Control Engineering
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Module - 08
Lecture - 04
Design using the Root Locus

Hello everybody. In this last lecture of module 8, we will still continue with the design of root or Design using Root Locus methods.

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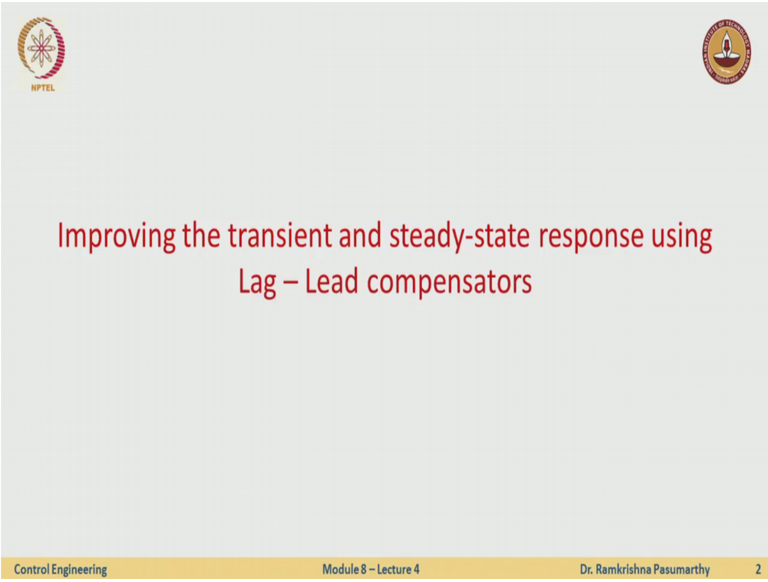

Now, by looking at improving both the transient and the steady state behavior which means we have to we have to design both the lead and the lag compensators. So, how do we go about doing it? So, far what we have learnt is that the lead compensator is suitable where the or in scenarios where the transient response is unsatisfactory.

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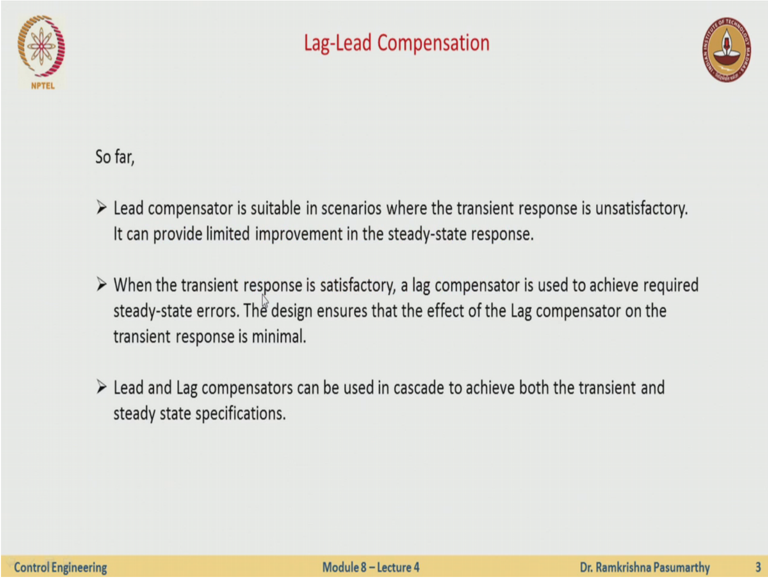

And we sp that this is essentially that you are adding 0 which we will pull the root locus to the left and then we had a compensating pole so that we could realize specifically right. So, once the transient response is satisfactory or in problems or very in specifications where we do not have to worry much about the transient response and we have to improve the steady state response, we or the or improve the steady state errors we use a lag compensator, right.

And in this and we will do this to design procedure. So, while designing the lead compensator we would like the effect on the steady state to be minimal or maybe sometimes improve. Whereas, even we design the lag compensator we would want to design it in such a way the other system transients or not affected. And therefore, we look at keeping the angle to less than 5 degrees. And we also saw the effect of adding a lag compensator you just is that it just shifts the root locus slightly to the right. In case of specifications where neither the transients specifications are met not the steady state is satisfactory we would go for designing both lead and lag compensators.

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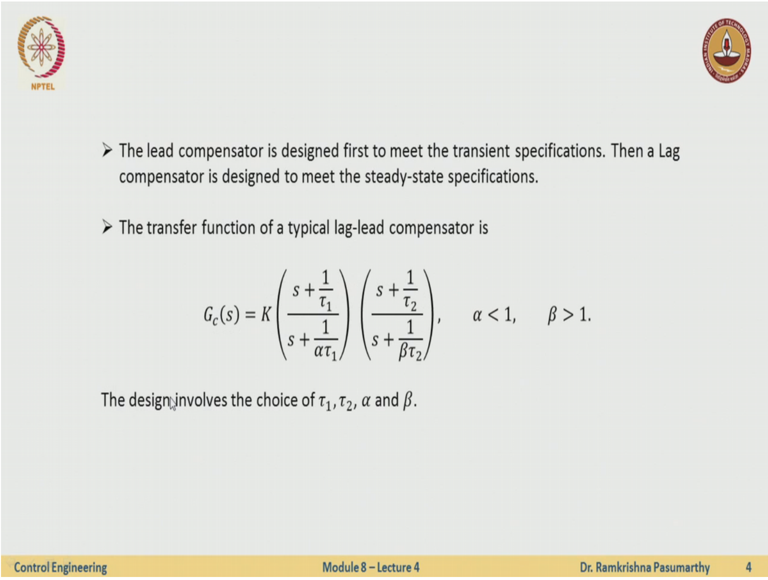

So, how would it look like? Well so, if I look at the construction we will you have general gain of the compensator, we will have lead component, lag component here in such a way that alpha is less than 1, this ensures that the pole is further to the left of the 0, beta greater than 1 ensures that the pole is to the right of the 0.

So, in a standard procedure is whenever we have to design a lead and the lag together we first design the lead compensator and then lag compensator. So, first we try to meet the transient specifications and then go to the steady state compensation. So, if I just look what do I need to know here well the announcer tau 1 tau 2 alpha beta and of course, they can be completed directly from the root locus once this specifications are made ok.

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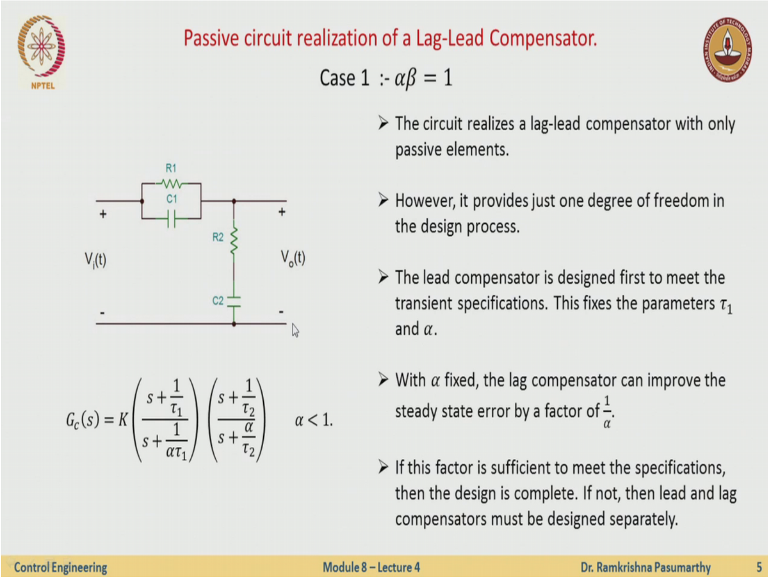

So, if I go to look at how does the realization of a lead lag compensator look. So, first case one where if you look at here. So, alpha is less than 1 beta greater than 1. So, I can always choose a combination such that alpha times beta is 1.

So, this will ensure that alpha is less than 1 and beta is greater than 1. So, this circuit here it realizes a lead a lead lag compensator just with passive elements, that is why we know all this guys here or passive elements. However, you have little less freedom right. Because, once he chose the alpha here the beta here is fixed. Or much ever you fixed beta you do not have too much control over the alpha. So, first we design a lead compensator which fixes parameters tau 1 and alpha.

So, with alpha fixed I can prove or what even So earlier, was at the lag compensator that this guy can improve the steady state error by a factor of 1 over alpha. Now we have this alpha which comes as a result of designing the lead compensator is not a, does not provide the satisfactory steady state response then I would go for the design separately.

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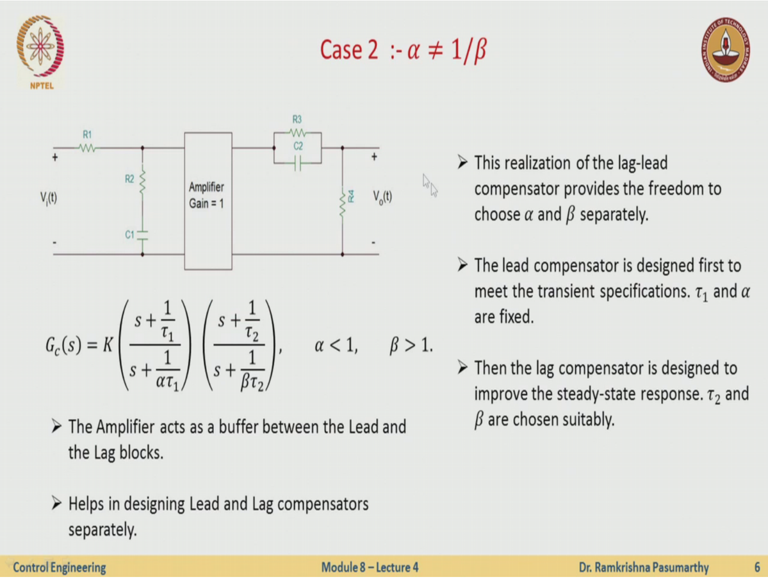

In the next case where I would want to choose my alpha and beta separately in such a way that alpha times beta is not 1 then So, first let us look at how the how the series how the how the cascade of these 2 compensators look looks like. To maintain this transfer function I would have to ensure that there is no loading at the output of system 1 right. So, if you see if I just remove this amplifier and I just plug this in here, this may not be this may not be the overall transfer function as we saw in one of our very earlier lectures even we were deriving transfer functions right. So, what I put here is a is a amplifier of gain 1 in such a way that the overall transfer function now is just the transfer function of this multiplied by transfer function of the lag compensator here.

So, this amplifier acts as a buffer between the lead and the lag and helps in designing the lead and the lag separately. So, here I can choose tau 1 and alpha independently of tau 2. And beta right and I said have to satisfy this one and alpha should always be less than 1 and beta should always be greater than 1.

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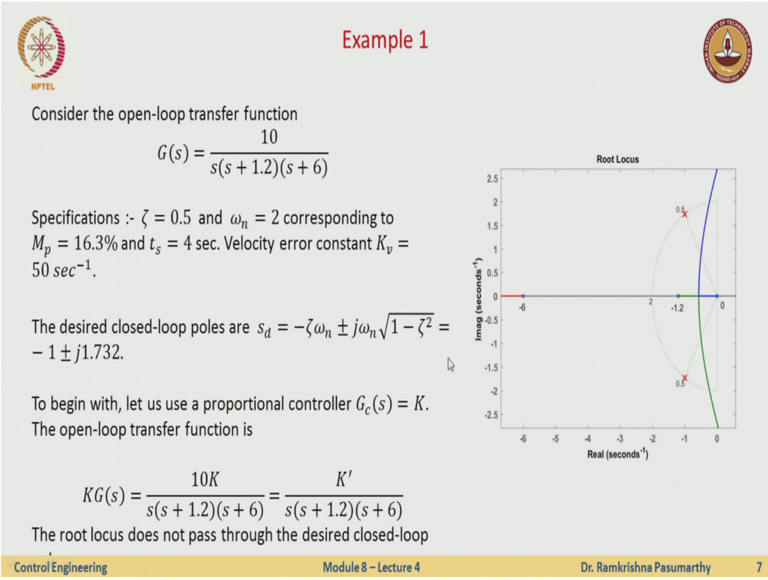

So, we will start with a problem directly right. Now, we know the theory of what are the steps involved when I where when I want to design a lead compensator and what are the steps involved when I when I want to derive a lag compensator. So, let us say I am given the plant this if like this G of s is 10 over s, s plus 0.2 s plus 6. And the desired specifications are the damping coefficient being 0.5 and omega n being 2. And this corresponds to a peak overshoot of 63, 16.3 percent.

So, I can I know the formulas to compute this and the settling time of 4 seconds. In addition I also want or a desired that the error constants should be 50. So, the based on these 2 specifications I can, I can compute what are the desired closed loop poles or where should be the dominant closed loop poles right. That is s d is minus theta omega n plus minus j omega and square root one minus theta square. Let turns out to be this 2 numbers and so somewhere here and you.

So, as a first test to see; what is an appropriate compensator I just plot the root locus of this guy. And just see if for any gain K if for, I will check if there is any gain case such that the s d lies on the root locus right. If I just move along this lines and I would see that the s d does not lie on the root locus met. And therefore, just minimum, just mere gain adjustment would not suffice.

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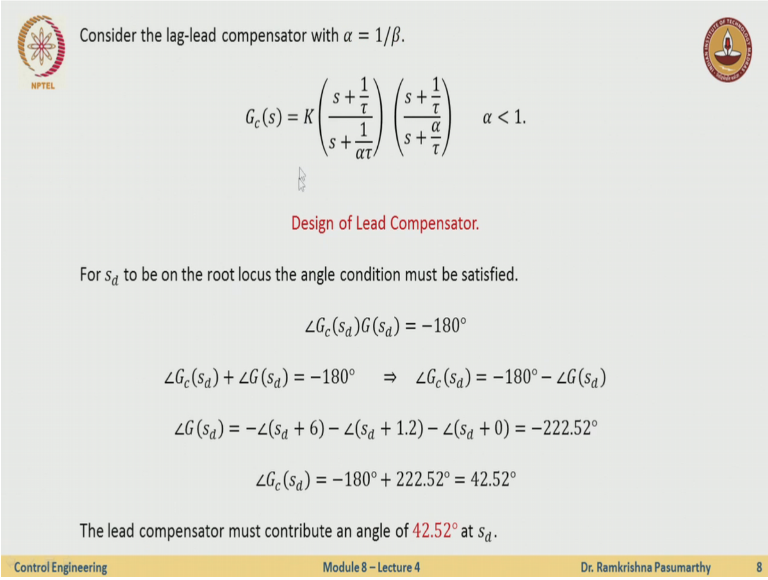

So, the root locus does not pass through the desired closed loop. When I am just considering a proportional controller or and you said adjusting the gain. So, what we conclude from the previous slide by just using a proportional controller is neither the transient specifications are made nor or the steady state specifications. So, we need to go for designing both the lead and the lag compensator. So, let us start with this simple case when alpha times beta is 1.

So, first I would do is to design a lead compensator. So, what are the steps right. So, if my root locus of the closed loop has to pass through these desired poles, then the angle contribution at that point which is which I call s d of the plant and the compensator should be minus 180. So, now, I know how to compute this G of s d right.

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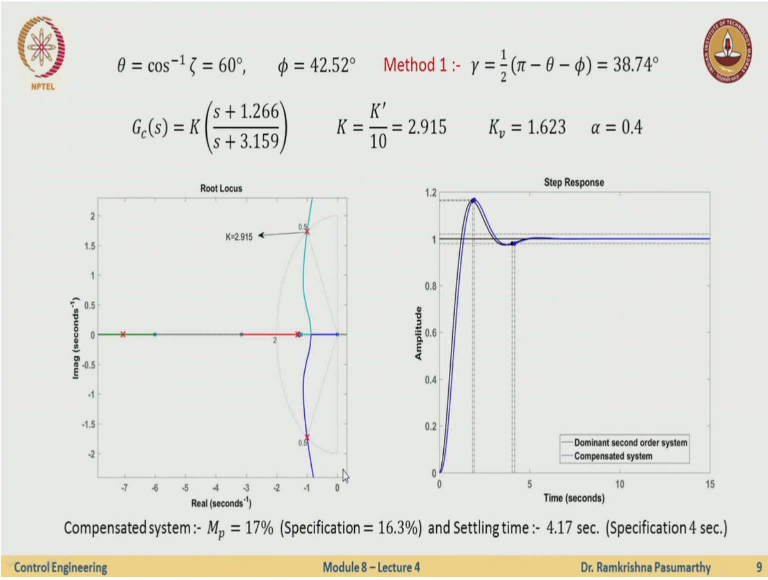

So, straight forward procedure; so what I find from all this computations is, the angle that the com compensator should contribute is 42.5, right? Very straight forward computations as we did when we were just designing a lead compensator. Now the second thing is well we are designing is, now I know that my compensator should provide an angle of 42.5.

Now we saw last time there are several of this pole 0 pairs that could achieve this one. And what we saw is the best one is the one which is computed by this one, let us if you remember from the diagram what was gamma right. So, once I know the gamma I can compute the locations of my poles and zeros exactly the same procedure right you just have to just write down the formulas on the left hand side in the certificate table on the right hand side.

So the method to choose or to find the poles and zeros was something like this right. And this gives me compensator which is essentially lead compensator with 0 at minus 1.2 and a pole at minus 3.15 and all these conditions are unsatisfied at the pole should be to the left of 0. And the gain at which this occurs on the root locus is at K is 2.915 and this gain also includes this factor here right. So, 10 K is what we call s K prime right, and therefore the gain s K prime over 10 and that is 2.915. Now what is the impact of this on the steady state error? Where if I look at substituting alpha as 1 over beta and computing the steady state error I get K v is 1.623?

So, let us first look at, what is how does the compensated system step response look like? So, the black one is one when I just look at these 2 dominant poles and these 2. And then I want the compensated system which is essentially the dominant poles plus the compensator to be as close to the desired as possible. So, this is what is desired the black one, at in terms of the dominant pole and analysis.

And this is actually fairly we are close to each other. And this is the max maximum this is the closest we could get as we saw earlier in the analysis based on the computation of this gamma. When you see that in the compensator system my M p is at 17 percent which is close by to what a desire. And 13 time is also very fairly what I, want I wanted around 4 seconds.

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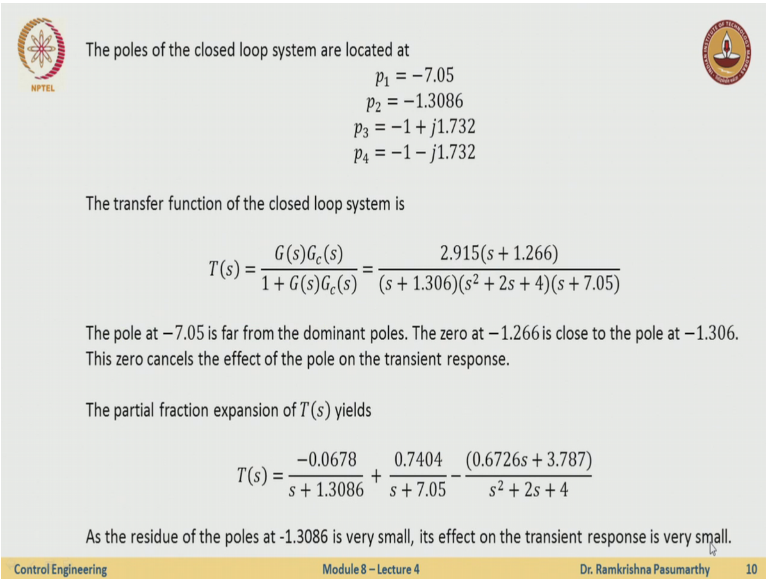

So, now just to do a little bit of analysis on the contribution of these extra poles and why we have this blue line and why do not the blue and the black line match each other. Might so have, the systems the poles of the closed loop system at minus 7 minus 1 and then minus 1 with the plus minus square root of 3. And then the transfer function of the closed loop system looks like this. So, look at the dominance condition. So, this is pole is very far from the dominant poles right.

And then there is a 0 closed by 1.26 and this is closer to the pole at 1.3. So, if you remember last time that we actually would it was desired that there should be a cancellation of us as far as possible between the 0 of the closed loop system and one of the poles here right. And see these are placed fairly close to each other and therefore, this minimizes the effect or the negative effect on the dominance of the remaining 2 poles right ok.

So, what we see in this when we do the partial fraction expansion is well there are these a pole at minus 1.3 there is a pole at minus 7 this we now is much further away from the desired or from the dominant pole. So, this effect is minimal whereas, the effect of this pole is getting multiplied by a very small number on the numerator and this is essentially takes care tells me that this pole 0 combination is almost like facing a facing a cancellation and then you just left with this part of the transfer function which essentially deals with the dominant poles ok.

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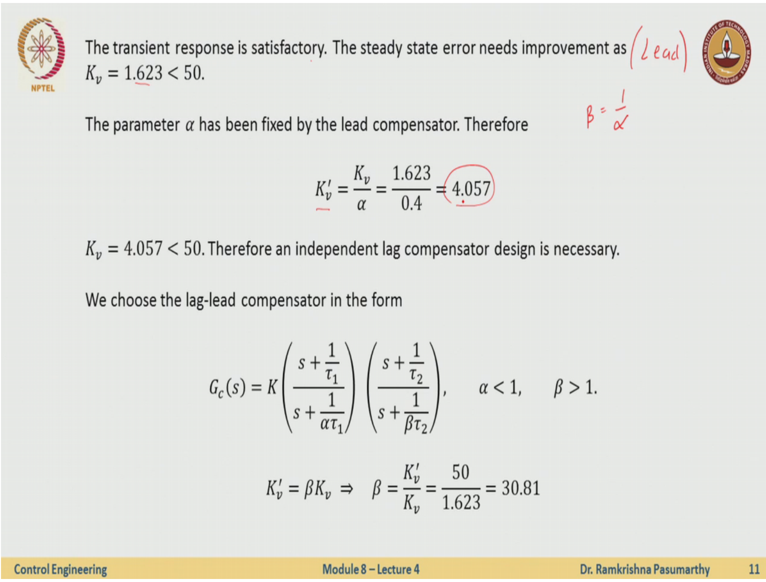

So, the design of the lead compensator ensures that the transient response is satisfactory. Now let us see; what is the effect on this steady state only by this lead compensator. Say if I compute the velocity error constant it turns out to be that is just 1.623 of this is must less than 50, this is just with the lead compensator.

Now, let me design a lag compensator with the beta being fixed by alpha as 1 over alpha. In that case if I compute what is my steady state error constant the new one with the lag compensator it just turns out to be this guy, 4.057. And this actually comes in the design procedure somewhere over here right. With alpha fixed the lag compensator can improve the steady state error by a factor 1 over alpha. And that is exactly what is happening here right 1 over alpha. Now this is still not good enough for me because, what I want is a is K v to go to 50.

Therefore, we need to design a lag compensator independently of alpha right. Or in other words we choose the lead like compensator which looks like this. So, we have the lead now separately and the lag separately in such a way that alpha and beta are chosen independently, but we still satisfying alpha being less than 1 and beta be greater than 1. So, what is this beta which I want to want is desired? This is the desired the beta is the ratio of the desired K v which is 50 and the original K v which I have.

So, what is the K v which I have now is whatever is left after I designed the lead compensator. So, I can now get the beta to be the ratio of 50 over 1.6 and then tends out to be around 30.8.

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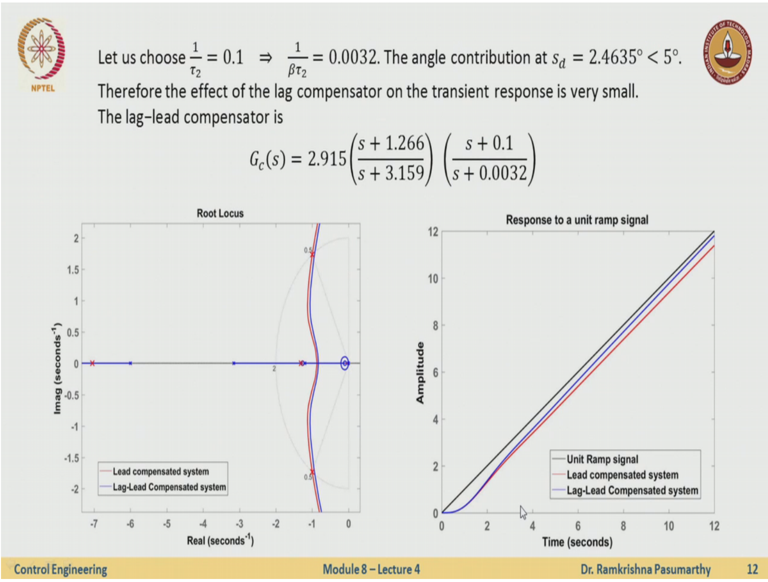

So, with this a design well now I have to fix where are the poles and zeros of the lag compensator. So, let us simply choose 1 over tau 2 be 0.1 and this gives me the location 1 over beta tau of 2. And what we also have to make sure is that the angle contribution of this lag compensator is less than 5 degrees. And that is ensured by this one that that the angle contribution at s d which is the dominant poles, pole locations is 2.4 which is very satisfactory. Therefore, the effect of lag compensator on the transients is very small. So, my overall lead lag compensator looks like this. So, this is the gain, this is the lead component and this is the lag component. And let just compare it with the help of some plots.

So, this is what I really want to track right the unit ramp. So, just by designing a lead compensator I see that you know the red line that there is a significant amount of steady state error. And by design the lead lag compensation and see when there the steady state error actually it is uses significantly right. This is it is obtained by the looking at the blue line.

Similarly, what is the effect of designing the lead lag on the root locus? So, first when I do my time domain specifications well the lead compensation the root locus looks something like this and you know the see that the root locus actually passes through the desired poles. Now if I do a lead lag compensation I see that well it is more or less preserved by just which is not very unexpected that the root locus shifts slightly to the right.

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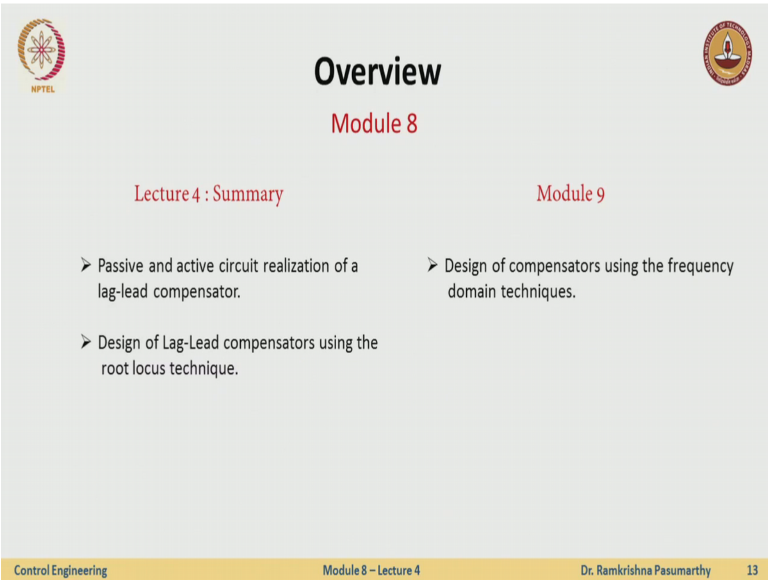

So, in this in this lecture we are seen both are passive and then active circuit realization of a lead lag compensator, where this active element came in terms of this amplifier with a gain 1 over here right. And we had seen also how to design a lead lag compensation base of the root locus method. So, we end this module here and so we just did one problem in each case, but more or less the problems will follow the same procedures.

There are some special cases which we need to be careful off, but we will post some extra nodes based on that and we could possibly discuss them on at a later stage if you have any more difficulties. It is a part of the assignment in this module we will give you a problem or at least a set of problems where you need to start from modeling of the plant you need to use matlab several times to check, you actually would be made to solve for design problem with the help of matlab. And we will fall we will give you as many instructions as you require.

And I hope it actually goes well when we when we, we learn a little more when we actually solve a problem by ourselves. So, that will be about the assignment of this module. And the next module we will look at the frequency domain compensations, where we will see; what are the specifications transient and steady state in terms of frequency domain parameters, like the gain margin or the phase margin bandwidth and several others. And similarly as we made use of the root locus in the time domain analysis we will make use of bode plot in while designing compensators in the frequency domain. That is coming up in the next lecture.

Thank you.