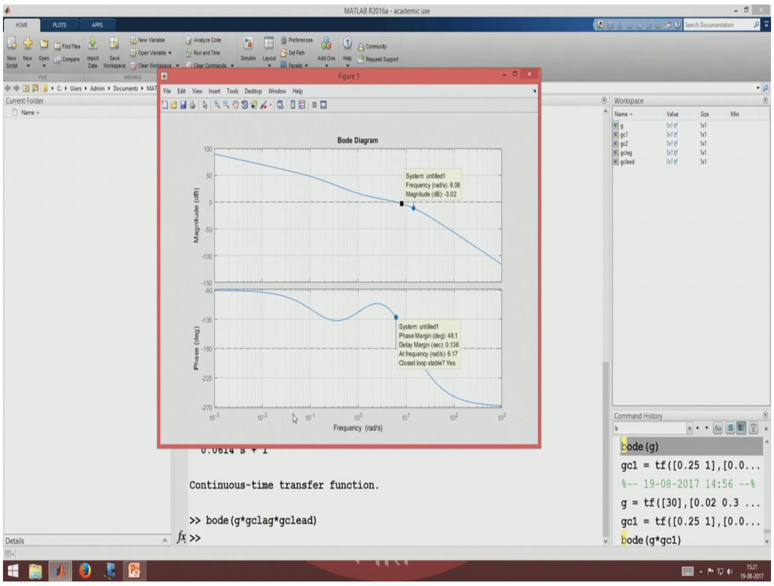

So, what are the characteristics now? First, phase margin is 48.1. Let us look at how is the bandwidth doing, well band with not doing too well, it is roughly about 8 radians per second. So, with this the phase margin is 48 to the desired was 50 was to be greater than or equal to 50 and the bandwidth is about 8 radians per second. Now see the first the steps involved. So, what did we do is with the lag compensator, we had some partial compensation where we at had the closed loop the gain compensated system with the lag compensated to be stable, and a decent enough phase margin. We still far away from what is desired and a some amount of bandwidth.

Now, I know that given this configuration I can now use a lead compensator to increase the bandwidth and also to push up the phase margin. So, here the design involved choosing an appropriate beta choosing a T 1 and we did not have much flexibility while designing the lead component because alpha was fixed. Now, what we could do is in a similarly what we did in the time domain compensation we could choose or design alpha independently or again recalculate beta not really recalculate. But now choose a slightly new different beta say maybe choose beta choose a different beta let us say beta equal to 9 or even 10 as was with other problems.

So, in the first iteration, what we observed here is that we do not really achieve what are the desired specifications; exactly. And therefore, we need to do a little bit of tweaking in with our parameters beta possibly alpha and so on. So, I will not run you through that process; I will leave that to you as an exercise to play around with different values of alpha or even designing the lead compensator completely separately.

However what I will do is I will provide you some nodes with different ways to do these things. So, can I just choose a different beta and keep alpha times beta equal to 1, can I well given this beta. Can I choose a new alpha independently, can I know that I could that alpha times beta equal to 1 is not a severe restriction in my design process. So, I will post some nodes on that, but what my the aim of this lecture was you run you through that process of the design well how to possibly use MATLAB appropriately or how to read the MATLAB plots and so on.

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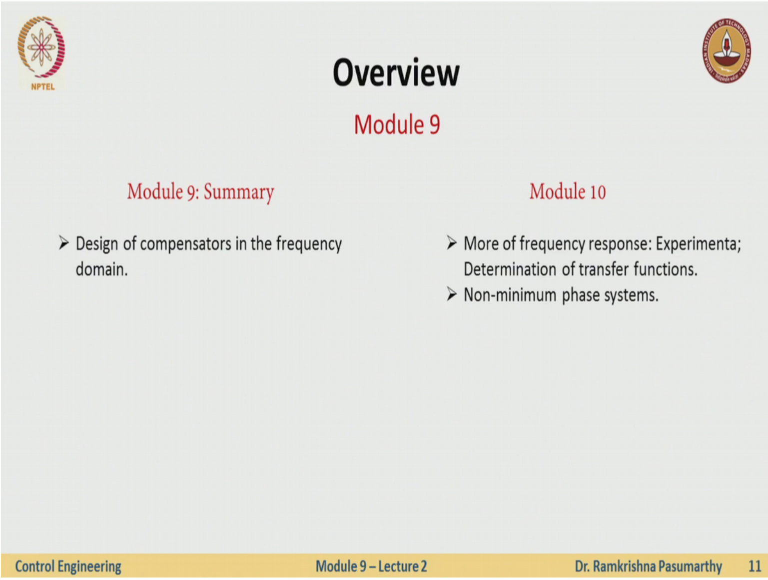

So, to summarize what we learnt today was to design compensators in the frequency domain or across this module. In the 10th module, we will do something more something little different. So, these are not necessarily a part of a regular control curriculum, but these are important and interesting and also useful things to know. The first is experimental determination of a transfer function.

So, given a system for which I have no physical model; physical model essentially means I cannot realize them in terms of any circuit components or mechanical components. Can I just look at its frequency response generator bode plot and get the transfer function we will do the later part of it. Given bode plot can I generate a transfer function and that leads to a very, very nice concept over we should be careful of is that of non-minimum phase systems.

So, these are essentially the two things which we will deal in module 10, might be a smaller module in terms of the length of the lectures than compared to the other models, but still very informative and useful enough.

Thank you.