Control Engineering

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Module - 09
Design using Bode plots
Lecture - 02
Design of Lead compensators using Bode plots

So, continuing on our discussions with respect to designing lead compensators in the frequency domain making use of the bode plots.

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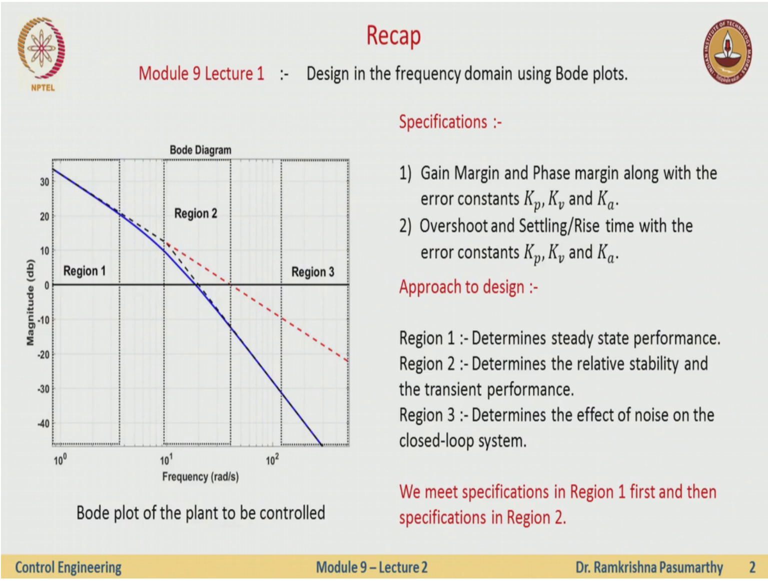

So, let us just recall what we had in the last lecture. So, specifications the performance specifications were usually in terms of the gain and the phase margins this were like what determine the transient performance together with the error constants in terms of the position, the velocity or the error the sorry the acceleration error constant. So, what happens in the frequency domain? So, we saw that the lower frequency region decides my steady state performance in terms of a certain K p, K v or K a region two where the gain margin. And phase margin kind of concepts come they determine the relative stability and the transient performance, and the higher frequency region determines how we are handling noise is it amplified or not and so on.

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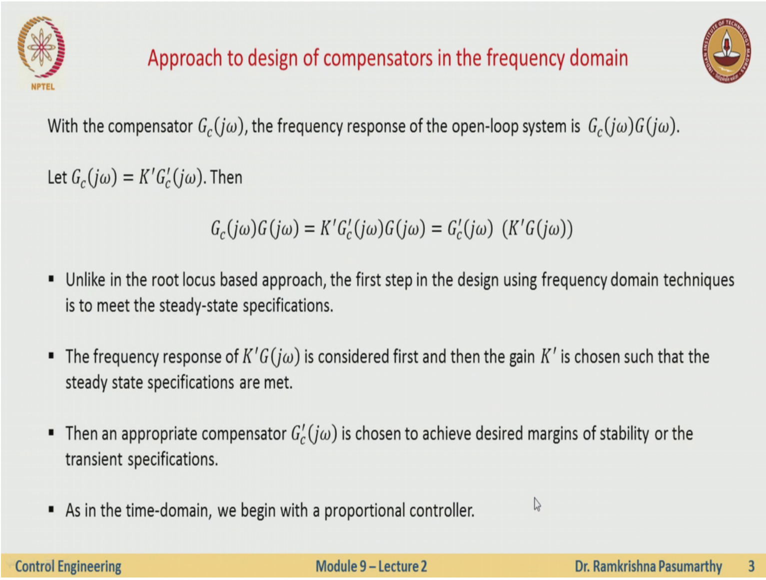

So, here we would be interested typically in region one and region two. And we will see how we go about looking at a design problem. So, just look at the sinusoidal transfer function and then we plot the response of G c times G. So, if G c can be written as K prime G c prime with a gain coming out explicitly then G c times G c j omega times G j omega can be written down explicitly like this right. And then this guy can actually go into what we also can say as the point dynamics. So, what we did in the in the root locus when we had a problem where we wanted to compensate for the transient performance and also ensure a certain steady state error or studied certain bounds of the steady state error, we first design the transient controller which is essentially the lead compensator. And then went onto design the steady state compensation which was a lag compensator.

So, here we do slightly different approaches. The first step would be to meet the steady state specifications and it will be obvious in the next few slides why we need to meet the steady state requirements first. So, in order to meet the steady state requirements, we look at the frequency response of this thing. And then we choose the gain K prime such that the steady state specifications are met. And once the steady state specifications are met we choose an appropriate controller which we denote here as G c prime to achieve the desired gain margins and the phase margins. So, this is how my transient performance specifications are seated to be they are given to me in terms of the phase margin and the gain margin and we know that their actually excess of one to one relation between the phase margin and the things like the damping coefficient and all.

So, the first step in any control design is to just start with a proportional controller as we did even in the root locus is there. So, we mapped that given transient performance specifications to a set of dominant poles. And if the dominant poles were lying on the open loop root locus, we just compute that gain at that location and say well I can just designed a proportional controller of that particular gain to ensure my behavior in terms of the transient performance. So, similar things we also do here right, we begin first with a proportional controller.

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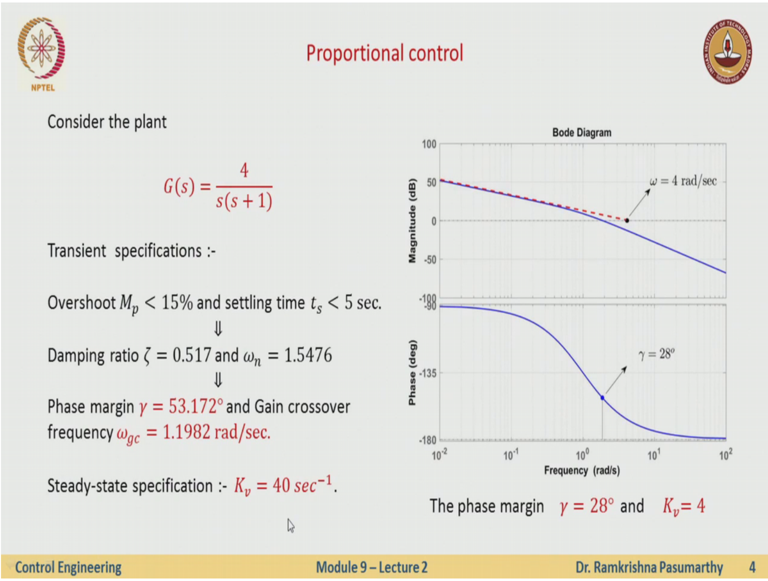

So, well just to make things a little easier, let us start with the help of an example. And I am given G of s the plant has four over s s plus one with specifications of this form that we overshoot should be less than 15 percent and the settling time of less than 5 seconds which translate to appropriate zeta and omega n when we do the formulas for this. Which in turn translate to a phase margin of 53 degrees and a certain gain crossover frequency of 1.19. And in addition I am given that the steady state specifications which I should meet are 40 per second.

Now, just look at the open loop bode plot right. So, here the open loop things would tell me that my phase margin is 28 degrees and I know how to find out the velocity error constant from the bode plot I just extend this line and find the frequency at which it intersect the zero db line and that frequency is actually my K v. So, this is what we did when we were discussing how to find out error constants just by looking at the bode plot.

So, the specifications of the open loop system has the following characteristic K v is 4, and the gain margin is 28. And we would like to have it something like this that K v should be 40 with a phase margin of 53 degrees.

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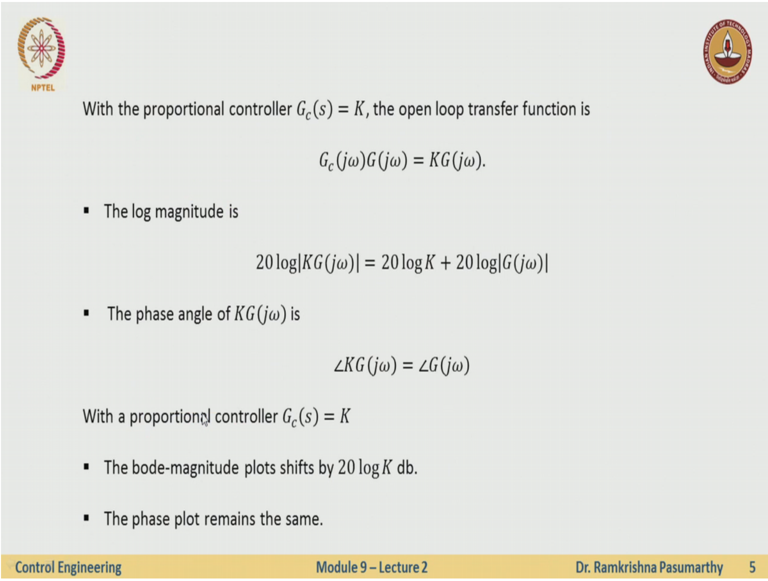

So, let just start with the proportional controller where G c is just K. How does what changes in the magnitude in the phase will the log magnitude changes according to this thing 20 log K times G of j omega. So, you see that this gain K this adds up this magnitude to my open loop frequency response or open loop bode plot, this is my plant. The phase angle remains the same. So, with a proportional controller, the bode magnitude plot shifts by this number twenty log K the phase plot remains the same, but this is well the phase plot remains the same, but something strange happens here.

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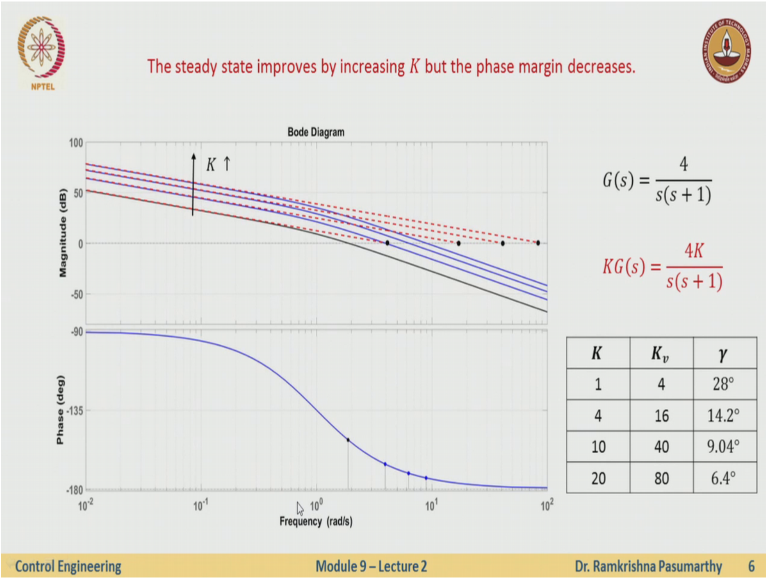

K	K_v	γ
1	4	28°
4	16	14.2°
10	40	9.04°
20	80	6.4°

Let say I start with this plant and I say I just keep on increasing my gain to see what happens. So, I keep on increasing the gain and I see that what happens is with a increase in gain, this bode plot shifts the magnitude plot shifts up. And when it shifts up, what we see is that this frequency which we call as the gain crossover frequency this keeps on shifting to the right. So, even though the phase plot remains the same, the phase margin changes because the phase margin is defined now at the new gain crossover frequency. So, if I just keep plotting some of this and I see that my phase margin actually keeps on decreasing as I arbitrarily keep on increasing the gain and that is essentially because this changes this frequency changes right.

So, this is corresponding to this thing I was keep going here; even though I see that an improvement in my steady state is visible. So, because this frequency at which the initial slope intersects the zero db line will tell me what is my K v. So, I really wanted to be as further as possible. So, for a gain K gain of 10 K a gain of K equal to 10, I have K v equal to 40, but the phase margin is very is highly unacceptable is 9.04 when the original system had a phase margin of 28 and the desired is 53.

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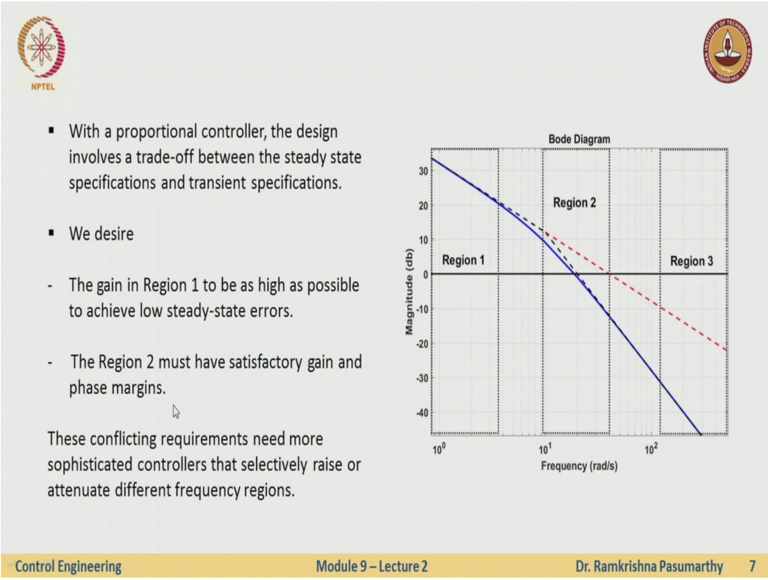

Now, what we observe is that well I just when I just do a proportional controller well I have a big trade off between the steady state and the transient specification. So, based on this plot here, what I see is that to achieve a desired steady state performance or if K v should be large this actually means that the gain in region one should be as high as possible to achieve low steady state errors, so if you see here right. This keeps on increasing, and this keeps on increasing my K v goes on increasing and therefore, my steady state error decreases. Now, the region one thing is taken care of. Now, I want to do something over here in such a way that my phase margin requirement is also met.

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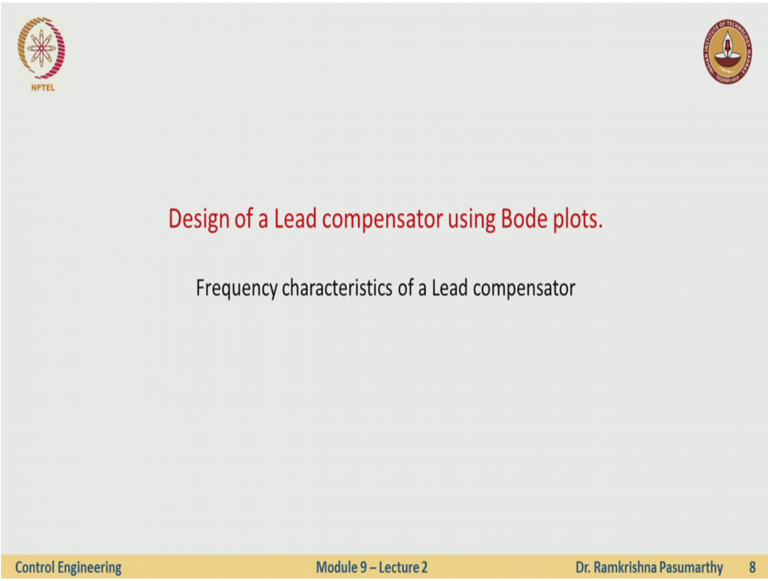

And this is where we come to this point of designing a lead compensator using bode plots.

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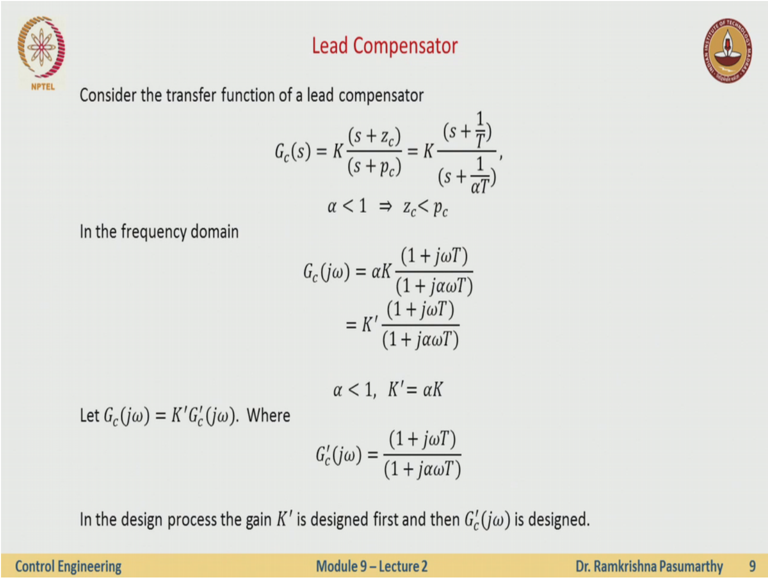

So, the transfer function of the lead compensator is exactly the same as we did in the time domain, where I have a gain zero a pole where pole was significantly to the left of the zero. I could write it in time constants in this way. And this is my a sinusoidal transfer function. And as usual alpha would be less than one. So, I am looking at designing G c j omega as K prime, G c prime where this is this guy is a just the G c prime and K prime just take care of the gain factor here that is alpha times k. So, first we design this K prime, the gain which is which ensures a desired steady state performance and then I go about finding what are these numbers here, the unknowns are the omega unknown is a capital T and unknown is the alpha.

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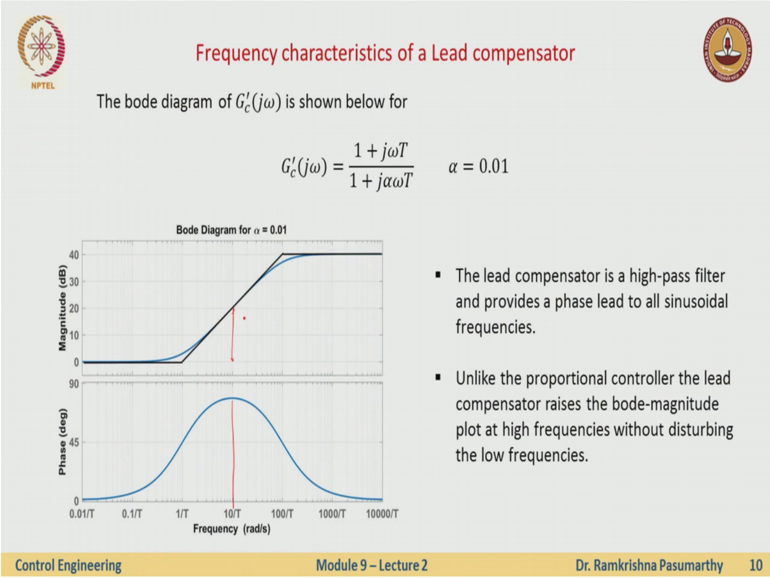

So, just by making alpha less than 1, well I know that the pole is to the left of the zero on my complex plane. So, a typical bode diagram they asymptotic plot is given in the black, it would look something like this. And when you see that the phase actually has a nice shape here. So, if I go back and compare to what was happening earlier, you see that the low frequency region right, so here, so this is now this is my region of interest right. So, nothing much should happen over here. So, what I see is at the low frequency region the addition to the gain is very small or almost zero, similarly with a phase.

So, I am not really interfering with the low frequency region; however, here I see that on the phase margin should increase which means this plot should be pushed to the right, so sorry to further a little up to ensure a bigger phase margin. And this curve here ensures that I push the phase margin or push the phase plot to exactly upwards, so that I have the desired phase margin. So now, the lead compensator is now essentially seen as the high pass filter that I can see for a higher frequency is have a higher gain and so on. So and unlike the proportional controller the lead compensator raises the bode magnitude plot at only high frequencies not at the low frequencies right so nothing is happening at the low frequencies.

So, I am ensuring something in the mid frequency range without actually disturbing my low frequency range. Now, if you see how this plot goes there is a point where the phase attains a maximum value and it again goes back close to zero.

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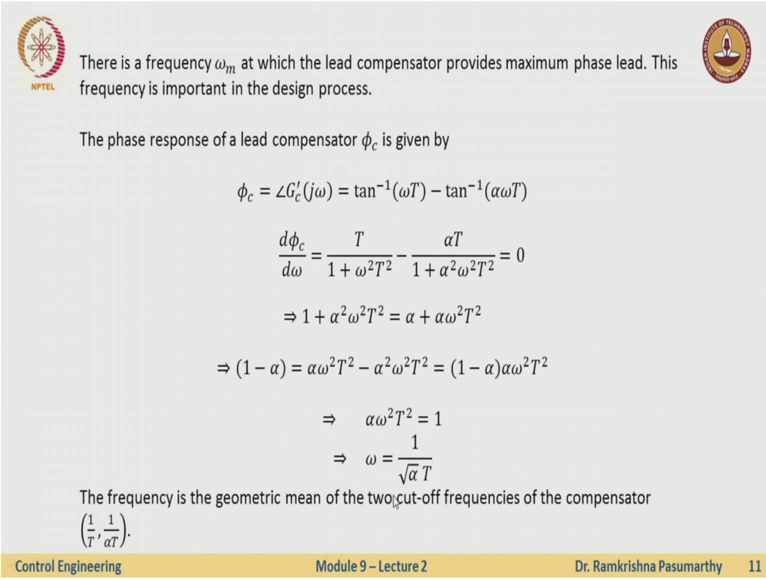

Now, there is a frequency at which this compensator provides a maximum phase lead right and this frequencies what we will we will use in the design process. Now, what is this maximum phase lag and what are we trying to identify here, we are trying to identify these two corner frequencies and this alpha. So, once we know these two control frequencies, we know the alpha then our design process is complete.

Now, the angle which the compensator provides is given by this one. So, have an tan inverse of omega T coming from the numerator and the negative of tan inverse of alpha omega T. And then you see that this angle phi which is angle contribution here, it varies with frequency and it has a maximum at a particular frequency. This is like a nice looking continuous function. So, I can just find the maximum of it. I can say what if I ask myself a question what is the frequency at which phi c attains a maximum value and calculus as mean that just do this thing d phi by d omega is 0, which leads to the to this expressions.

And I say the omega at which the maximum phase occurs is just this one omega is 1 over square root of alpha times T. Now, this frequency is a geometric mean of the two cut off frequencies as you see this is one cutoff frequency at 1 over T and this is a cutoff frequency of 1 over alpha T. And this is just a geometric mean of both of them right. So, the frequency at which the maximum peak occurs is a geometric mean of these two cutoff frequencies.

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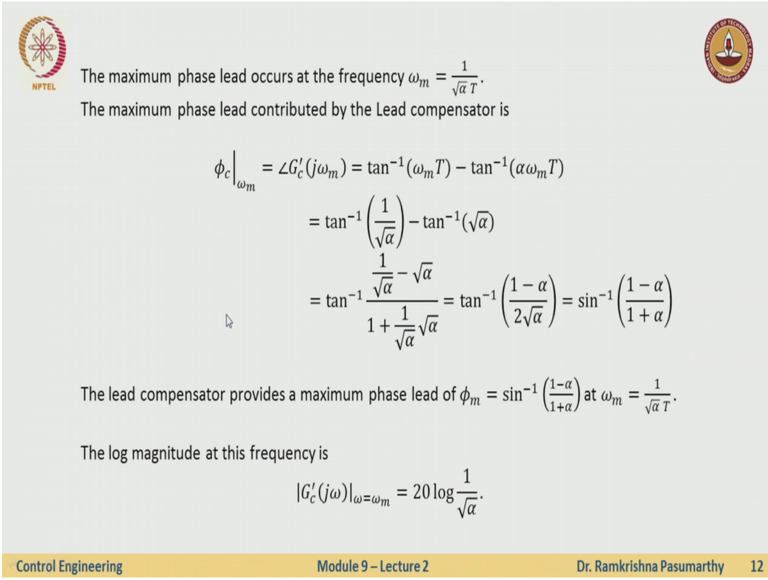

And then we conclude that the maximum phase lead occurs at this particular frequency. Now, at this frequency what is how does phi of c look like. Well, in this expression, now I know what is this omega right where maximum phi of c occurs that is this one. So, I substitute this expression into this into the computation of the angle, and I do all these things. And I see that the maximum phase lead is related to alpha via this expression, just a simple computation. In the first I find out what is the frequency at which the maximum phase lead occurs I substitute it to into this expression and I get what is the maximum value of phi c.

So, two questions what frequency well it occurs at this frequency the maximum phase lead. What is the value at maximum phase lead well that value of that angle is written down in terms of alpha with this, this is little steps here. So, given alpha I can find a phi m or vice versa. So, if you given how much phase lead I need to add to the system or how much should be the extra angle contribution I can find out what is alpha. So, in this expression, now I find out alpha let say phi m is sin inverse of 1 minus alpha over 1 plus alpha and this occurs at this frequency.

Now, what is happening to the magnitude at this frequency so somewhere here. In this magnitude is at omega equal to omega n is 20 log 1 over square root of alpha this also can be completed pretty easily that I do not really need to derive expressions. So, we already know where bode plots of how we do this and I am just substituting in this expression these values which I compute right. So, substitute omega equal to 1 over square root alpha T in this expression find out what the magnitude is and I just get this one 20 log 1 over square root alpha is the magnitude at this frequency. And what is this frequency this is a frequency at which the maximum phase lead occurs due to the compensator.

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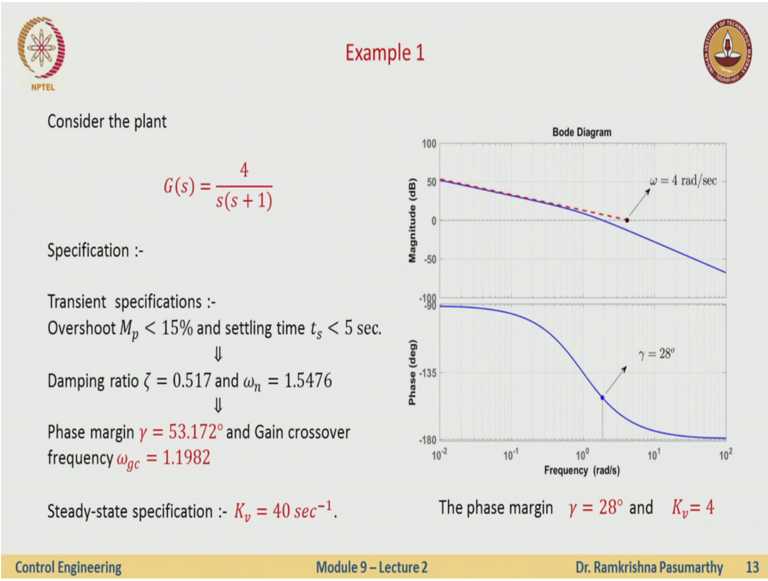

Now, let us come back to a design problem. So, I have G of s is 4 over s s plus 1 and then I translate all this again to specifications to such a way that I want gamma or the phase margin to be 53 degrees together with K v of 40 per second. So, the open loop characteristic is at gamma is 28 and K v is 4 as we have seen even earlier.

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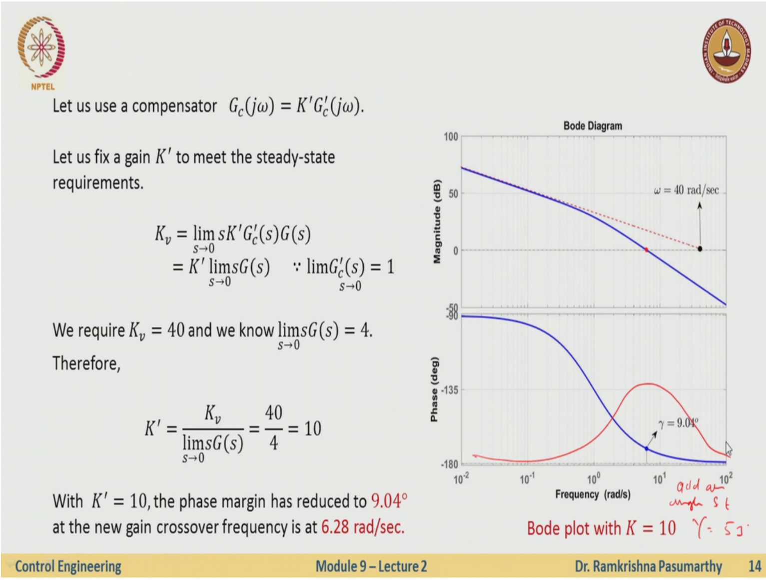

So, first is well can I design, if I can at least first take care of my study state requirements. Well, how is the steady state requirement. So, K v the velocity error constant is s times the entire transfer function. So, entire transfers function. So, entire transfer function consists of this K prime G c prime G of s. So, what I know is that if I look at this expression limit s tends to 0 for my G c prime is always 1. Why is that obvious? That is obvious because this substitute omega equal to 0, here you always get 1. So, this really does not contribute anything to that. So, on indication that no nothing really is happening with this with this compensator for this steady state error, it is really contribute anything either positive or negative.

So, I am just left with this expressions K v is K prime limit as tends to 0 s times G of s. What is required K v is 40. And I know that limit s tends to 0 s times G of s is 4, this is given to me right from here limit s tends to 0 s times G of s is 4 and then so I can just find out the new gain K prime this is 10, 40 over 4. Now, this is a then the additional gain that my compensator should provide with K prime equal to 10, I can get K v is 40. Now, I just simply at this compensator and I just plot K times G. So, this is my controller the proportional controller which gives me a steady state error constant of 40.

And what happens to the bode flat well this is fine. This initial slope intersects at 40 radiance per second and this is the value of my K v I look at this gain crossover frequency. And if I something terrible has happened here is at the gain. The phase margin has reduced to 9 degrees at with this gain K. So, I want this guy here the phase margin to be like 50 degrees. So, if I just say when I want to put a controller here such a way that you know say if I do something like this then the phase will increase to give me a desired phase margin. This is essentially what I want to do, I want to add some angle here; add an angle such that the new gamma is 53 degrees. Now how will we do this? So, here the gain crossover frequency is roughly about 6.2 at you can just see from this plot here.

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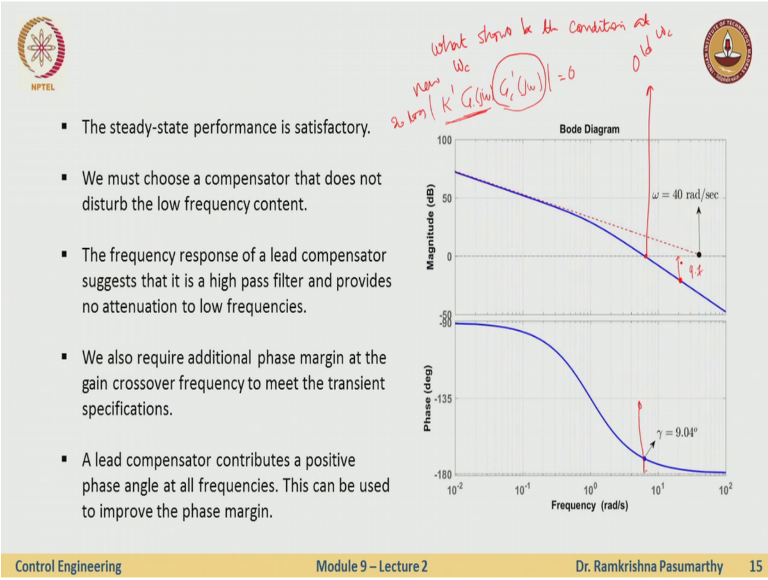

So, I have to perform this operation in such a way that they affect here is minimum so that is what we even said that. So, now, given that the steady state performance is satisfactory I must choose the compensator that does not disturb this low frequency content. And then if I look at a lead compensator as I said earlier that it is a high pass filter and provides no attenuation to low frequency signals no gain, no attenuation also. Now, we also required that the additional phase margin at the gain crossover frequency to meet the transient specifications and then this lead compensator provides me that additional thing over here. So, this actually should not be 9, but this should be somewhere like close to 50. Now, this is what we will achieve with the help of a lead compensator.

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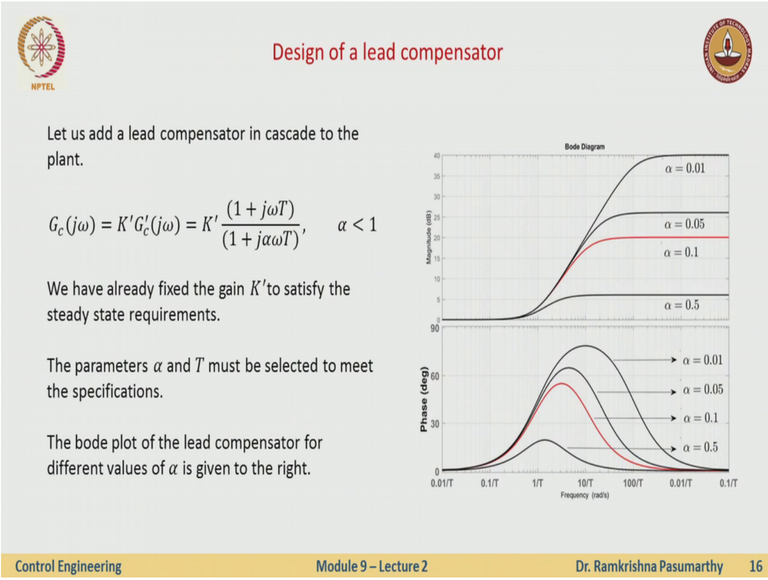

Now, let us just do some experiments right to say well now I want to design a compensator, which looks like this. The only information I have is alpha is less than one and I know the K prime which gives me a K v of 40. So, we want to shall it alpha and T in such a way that I should meet the desired specification. So, I just play around with different values of alpha. So, for different values of alpha the bode plot of the compensator looks something like this.

So, that is an angle here and then the frequency at which the maximum angle occurs keeps on shifting to the right, and you have the magnitude plot going this way. And you see the low frequency behavior more or less remains the same, there is nothing happening in this region. So, we are not really playing around or we are not really disturbing what is happening to the K v.

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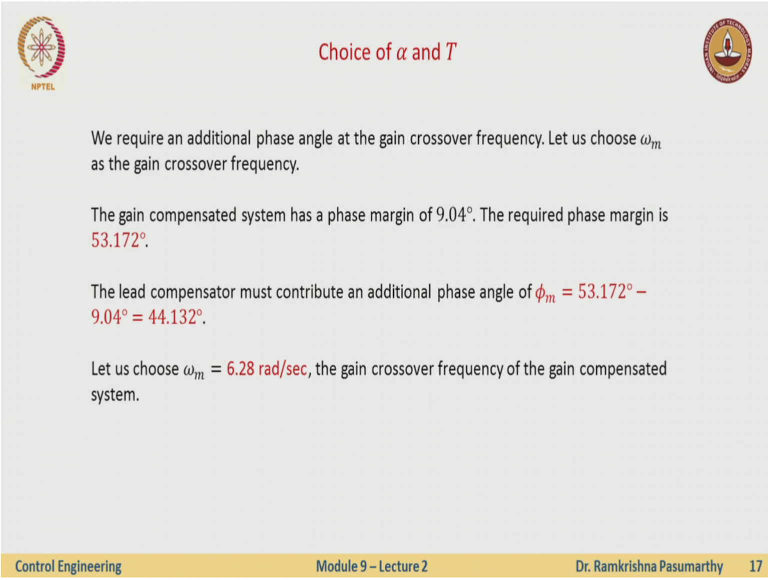

Now, we require an additional phase angle at the gain crossover frequency. So, this is how the phase margin is defined, the amount of phase of 180 minus the formula. Now, where do we compute this we compute this at the gain crossover frequency. Now, at this gain crossover frequency the angle should be 53. Now, I already have an angle of 9.04 now how much should the lead compensator give me well I have through this computation or the lead compensator should give me an additional angle of 44 degrees at this point.

So, this is this angle plus 44 then I am fine right I am my transient specifications are made, but my study state specifications are made and I am happy. So, how do we go about this? So, here well this is my gain crossover frequency 6.28 and I add this much of angle at this frequency and that is what I have naturally do. So, that this is my gain crossover frequency and what I need is angle the overall angle here to be like 53 degrees.

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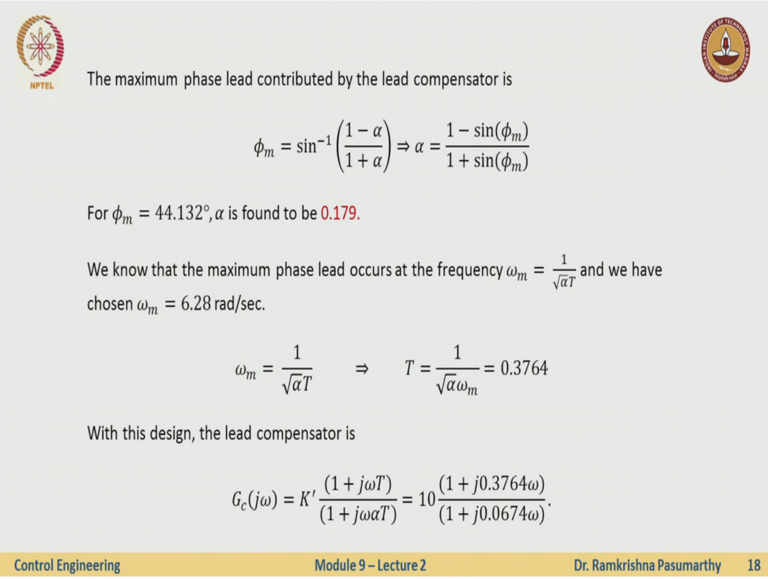

Now, let us just do this right arbitrarily. Now, what I know how do I compute this thing well I know that phi m is related to alpha via this formula. So, phi m is the additional amount of phase my controller or the compensators will should provide. So, for this phi m, it turns out that the alpha is 0.179. Now, this phi m the maximum phase lead, I know that it should occur at the gain crossover frequency right here, I want this to be 53 over here at this frequency. Now, this frequency will be my omega m.

Now, once I know omega m I know alpha I can find out where what is the T, because this omega m occurs at the geometric mean of these two cutoff frequencies. So, now, I know alpha, I know what is the omega m, so I can compute what is T based on the formulas which we derived. Now, I know K prime, I know omega I know, so I do not need to know omega right, I know the omega m based on this omega m I compute what is alpha and what is T. And this is how my compensator would look like. Now, let us verify if this is actually helping me get the desired phase margin or not.

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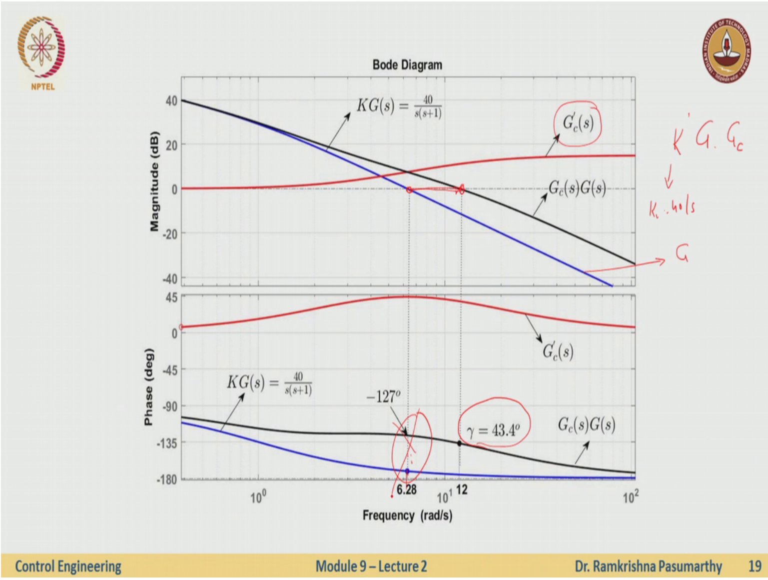

Now, the bode plot I would now plot see the plot of not just G of s, but it will be K prime with this K prime gives me a K v of 40 per second. Now, I have G and I also have G c. This is my G c, G is given to me as a part. Now, I just draw this things and I find out well what is the phase margin, the phase margin actually turns out. So, I should look at the phase margin not of G, but now of G c times G of s G c times G of s. This is my, this is where I will measure the phase margin this is the gain crossover frequency of the compensated system. I go down and I check well it is not 53 that is actually 43. What has gone wrong here?

So, if I just compare the bode plot of the uncompensated system, what I see is that there is a change in the magnitude plot in such a way that my crossover frequency is no longer at 6.28, but it will somewhere here. And we design our compensator such that the maximum phase lead occurs at this frequency, but that does not really satisfy the close loop conditions because this point has shifted to the right. And therefore, we should have gamma at this new crossover frequency to be 53 not here; this is not useful to me.

So, just by looking at this crossover frequency and designing a compensator does not help me much because the gain crossover frequency of the compensated system shifts to the right that is obvious because you see there is a little bit of a magnitude change also in the characteristics of the compensator, not only the phase changes, but also the magnitude changes. Therefore, the natural tendency is at the gain crossover frequency will shift to the right because we actually adding up some magnitude.

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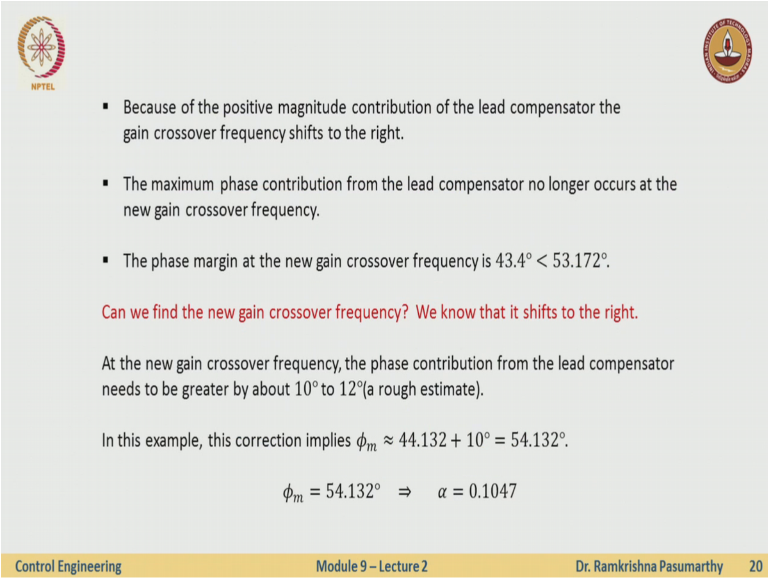

Now, how to fix this problem? Because of the positive magnitude contribution of the lead compensator the gain crossover frequency naturally shifts to the right. And the maximum phase contribution from the lead compensator it no longer occurs at the new gain crossover frequency it occurs till at the all the gain crossover frequency, but we want the 53 degrees to be at the new gain cross over frequency. So, add the new gain crossover frequency it is 40 three, but I really want 53.

Now, well how do we know? So, now, I know already that that the bode plot shifts to the right. So, can I appropriately find a new gain crossover frequency? Well, one technique is so instead of 40 three this should have been 53. So, you can I actually convert this 53 which I was designing to be here to be some 63 for example. So, this is I push it a little further up this will naturally go up right, so that is one technique. The new gain crossover frequency the phase contribution should be greater than by 10.

So, what I should then do is I should shift this by some extra 10 degrees which means that even though this is going down here this can be 53, and the phi m here can be much higher. So, I just add up 10 degrees here, so that I compensate here. Now, if I design for phi m equal to 54, I can find the appropriate alpha again from the same formula nothing changes here; instead of 40 four now just to compensate for this shift and adding some 10 here.

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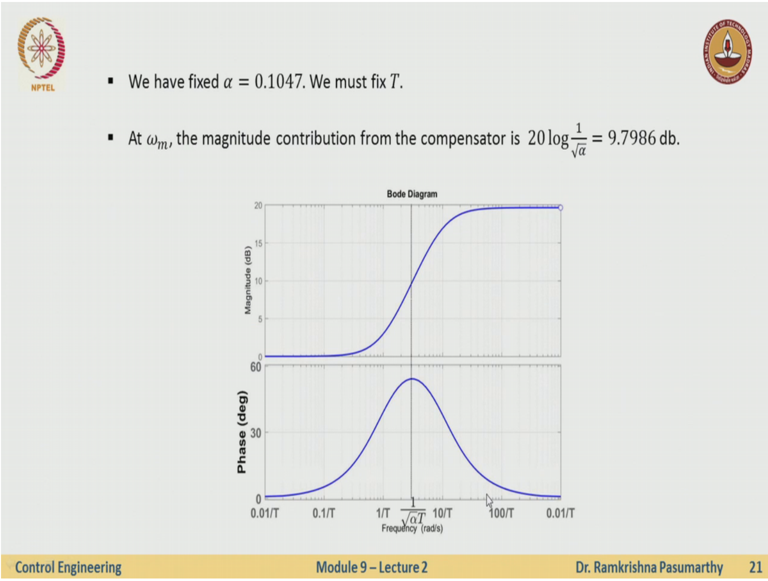

So, with this new alpha, I can find well now we must fix the T. Now, how do we fix the T, this is the question. Now, look at what happens at omega m. At this omega m, if I go back to what my bode plot does, we just take a much simpler one the first one here. At this omega m, there is a certain substitute magnitude here. So, this is a extra magnitude contribution right to my close loop bode plot and therefore, it shifts to a little to a right and therefore, I select omega m at omega m I do not know this omega m, but I know what is the contribution of the magnitude.

So, based on what we had derived earlier this one. I do not really need to know omega m, but I know that at a particular omega m where the maximum phase leads occurs. The contribution of the magnitude plot is 20 log one over alpha. Now, in this example that turns out to be 20 log 1 over alpha with this new alpha turns out to be 9.79.

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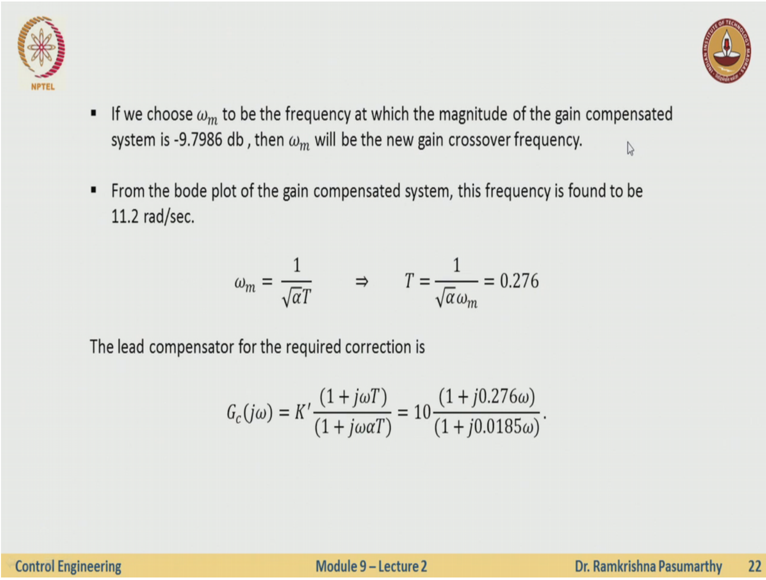

Now, what do I do? If we choose omega m to be the frequency at which the magnitude of the gain compensated system is the negative of this then omega will be the new gain crossover frequency. Let us again go back to our plot and see what this means. Now, this is the old gain crossover frequency. This does not help me right. Now, what should be the condition at new gain crossover frequency call it omega c. At this for the new frequency should be such that G put a K prime G of j omega G c of j omega the 20 log of this should be 0, which means that now I know what is this at the new crossover frequency. And I can just this computed by 20 log 1 over square root of alpha. So, it should be such that. So, if this is my 20 log square root of alpha. So, what this is 9.7 here.

Then this is the new frequency which I will choose because at this frequency right here the magnitude of g by itself is negative 9.7 this entire guy K times G. And what I am adding to this to this negative of 9.7, I am adding now a positive of 9.7 which comes from the compensator so that the overall contribution become zero. Now let read this statement again. Choose a omega m to be the frequency at which the magnitude of the gain compensated system that is K prime time G is minus 9.7 right, then omega m will be a new crossover frequency.

Now, from the bode plot I can find out what this is right, this frequency is found out to be 11.2 radians per second. Now, once I know the omega m I could compute easily what is the T. Once I know the T, I can get the structure for the compensator. And what this gives me is well this is kind of very nice here.

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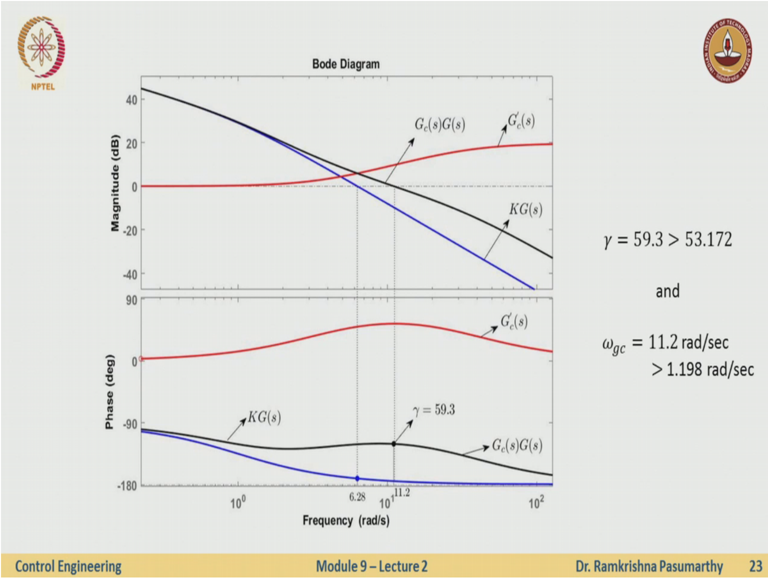

So, this gives me again margin of 59 just (Refer Time: 31:52) exactly what I want right, and well the gain crossover frequency here is 11.2. So, this is this is how we just do it a little bit by trial and error to find out to compensate for the shift in the bode plot to the right. So, there was when we added a lead compensator to the root locus it was pulling it to the right, here the magnitude plots shifts slightly to the right. And therefore, I need to find out what is a new gain crossover frequency. This new gain crossover frequency is just a frequency at which the gain compensated system has this much of magnitude contribution.

So, this is how it is done right. So, with the help of a very simple example; other example would also follow the same nature of it right. So, this is nothing really complicated about taking a system which has four poles and five you know 530 something like that.

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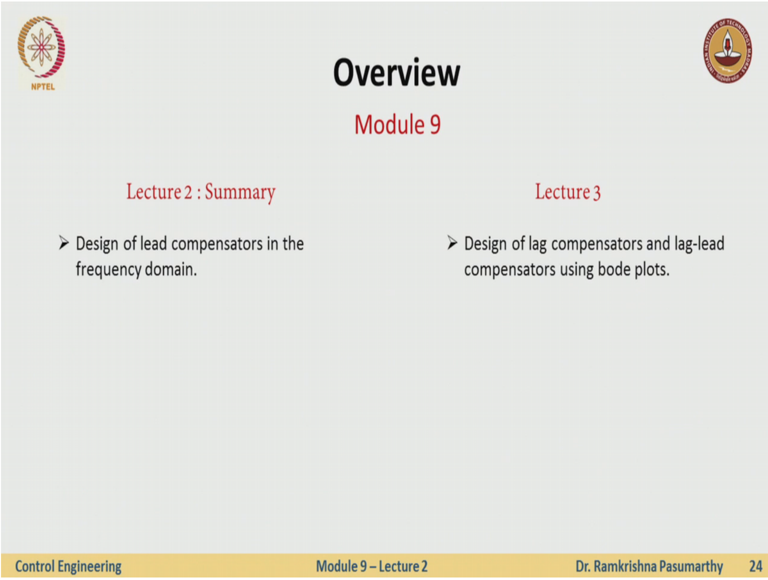

So, this was a very simple and yet powerful illustration of how we design lead compensators in the frequency domain. So, the idea was just ok, so then the thing was first I need to find out what are the locations of the poles and zeros. First unlike the root locus, I here first compensate for the steady state behavior and then come to the transient behavior. So, I again as usual compute what is a phase lead, I compute the new gain crossover frequency and I have no techniques how to compute the new gain crossover frequency and based on that I can find out what is my alpha and T which will give me the location of the poles and the zeros.

So, similarly next time, we will see how to design a leg lag compensators and eventually lead into lead and lag compensators using bode plots.

Thank you.