Control Engineering

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Module – 08
Design using the Root Locus
Lecture – 01
Introduction to design in the time domain

Hello everybody. So, in this module we will focus on explicit a design methods, what we saw previously was suppose controller components like there was a proportional controller there was a derivative controller there was an integral action and we had analyzed when under what situations do we need a derivative component, under what situation do we need to add an integral component what are the affects of adding zeros to the system, what are the effects of adding extra poles to the system like for example, in terms of the root locus what we had seen was adding a 0 has the effect of pulling the root locus further to the left, and the more the root locus is further from the imaginary axis the better the transient response right.

So, to peak and there are also methods where if I add an integrator or a pole at the origin I could derive that the steady state errors go to 0, but we need to be carefully because what we saw with a help of a examples is just that just blindly adding integrators might even lead the close loop system to be unstable. So, we will formulize those things a little more.

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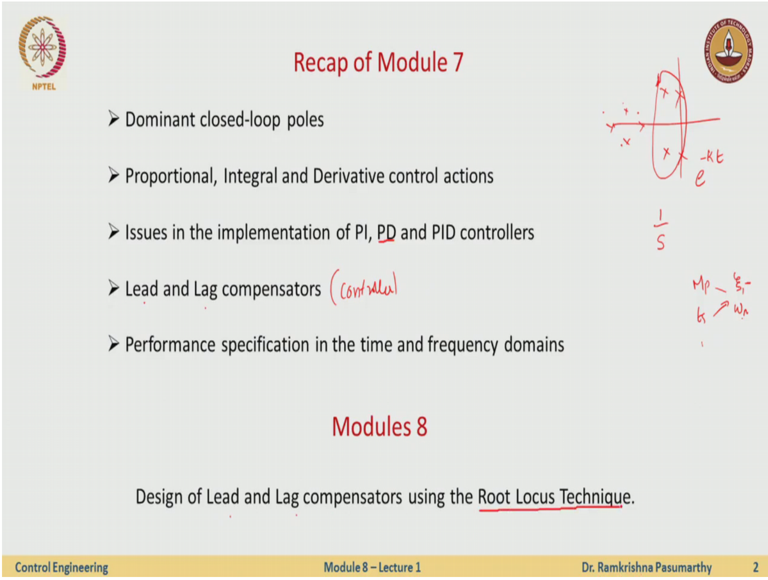

So, what we studied earlier was the entire analysis based on dominant close loop poles. Say for example, if I have 10 poles in my system all the analysis could be based on the poles closest to the origin, because that is what really decides the behavior of the system in the long term because the other poles which are to the left, there their response actually dies strong pretty fast. So, for example, if I just take a system which has two poles here some guys sitting here, here, here and so on. So, the pair of poles will decide how my system behaves more or less can be approximated by just looking at these two poles; because this transfuse diode pretty quickly because we are looking at something like exponential of some number K times t; bigger the K faster there would decays smaller the K this lower the decay. So, this actually corresponds to smaller k, and if you see that if they are on the imaginary axis then this guys never decay. So, constant oscillation process ok.

So, we also looked at issues in implementation of the nominal derivative or PID controllers right. So, we have in a way that if I just take the P and D controller, we saw that it cannot be realize physically as it is a non causal systems. So, this we need us to have slight modification of these components in terms of physical realizability like for example, I can never have a perfect integrator right. So, this kind of systems are very difficult to realize in real world. So, an approximation could be to classify PD controllers as lead compensators, and PI as a lag compensators. Compensators also needs controller. So, whenever I say compensator it also means controller and vice versa right and. So, given a system how are the performance criterion specified, how is the performance criterion specified. It essentially specified in terms of some kind of peak over shoots, some kind of settling time and so on and this loosely translate with some you know you know the formulas these are specifications of zeta and omega n, and I can construct the dominant poles based on this right.

And we see that these are how these are related both in the time domain and the frequency domain like there is a direct one to one relationship between the phase margin and the damping coefficient. So, here we will see explicitly how we construct this lead and lag compensators, and to we will use for a for our analysis is the root locus technique. So, we will use make use of the root locus plots, which we learn to design lead and lag compensators to made a certain performance specification ok.

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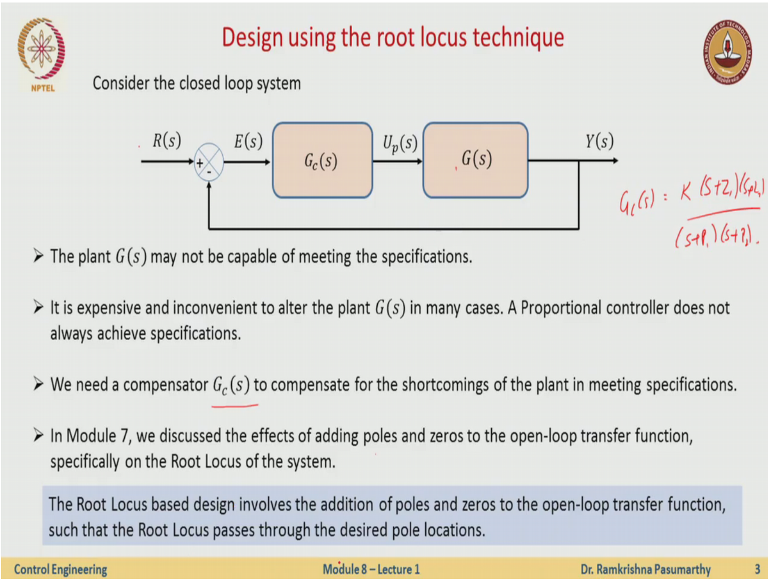

So, in a typical close loop system looks like this. So, I am given a plant G of s and suppose G of s does not behave the way I want, well I the constraint is that I may not be able to alter the plant or it might be expensive. In some cases it may also happen that a proportional controller may not achieve specifications, in that case we need to add. So, these are like. So, exotic controllers right this I am just increasing the gain.

If increasing a gain does not help me I need to add a dynamic compensator right which are some poles and zeros right to overcome the shortcomings of the plant, in meeting a desired specification as simple as tracking a reference signal. So, when I say I am adding a compensator, it does not mean that I would have a system which possibly some gain some zeros so on and some poles and so on. I will make sure that the number of poles is greater than or equal to the number of zeros to maintain the causality condition. So, we had earlier discuss the effects of adding poles and zeros to the open loop transfer function right. So, now, how do I make use of the root locus to achieve this a desired a performance specification, that is what we will analyze and all this analyze, analysis will be in terms of the dominant pole pair ok.

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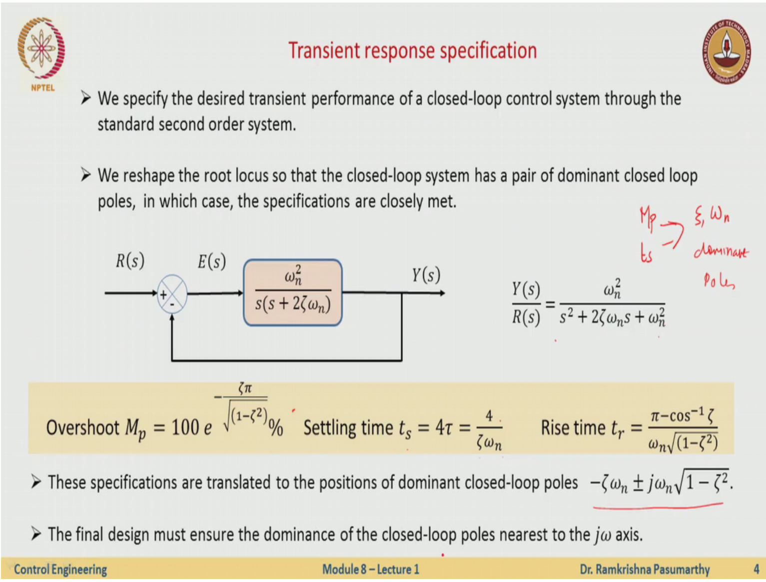

So, when I talk about dominant pole pair it is easy to specify this transient performances through a standard second order system right. So, how does the second order system look like I have the transient function as omega n square, s square plus 2 zeta omega n s plus omega n square and we know what zeta means zeta is a damping coefficient, that is which decides the system is under damped over damped critically damped and so on omega n is the natural frequency of the system.

So, a certain step response to a system lead says to these expressions, at the peak overshoot is directly related to a zeta and the settling time depends on zeta and omega n. Similarly there are on for the rise time the peak time and so on ok.

So, now given M p given p s this guys would result in certain zeta a certain omega n. So, this formulas and how would they be related to the dominant poles. So, given zeta and omega n can I actually construct domain poles (Refer Time: 07:28) just by this formula right. So, how just the poles to the solutions of this system right for expectably for an 100 and case. So, why. So, when why do all this design procedure and if I just design my system based on these numbers here, I must ensure that this design procedure which I follow in terms of the second order system these are actually the dominant poles that these are the poles which are closest to the j omega axis.

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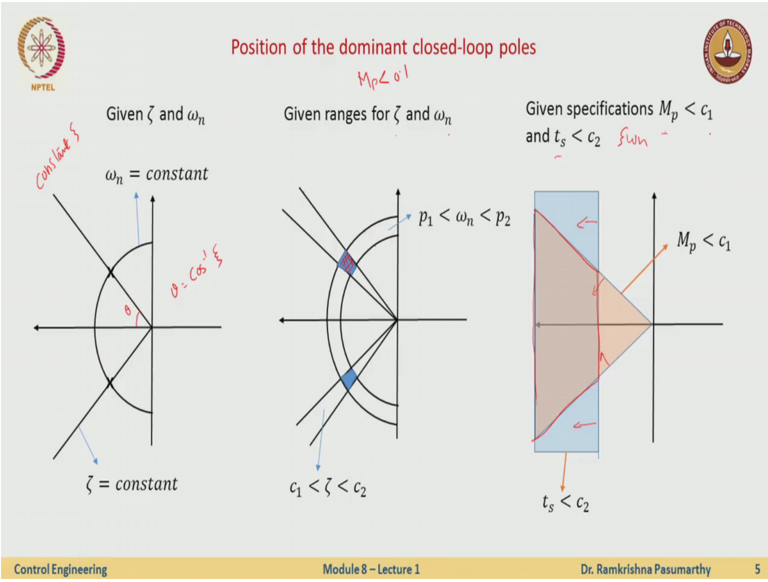

So, first question is there are any dominant poles if I look at it graphically. So, given a zeta and given a omega n I can look at this (Refer Time: 08:12) So, these are lines which are constant zeta lines right, and how is this angle theta determine theta is cos inverse zeta and these are lines which have constant frequency. So, for different frequencies are (Refer Time: 08:34) circles here or here and so on and we had we had seen this in one of our earlier lectures. So, if I am given a certain range of zeta and omega n, say my performance specifications if they are say let the peak overshoot be less than 0.1 or 10 percent that will given in a range of values right, it is not strictly that it should be 0.1. similarly for omega n in terms of the settling time. So, if I gives give some ranges that the zeta should be between this number and this number or the omega n should between this number, I will have a range here right I can choose any pole within this blue region right.

So, in if I look at specifications in terms of M p and t s this translate to zeta being within a certain bound right. So, these are like the zeta lines. So, any zeta within this will give me m p less than c, c 1 right and because we can just you can go anywhere all the way till infinity on left hand side.

Similarly, if I look at the settling time which is again depending on zeta and omega n these are the lines is zeta omega n, and then anything here would be this specification how both of this are met is so the area here. So, it choose a pole anywhere here right somewhere here or here it will be this specifications ok.

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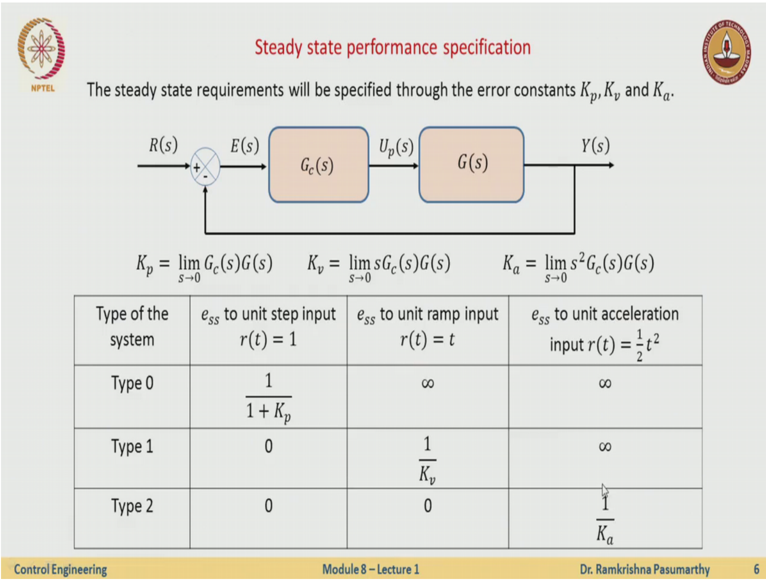

So, the transient performance specifications are usually in terms of zeta and omega n, and the steady state performance specifications are usually in terms of this steady state error, depending on if I am tracking a step or I am tracking a ramp or a parabolic input and based on this specifications and or this definitions and the type of the system we had characterized several error constants. So, what we concluded from that analysis is that if it is a type 0 system, which means G will not or G times Gc will not have a pole at the origin, then it will be able to track a step input, but with some error and we know how to compute this Kp right. So, with this simple formula.

Similarly, a type 0 system will never be able to track a ramp signal and parabolic signal and other higher order signals. The system is of type one which means G times G c has a pole at the origin the rate can track a step input with 0 steady state error. It is able to track a ramp with some steady state error and it will never be able to track a parabolic signal and similarly for type two systems it is the steady state error for step in step reference is

0 steady state error for a ramp input is 0, but for a parabolic input it some infinite number right.

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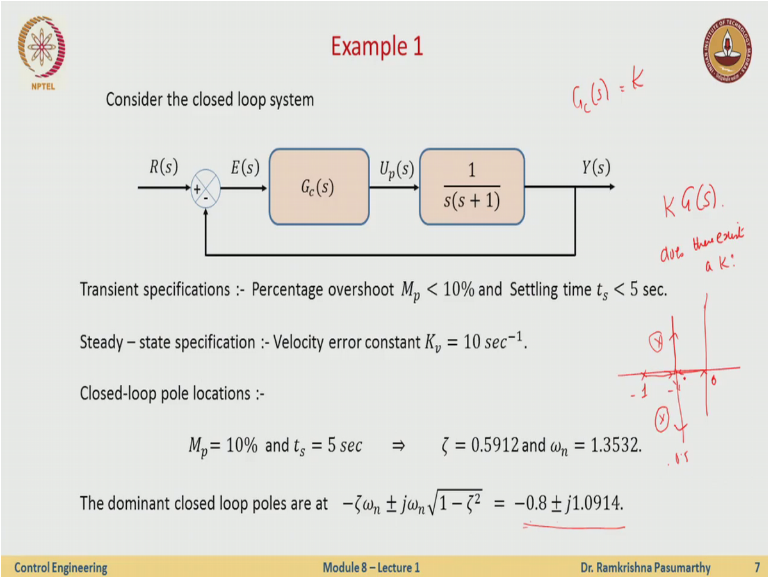

So, this analysis is what we did mush earlier in the course. So, let us start with an example to motivate how we are going how we will build our case to design a controller.

So, let us start with a simple plant 1 over s, s plus 1 and I interconnect it to a controller in the standard feedback setting, and the transient specifications are that the peak overshoot should be at best 10 percent or at most 10 percent and the settling time should be less than 5 seconds right. And the steady state specification or the is in terms of the velocity error constant that should be less than 10. So, let us first translate this into the locations of close loop poles in terms of the dominant poles. So, M p of 10 percent according to the formula t s of 5 seconds, I put it in the formula and I get this number as zeta is 0.59 and omega is 1.35.

With this two numbers I get the location of the determinant close loop poles at minus 0.8 plus minus j 1.0914. So, the first thing to do here would be to see look at this in terms of the close loop specifications in terms of the dominant poles. So, first what I would do you see what is the root locus of the system right. So, you see is this kind of a very straight forward to plot we have drawn much more complicated plots and this is a 0.1 over 2, this is my pole, this is my pole. So, this guy was here this guy goes here and then they we will split this way ok.

Now, I want my compensator to be such that the close loop poles or the dominant poles should lie on the root locus this is what the system is if I just look at well. So, this is the root locus of G times s, I can just say well can I just add a controller K proportion controller and check if my requirements are met how do I do this I just move along the root locus and see if for any value of K for any value of K am I reaching this point or are these points on the root locus for some value of K now does there exist a K right. So, if there exist a K then my Gc would just be that K right. this proportional controller would satisfy that. So, here I can easily see right. So, this line is the this minus 0.59 this is minus 1 this is minus 15, and your dominant poles are somewhere here.

So, what I see here is the matter how much I increase or decrease the gain I will never be able to reach this points. So, the thing is that or the or the concept here is that this point or this dominant poles should lie on the root locus of the close loop system that is how we will make use of the root locus plot to design controllers. So, more on this we will come in shortly.

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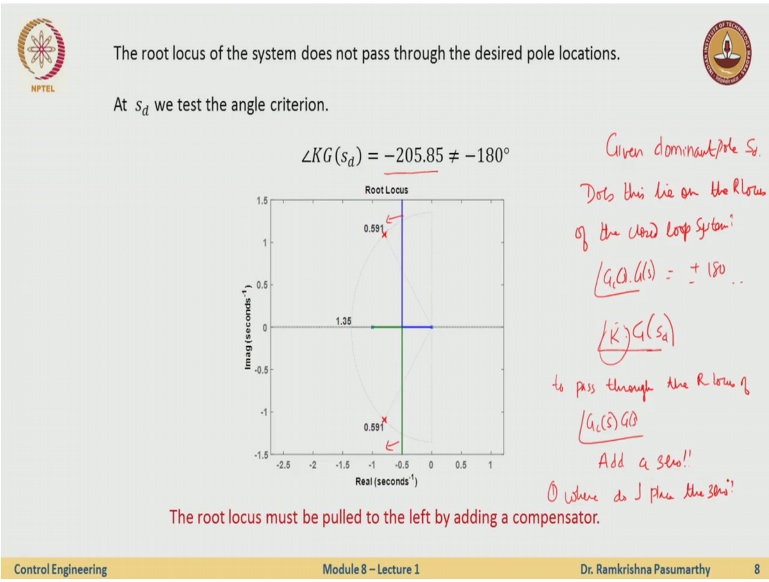

So, what you should check is that does there would locus on the system pass through a desired location the answer is no, another we have to check is also the angle criterion right if I just do not know how to plot I do not have mat lab with me I will just check what is the angle right. So, I am looking at the given dominant pole, and I will always denote this as Sd. Now does this lie on the root locus of the close loop system, and I know that way to check this is at the angle of the compensator or the controller plus the plant should satisfy this condition plus minus 180 blah blah blah. So, if my controller is just a K right and then G, I know what is at which point I am looking the angle this was. So, this kind G times Sd or this tens out to be minus 2 of 5.85. So, that 180 therefore, they will never exists a gain K such that K times G at Sd is 180 degrees. So, just by at this time the gain we are not able to meet the desired specifications right.

Another way to find out is just look at the intersection of the constant zeta and the constant omega n 9 and that intersection turns out to be this one and this will find graphically the dominant or the or the desired close loop dominant pole pair. And it is easy to check these are not on the root locus right that what we would need is this points the one in the rights here, the pass through the root locus of the close loop system which is G c times G of s. So, which essentially means I want to pull the root locus to the left and if I remember what I had learnt in module 7 is add a 0 add a 0. So, that it will be to the left now how much where to place this zeros was question. Say add a 0 do I really add this blindly anywhere. So, this is the question which we will try to answer ok.

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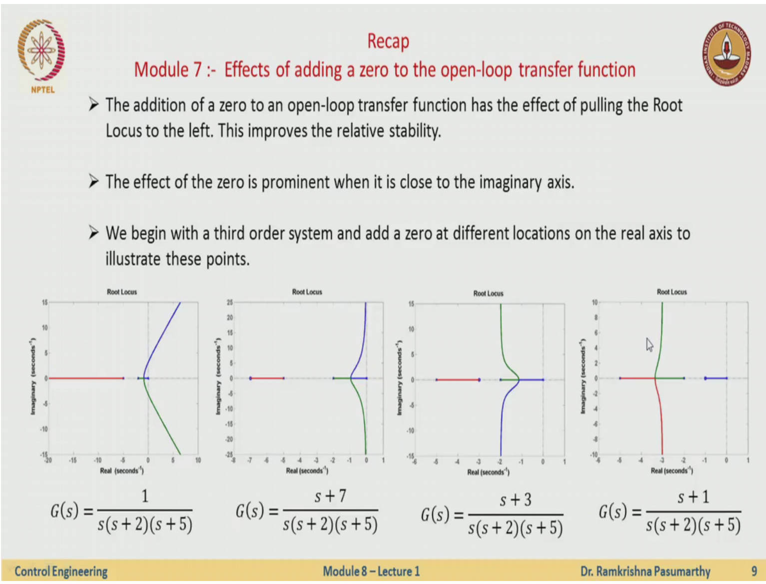

So, now is a before I know where the 0 will be place, I just recollect what I observed earlier in terms of the effects of adding a 0 to a system right. So, I just have this standard system where this is my given plant 1 over s, s plus 2, s plus 5 has a root locus like this right. So, therefore, certain gains it should be goes to instability ok.

Let us say I add a 0 at s equal to minus 7 somewhere here, as I see that the root locus is pulled slightly to the left that I am actually ensuring stability here for certain values of K or for certain values greater than s than this number my system is becoming unstable here my system is stable right for all values of a and K all right goes it is marginally stable. However, this is a system which has a very lower relative stability slight changes in system parameters can actually push this to the right and lead to the verge of instability. So, I will I add 0 further to the right at say at s equal to minus 3, that I will see some you know myself behave here at the this is a system which has a bigger stability margin than this one hence finally, if I add 0 at s equal to minus 1 you see the rate as something more. So, something much better in terms of a the relative stability. Not only that if I say that the transient response of this system would be much poor compared to this system, and this root locus is to the further to the left of this root locus. So, this will have a better transient response.

So the effects of adding a 0 is to pull the root locus to the left to the left to the left here further to the left, further to the left. Now thing is how much to pull the how much to pull is answered in way that you pull it just enough. So, that this Sd sorry a dominant pole lies on the root locus of G times Gs or this e times Gs ok.

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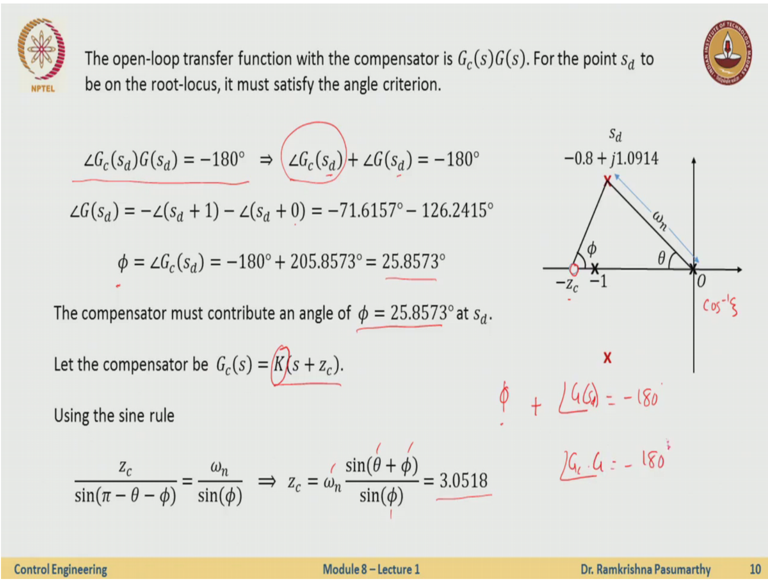

So, the first thing which we would like to see is how much do I pull or you pull just enough such that it meets the angle criterion. If the angle of my compensator evaluated at

Sd angle of my plant evaluated at c should be minus 180. Now what is my plant my plant has a pole at the origin and a pole at minus 1. So, now, I now I compute the angle contribution to Sd through which of these poles at pole at 0 pull to the minus 1 and I get a certain number. So, the difference. So, what should this G c do such that the total is 180, that it should met the required at just met in the angle of G of Sd such that the total is 180. So, let me call this angle as phi right. So, phi plus angle of G Sd should be minus 180, if the angle of gsd by itself is minus one eighty I do not need to do anything here and the compensator need not add an extra angle right.

So, in this example it turns out that the phi should be 25.8 degrees, if you just these are very simple computations. So, this compensator or controller must contribute an extra angle of this one and therefore, I call it a lead compensator it adds an angle to the to the root locus. Now let us let we just assume without really worrying about if I can realize this compensator or not, let me just do some analysis before I really have a conceptive way to determine how the lead compensator looks like. So, let us say my compensator just is a proportional plus the derivative controller the same constant right. Now let us let me assume that I just put my 0 at some arbitrary location Zc and this is what I have to find out now the total angle of Gc does G should be minus 180. So, I just use this little triangle here right. So, this phi is given to me phi is this one now theta is also known to me is comes from cos inverse zeta I also know what is omega n ok.

Now, I know several things here I know theta I know omega and I know this guy now can I calculate what is my Zc, but I just use a very simple trigonometric rule right. So, sin of this Zc over of these numbers would be omega n over sin phi and just this computation. So, theta is known to me phi is known to me omega n is known, phi is known I just get the location of Zc is 3.0518 somewhere here right. So, choosing a 0 at now I am not really worried about what is this K because this K does not really contribute to the angle right or it does not contribute at all. So, if I place my Zc at this value then my contribution of angle Gc times G is minus 180 degrees.

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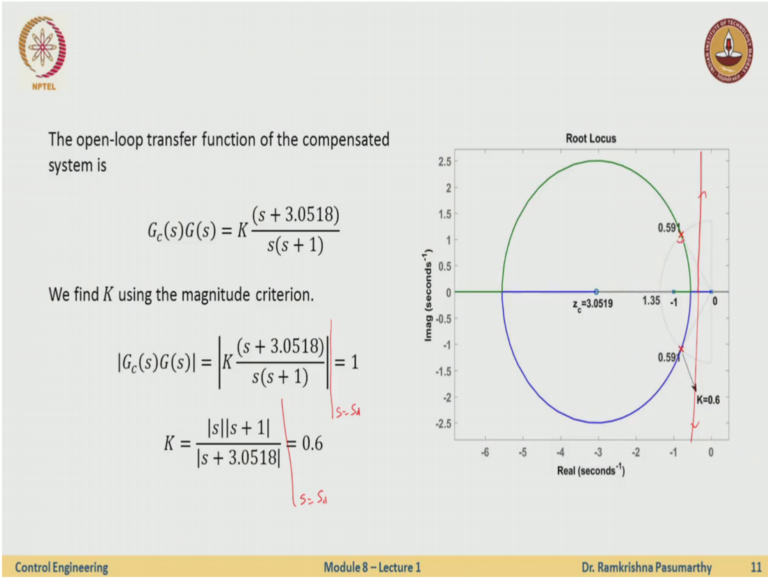

So now how does Gc times G look like Gc times G looks like this K, the 0 which I added because of the compensator and the poles of the plant. Now what is this K and I just simply cannot chose any K this K is chosen by the magnitude criterion. So, Zc overall magnitude of K times the 0 and the poles should be one and this K can be computed by again looking at what is the gain contribution at sd. So, all these are evaluated at s equal to Sd this is also evaluated at s equal sd. So, can I can these are just simple computations ok.

Now, let us see what is happening with the root locus, well the root locus it has a pole here a 0 at minus 1 sorry a pole at 0 a pole at minus 1 and the new 0 which is a compensator this is my controller. And I see that the root locus now which was earlier we do something like this we do this been up here will down here as now been pull to the left in such a way that the close loop root locus actually points possess to this point ok.

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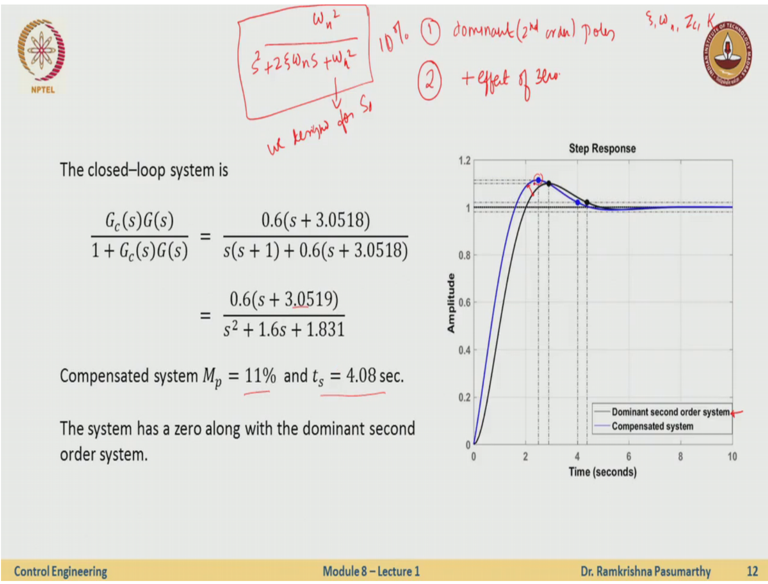

So, how does now the plot look like or the rest all looks like well this is my compensated system with a close loop transfer function Gc times G 1over Gc plus c we know how to derive this point the compensated system has a peak overshoot if I compute of 11 percent and the settling time of 4.08 percent which are more or less what we what we wanted right

So, the system has a 0 here and this along with the dominant poles now let us see how close are these two plots first all my specification are in terms of dominant second order these are in do not have second order, but this full computation that dominant second order poles right. So, what I designed in terms of zeta omega n and Zc and the appropriate K , were only was in in terms of just taking the response of a standard second order system now in addition I add the compensators. So, I have in reality this is not true right. So, my dominant pole analysis is based on this kind of transient function two zeta omega ns plus omega n square.

Now in addition I have a 0 here right. So, I have the plot will consider; we will be the plot will include the dominant second order poles for sure plus the effect of 0. Now well what is the effect of 0 well there is some not very good thing happening here right if I say if I just draw a plot of the dominant second order system which is without considering the effect of 0 this is how my how my design goes right all my specifications are translated to a second order system. This is how I would want the look like in the black line.

But however, the compensated system has a 0 0 at this number 3.05 and so on. So, the compensated system now has the responses like this at the blue line; and therefore, we will see there is slight increase in overshoot than what I actually wanted. So, what I design is for this, we design for this dominant poles thinking that the close lop system would actually have only this kind of dominant poles and the other guys wouldn’t matter much; however, the actual response has a 0. So, what we, so this is not the completion of the design process because there is some gap here right that is not really ensure complete dominance of the desired pole locations.

So, we would want to select those 0 such that these two guys come as close as possible right and this will be clear in a. So, this is similar to a consecutive procedure to design a lead compensator, this just by following steps of what we had learnt earlier. That if I want the transient performance to improve, I need to pull the root locus to the left I can pull the root locus to the left by as adding a 0 where do I add a 0 is given by this this angle and sorry the angle criterion and the magnitude criterion now once I add a 0 is it really dominant we will see a (Refer Time: 28:59) close to dominance, but just that now there is a little little error right.

So, the actual system this has 11 percent. So, this has 10 percent overshoot, but this guy has eleven percent. So, we will try to see how we how we minimize this gap right.

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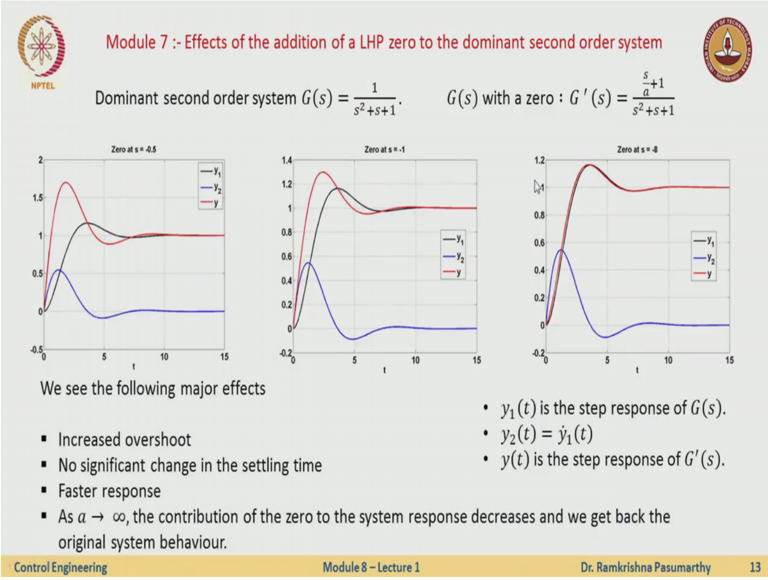

There is also we should be careful off. So, now, we will see. So, the response so far is we looked at dominants second order systems, which should say this is for example, consider a system like this, with a 0 somewhere here now we will see. So, what we wanted we wanted this plots in blue and black to be as close as possible Now are they related in any way to the location of zeros if they are closer to the origin is it good for me as they further away away from the origin are they good for me. So, this is a kind of a response I am looking into. So, let us start with this system and say I am adding a 0 at s equal to minus 0.5 ok.

So, this y 1 is the step response of this kind, and y 2 is this adding a derivative right y one dot and y is the total response and this this y is a total response. Now I am adding a 0 at minus 0.5 and. So, now, I add a 0 at s equal to minus 1 and we see that the red and the black lines actually are coming closer to each other right this. So, the red is the response of G prime, and the black is the response of G and I want to place the 0 such a way that these two are closer to each other because put further further black at s equal to minus 8 when actually see that the s they are very close to each other. So, depending on the location of this zeros my red and black curves conclusive or go further away ok.

Now, another effect of adding this 0 is that well there are no there is no significant change in the settling time. In addition to say that well this actually the response here is faster right this is slower to respond the black line is slower I pull the root locus to the later you see the faster response. the faster response will also lead to increase in overshoot right and the limiting case as a goes to infinity the contribution of the 0 decreases and we get back the total system behavior, but we really do not want a to be. So, large enough that it does not really matter right.

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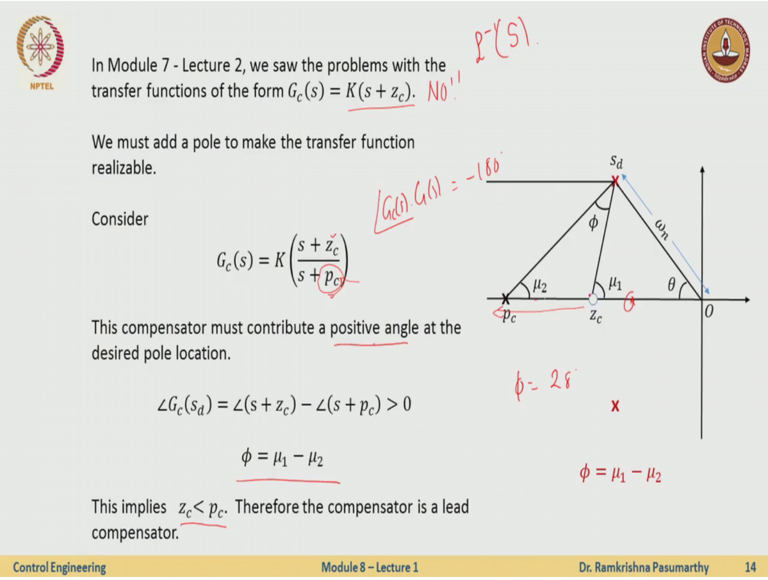

So, we need to come to a nice trade of what is a good a to select. So, once I know the effect of adding this 0 there is some drawback here that can I just go to the market and buy a controller which looks like this and the answer is no we had seen earlier because this is a non causal system right ok.

So, somebody I think in in the research forum also ask you know what is inverse Laplace transform of s and the answer is really does not exist. So, therefore, to make it realizable we add a compensating pole in such a way that the effect of adding this pole is just for construction purposes just for practical purposes, but this guy should just keep quite should not really try to interfere too much in the process.

So, now, this compensator, so this again the angle of Gc and angle of G should always be one180 all those condition should be should be satisfied. So, if I say that now earlier only the 0 had to contribute certain angle now I have a 0 and a pole. So, these two together should contribute an angle phi right. So, this is my Sd mu 1 is the contribution and. So, of contribution of Zc mu 2 is the contribution of. So, the this is Gc will now have angle contribution because of the 0 and because of the pole and the total thing should satisfy this one now this should more should always contribute a positive angle right which means the angle this guy should always be greater than 0 earlier we see in in an example phi was some 28 degrees, this always contributes a positive angle for it to contribute the positive angle, the pole should be to the left of 0, I can have a 0 here and I have a pole here now this will not work for this reason ok

Now, this first conclusion here before we go further is that I just cannot have only a Z I need to place a pole ,where do I need to place the pole is to the left of this 0 why because of this condition here positive angle condition right.

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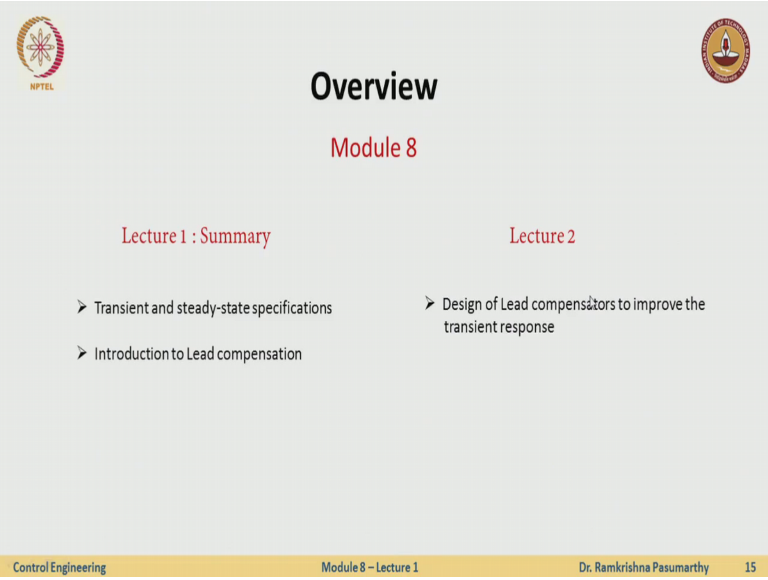

Now, we slowly see where should this guy sit and where should this guy sit right and that we will do in the in the next lecture right we will so far what we have learnt is the effect of adding a 0 helps me in transient response or improving the transient response of the system I cannot re arise is 0 by itself. So, I add a pole to make it physically realizable now we will try to ask a question where to place this poles and 0s given a certain performance metric so that we will do in a next lecture.

Thank you.