Control Engineering
Dr. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 09
Design using Bode plots
Lecture - 01
Introduction to design in the frequency domain

Hello. In this module we will learn how to use boded plots for meeting our design purposes. So, we will call it as design in the frequency domain. So, so far what we have done in the previous module was to learn how to design compensators and our main analysis tool there was a root locus plot. So, we had the issue of addressing transient performance specifications via a lead compensator, addressing steady state performance specifications via a lack of compensator and in cases where neither of these are met in the open loop be go to design a lead lag compensator.

And, so here will see it from the slightly different prospective: from the prospective of the frequency domain. And we saw that the that the lead compensator was used to improve the transient response, like was the steady state and in case we want to improve wanted to improve both we use a lead lag compensator.

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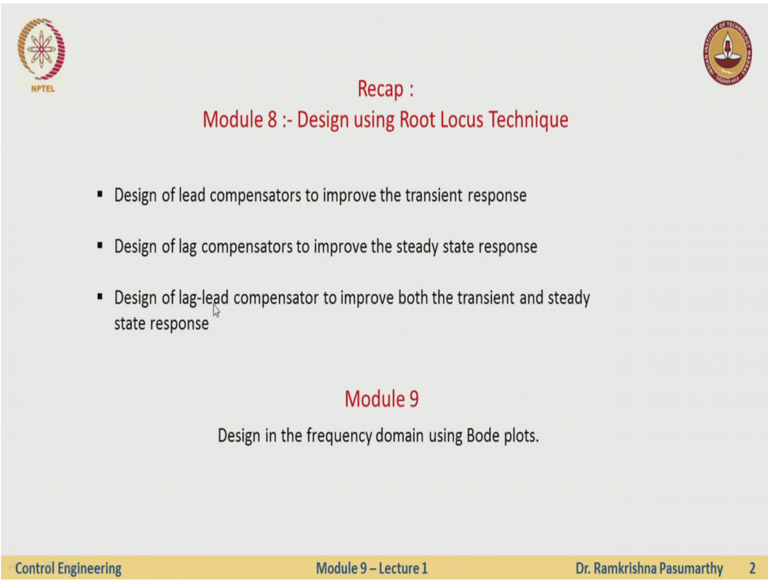

So, here the tool which we used was the root locus. And now in the frequency domain design techniques we will make use of the bode plots to verify our design or even for the analysis process.

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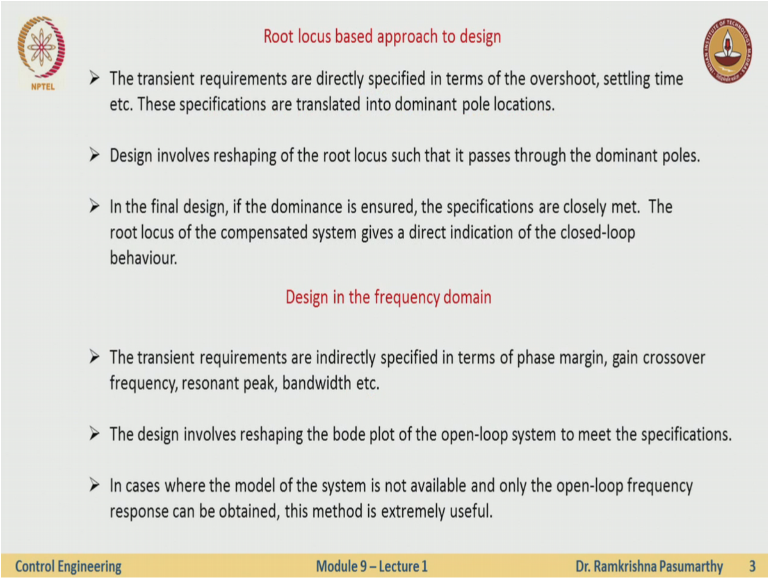

So, just quickly again recollect, what was the root locus they suppose to design? So, the transient requirements were given to me in terms of the peak overshoot, in terms of percentage and I had the settling time. And these 2 specifications were translated to the location of the dominant poles. And the root locus was used in such a way that I would want the root locus of the compensated system to pass through the dominant pools all, right.

And then in the final design we ensure that the dominance conditions are met very closely. We may not really get an exact overlap between the dominant poles and the compensated plant, but we know techniques of how to get them as close to each other as possible. So, what will be do or what will be our philosophy in the frequency domain techniques? So, in this case the transient requirements are specified in terms of the phase margin, the gain crossover frequency, sometimes the resonant peak and even quantities like the bandwidth.

So, what we saw while we were analyzing the frequency domain plots or the frequency domain response was that there is some relation between the specifications in the frequency domain and the specifications in the time loop gain. We will again recollect those.

So, in this case the design would involve reshaping of the bode plot to made this specifications. So, as analogous to what was happening in the time domain that we wanted the root locus to go through or pass through the dominant pole location; similar specifications will have to be made even if the frequency domain by the bode plot and we will see how we do that.

This also advantages to me in more than one way set in cases where the model of the system is not available. And, and in cases where I could get or I could construct the open loop frequency response this method is extremely helpful. So, anyway it means that I could actually construct the transfer function with biasing the response of the system for various frequency signals I could, once I could consider the transfer function I will have a model of it and from the model I can go to design the frequency domain compensator. So, this is a part which you will do in the in the next module which will be titled, how to? So, title experimental determination of the bode plots.

So, given a frequency response or given a bode plot, how do I obtain the transfer function? That will we will do in the next module, but at the moment it is safe to assume that if I do not have a model, I can actually construct a model by using the frequency response techniques.

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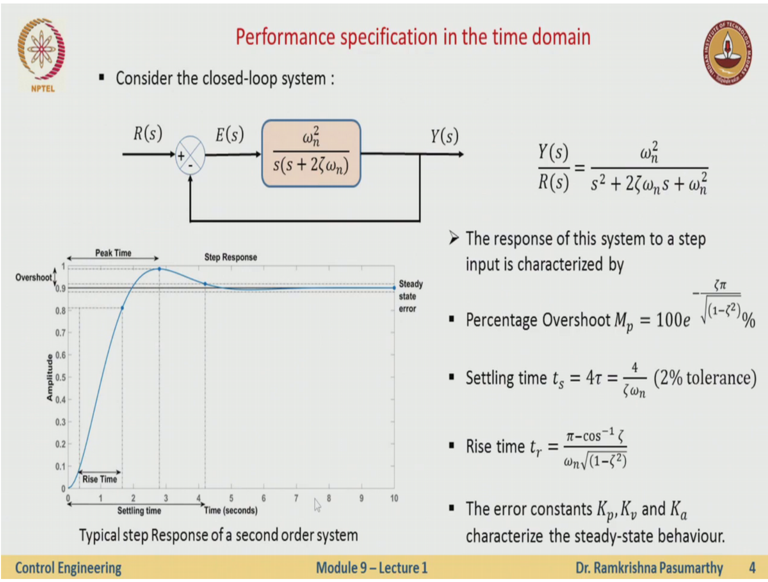

So again this has very quick recap. So, a standard second order plant in closed loop looks like this. Where we have the damping coefficient, the natural frequency, typical step response for an hundred and K case loops something like this, where the overshoot was calculated as a function of the zeta settling time dependent on both zeta and omega n and similarly we had other quantities like the rise time.

And in addition to these 3 specifications we also had the steady state specification in terms of the position error, the velocity error and the acceleration error or error constants of this peak.

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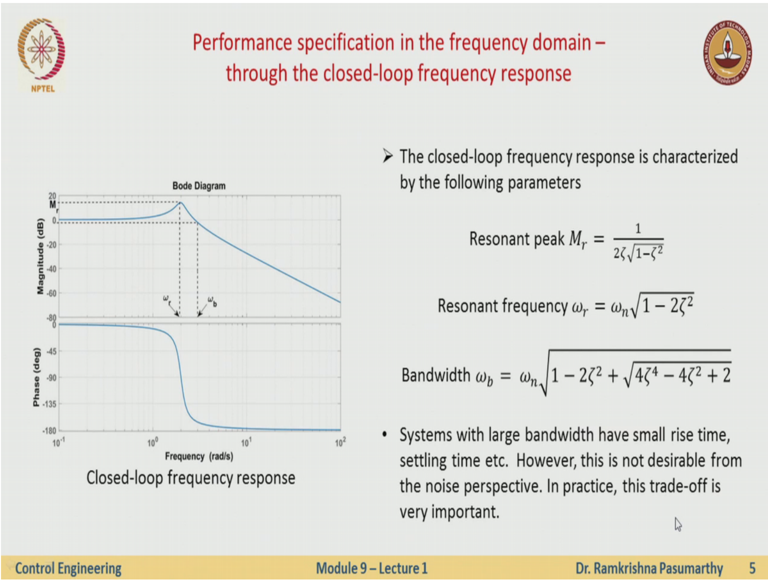

These close 2 characteristics have few components. So, first is see if you see this is the response to different frequencies. So I will, from here will have a peak and are they again go down right. So, this the earlier term bit as the resonant peak and the frequency at which this occurs is the resonant frequency and we computed that this was directly depending on the damping factor. And the resonant freq frequency was depending on the damping factor and also the natural frequency.

Similarly, for the bandwidth right, so bandwidth was computed in the following way that is depending on omega n and also the zeta. So, what is observation from these 3 things? That systems which have large bandwidth or bandwidth close to m which means zeta would be close to 0; sorry, bandwidth close to omega n which means the zeta would be closer to 0 and smaller the zeta would mean we will have a smaller rise time and so on. Of course, you will have to trade off in terms of the overshoots.

However I would from the filtering perspective, my noise signals appear as high frequency signals. And I would want to filter them out as much as possible therefore, just by nearly increasing the bandwidth or designing my system to have a very large bandwidth is sometimes no desirable. And therefore, we need to have a tradeoff between the bandwidth and the rise time or even the overshoot.

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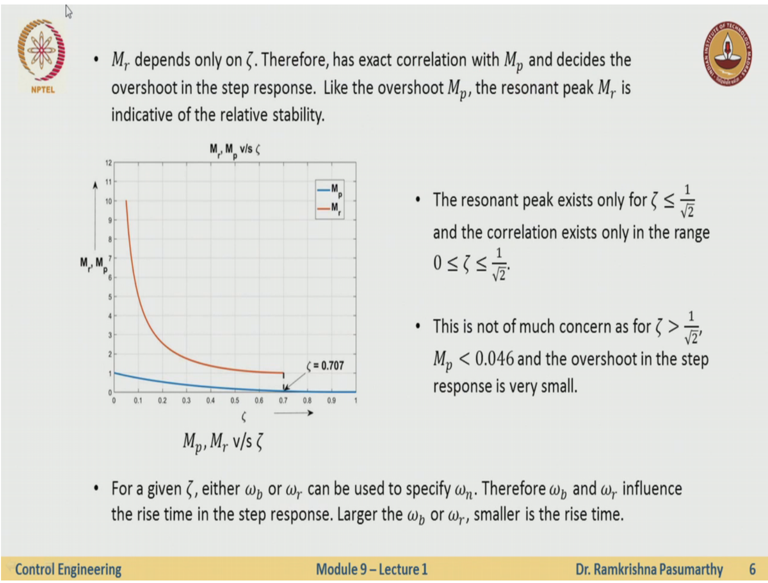

Now a little bit more on the specification. So, M r the response of the freak or the resonant peak here depends only on zeta. Therefore, if you look at even the formula for M p in a time domain it also dependent only on zeta. And therefore, we can say there is actually a very nice correlation between M r and M p and therefore, given a specification on M r I can directly the have translated into a specification on M p or in a specification on zeta.

So, in the same way as peak overshoot the resonant frequency is also an indicator of the relative stability of the system. So, the expression for the resonant peak tells me that is it exists only for these values of zeta and therefore, the correlation between M p and M r exist only for zeta between 0 and 1 over square root of 2 hence given by the plots like this here right. So, at zeta equal to 0.02 I have point 0 0.707, I have this number that M p goes to say 2 one and if sequent get from this expression.

So what happens for zeta which is larger than this value? Well for zeta larger than square root, zeta larger than 1 over square root 2 the peak overshoot is very insignificant if you just computed from the formula for the M p. And therefore, I do not really need to worry about what happens for larger values of zeta because, the overshoot is very minimal. Now for a given zeta right, either omega or the bandwidth or the resonant frequency can be used to specify omega n right which is apparent from these 3 formulas. And therefore, you can say that the bandwidth and the resonant frequency they in influence the rise time in the step response.

Again this by this 3 formulas and I am deleting it to the formulas in the time domain, given the rise time as zeta and omega n. And therefore, we can conclude that larger the bandwidth or omega r, which also I have give here, larger the bandwidth or the resonant frequency smaller is the rise time.

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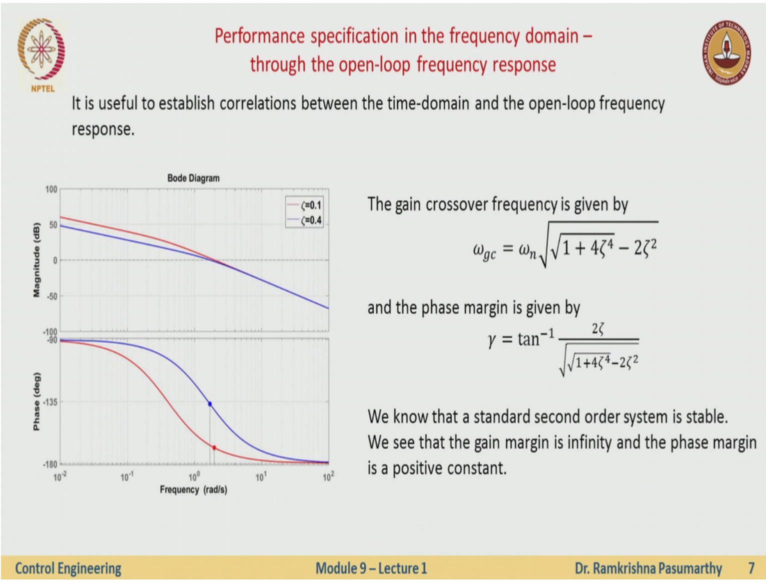

Now, further how do we translate performance specifications from the time domain to the frequency domain? Or are there any direct correlations? So, let us let us see from with the help of the plots here. So, the expression for the gain crossover frequency is depending on omega n and zeta.

And similarly, the phase margin has a direct relation with zeta given by this formula. And moreover a standard second order system this system is always stable at has a infinite gain margin and the phase margin is always a positive constant right.

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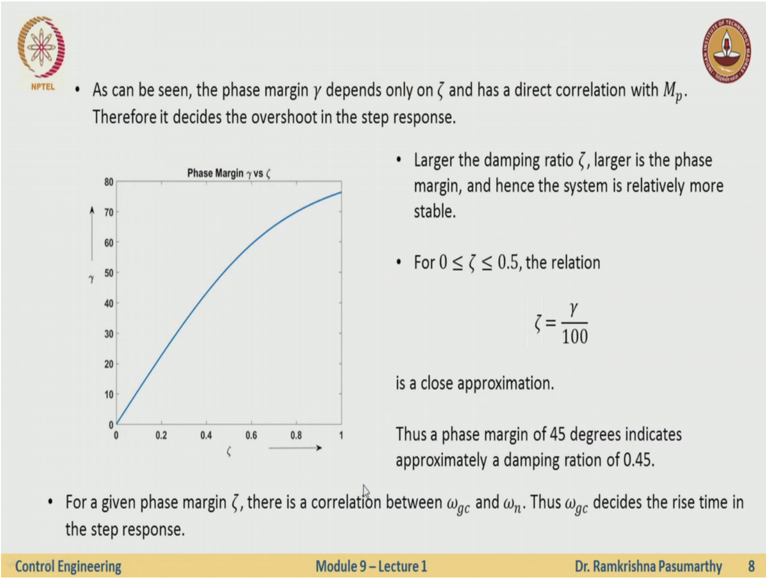

So, that is obvious law that the phase margin depends only on zeta, at this one. And defining the zeta has a direct correlation with the peak overshoot in the time domain. Therefore, specifications on the peak overshoot directly translate to specifications on the face margin of the system right. So, this expression now, again larger the damping ratio larger is the phase margin and therefore, the system is relatively more stable. Larger the damping ratio more the pole shift to the left in the root locus or in the in the time domain, and therefore, we can say the system is more stable.

Now if you look at the plot of zeta versus gamma the phase margin which is essentially a graph of this function. What we see is at for lower values of zeta between 0 and 0.5, I can get this in the relationship and zeta is gamma over 100 and this is very to the approximation if we just look at this plot. So, this actually means, that if I design a want a phase margin of 45 degrees it approximately means that I am looking at a damping ratio of 0.45.

And for a given phase margin I can always related to the gain crossover frequency and the bandwidth, again just go through the expression before. And therefore, omega gc or the gain crossover frequency at way this form, decides the rise time in the step response, will again come to come to this things when we analyze the low frequency regions the middle frequency regions and the high frequency regions. So, at the moment it is important for us to observe that if zeta is given right here.

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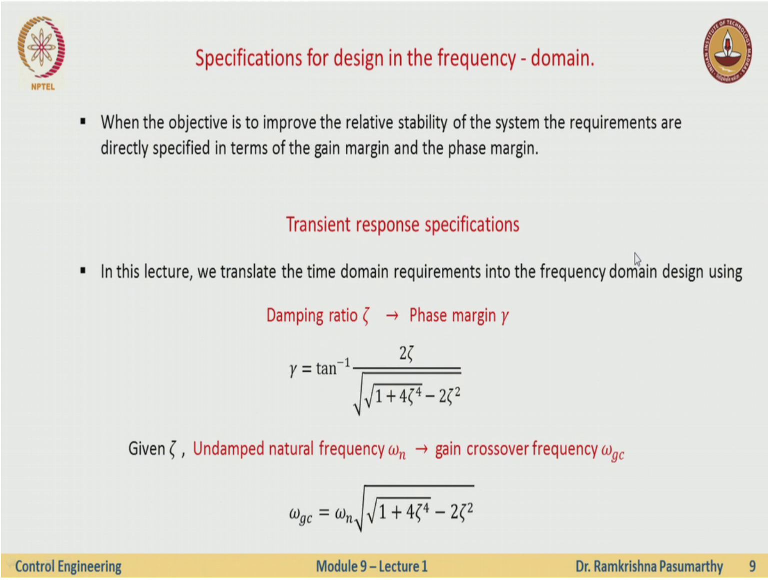

Then omega gc decides the rise time in the step response right because, this is because of this noise correlation over here. Now what are the exact specifications, right? So, when the objective is to improve the relative stability of the system, right? Then we will specify the requirements directly in terms of the gain margin and the phase margin now.

However, they related to transients well the damping ratio was related to the phase margin where this expression and the given zeta and omega n the gain crossover frequency was rated via this one.

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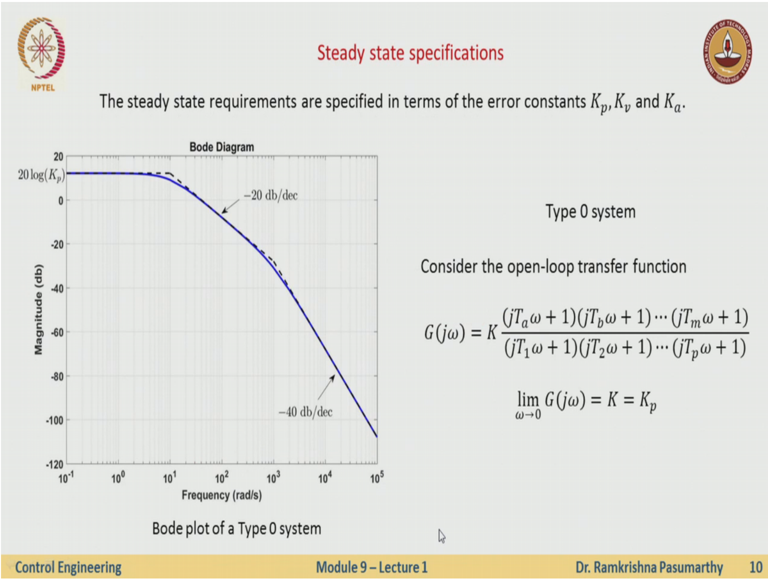

Now, we go now to the steady state specifications. And the steady state specifications are usually in terms of the error constant could be either the position error constant, the velocity error constant, or the acceleration error constant. So, we had in our in the lectures of bode plots learnt how do we get this error constants directly via the bode plot, we will just try to try to recollect those again.

So, given this bode plot what is K p? And K p will exist only for a type 0 system which means that I am actually having a transfer function of this form. So, if I take the limit of omega no limit of g j omega as omega goes to 0, I get that the intersection here is 20 log K p in decibels or K p if I just look at in terms of absolute numbers.

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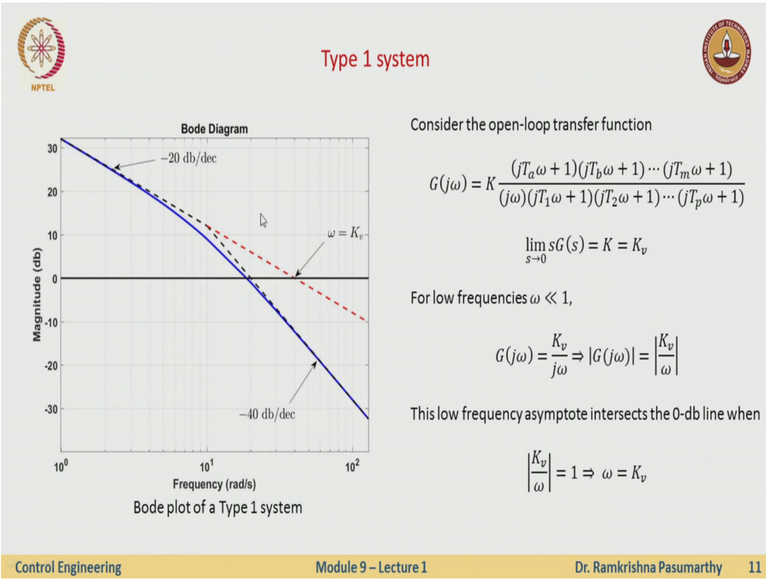

Similarly, I look at a type 1 system to compute the velocity error constant. So, type 1 have a pole at the origin and I know that this formula here limit as going to 0 s times G s will give me the value of K v. Now how do I compute the value of K v from the bode plot? Well if I look at very low frequencies which means omega is very less than 1 or much less than 1 I just ignore these terms.

So, for low frequencies this expression would hold that G of j omega can be approximated simply as K v because of this see by j omega. And the K v can be obtained by looking at the intersection of the initial slope of this line, of this guy with the 0 db line or at which point the magnitude of K v by omega which essentially this guy, the bode plot for low frequencies becomes 1. And the frequency at which this initial line intersects the 0 dB line is actually my K v, let on this before.

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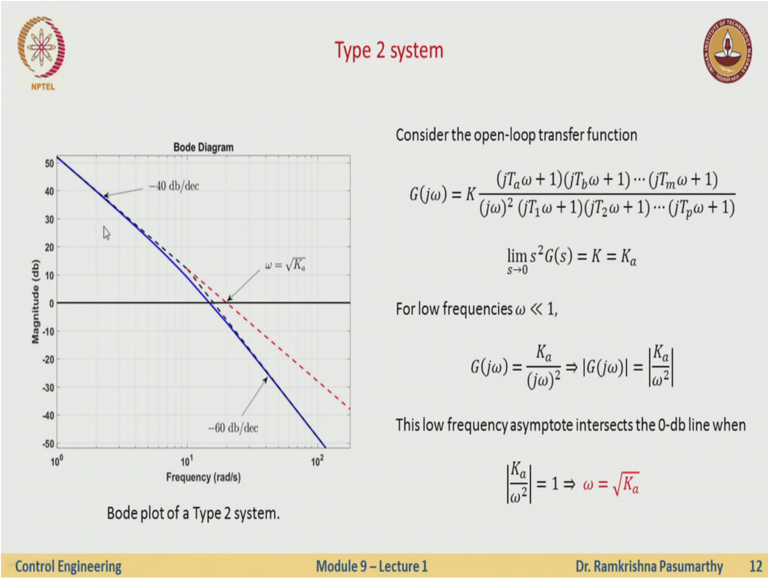

Similarly, with the acceleration error constant; I am essentially now looking at the type 2 system and I know that K a is given by this formula of limit s tends to 0 s square times G s. I look at very low frequencies omega much less than 1. In which case I can approximate my transfer function G of j omega as K a over j omega square. And if I this keep doing all those same exercise I look at the intersection with the 0 dB line or in which or in the absolute terms K a by omega square which is a magnitude of this becomes 1 well, it becomes 1 here exactly at the point where omega is square root of K a and which is from this formula.

So, what is K a? Well it is we can just recomputed by this form right, it is a square of the frequency at which my initial line intersects this 0 dB line.

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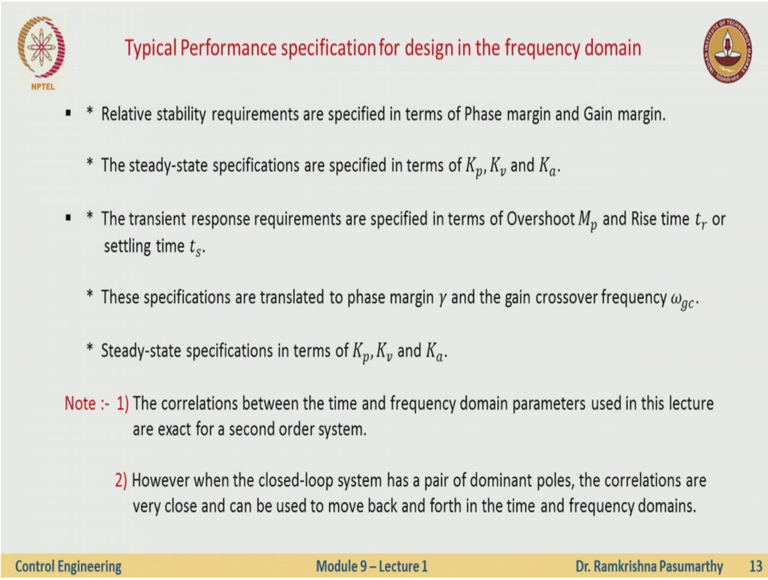

Now, yes talk more in terms of now this the performance specifications in the in the frequency domain. So, the relative stability requirements are in terms of the phase and the gain margin. And steady state are usually again in the same terms of K p K v and K a right. Similar to time domain again we recollect this in terms of overshoot the rise time of the settling time. These are now translated by a appropriate formulas; so relations to specifications on the gain margin and the gain crossover frequency.

And steady state specifications remain the same right. And what we have to be careful here is that the correlations between the time and frequency domain parameters used in this lecture or exact for a second order system right. So, we started our analysis all the M p, M r these 4 formulas derived only for second order system. And in the time domain we translated this specification in terms of the dominant pole, where we wanted the exact plots and the plots with the compensator and the dominant response to be as close to each other as possible.

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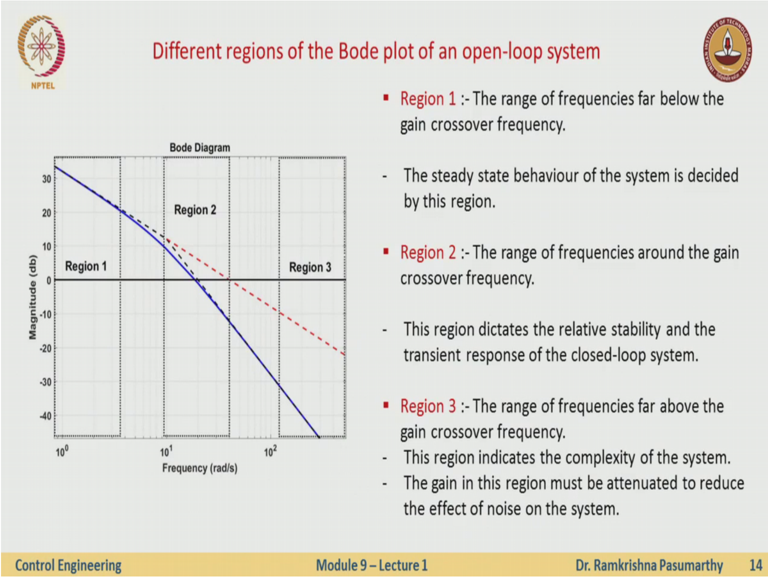

So, same thing we could do even in the frequency domain. Now depending on, So far we saw relations between you know; how are the phase margin and the gain margin related to the time domain specifications in terms of zeta omega n, the bandwidth the resonant frequency and so on. Now this one what we had argued So far, if I look at the low frequency region I can look at this plots, the low frequency region here, the low frequency regions here no omega being less than 1. Or the low frequency regions here they were deciding if I call them as region 1 they were deciding the steady state response right. So, the region 1 will decide how my system behaves at the steady state right, what is steady state? Specifications either in terms of K p or K v or K a.

Now, look at this region here in terms of the gain crossover frequency. And if we go back a few slides, the gain crossover frequency was related to the transient response of the system. So, this frequency range here which I call the region 2 will decide the transient behavior of my close loop system or also the relative stability. Region 3 which is much ahead or much after the region 2 or much after the gain crossover frequency, this indicates in some sense the complexity of the system what are the extra poles and zeros in the system.

And at this region what we must be careful is that the gain at this in this region should be attenuated to reduce the effect of noise. So, we the design specifications can be can be categories essentially into 2 regions, region 1 the blue frequency taking care of the steady state requirements, region 2 taking care of the transient response which is essentially the region slightly to the left and sited to the right of the gain crossover frequency. And only a specification or only thing which we need to be careful in region 3 is at the gain must be attenuated at in order to reduce the effect of noise on the system.

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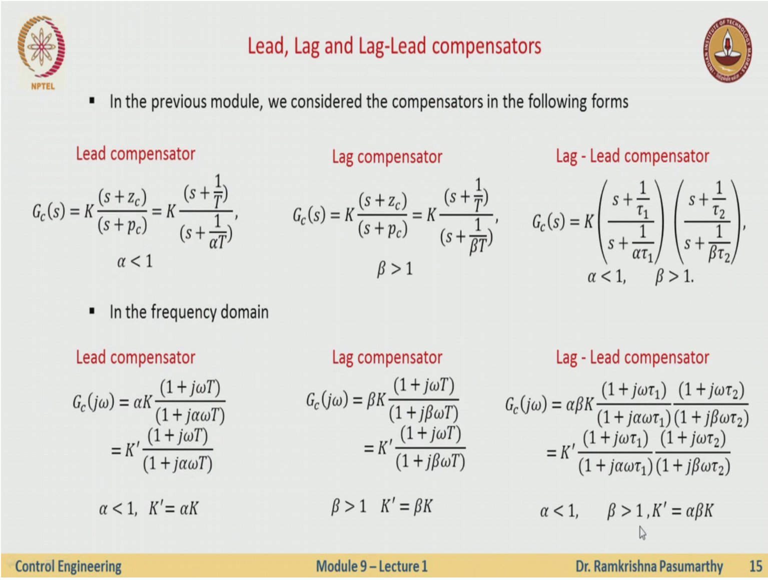

So, how do these things look by construction? So, in the time domain I know that the lead compensator had 0 and a pole play such that alpha was equal to 1 and similarly here I can write down the sinusoidal transfer function. The lead compensator goes exactly the same way. And similarly for the lead lag compensator, I am just writing down the sinusoidal transfer functions here. Here I just call the gain K prime to be just alpha times K and here to be beta times K and so on.

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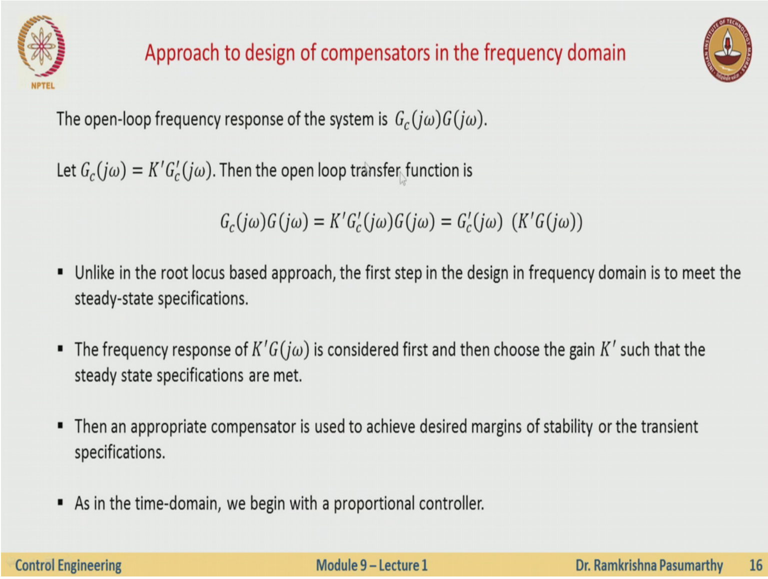

So, what how do we go about doing this? Now we have a nice correlation between the time domain specifications and the frequency domain specifications, you also have nice explanation of the low frequency region corresponding to the steady state requirements, then the mid frequency range around the gain crossover frequency corresponding to the transient specifications right. So, the first thing as usual we would do is to see in terms of the gain adjustment, just by adjusting the again do I get the appropriate gain margin of the phase margin. And by the root locus plots I actually sorry, by the bode plots actually know how to compute the gain and the phase margin.

And once I design or look for an appropriate gain K and if it does not exist, this by nearly adjusting the gain if my close loop specifications are not met then I would use an appropriate compensator could be again in terms of a lead compensator and the lag compensator.

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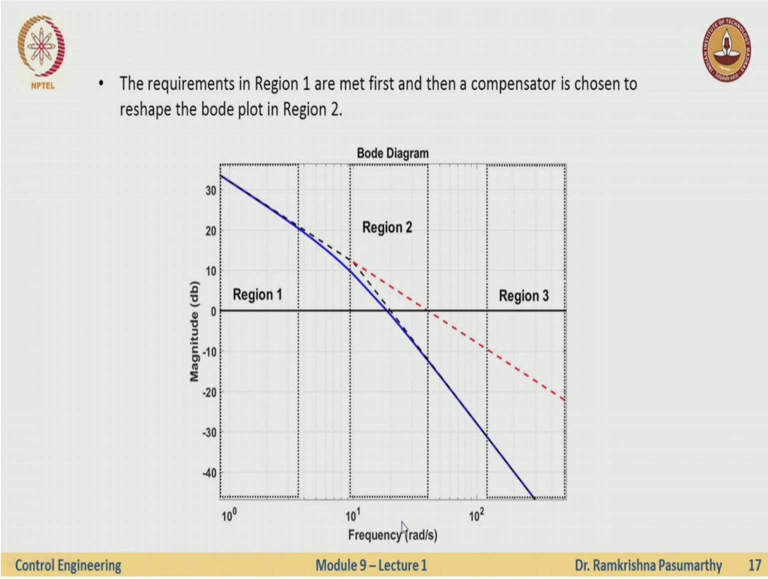

But as usual the very simple first step is to is to just look at a proportional controller or just look at adjusting the gain. And then if they are not met then we look at appropriate regions for the compensation.

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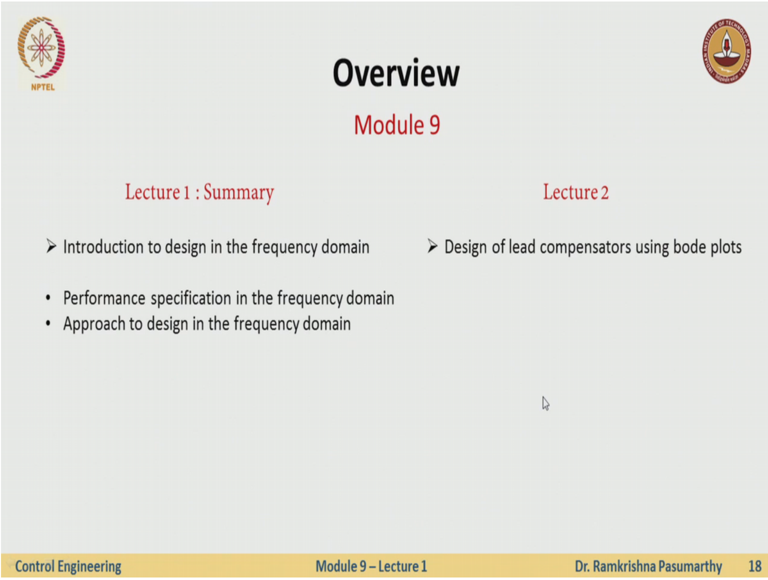

So, in this lecture would be. So, is how the time domain and the frequency domain specifications are related to each other, the various regions how they are categories in terms of frequency? And in the next lecture we will see exact construction of the lead compensator using bode plots.

Thank you.