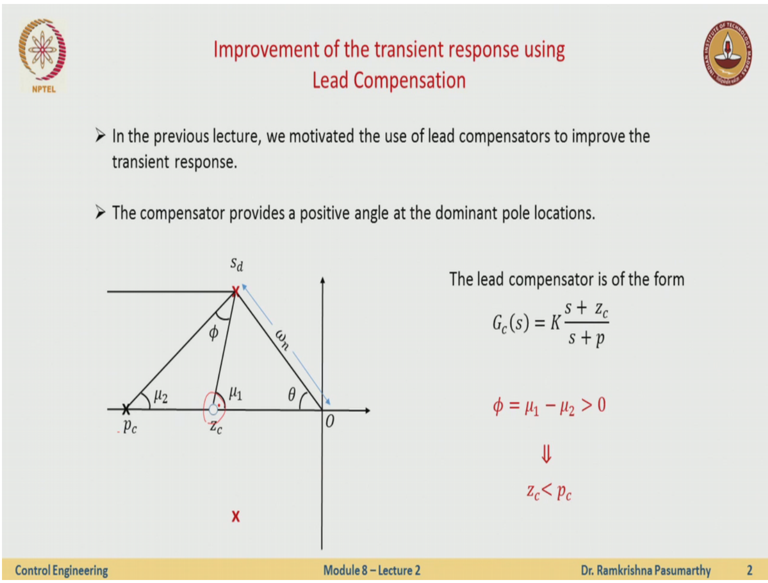
Control Engineering
Dr. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 08
Design using the Root Locus
Lecture - 02
Improvement of the transient response using Lead Compensation

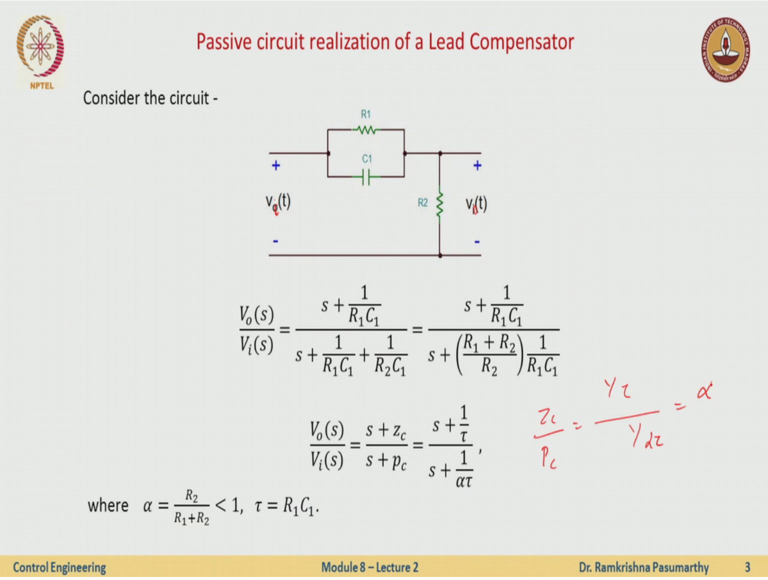
Hello everybody. So, in this module, we will continue with the design of the lead compensation and see what are the techniques or how do we place the poles and the 0s earlier we had a little formula to place the 0, but we saw that 0 by itself is not realizable. So, we need to add a compensating pole.

(Refer Slide Time: 00:41)



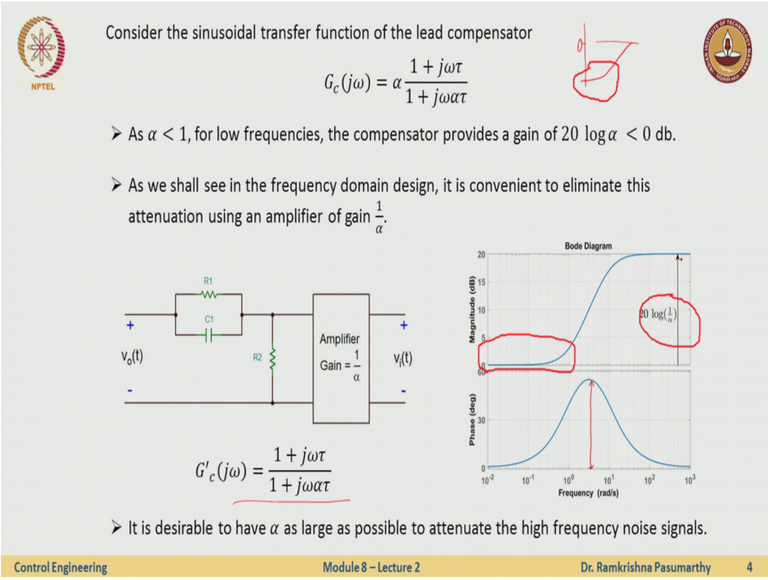
So, let us starts slowly with this analysis. So, what we concluded from the previous lecture is that well this is the compensating 0 and I should have a pole for practical reasons to realize. And we also knew why the pole should be to the left of 0. So, what we will now try to answer is where should the 0 b and how far left should the pole b from the 0 and that will actually lead to some nice exiting observations.

(Refer Slide Time: 01:17)



So, before we proceed we will see how we can realize or construct the lead compensator with help of some basic circuit elements. We turns out that if I take distance R 1, R 2, C 1, I apply input voltage and measure the output voltage across the resistance R 2 which are connected in this way. So, V o - the output voltage by V i which is has the construction of a 0 and a pole with these numbers, and this numbers can be computed directly. So, where alpha here is R 2 over R 1 plus R 2 and clearly say this is lesson one and tau is R 1 C 1. And this alpha being less than 1 ensures that the pole is to the left of 0. So, this is how we will construct a lead compensator. And this guy you know this guy we will have some angle contribution some gain contribution and so on, if some dynamic element here it terms of C 1.

(Refer Slide Time: 02:22)

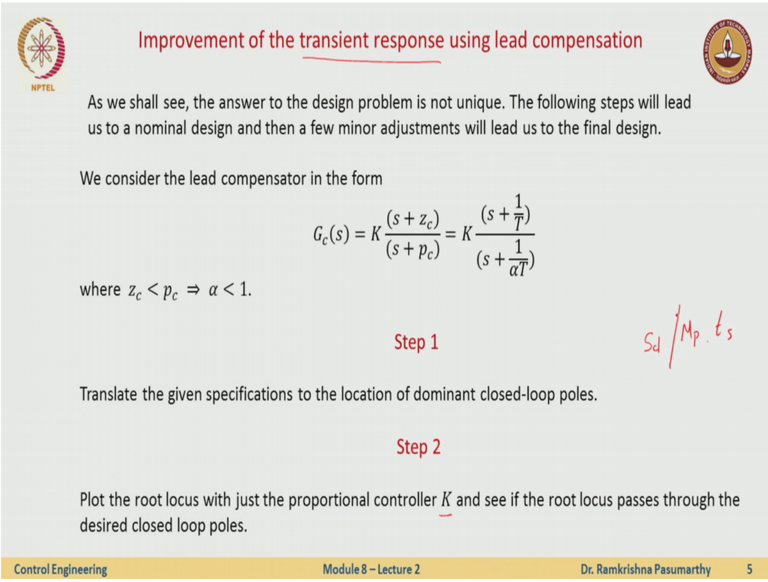


So, now how does this look like? So, if I look at the sinusoidal transfer function of the lead compensator, it looks something like this. I have alpha here I have 1 over j 1 plus j omega tau and 1 plus j omega alpha tau, this is rewriting this in the in the sinusoidal form. So, this is guy goes here this guy goes here and so on. So, when alpha is less than one for low frequencies the compensator provides the gain, which is less than 0 db. So, if I look at the bode plot here and if I were just to draw with the regular alpha my plot would sort somewhere from not from 0 this is the 0 db line my plot would sort somewhere from here depending on the values. This is not really desirable because the control system is essentially low pass filter and I really do not want the low frequency signals to get attenuated. So, this low frequency it seems I should preserve.

And therefore, what I do is I just add a gain of one over alpha and call V i t as the output of the controller where my compensator now becomes this one and the alpha sink is gone now. So, when I do this my bode plot now starts at the 0 db line or on absolute gain of one and it increases this way. I should also be careful that I do not, so I wish I would not want to attenuate the low frequency signals and at the same time also would not want to amplify the higher frequency signals. So, I tried of would be to said this alpha smartly enough such that this is preserved; whereas, this length here is not big enough. So, I have to maximize this alpha in such a way that the high frequency signal attenuation is as minimum as possible.

And just look at the phase protect this will we were always talking of a phase lead. So, this is the phase lead at that it provides, so this guy it is adding an angle this thing here. We will do little bit of more of where to this angle and all this when we do the frequency de design compensation, but at the movement it is we will just concentrate on this thing here this thing here and what to do with this alpha. So, it is desirable to have as large as possible so that this length is as small as possible which means the higher frequency signals are not amplified beyond the reasonable limits.

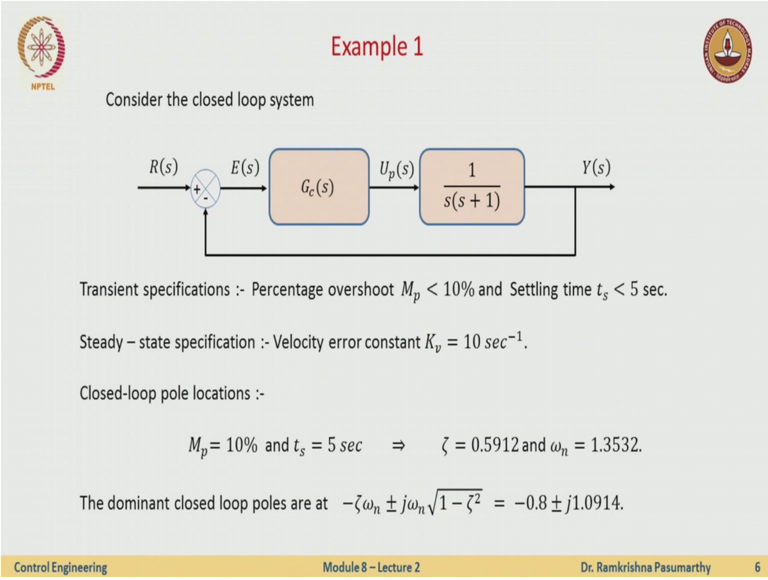
(Refer Slide Time: 04:57)



Now, we come back to design. So, warning before we do this design procedure is that if I follow the procedure it is not guaranteed that I will get the exact solution. I will be very close to the exact solution after which I may have to do a little bit of trial and error, just to may be just save the pole slightly to the right or the 0 to slightly left and so on. So, this is how the construction looks like. So, given a certain performance specification in terms of the peak overshoot and the settling time; so again just to just to remind about ourselves. So, these are all transient response specifications right I am improving only the transient response. What will happen to the steady state is something which we will see parallel as we go through these steps.

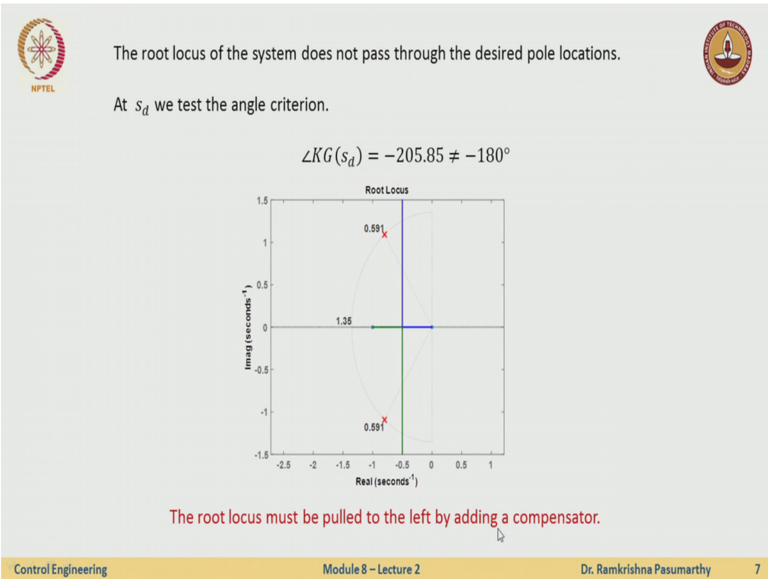
Step one is again find what are the dominant poles given M p and t s. Now, step two is and designing a controller is plot the root locus and see if there is a gain k which passes through this s d. If it passes through the s d is and then it is perfect too much, but usually it does not as we saw in the previous example.

(Refer Slide Time: 06:18)



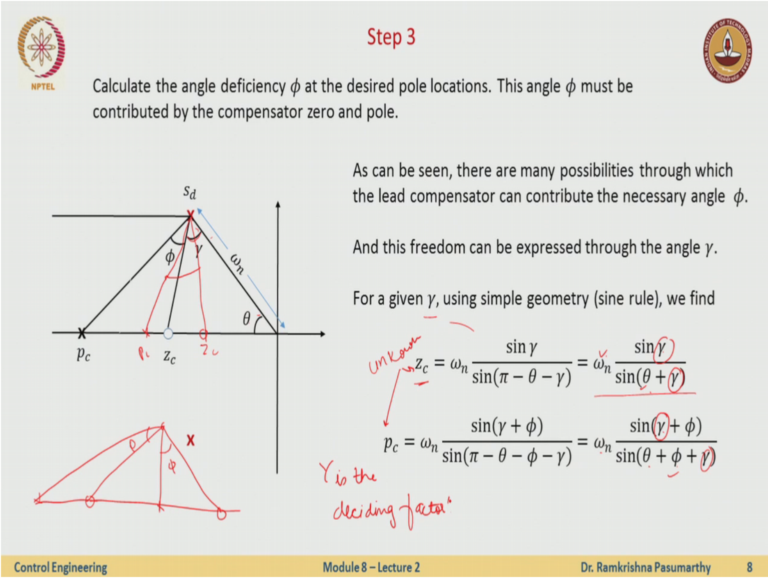
So, we will again start with the same example and see how we go about constructing a compensator which looks like this. So, this is what we had in the example in the last lecture where which where M p being 10 percent and settling time being 5 seconds translates to close loop pole locations at this numbers.

(Refer Slide Time: 06:44)



So, the angle deficiency is obvious by this computation, and we know that we now conclude that the root locus should be put to the left also.

(Refer Slide Time: 06:55)



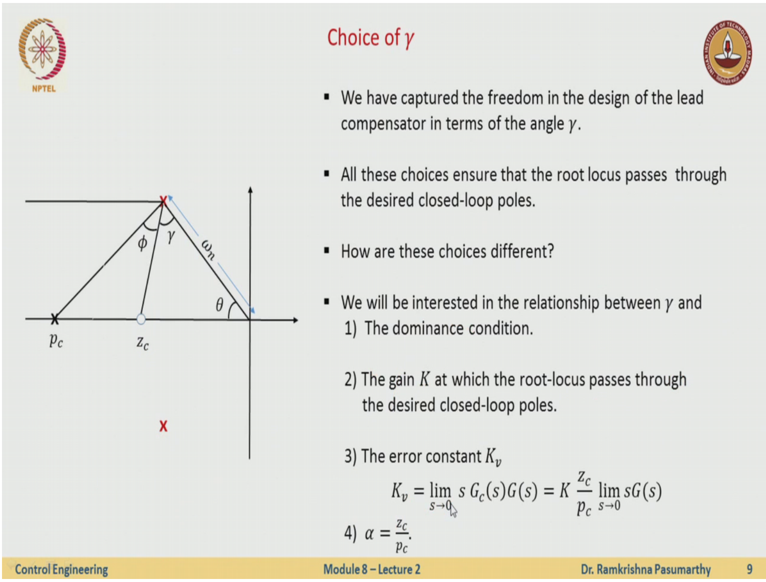
Now, earlier construction just focused on this 0 right. So, we do not have a p. Now what changes when we have this p here? So, first let me just say that this is at some arbitrary location z c and p c is some arbitrary location. What I know is just the phi this I have already computed from here right that the compensator should provide an additional angle of 28 degrees with this formula it also done in the in the previous lecture. So, I know this guy this is 28 degree something this theta is cos inverse of the decide damping.

So, the location of z c now depends on where is on you know how where is the gamma. For example, this phi when you say that the location that this is may not be unique if I just choose a z c here and this is draw line here. I can just add some number five, the same number it this is said 25 degrees I can add 25 degrees and I will say that this is also a p c right which also gives the same five. There could be several possibilities right, if this is my s d, z c, p c with phi could even be when this is z c much further p c and so on, this is also phi. This is my 0; this is my pole.

Well now, what is a good location because there are infinite possibilities of z c and p c which gives me this lead? Now, let us draw a line from here to here, the omega n line and call this angle gamma, which is at the locations now depend on how much is the angle gamma. So, for a given gamma, so this I can use this phi is theta to write down this formulas with the omega n. I am really derive this formulas, but given this configuration this angle is phi, this is gamma, this is theta and an omega n, z c is related to gamma, theta and omega n why are this formula; p c is treated again to omega n gamma theta phi and this formula. So, this is unknown because my problem is where to find where to place z c where to place p c. On the right hand side well omega n is known, theta is known, phi is known, what is unknown is this gamma.

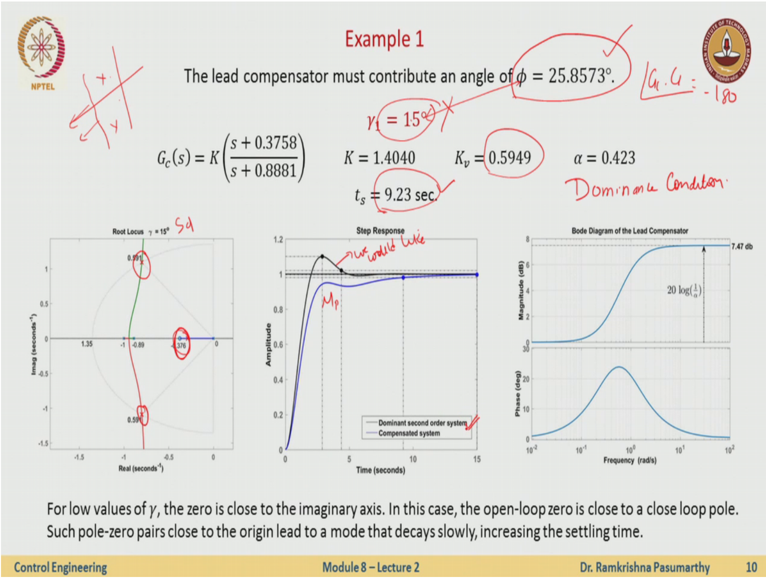
So, the location of poles and 0s depends on this gamma. So, gamma is the deciding factor.

(Refer Slide Time: 10:04)



Now, let us just arbitrarily select few gammas and check. So, let us see what is important when we choose this gamma. First and the most important is the dominance condition, second is the gain k, third are the error constants and four is the alpha right, because here we wanted alpha in such a way that this length here from here to here was as small as possible. So, let just to again some trial and error start with gamma 1 of 15 take this gamma 1 of 15 I put in this formulas and I get appropriate locations of 0s right and the pole.

(Refer Slide Time: 10:46)



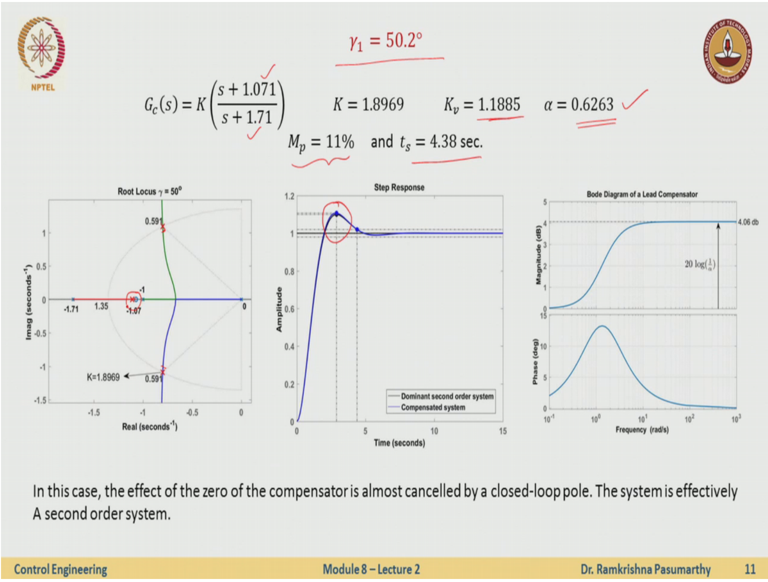
Let us see on the pole is fairly to the left of 0 so that condition is trivially satisfied. The gain k is 1.4 the velocity error constant is 0.5, alpha is small enough 0.423 and the settling time is 9.23 seconds. So, with these numbers, what we see is the gain is 1.4, the velocity error constant has decreased from 1 to 0.5, which means the steady state error increases alpha has some number 0.423 which is less than 1, and it is again trivially satisfied. Settling time is pretty poor at what I want is a settling time of less than 5 seconds.

Why does this happen? Say well even though my lead compensator is successful in giving me this lead or this ensures that angle of G c times G is minus 180 why are these things really bad. The errors have increased settling time is bad. This is something to do with the dominance condition that the dominance condition is not satisfied which is obvious on this plots. So, this is what we would like to decide, we would like our closed loop system to behave, but actually it behaves like this of course, it is a little strange behavior here then there is no (Refer Time: 12:29), but there is say the little bit goes down and then it will result something like this.

The M p criterion is made, but we are not really happy because does not just the objective, you settling time is fairly large and the errors have increased. So, gamma equal to 15 is a wrong choice even though this is satisfied. So, this plus this is not the sufficient condition. So, why is this happening? Why is a dominance condition failing because very close to the closed loop poles is a 0, and that is here. So, if you look at so this guy, so the how did we define the dominant poles that they should be here such that well there is no nobody sitting over here and the other guys should be like father away from some limit. But here when there is this 0 here which loses or which helps losing the dominance.

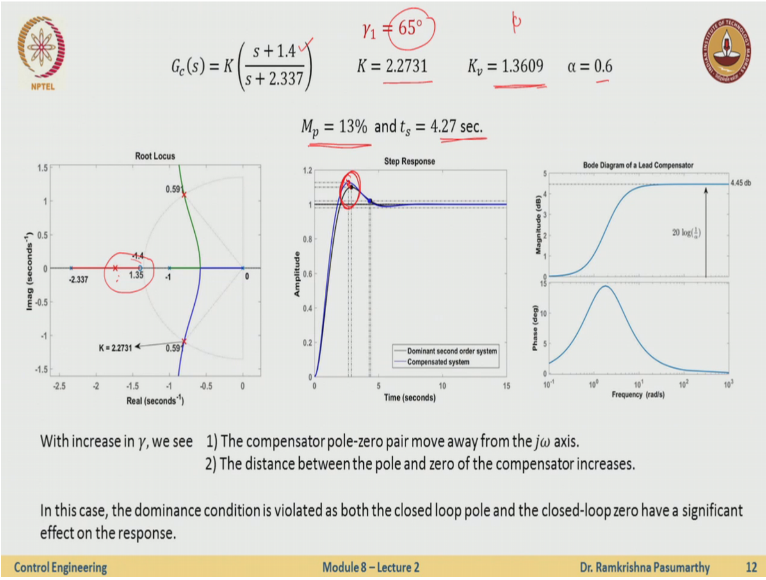
So, these guys are no longer dominant. And therefore, even though the root locus passes through s d, now this is very important. The root locus is actually passing through s d, but the settling time is not guaranteed, because these are not the dominant poles anymore, because there this guy sitting over here. And therefore, this combination does not work even though the root locus is passing through this point because the nominal design what we do is just for a second order approximation this one.

(Refer Slide Time: 14:05)



Let us chose another number gamma is 50.2, gamma is 15.2 then I get some with the same formulas I get some location of the 0, some location of the pole the pole being to the left of 0. Now, it is kind of quite good rather that the velocity error does not change much. Alpha is good and well the peak overshoot is more or less which I wanted it was 10 percent was the desired; now I have 11 percent settling time is fairly good enough. And you see that well the some kind of dominance is ensured because both this blue and the black lines are very close to each other. So, this works well right as a this is actually something nice for me.

(Refer Slide Time: 14:49)



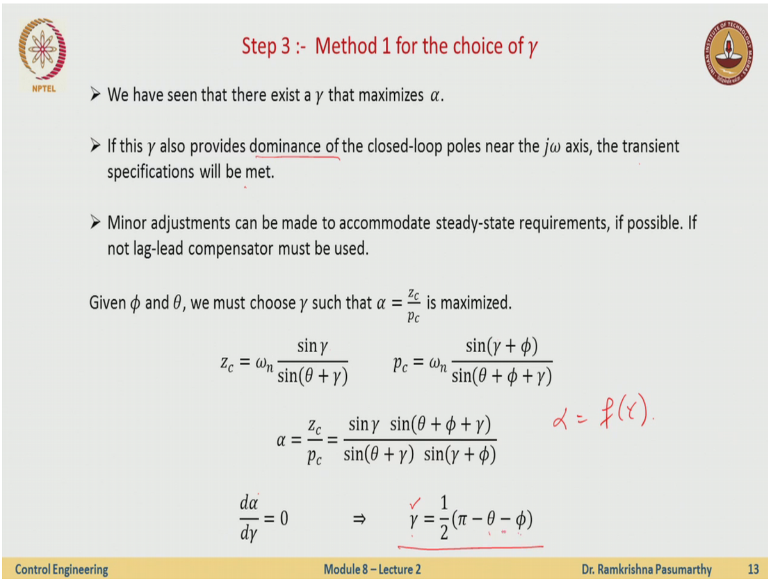
Now, I become a little greedy and say that from 15 to 50.2, I had this I could get this blue and the black lines close to each other. Now, let me get a little more greedy and say well I will chose the 65 degrees again the same phi and or would be ensured I choose gamma want to be 65 degrees and them 0 comes at minus 1.4 pole comes at minus 2.3 and some. So, this is certain gain the velocity error constant has changed again from 1 to 1.3 and the alpha has now decreased from 0.6 to 0.6.

So, now let us look at the numbers well the peak overshoot is not really good right it is 13 percent it has actually slightly increased. Settling time is still right 4.27 seconds. And you see that the dominance is slowly have started you know the system has the or the this poles as slowly started to lose the dominance because well the distance between the red and sorry the blue and the black curves is increasing. So, so why is this happening with the increase in gamma? So, now, let us look at what is happening with the location of the poles and 0s right. So, in the previous case the value of gain k at 1.89, which results in the closed loop poles to be here and here you see that the distance between the pole and the 0 here for this value of gain k is very minimal in such a way that they actually cancel out each other.

So, the closed loop system will just have like two poles one pole here and one pull here that is like dominance is ensured by these two be in very closed to each other. Ideally, we would want these two cancel you know these two guys to get cancelled whether that that will possibly happen only in the limiting case and case very large. So, for this value of gain k these are my dominant poles and there are very closed to each other. So, they will almost nullify the effect of each other. Whereas, in this case I see that there is some distance between these two that this do not cancel out each other; and they in some way affect the performance or you know they contribute to the plot here to the step response. And therefore, we see that as a distance between these two again increases, the distance between these plots also increases.

So, this is a little this is nice thing happening here and nice observation that for this gamma the effect of this pole 0 pair is the minimal.

(Refer Slide Time: 17:48)



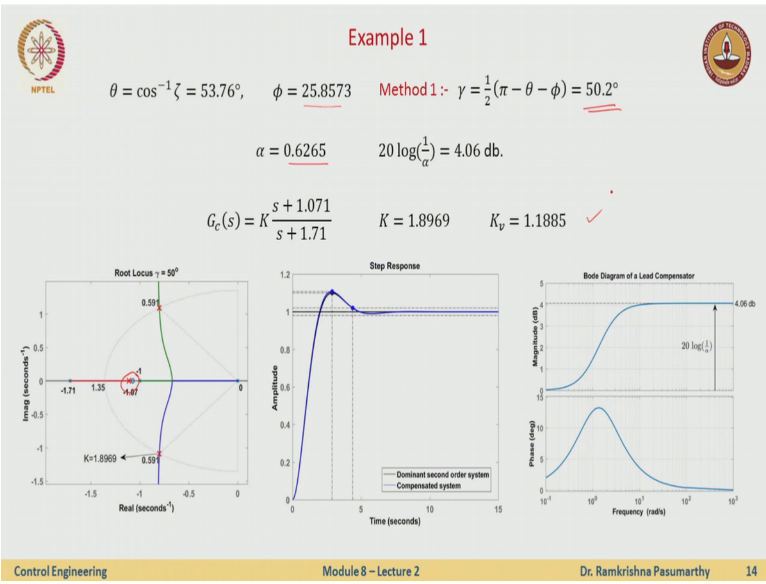
Now, is there really method of finding, what is a good choice of gamma? I just cannot do a trial and error all the time. And if you look at the relation between gamma and alpha first case I had gamma of 15 and alpha of 0.4 gamma of 50 alpha increases 2.62, gamma of 61 alphas are decreasing. So, there is one gamma which maximizes the alpha and this is the gamma which I am interested in right and maximizing alpha is also good for me in terms of the attenuation because if you see this is this is 4.45 db here this is 4.06, this is 7.47. So, this is I want as a said earlier I want to keep this length to the minimum. So, good gamma is the one which maximizes alpha right and not only that this also provides us or helps me satisfies the dominance condition in such a way that the transient specifications are met.

What is the effect on the steady state error we not really worry about that, but some adjustments can be made to accommodate steady state requirements. So, here I am at prime I would primarily be interested in just improving the transient response without having too much effect on the steady state as we said. So, so here the effect on the steady state is not really too much there is some effect but we could possibly try and keep this to a minimum.

So, now the gamma was the variable. Now say in this expression for z c gamma was unknown, p c gamma was unknown. And alpha is related to z c and p c from the construction of the compensator. Let us go back to the slide and say well how is z c and p c related why a alpha here at this is z c this is p c. And z c over p c is 1 over tau 1 plus sorry there is no 1 plus 1 over tau and on the denominator I have 1 over alpha tau, so this is z c by p c is alpha. So, how to maximize this?

So, alpha which is z c by p c alpha is now like in a like a some non-linear function of gamma, then I can just use standard calculus what calculus teaches me that they maxima occurs when d alpha by d gamma equal to 0. I do all the computations and I get a gamma these gamma maximize the alpha. Now, the unknown when I was computing the location of the 0s and the poles were the gamma. Now, I get the gamma in terms of theta and phi, these two are known to me. So, I know where to place these poles and 0s right given a gamma.

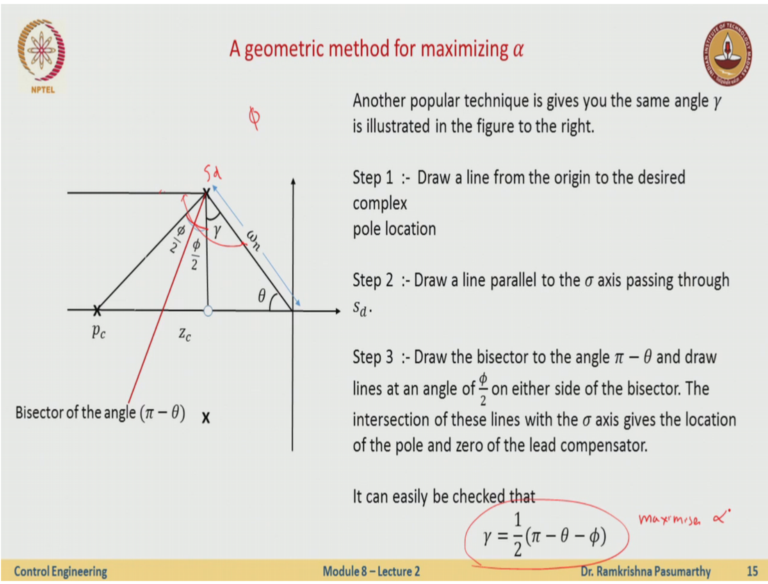
(Refer Slide Time: 21:11)



So, I just do the computations as before I get that sort from theta I found I find what is theta from cos inverse of zeta I get how much is the gain and then gamma turn out with. Therefore, mirror to be 50.2 degrees exactly is what we what we had guessed in the second guess of hours which are the good alpha of 0.62. And this plus are exactly the same as what we saw right at the poles and 0s are very close to each other. So, then nullify each other effect and this and these two guys then ensure the dominance of the closed loop. So, this is exactly what we had before.

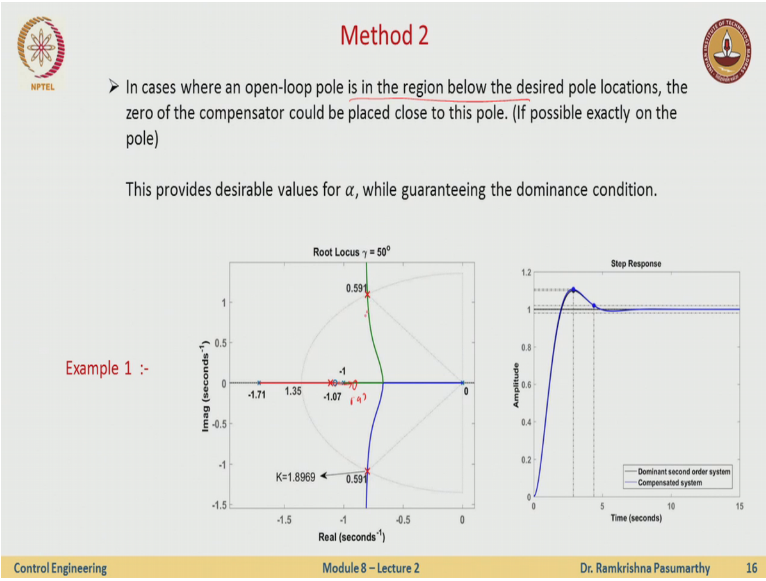
So, what we have answered here so far is how to place the z c and p c. First is that the angle of g c should be phi. Now, next is, is there a unit location well the answer is no. The z c and p c were depending on the gamma and which shows the gamma in such a way that it maximizes alpha so, that is that is method one and it gives us some beautiful results here.

(Refer Slide Time: 22:35)



Now, if I am really lazy go you know through all this calculus and you know these are another method of doing this well it is a nice graphical method that I know the location of s d. So, this is s d a right and then I know theta and omega n I just draw a line to the left here. And what I do is I bisect this entire angle this entire angle right I draw a bisector to this angle which is this one. So, this is this is a bisector right and then I know phi, phi is got is derived by easiest competitions. So, what I do is I will go phi by 2 to this side right draw a line this is z c, I go phi by 2 to this side draw a line and this is p c, this is exactly the same as what we are done before. All that analysis which we talked in terms of maxima and minima would lead to the same thing if I would just do this bisector thing. So, there are two ways of understanding this. So, this gamma which I ob obtained by this geometric method actually maximizes alpha as we saw earlier.

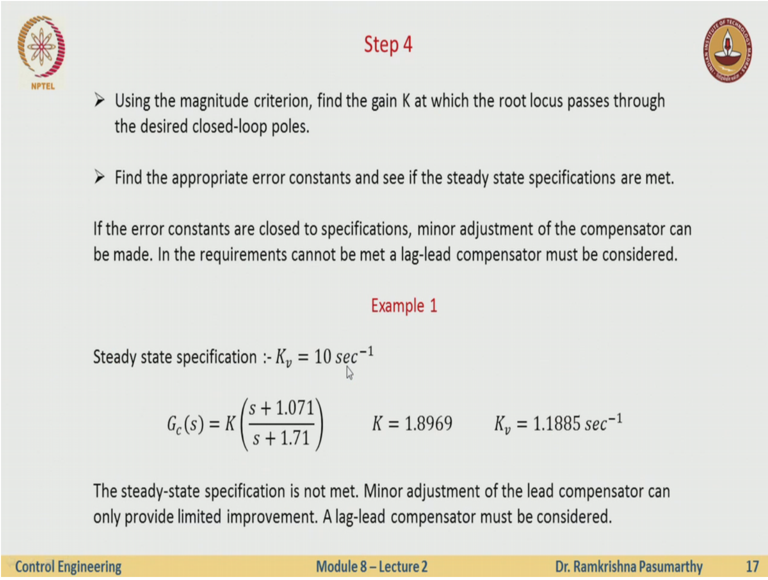
(Refer Slide Time: 23:46)



Now, method two, is there another way right, if I really do not want to compute gamma, I forgot the calculator to the exam and thinks like that well what could I do. So, what we had seen here is the following. So, look at again these distances, this and this. And things were depending on how this is where located from each other. So, from this observation, I can say that in cases where an open loop pole is in the region below the desired pole locations. So, this is not really for right this is pointed and this is one in cases where the open loop pole minus 1 is in the region below the desired locations in the in the close vicinity of it then I just place 0 just slightly to the left of that pole. So, here I have a pole at minus 1 and a place is 0 just say at you know 1.07.

Let once I know this 0, I can easily calculate where is the pole, this by based on the phi or the lead angle which is should contribute. So, this is also provides desirable value of alpha while guaranteed the dominance condition. In our condition this is kind of easily if you see here this is kind of met easily. So, this 0 is placed slightly to the left of this minus 1, I would not want you can you may even ask why not instead of 1.07 place it at point nine three if I place it at 0.93. So, this pole according to the root locus condition we will move towards the right and I do not want anybody to move to the right. So, to ensure dominance of these two poles here, so I want them to go to the left, but this is the very restricted condition only in this case; in the case where an open loop pole is placed just slightly now just in the vicinity just below this dominant closed loop pole.

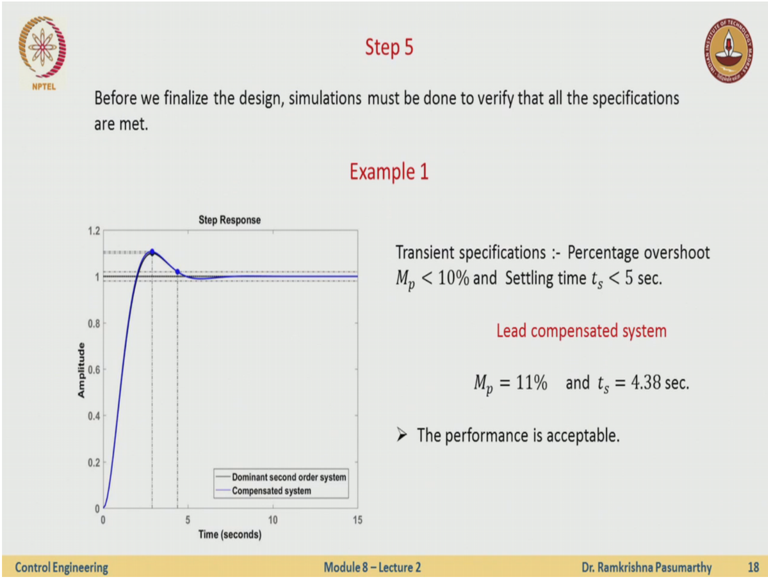
(Refer Slide Time: 26:00)



So, now, I found out what is the z c and what is the p c. Now, the unknown now is; what is the closed loop gain k. This I can easily find by the root locus magnitude criterion and use the magnitude criterion to find the gain k at which the root locus passes through the desired closed loop poles. So, we could see also; what is the effect on the steady state specification, see we know that is not really the objective of this of this controller. And if the error constants are close to the specifications we can just do some minor adjustments. So, in our example 1, where the locations of 0 was at 1.07 pole at 1.7 resulted in a gain k of 1.89 this can be computed by the by the magnitude criteria criterion. Now, this is very straight forward and we had also seen the formula for this K v was 1.1 per second.

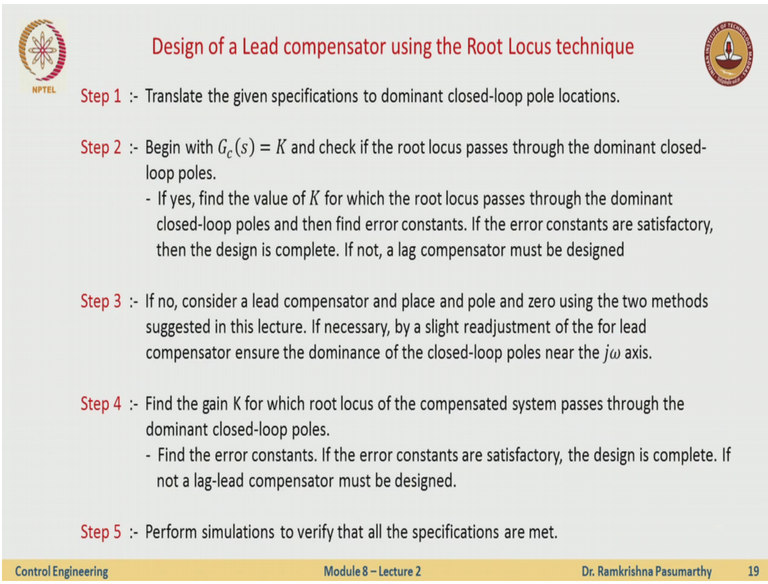
The steady state requirement here is not met right then can we actually do a minor adjustment of the lead compensator. Well this can only provide a minimal you know some very small adjustment, but that can somehow even alter with your peak overshoot and all. So, in those cases where we want to alter both the transient and the steady state we would do also look at what we would call a lag compensated which we will steady this lag compensator in the next lecture.

(Refer Slide Time: 27:37)



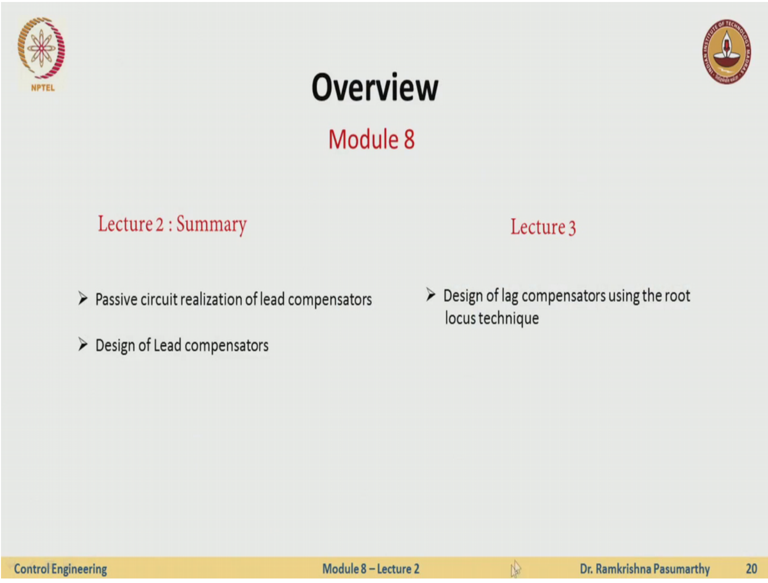
So, before we finalize a design we just check with simulations to verify if all the criterion or met or not. So, where did we start with we started with specifications in such a way that M p was less than 10 percent settling time was less than 5 percent and I do all the design cha choose the best alpha it gives me 11 percent of M p and 4.38 seconds of the settling time. And these are kind of within acceptable limits.

(Refer Slide Time: 28:07)



So, I could just summarize the design process into the following steps. First is find the dominant poles; second is check the gain condition just go through the root locus and check is there any value of gain k such that the dominant poles lie on the root locus; if no then I consider a lead compensator and then calculate where to place the poles and the 0s. Next step is to calculate the gain k that and then see is there too much effect on the steady state error constants; if they are then I would go further I would go for a further design procedure which would also include a lag compensator and I would verify all this design to by appropriate simulation. So, see all to see if all these specifications are met or not; if not then I could just slightly push my pole or 0 slightly to right or left to see that the appropriate specifications are met.

(Refer Slide Time: 29:07)



So, in the next lecture, we will see how to meet specifications on the steady state error or the error constants via lag compensator.

Thank you.