Control Engineering
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Module - 09
Design using Bode plots
Lecture - 03
Design of Lag compensators using Bode plots

Hey guys. So, in this module this is rather smaller lecture, where we will learn how to design lag compensators via bode plots. So, suggest to recall Where we started of all this.

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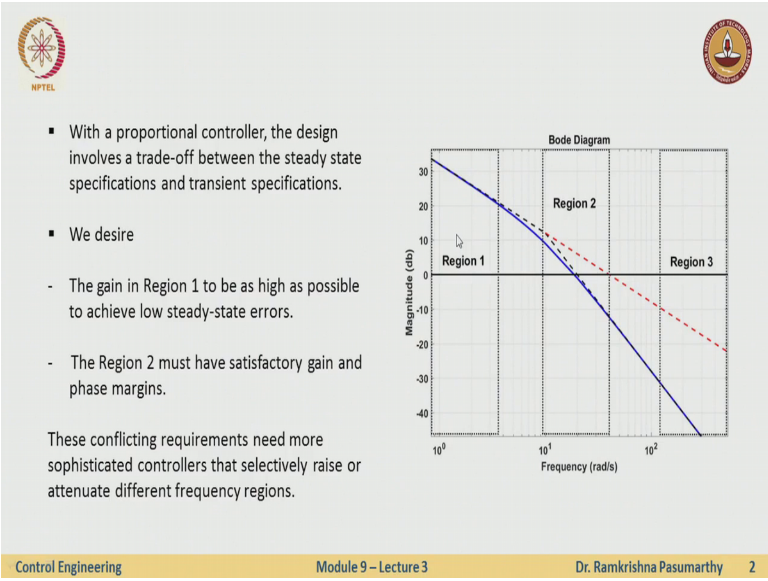

So, in the frequency domain we classified the entire frequency range into 3 regions. As the low frequency, the middle frequency, and the higher frequency range. And what we observed was, what we were desired that with the gain in region one should be as high as possible so as to achieve low study state errors. Region 2 must have satisfactory gain and phase margins. And if there are conflicting requirements we need sophisticated controllers and so on. And then there are there are higher frequency region takes care of attenuation of unwanted noise signals, which usually occur at higher frequencies ok.

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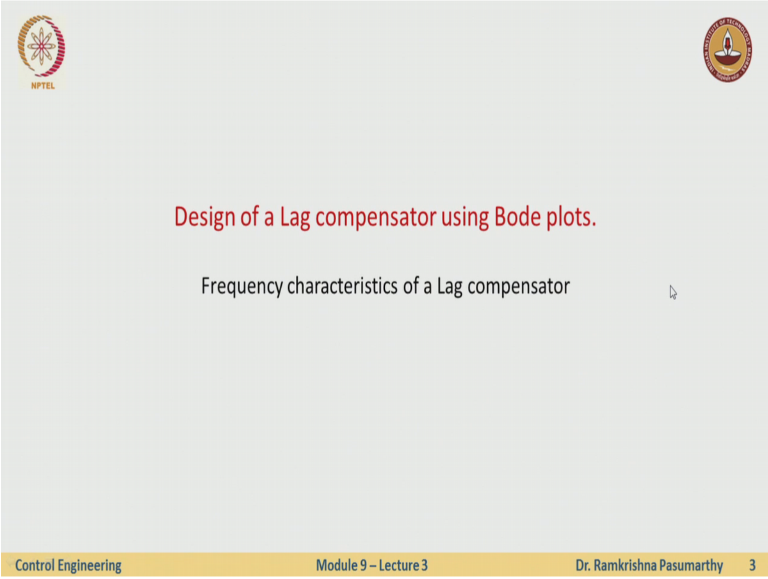

So, how does this lag compensator look like? Well, by construction well I have a again K, I have a 0 and a pole placed in this manner. Exactly the same as we did while we were doing compensation via root locus right, in the time domain.

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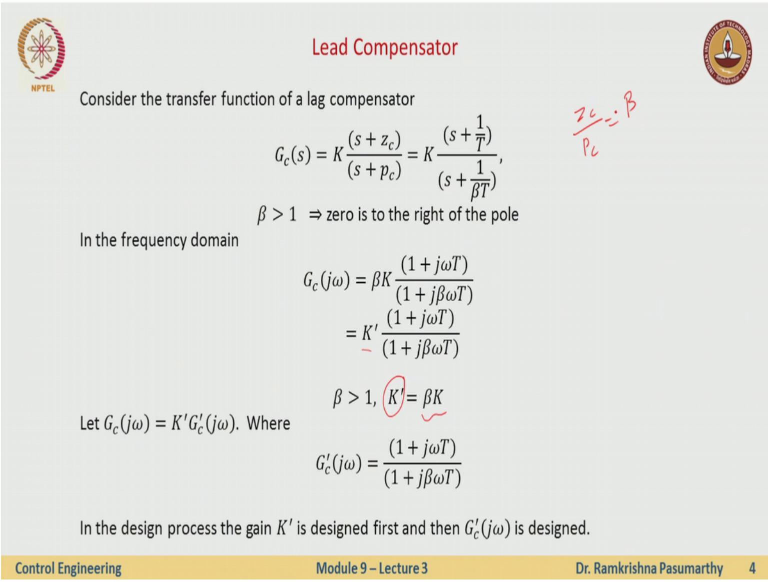

The value of beta is chosen such that it is greater than 1, which means that 0 is to the right. Of the pole as we did in root locus. Now in the frequency domain I can write down the sinusoidal transfer function in the following forms. So, I have just re arrange this betas and write the sinusoidal transfer function. So, I have beta times K 1 plus j omega T and similarly, the denominator with a beta. And I call this number K prime to be beta times K and then these terms remain. As it should be beta is greater than 1 and K prime is beta K.

So, the compensator now I split into 2 terms. One is purely to do with the gain K prime here. And then G c prime, where G c prime is just this transfer function without this gain K which we already accounted for over here. So, this So, what is important to note is that this K prime also contains this design parameter beta; however, when we designed it will not really be explicitly is mentioned, but we need to keep in mind that we need to design K prime in such a way that it also accounts for beta. So, this K and this K prime are different in such a way that they are related via beta.

So, some books may say that may once they end up computing the controller they say desire you know divide the original gain K by K prime by beta and so on, but once you just do it in this steps we will avoid that complication right. I just designed K prime and I say. And then the compensator the dynamic part of it will just be this one. So, we really eliminated again you know dividing or multiplying by certain number. So, in our process we first design the gain K prime, and then we go towards designing this compensator with the dynamics part ok.

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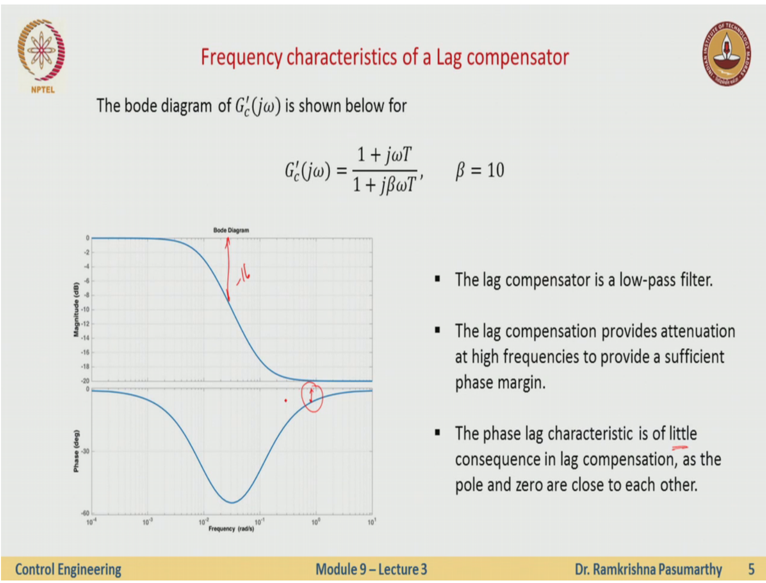

So, how does it look like when I do the bode plots for a typical value say, beta equal to 10 my bode plot looks something like this. So, I am not really accounting for the gain here, I am just looking at this part. And therefore, my magnitude starts from the 0 dB line. And then it goes down and just like this. Similarly, where the phase lag right, this is this is essentially why it is actually called a lag compensator.

So, the first observations here is that it is a low pass filter right. So, the gain is 1 or 0 dB for low frequencies and then it keeps attenuating the higher frequencies. So, first it is a low pass filter because it provides attenuation at higher frequencies. And we will see how this actually results in providing a sufficient phase margin.

And as see here this phase lag characteristic is of little consequence as the pole and 0 are close to each other and then, even if you look at in the root locus design in the like compensator, we wanted the angle contribution of the lag compensator to be very small. And therefore, we said that while designing I should make sure that the angle contribution of the lag capacitor is less than 5 degrees. Many text books say it is of low consequence, but that is not completely true and we will say we will see why that is important. And I say and I will also tell you why I use this bode little. It has some effect, but a small effect.

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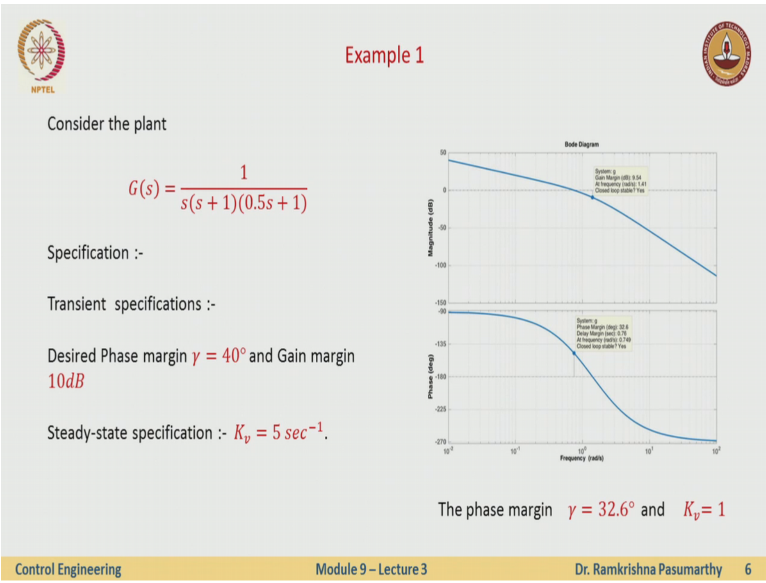

Now I will teach you this thing with the help of an example. So, that the steps are clear right. So, if I just write down the steps and then do the example later it may not really match, but we will motivate ourselves and learn the procedure with the help of an example. So, I am given this open loop transfer function. And my closed loop design specifications are the phase margin should be 40 degrees the gain margin should be 10 dB and together with a steady state specification of 5 ok.

So, I just look at what the plan by itself means when it has a gain margin of 30.6 degrees. And the gain margin of 9. So, gain margin is fairly phase margin is little out of a robot I want to study 2.6 and what I want is 40 K v is really bad you know I am I am having one, but I want to change it to 5.

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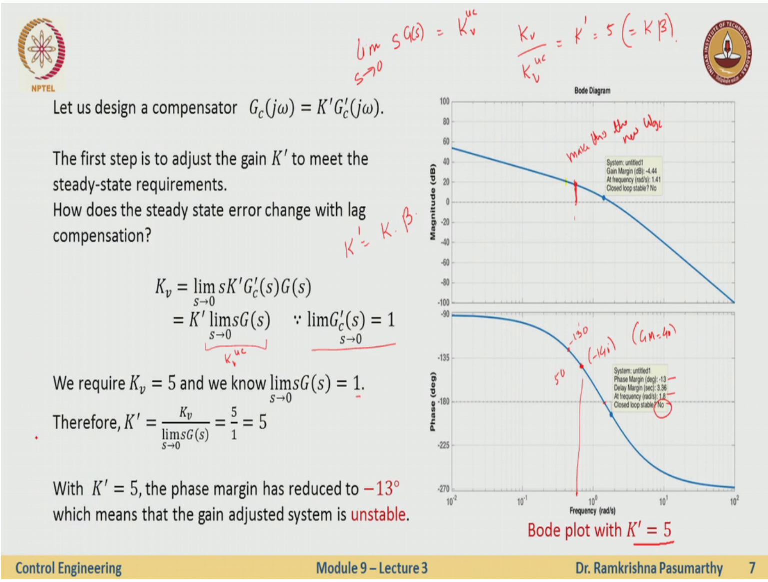

So, the first step while we design in frequency domain is to adjust the gain. So, let us again look at the standard structure of the compensator of the lag compensator of this case where G c j omega I separate that into the gain part and then the dynamics part. So, first is to adjust the gain K prime to meet the study state requirements. Now how does the steady state error change with lag compensation? Well, I look at K v is so; this is my formula to find out K v. And this K v is K prime with this K prime here times s times G s. This K prime is K time's beta. So, this is actually K time's beta.

So, now while I do this things what I find is that limit s tends to 0 G c prime of this is 1, because of this construction I put 0, 0 here and what I am left with this 1. So, on the left hand side I have K v, now this guy is K prime limit s tends to 0 s times G S. Now this is what is this? Limit s going to 0 s times G s is the K v of the un composite system right. Let me just say uc ok. So this is K v of the uncompensated system in general.

Now, here it turns out that this is 1 and the desired K v is 5 and therefore, K prime which is the ratio of the desired K v to the original K v turns out to be 5. Now again let us just see through the construction also right. So, this is my compensator design here. G c K and then the K also include this beta. So, this is K prime and so on. Now when we do the design later I will tell you what is what is exactly the relation between you know, K prime because we are actually not yet calculating the value of beta, but you are only calculating the value of K prime, which is actually beta times K ok.

So, here what I have is, this is the K v desired over K v of the uncompensated system is K prime, and this is 5 and this 5 is actually equal to the K times beta. Now with this K prime while I just plot the bode again, and I see that something strange has happened. Well, K prime is actually 5 that is perfectly all right. But well the phase martin a negative of 13, and the closed loop is unstable. And I really wouldn't this right. So, a steady state error improvement means nothing if the system is unstable ok.

Now, what I would like to do and you see also these frequencies are typically not very high frequency; it is like 1.8 you know 1.88, I think so. Even the cross over frequencies one this is so like this cross over frequencies around one point 1.5 for something else. So, what I would want to do is well, you might think why not just use a lead compensator over here and push it, push it upwards right. So, that my I will have the desired phase margin. Well, first is this frequency ranges which we are talking about or very if is low frequency ranges, and therefore this kind of thing may not necessarily work ok.

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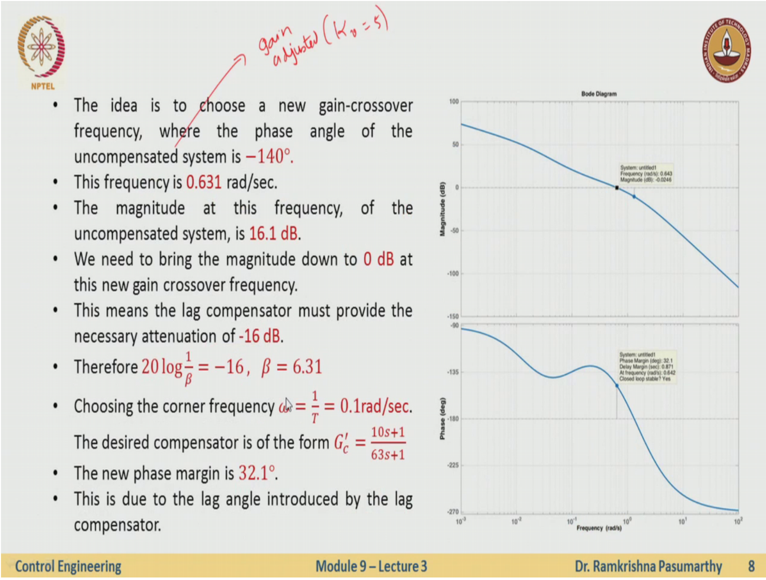

So, how would I do this? So, the idea is to So that the other way of looking at this is also, that if I look at a point say somewhere here, at this point where the angle is minus 140 degrees. Which means if the and at a at a certain frequency right, I can actually map that frequency it is somewhere here may be between 0.1 and 1 and I will actually compute what the frequency is. So, if I pick up this frequency I go here and I make this frequency at which the angle is minus 140, this will correspond to a gain margin of 40. If I pick this frequency go here and I say make this the new gain cross over frequency omega G c, then I am fine right. Ok

So, all I need to do is to identify this frequency and pull this magnitude to 0 here, such that this will be the new gain process of frequency. Now that is what is happening in a in this lag compensator right. And actually having a negative number in the in the magnitude right. So, if I super impose this over here it could have an effect of pulling down the overall magnitude. So, that is what we will do.

So, the idea is to choose a new gain cross over frequency where the phase angle of the uncompensated system is minus 140. This uncompensated is a I am also looking at uncompensated system with the gain adjusted one. They adjusted in such a way that I have my K v to be a 5 right. I will talk everything after I adjust the again. Now this frequency turns out to be 0.631 like here right. So, some where here this is about 0.631 ok.

Now, I will find out what is the magnitude at this frequency of the uncompensated system, which is essentially I am looking at interested in this magnitude, now this one. Now we at this frequency, we need to bring the magnitude down to 0 dB. At these new gains cross over frequency. This means that the lag compensator must provide an attenuation of minus 16 dB. And by the construction I will skip the steps here it is exactly the same as what we did in the lead compensator, but the construction of the lag compensator. So, the beta can now be designed in such a way that well the magnitude 20 log 1 over beta is minus 16. Well that this and this is magnitude right.

This is the magnitude which I should compensate via lag compensator. So, essentially I will I just find out a point where if this is actually a minus 16 right, in respect to that point. This is minus 16 at that new cross over frequency then I can say well this like here, I can I just want to make the magnitude here to 0 dB that is all ok.

Now, it turns out that well if I just do this that beta is 6.31. Now if I choose the first corner frequency to be 0.1 radiance per second or T equal to 10. If I look at the construction what are my unknown is the beta and unknown is the T. If I contrast with the lead compensator there is no real constructive procedure to exactly find out what this T and beta are. I can explicitly find out what is beta that is that is much easier for me. T I will just choose to be closer to the original, that is what I am all doing these guys should be closer to the origin, even from the root locus. So, I select T to be 0.1 and then the desired compensator will now be something like this, this one. T is sorry; sorry T would be 10 and then I may send multiply this 10 by 6.31 which is the value of beta and I get this to be a G c prime and I see that the new phase margin is well where am I here at this new gain possibly I am at 32.1.

And this is the gain margin, and given in is I do not really. So, much is this, but it is like may close to 10 to 11 that is fine. But what I what do I really want is this to be 40 degrees not 32. So, where am I going wrong in the design procedure? Now this error is due to the lag angle introduced by the compensator. If I go back to the construction you see that well there is some bad effect going on over here. So, maybe for a simple construction I am looking at a frequency of 0.6, you see there is some 8 to 10 degrees which is lost because of this one. This is not significant, but still it is it is small, but it is it is of consequence. Therefore, I say it is of little consequences it is not of no consequence. So, somehow I have to compensate for this.

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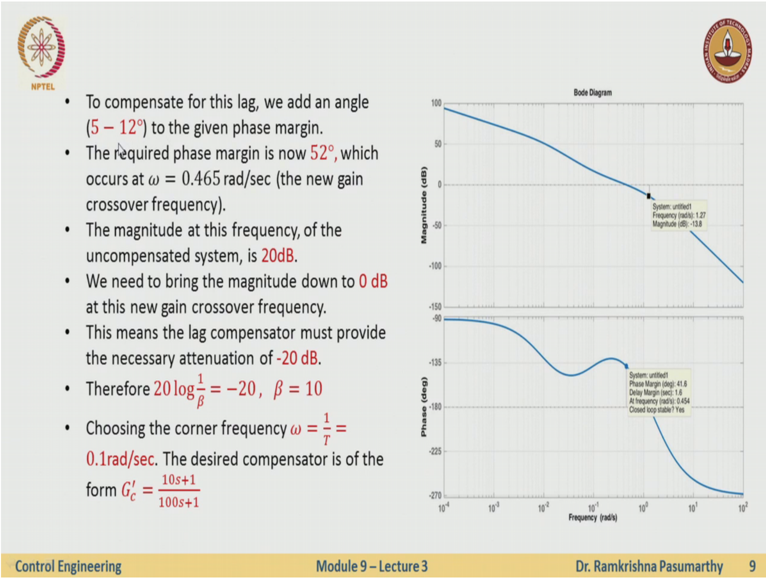

So, in order to compensate for this what I do is to add an angle 5 to 12 degrees to the phase margin. There is there is some trial and error in such a way that I do not really look at this number now. But I aim to a matters slightly bigger number, so not minus 140, but say somewhere around minus 150 sorry, minus 130 So that my phase margin is 50 degrees. Just to compensate for the loss of angle because of the lag compensation, because of the angle criterion of the line or the phase nature of the lag compensation. So, I just say instead of 40 I choose it to be 52. And this 52 I will find out at which frequency occurs somewhere here and it turns out that this 52 occurs at a frequency of 0.465. And at that number other frequency 0.465 what is the magnitude that is exactly the same procedure. I found that what is advanced here and then I go here right.

So something is error, ok whatever. So, here I say that the magnitude is 20 dB it was 16 earlier now it is 20. So, I need a different beta than earlier. So, beta would be 10 and I just now just against choose the first corner frequency to be 0.1 and I have this G c prime. And let us see how the closed loop responses right. Now the close loop response has a good gain margin. So, somewhere over here and then, now the phase margin is quite satisfactory 41.6 and of course, the closed loop system is stable right.

So, the design procedure is straight forward I do not now really need to write down the rules for you. And the rules are pretty straight forward right. So, you start with adjusting the gain first, and you see well what is happening. So, I want the signal to be attenuated, such that I have a new gain crossover frequency, which is lower than the original one. You know I am just you know shifting this gain crossover frequency to the left so that I will have a better phase margin right. And of course, this is just now design procedures I add an extra angle between 5 and 12, to compensate for the lag introduced because of the phase nature of the lag compensator ok.

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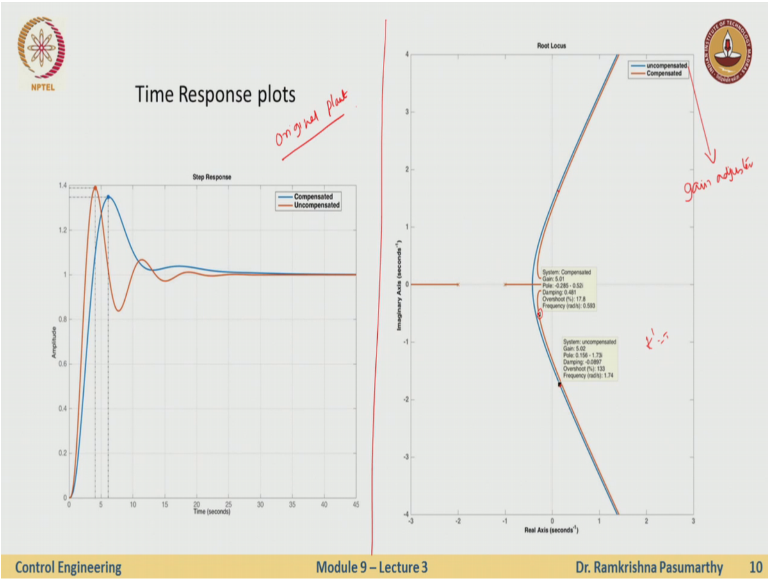

Now if I compare the time response plot I see a decrease in the overshoot right. And this uncompensated essentially means that I am not even looking at the gain adjusted system; I am just looking at the original plant. So, this in the left hand side I am just looking at the original plant right.

So if I can go back to things I have ever had said that well with K prime equal to 5, my a system turns out to be unstable. So, here when we look at the root locus plots and what happens to the root locus plots. If I look at the uncompensated system like this now is the gain adjusted system. So, these colors here and in these colors here are they have no relations, to suggest 2 different plots.

So, if I look at the blue plot which is the gain adjusted system at a gain of 5, you sort we found at that K prime was equal to 5, my see that my root locus shifts to the right, and therefore, my system is unstable. Now I introduce a lag compensator and with the same gain I am now here right. My system is now stable and I see the effect of lag compositor it just shifts the root locus slightly to the right, but then the things are taken care of here and that I am actually ending up with stable system.

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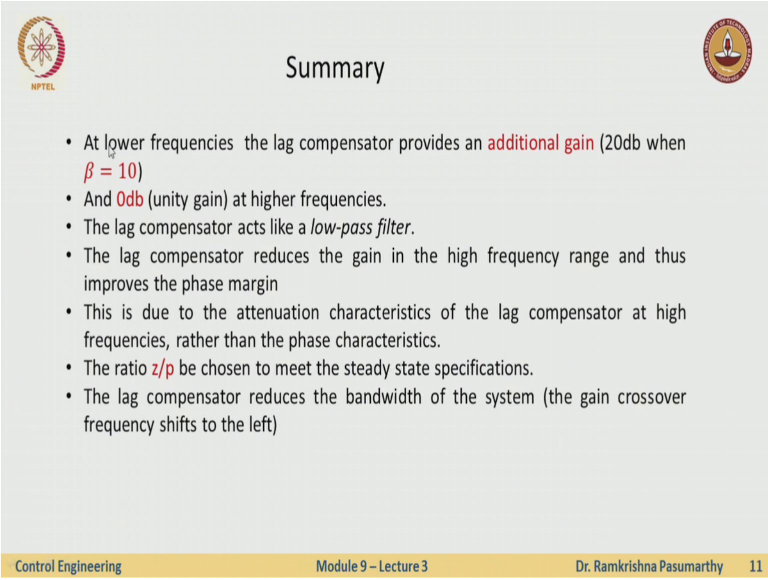

So to summarize at low frequencies the lag compensator provides an additional gain. So, I when I am look saying of these additional gain I mean that I am looking at the plot of this is my entire compensator right. K prime times this thing right. This is this is the entire compensator I just spirit into the gain part and the dynamic compensation part right. So, if I look at the entire plot this will just go up with 20 log K depending on and again what is the what is the value of beta for example.

So, the So, this is how it goes and at a higher frequencies it has the smaller gain. And therefore, the lag compensator acts like a low pass filter right. In such a way that it reduces the gain at the high frequency range and improving the phase margin right. Does what the example also told us. And the ratio z by p is chosen to meet the steady state specification. So, right so again these steady state specifications were through this gain and you see that this gain was equal to K time’s beta.

Now, what various is beta coming from this? Beta is coming from the construction or even from here that is the ratio of the 0 to the pole. But if I call this z c by p c this is beta. So, these are not disassociated from each other at this beta actually he has to do something with here also. But just at I make my design procedure e c 2 look at it as at the gain independently and this part independently, but this has also to do with beta, this directly sitting in over here.

And of course, the lag compensator reduces the bandwidth of the system because the gain crossover frequency now shifts to the left. And we always claim that the poles and zeros should be placed close to the origin ok.

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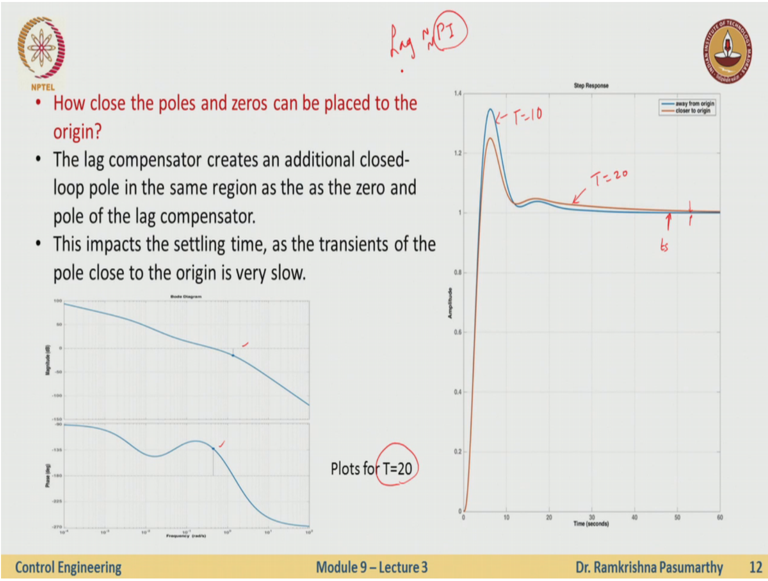

So, how close to the origin? So, and what happens when I place poles and zeros close to the original or even must closer to the origin, is at the lag compensator it creates an additional closed loop pole 0 in the same region. So, it So, it creates an additional closed loop pole in the same region as the 0 and the pole of the lag compensator. And which and this pole if it is closer and closer to 0 we will create some additional dynamics, for which the settling time might be higher as we go close and close to the origin.

What is at mean right? So, without going to merge into the details, say if I just for trial basis select another value of T right. With to be 20 with this actually we will place the poles and zeros a little closer to the origin, in that case well I think my gain margin and phase margin are good, but now look at this right. So, if I am away from the origin I see that here I am I am I am settling down little faster right. So, this is this is what I had I had with earlier with T equal to 10 right. And I say I call this away from the origin the relatively away and this plot is 40 equal to 20.

And you see that the settling time is actually higher if you just zoom in here you could have a bit of a difference in the settling time. And that is due to the additional pole which is created because of the lag compensation, you can just check for the closed loop poles in matlab and that will actually give you the exact locations of the poles. That is not really important, what is important is to just understand this phenomena, how close do we go to the origin.

Now, closer to the origin also means that I am actually looking at a PI kind of controller right. So, this lag is an approximation of a PI controller. And one drawback of this PI controller was at it could lead to instability, has we have seen right. In the module number 7 and even in 8. That this could actually we have to a closed loop unstable system.

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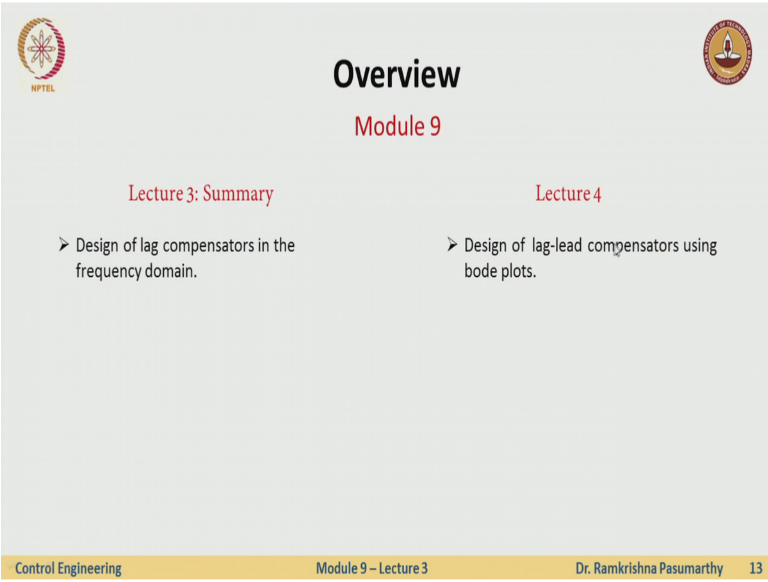

So, these are the things which we need to be careful of, apart from that the process is very simple. So, next what we will see is to design lead lag compensators and this is a little philosophically different than when we do in the root locus or in the time domain, where an obvious interpretation was if I were to improve the steady state performance, I design a lag compensator if I were to improve the transient performance, I go to a lead, if my specifications or a combination of both then I directly go for a lead lag compensator.

Over here it is a little different because, we have additional things introduced into the system like the bandwidth. And we will see how we make use of lead and lag compensators to achieve little more objectives then what we had in the time domain, and see if it is straight forward or what are the pros steps involved with that.

Thank you.