Control Engineering

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Module - 08
Lecture - 03
Design using the Root Locus

So, continuing with our design methods using root locus, what we will see in this particular lecture is design criterion to improve the steady state performance.

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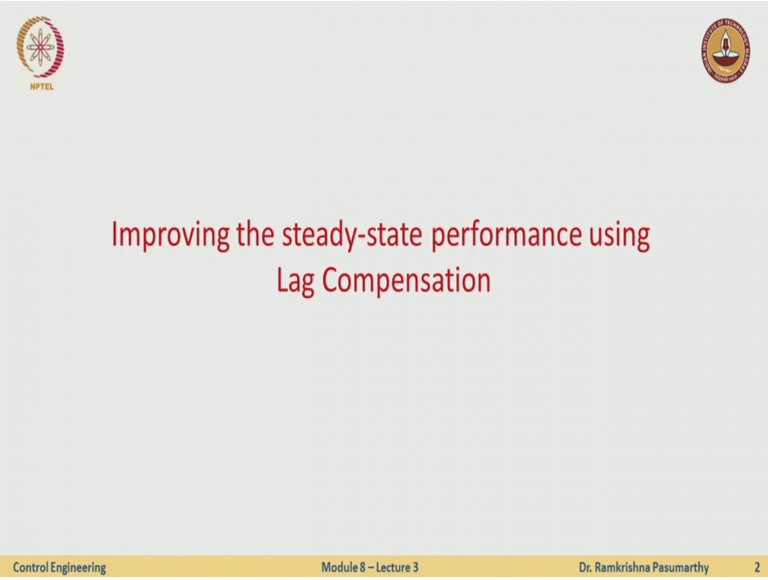

And steady state performance essentially means reducing the steady state errors either getting into 0 or having them below some predefined values. This is done with something which we call a lag compensation. Lag compensation is an approximation of an integral control. We saw how adding an integrator increases the type of the system increasing the type of the systems, help me helps me track signals in a better way right. So, we had this in relation between, what is the relation between a type 2 system and the acceleration error constant the velocity are constants and so on, right.

So, this lecture we will focus on the lag compensation which is an approximation of a p I controller. So, let us start with an example.

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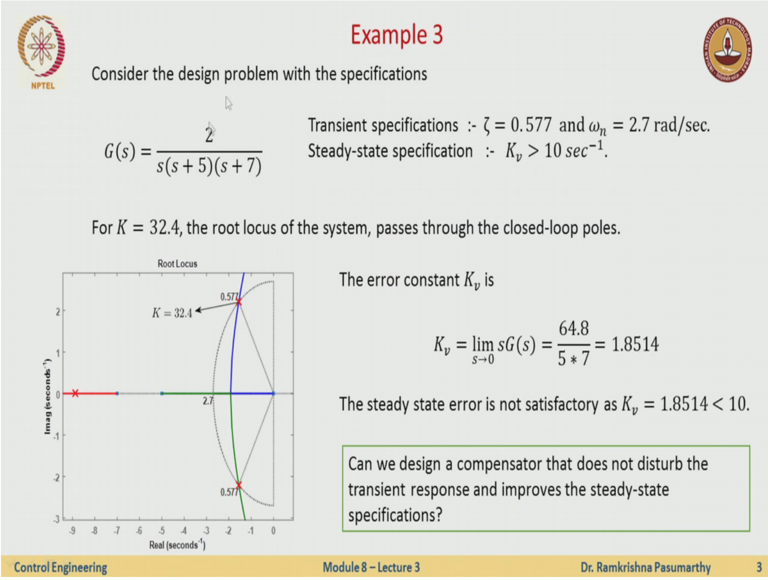

So, given plant sudden transient specifications which translate to zeta being 0.5 omega n being 2.7 these are may transient specifications and the steady state specification says that my K v should be greater than 10 right. So, first I draw the root locus and I see that at a certain value of gain K which is 32.4. My root locus passes through the dominant poles and root locus passes through the poles which are defined by this zeta and omega n. So, I do not, I do not really to worry about the transient specifications because my root locus actually passes through those points by just mere adjustment of the gain.

Let us now worry about the steady state error. The error constant K v here is 1.8. And I want to through make this larger than ten. So, the question here is, can we design a compensator in such a way that it has K v greater than 10, but not really alter this? I do not want this thing to be mixed up right? I want that to design a controller such a way that this steady state specification is made by not disturbing too much of what is happening here in terms of the transient specifications.

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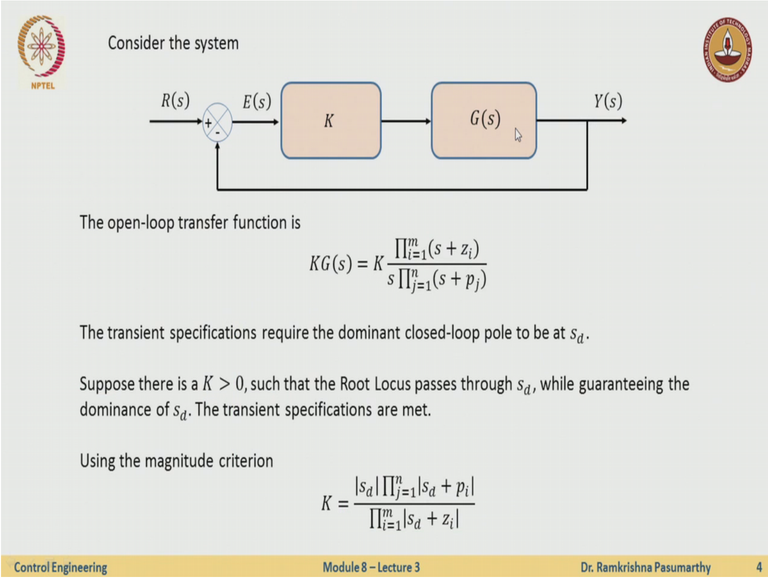

Let us see how we can do that right. So, consider again standard close loop system right. So, I have the steady the error again K, a plant G, the open loop transfer function is given by this no where G has a set of poles sorry, a sets G has set of zeroes; a polar G origin and set of other poles.

So, the transient specifications required the dominant closed loop poles to be at some s d, right. Which are completed by these 2 numbers? The first thing is assuming there is a K greater than 0 such that the root locus passes through s d. We compute the again K in the following way. Again this is just the magnitude criterion right G times H was equal to 1 or in this case K times G s would have it is magnitude to be equal to 1.

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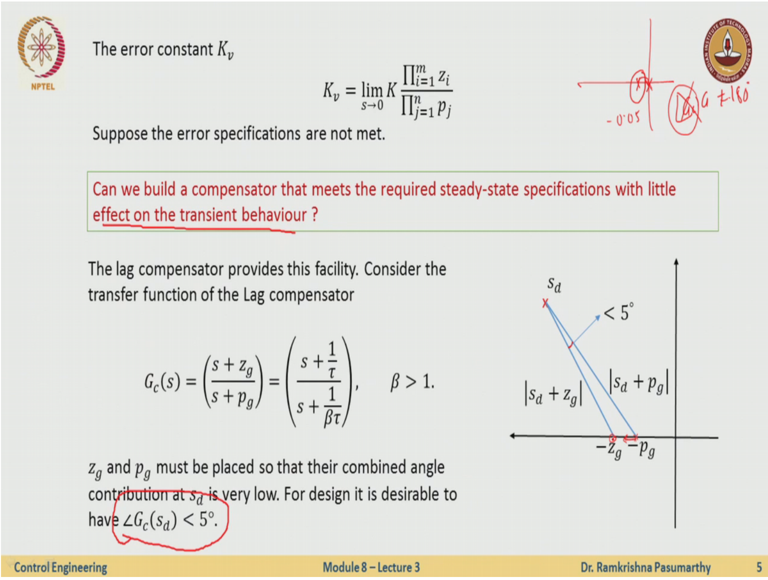

Then we look at the steady state error right, K v is limit s tends to 0 of this entire number and I am, I am deliberately looking at a type 1 system right. Because a type 0 system cannot track ramp input whereas, for type 2 system the steady state error is 0, K v is 0.

So, if K v is some finite number I am, I am essentially looking at a type 1 system. Again we can refer to the stable. So, how would this look like? Well, the lag compensator provides these facilities. So, the first thing I would, if I if I look at the ideal you know integral compensator I would say that I should place a pole at the origin. Which means well, which we should eliminate the steady state error completely. But what I told you is that an ideal integrator cannot be realized. So, what you will say well just put it slightly to the left to this can be realized. And that would be the first thing that in the lead compensator we just said just add to 0, because it poles my root locus to the left. And we see that it has it is own set of troubles.

Now, if I just add pole slightly to the left say inside of 0, let us say I had at 0.05. Now does this meet the steady state conditions possibly, but what this we will do is it we will contribute certain angle right. So, this G or G c we will contribute certain angle to the plant and then this condition would be violated. That is easy to check. So, we want to add this pole here, in such a way that it does not contribute to the angle. Or the angle contribution is as the numerous possible and therefore, I start with this pole and I add a 0 slightly to the left. I can say, why not to the right? Well, I do not really what, So I want again the dominos condition to be insures. So, that the pole moves to the left and by construction also it is kind of obvious.

So, I the lag compensator by construction looks like this, in such a way that the beta now which is analogous to the alpha earlier. This beta is now greater than 1 and z and p must be placed So close to each other, that the angle contribution is less than 5. I cannot make it 0 because, if I make it 0 the pole would sit on the 0 they would cancel out each other and there would be no compensation right. So, it is desirable to have G, the angle of G c to be as small as possible to have the least effect on the transient behavior. So, that is what we have to be careful in this, in this design procedure. So, here in this case this is the compensating pole the 0 should be to the left of the pole right. Contriver to what was happening earlier at the 0 was to the right of the pole.

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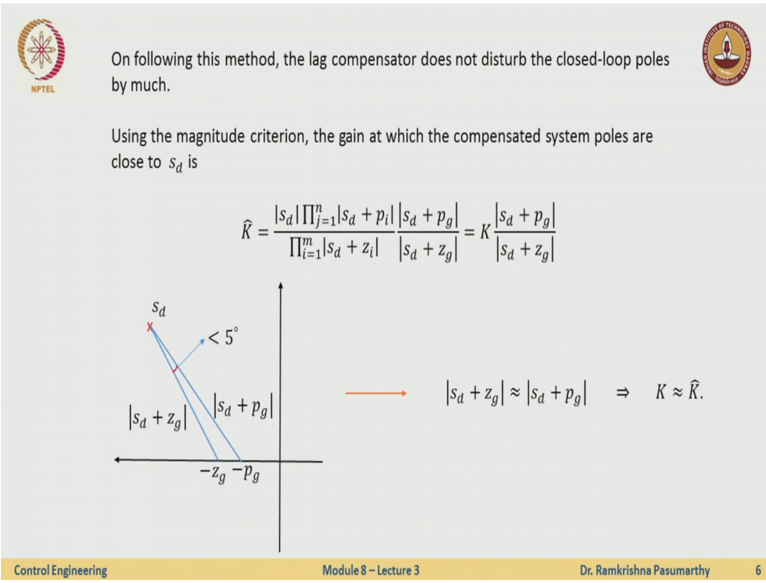

First let us see what is the effect of the closed loop gain K while be at the lag compensator. Earlier we had computed the gain to be something like this right K this in such a way that s d was passing through the root locus. Now how much does again change when I add a lag compensator? Say K cap which is the total gain when I add a lag compensator is the original gain K plus this extra factor. Now the reason why we place these 2 close to each other or also that this the K cap should be as close to K as possible, and this magnitudes if you see, their kind of almost equal, because these are very close to each other. So, K cap for K cap should to be equal to K this z s d plus z g the magnitude of this should be the same as this one.

So, this is also what is decided, in such a way that we really do not disturb the transient requirements.

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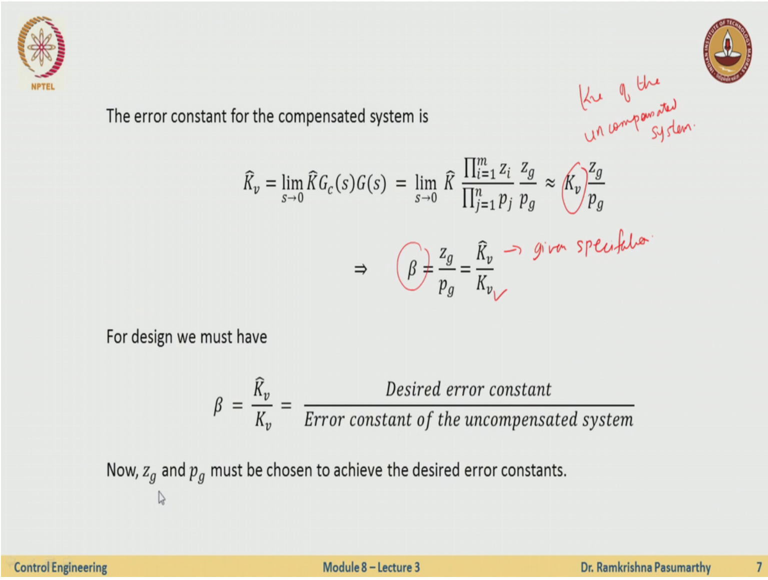

Now, what is the error constant for the compensated system? The velocity error constant is K tends to 0 K cap G c times G. And in this terms out to be this is K v z g by p g and this K v is the K v of the uncompensated system. That is right, K v of the uncompensated system. Now when I were to design this compensator, I am now worried about what is the location of this poles and zeros. And the unknown one of the factors that was deciding was this beta, beta is an unknown and tau is the other one known. K v of the compensated system is related to the K v of the uncompensated compensated system why are these 2 numbers inside G and p g. And base on this construction of the compensator z g by p g is beta ok.

Now, I know how to find beta right. This is known to me right, this is from the given specification. So, I can find out what is beta. So, for the appropriate design we must have beta as the ratio of the desired error constant, to the error constant of the compensated system which helps me in finding the location of z g and p g appropriately. So, how were these guys place?

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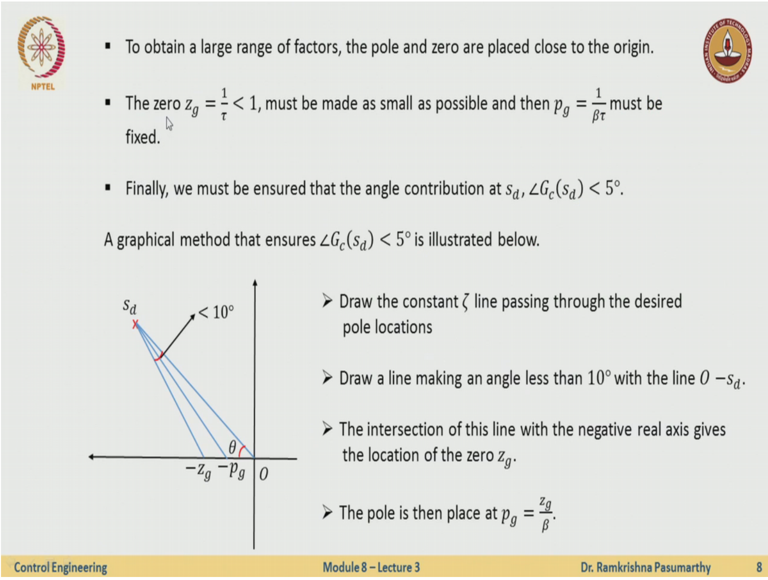

So, first thing is to obtain a large range of factors I would have them as close to the originals possible say at 0.1 or point not 1 and so on. So, the factors can be increased by just you know. So, so if add I am at 0.1 or point 0.01 and I want to have a factor of 10 I can easily go to 0.1 right and so on. So, the 0 must be made as small as possible and p g must send be fix wise, first select the 0.

Once I said the 0 since I know beta I can fix what is my p g right. So, what must also be insured is at the angle contribution is as minimal as possible right, less than 5 degrees. Unlike the lead compensator, there is no constructive way here neither a graphical way of choosing z c and p c here right there; we had a beautiful way of finding the gamma which minimize which maximizes the alpha and we could explicitly compute the formulas. So, here what we will do is we will just look at graphical method with without much proves. So, what is happening? But what this method we will ensure is that the angle contribution is less than 5 degrees.

So, what do I do is I take this constants zeta line or the line joining the origin to s d. From this line I draw another line which is at an angle less than 10 degrees should be 7 or 8 ready you can do a little modification once we have the first iteration of design. So, once I have this say let I fix this angle at 9 degrees, I just draw another line and this will give me the row the location of the 0. Once I know the location of 0 I can easily find; what is the location of the pole right. Now with this with this formula because, I know beta and beta is derived by comparing the decide error constant to the, to the, to the error constant of the uncompensated system.

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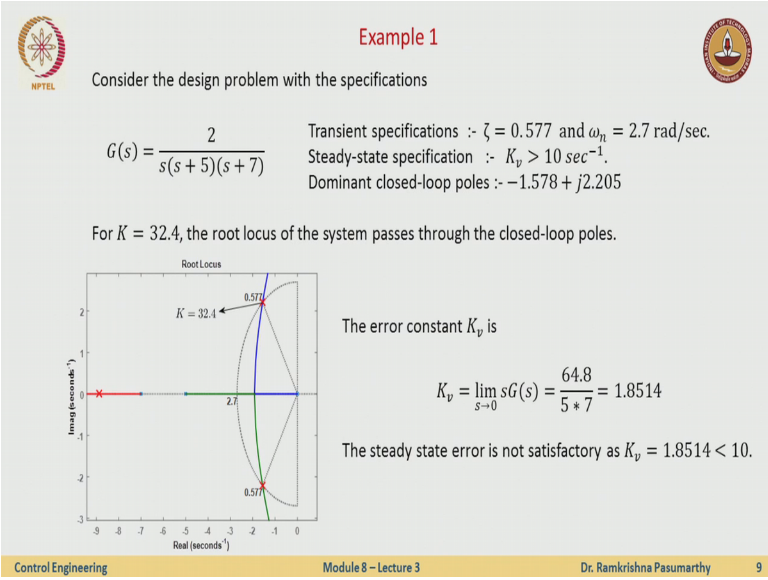

So, again back to the example as so we knew that just by this thing again 2, 32.4 the transient specifications are met. Now I am interested in this steady state specification right. The K v of the open loop or the uncompensated system is 1.8 and this is not satisfied is quite obvious.

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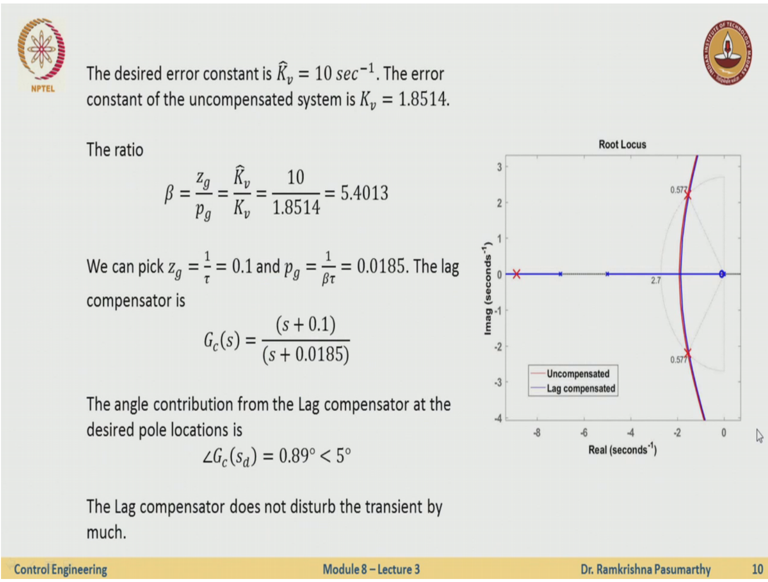

Now, this is the desired error constant. So, beta is K v hat by K v is 5.4 and beta is always greater than 1 right. So, now, just by doing the method as suggested earlier if I put my 0 at 0.1 the pole location turns out to be at 0.108 and the lag form of the lag compensator is something like this. I will not really have again K because I do not really want to it to do. So, the gain contribution of this entire thing should be as close to one as possible right. And what is the angle contribution where is just 0.89 degrees is less than 5.

So, the lag compensator does not really alter the transient performance too much. As we see here right. So, the uncompensated system the red one which was, which was the root locus of the un-compensated system we just the gain adjustment, is the red line and we see the blue line. Is just slightly to the to the right now this is this is not very surprising because I am adding a 0 adding a pole at the origin and this has the effect or pole very close to the origin this has the effect of poling the root locus slightly to the right.

But we say we would be so and we are improving the steady state performance by having just very little effect on the transients. And we could you know maybe choose alter the slightly now 2.09 and 6 said the effect is minimal. It will have it will never be a condition that the red and the blue lines match completely.

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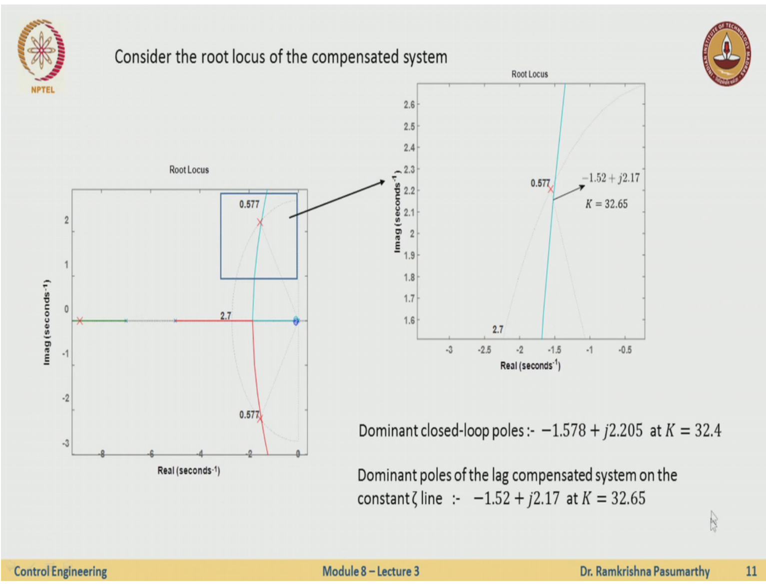

So, what we would see from this plot is that the original gain of 32.4 would not where because it does not really lie on the root locus. So, root locus has more slightly to the left.

So, what is a new gain K that we would choose? One method is to choose is to fix of the constants zeta line right and then see the intersection of the constant zeta line with the new root locus which is slightly shifted to the right and then compute the new gain right and that would turn out to be it is 32.65. So, that that is what that is what we would do here right and if, so a constant zeta line would ensure that the peak overshoot does not change at all. But if you were, are more interested in the settling time then I would look at the intersection with the constant omega n line and then choose again appropriately.

It depends on what kind of specifications I am looking at. So, this slight gain adjustment it is done by the root locus here by looking either at the constant zeta lines when my transient when my peak overshoot is more important or the constant omega n line when my steady state or the settling time is more important.

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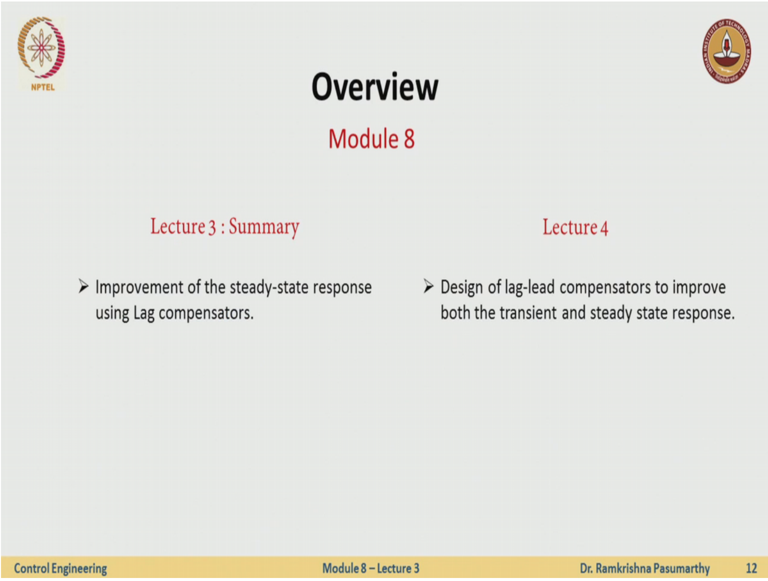

So, this is just like the basic design procedure for a lag compensator. So, we have seen how lag compensators helps in improve the steady state response in terms of the steady state error, the next slide.

So we will see given a certain system when either the transient specifications are met, not the steady state performance requirements are met. In that case how we will compensate the system, where we would need combination of both the lead and the lag compensator. And we will see how to go about the design procedure in that case.

Thank you.