**Control Engineering**
**Dr. Ramkrishna Pasumarthy**
**Department of Electrical Engineering**
**Indian Institute of Technology, Madras**

**Module - 07**
**Lecture – 03**
**Basics of control design – Proportional, Integral and Derivative actions**

So, in the last lectures, in the last couple of lectures what we saw were designing of basic controller or compensating elements. The basic elements were a proportional element, a derivative element, a integral element or a combination of proportional and a derivative or a proportional integral control or a combination of all of these three these elements. And through some problems we saw well when can we add a PI, when can we not add a PI, what are the affects of adding just a proportional controller, what are the affects of adding just a derivative controller and so on and those problems gave us little idea of what happens actually when we try to design controllers, to do it blindly without worrying about stability or not is what PI controller asks us to be careful of right. The derivative control we saw had it is in the transient response of the system in case when a proportional control is not good enough to stabilise my system. So, what we will do here is to do a further more analysis of those components.

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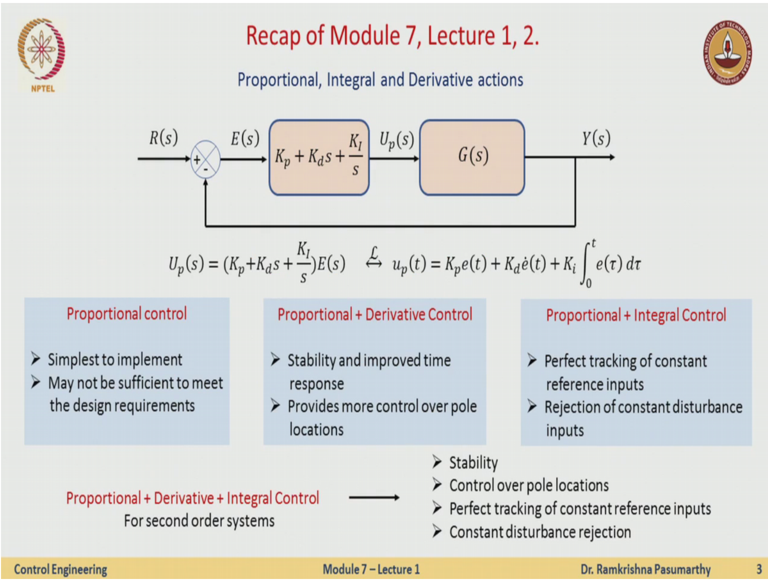

So, what we did so far is well just to sum up. So, we had a proportional element, a derivative element and an integral element, the standard thing is just to be the objective being to track a certain reference at the output. So, well proportional controller its well what we saw was it very very simply simple to implement, but it may not be able to meet some specifications sometimes it is not even good enough to stabilise the system.

Stability, when I look at proportional and the derivative control we saw that it actually had a very good transient response it could stabilise the systems and then based on the value of the K d which added damping into the into a simple second order system it provides more control over where I should place my poles of the closed loop systems. Integral control well it had well perfect tracking of constant reference inputs because it has a 0, adds a 0 at the origin and I know that of type one systems if the reference signal is a step then I could track it perfectly. And it also had good affect of rejection of constant disturbance inputs where we saw that if the plant had some 0s at sorry some poles at the origin they were not good enough to neglect or reduce or reject the affect of disturbance in that case we actually needed to have a controller which had a pole at the origin.

So, for second order systems all these proportional derivative and integral control they add to stability. I can also look at where to place the poles placing the poles I know has lot of affect on the relative stability of the system perfect tracking of constant reference inputs and also I could reject constant disturbances with the help of an integral control.

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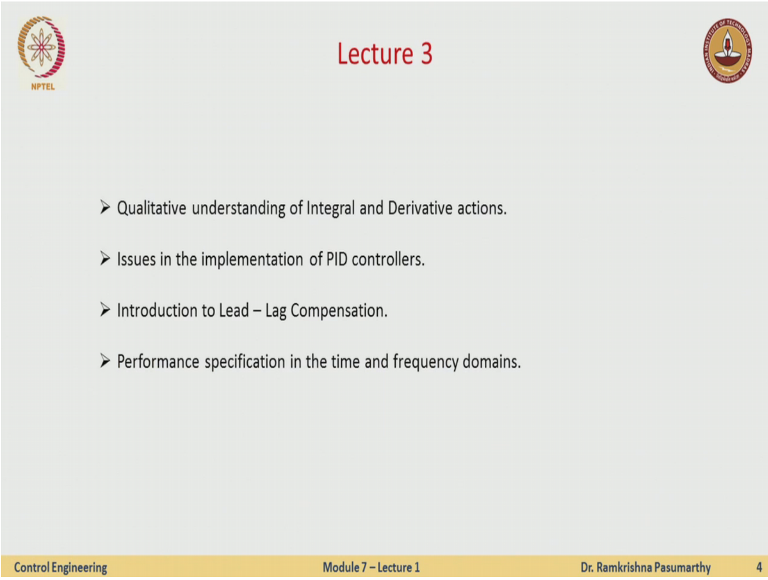


So, what we will do here is to just get little more understanding of this integral and derivative actions and what are the practical issues while I implement PID controllers and this would lead us to some kind of approximation of this PID components called the lead and the lag compensation right. And then of course, we will also look at this performance specifications both in the time and the frequency domain and see how they actually like connect to each other.

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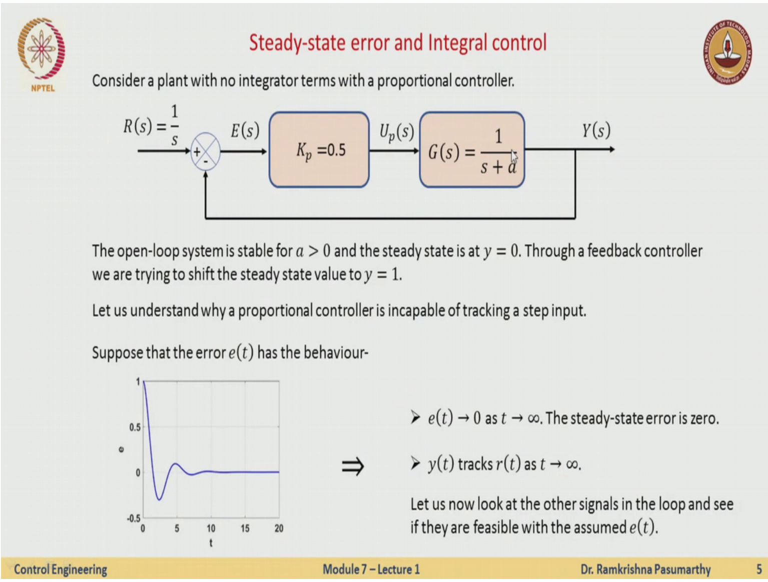

So, let us start with a very basic first order system with a proportional controller what do I usually want to do is to track a reference at the output. So, as long as a is greater than 0 I know that this system is stable and it will have a steady state value of y equal to 0 right some response like this. And through a feedback controller I just want to see if I could try to shift this steady state value to 1 right for example, if I am just something is lying on the table, if were to lift it I will just need some external force right can I just lift it to some 1 meter above the ground right. So, here as we were through a feedback controller can I try to shift the steady state position to one.

So, let us try to understand does a proportional controller help me help me achieve this. So, what is the structure so I have the reference? I have the output the error goes via K p and the U p is K p times E s and this goes as a input to the controller and then generates an output. So, typically what I would like is e t should go to 0 as t goes to infinity right or this steady state error should be 0 or which means that this y should track this reference asymptomatically right.

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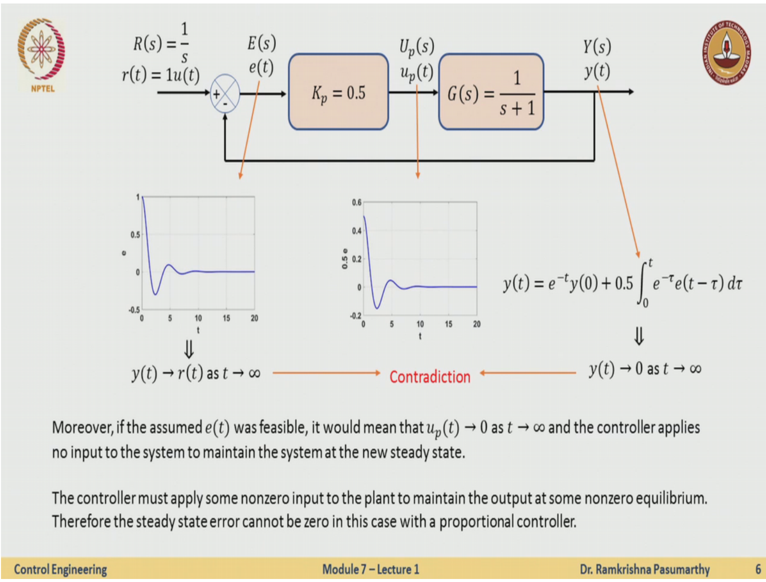


So, let know this try to understand this with the help of the block diagram here right. So, let us just see with this help of this block diagram just what do I want I want e to go to 0 as t goes to infinity or y to track R as time goes to infinity let us see how this propagates. So, once e goes to 0 then I will have that this is 0.5 times 0 this is also 0. So, this will be 0 now, what is this actually is - this actually is computed to be this one which is actually

0, now this is 0 I go back here and I compare this with this I see that the error is actually not 0.

So, there is there is actually a contradiction here I cannot have the steady state error to be 0 and in this setting it could be something else. I am not really worried about that something else I am just worried about keeping my steady state error to 0 or y tracking R asymptotically right which means that if this was assumed to be true or if this was feasible it would mean that this actual input to the plant would be 0.5 times 0 and the controller applies no input right its natural state is 0 without applying any input I cannot get any output here right. So, the controller applies no input to the system to maintain the system at the new steady state which contradicts this is contradiction right because if I want were to hold this at some nonzero steady state the controller must supply some input right the controller must apply some nonzero input to the plant to maintain the output at some nonzero value or a nonzero equilibrium if I may call. So, therefore, in this case the steady state error cannot be 0 you are only worried is the steady state error 0 when I have a first order plant with a proportional controller in the closed loop tracking a step well the answer is no. We also know these answers via the time domain analysis, we also know the answers via the error constants which we had found out.

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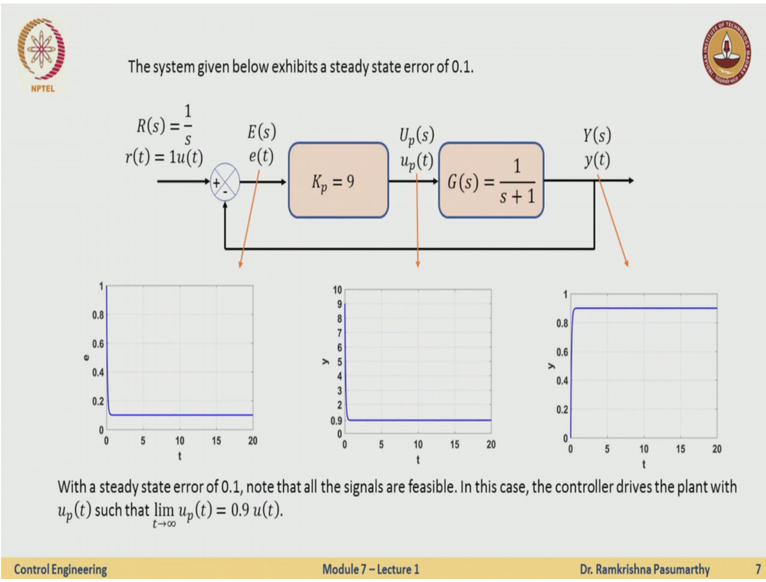


Now, let us say well I am not really worried about having a steady state error of 0, but some number. So, if I take this plant controller configuration then I can compute that the error is point one right. So, this is again for follows the first order system analysis which were in the earlier course I think module four or something like that.

Now let us see how is this consistent. So, there is a steady state error of 1 this error gets multiplied by 9 and I have a input at 0.1 input signal here as 0.9 and then the output because of this input will be 0.9 at steady state. So, I am holding this guy at 0.9, but to hold this guy at 0.9 I am actually having a nonzero input right which is actually also 0.9 right. So, I can with some adjustment of this guy reduce it reduce the steady state error, but of course, for the steady state error go to 0 this has to go to infinity and again it will lead to some other contradictions. The steady state will error will never go to 0, but it we can reduce it to some small number depending on the choice of the K p right.

So, the proportional controller it reduces the steady state error, but does not eliminate it completely. Of course, there are practical issues of noise amplification and things like that when I choose K p to be very large it may not be practically feasible sometimes and could come up with some other issues ok.

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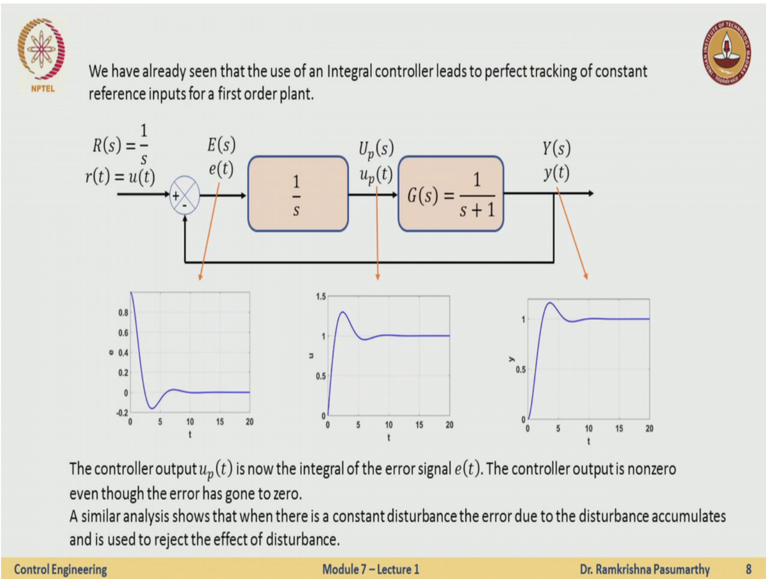


So, can I had an integral control? Well integral control would mean that this had an integrated here this would increase the type of this system to 1 and I know by the error constant analysis that type one system if I have a step here I will track it perfectly right.

So, what does it mean again in terms of this signals? So, if I compute the error signal then the error is something like this and it goes to 0 asymptotically now the input has some value like this right it has some input value even at steady state and based on this input value the output is tracking the reference asymptotically. So, U p this guy is now one over s times e t or in if I talk in terms of the time domain it is a integral of the error right this signal you integrate it and you get U p earlier it was just getting multiplied if it was 0 it was 0.5 times 0 plus 5, 0.5 times 5 and so on right. So, the controller input is now the integral of the error signal right. And when I do this what I also observed is the controller output is nonzero see it is all always nonzero. So, there are some points here, the error goes to 0 error goes to 0 and whatever, but the control input is nonzero even though the error has gone to 0.

Similarly, I could also show we also did this in some problems earlier that when there is a constant disturbance which could possibly come from here the error due to disturbance accumulates and it is used to reject the affect of disturbance. So, in this case the output of the controller is the integral of the error soothe controller output which is the input to the plant right this guy is nonzero even though the error has gone to 0 because you see here the error has gone to 0 the input to the plant is nonzero right and then for as it goes to 0 there is some steady state value for the input right. Similarly or else I could also do for the disturbance rejection as we have seen in some problems earlier. So, is that all, is life now easy that just add an integrator and then steady state errors are gone well it also comes with one set of rules.

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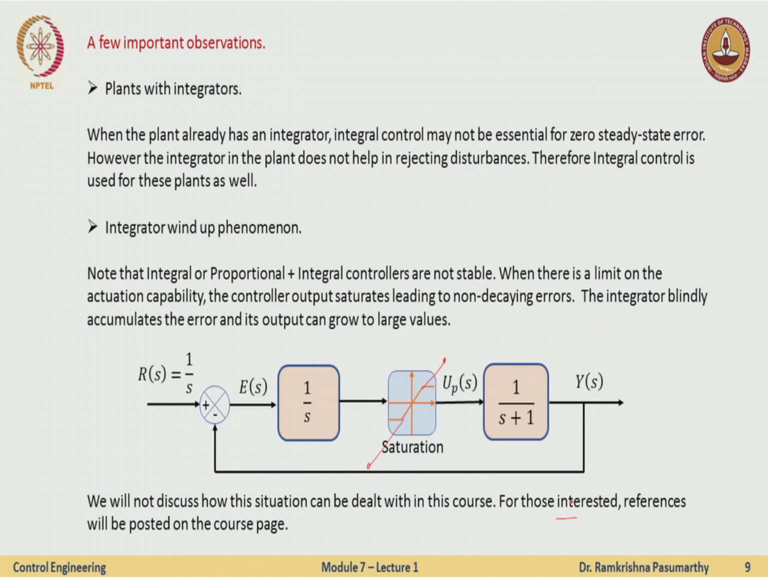


So, what are the important points to observe when I have with integrator? When the plant already has an integrator the integral control may not be necessary for 0 steady state error that we are also seen in problems earlier that I had a second order system it was tracking a step signal perfectly asymptotically right.

So, and we also saw that the integrator in the plant does not help in disturbances rejection right and therefore, for this kind of scenarios I would need to have an integral control via the compensator or the controller.

Now second thing is called the integral wind up phenomena and this is very very practical issue right. So, first is well these guys are not always stable right I have a pole at the origin right and if I just it could add best the marginally stable right. And then not only that well there is a limit on the actuation capability let us say my actuation signal is some kind of force right which I cannot just apply infinite force right if I just buy an equipment which could give me some force it is bounded by a above and below by some values right that phenomena is usually called as the actuator saturation right. So, in this case well this is my controller input n. So, the input to the controller from the from the error it goes via this and this U p is now has to go through some functions like this because my control action or my actuator has a limit here which does not allow the input force or the input voltage to go beyond a certain value.

So, what happens here is that I may need if I just you know roughly draw a graph I may need an input which is somewhere here or a input which is somewhere here n to drive the error to 0 or to meet some performance specifications and so on, but well my thing my controller via this actuated could just give me you know may be four volts even though I need 10 volts right. So, in this case what happens is that the integral, there will of course, an error. So, I need 10 volts, but I am getting is 4 volts the error will still keep on accumulating and then the output could go to very large values right. So, it does not really work in cases where I have some kind of an of limitation in my actuator capabilities.

So, that also I need to be careful of right we will not really discuss on how these kind of situations will be will be handled in practice right, but this is good to know that it is it just that the integral control action even though it looks extremely beautiful , it looks very simple, just 1 over s, it eliminates lots of steady state errors, it rejects disturbances, but it comes with two you know two of these this not really drawbacks, but something which we should be careful of while we are deciding a system.

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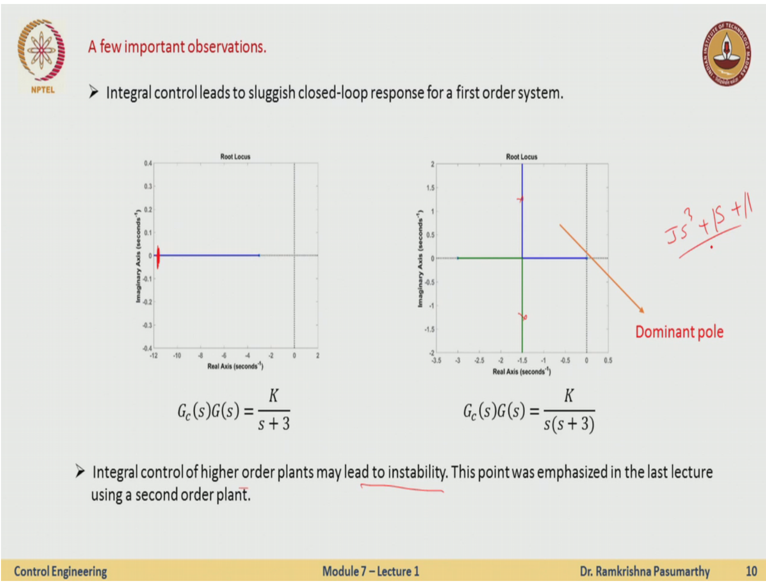


Also what we could observe is that an integral control leads to sluggish close loop response for first order system. What does this mean? Just take well normal proportional controller and a plant as 1 over s plus 3, the root locus will tell me that as K increases the changes I just transfer way to infinity. Now if I add an integral control which means I am adding a pole at the origin then my root locus is like somewhere here right here and here. So, I am actually having my dominant say for a here for a larger value of K and somewhere here if I just put the same value of K here I might just end up somewhere here. So, this means that for that value of K where you know my pole in without the integral action was say at minus 12 or minus 10 or somewhere in between this response is faster than just having a poles or this complex poles at minus 1.5 plus minus j something right which means the response has becomes sluggish which means my poles or what I would now define as the dominant poles are now closer to the origin and that might. So, here this system might be oscillatory here they are not oscillatory ok.

So, and then if I look at higher order plants then we lead to n instability as we have seen in the problems slide. So, we just had some cases where I just had j s square plus something times s plus 1 and you get some constants here, and no matter what the volumes of the constants were this system is always unstable right.

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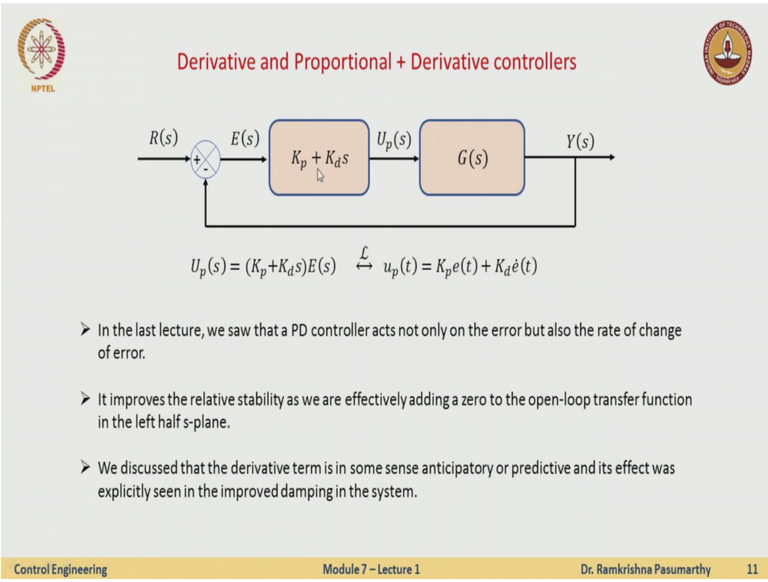


Now, what about the original plus derivative control action? The input to the plant now comes out later looks K plus K d s times E s K p times e by e t plus K d times the derivative of the error right. So, what we had seen earlier in the first lecture of this module was that a derivative control not only acts on the error, but also on the rate of change of error which improves the relative stability right. So, we are adding a 0 to the open loop transfer function in the left half plain and this 0 has the affect of pulling the root locus to the left right away from the imaginary axis in the stable region of the more away I am from the imaginary axis the more I can say that I am more, more and more relatively stable right.

And not only that right the derivative term is in some sense it has some anticipatory affect or some anticipatory behaviour it predicts how fast the error is changing and the and therefore, it could take corrective actions much better than if I would not have this e dot term.

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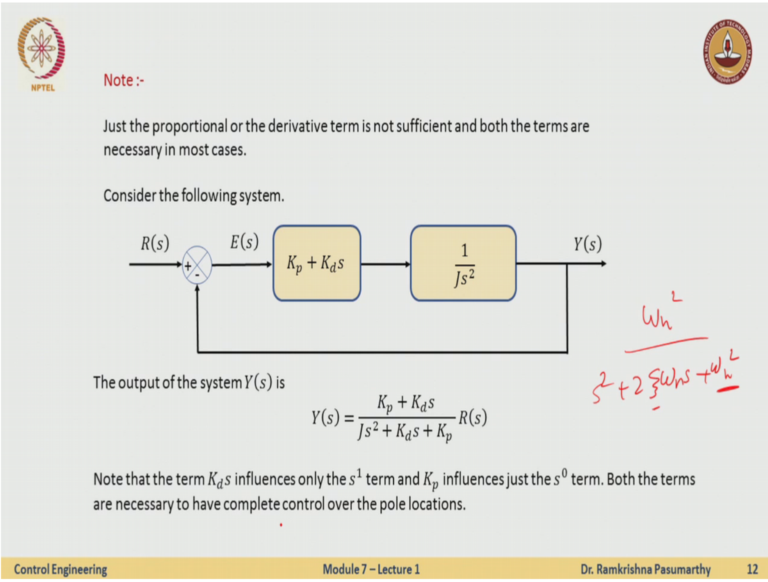


So, what is the affect of adding this proportional and derivative terms right? We have seen this again in earlier problems, but it is its good for us to remind us a (Refer Time: 18:05). So, this j square if it just had a K p without a K d I now that it is not stable that the roots will always be on the imaginary axis as soon as I add a K d I have I introduce a damping term and by proper choice of this K d I could choose this system is under damped over damped cortically damped or I can play around with the response of the system right.

Now K d influences only the s power 1 term p influences s power 0 term right, but then well to have control over the close loop pole locations I need two things right. So, the standard thing was I had omega n square s square plus 2 zeta omega n s plus n square and this omega n together with zeta characterise the complete behaviour of the close loop system right. So, therefore, here both these terms K and K d is necessary to have complete control over the close loop poles.

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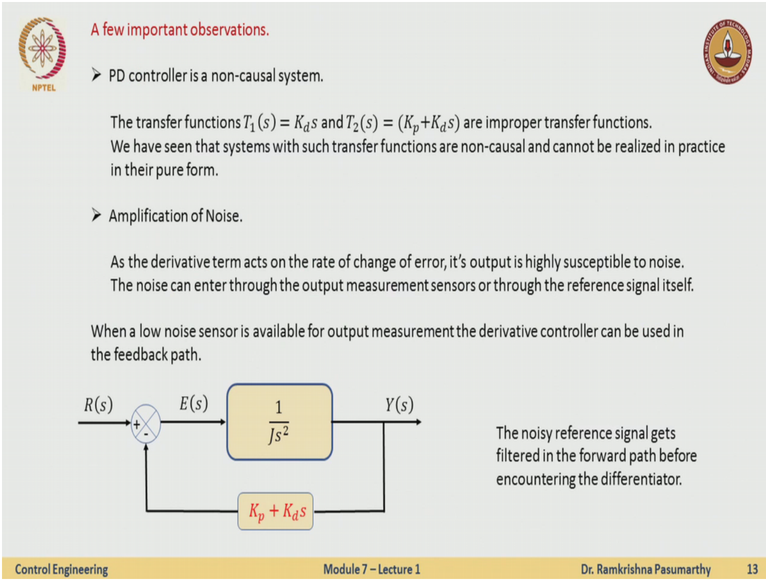


So, in the previous slides we had said well the derivatives term in some sense anticipatory or predictable it can tell it was is going to happen in the future and these systems we called as non causal systems moreover while defining transfer functions we said. Or we defined a proper transfer function in such a way that the number of poles should be equal to the number of 0s or the number of poles should be greater than the number of 0s where the number of 0s were great, were greater than the number of poles as in this case there is one 0 and no pole this were improper transfer functions and I could not realise or I would not be able to realise in practice right or I can just cannot just go to the market or say amazon dot com and say well I am was looking for a derivative control that will not happen right.

And amplification of noise when I am looking at a derivative component here and say I have first you know like where you have frequency noise the affect of the derivative could be very bad in terms of the amplification of the noise right. So, the derivative term acts on the rate of the error right and therefore, the output is highly susceptible to noise if it is changing or an higher frequency and this noises can cancel can come either through the measurement censors or through the reference signal itself. So, just for practical purposes sometimes it is good for us to add the derivative control in the feedback loop if

I have no noise censor right. So, I can then affect, so I can reduce the affect of noise which are evaluating through this right. So, in this case the noisy reference signal gets filtered before or it gets filtered in the forward path before encountering a differentiate it does not the noisy signal does not really affect or encounter the differentiator right, but of course, still I have do take care of the noise which is coming from y of s that is a bit of a trade off here, but just some issues which we have to be careful of while dealing with this censor practice.

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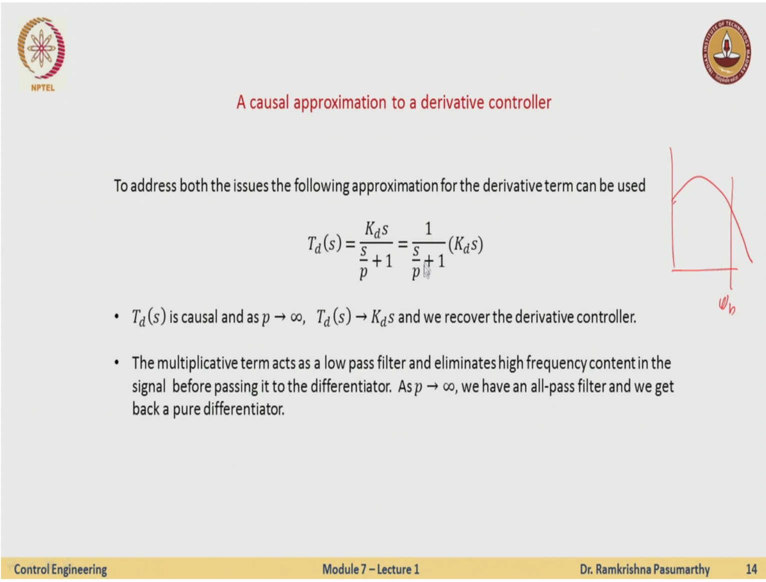


So, the point is now well I have this derivative controller I design nice properties and I also told you that cannot buy it in the market well. So, what do I do? Well can I make an approximation, to address both issues I will tell you why I am addressing actually both of these issues amplification of noise and I can also take care of this non causality to address both these issues I have an approximation of a p d controller now right. So, in such a way that as so I just if you see here I am just taking K d s which is the actual derivative term and I am adding a pole here and I do this s over p just to maintain the d p gain. So, t d s is causal I have one pole one 0. Now and as p goes to infinity t d goes to K d s and we recover the original derivative controller again which is non causal ok.

So, what does this guy do? 1 over s over p plus one this acts as a low pass filter right as we saw in that the frequency response well it goes something like this right and you have a some kind of bandwidth right a cut off frequency as we had called. So, this multiplicative term acts as a low pass filter and eliminates high frequency content before passing through the differentiator right and usually noise is a high frequency signal so there. So, if I just add a add a additional pole here that will take care of the high frequency signals right and of course, again as p goes to infinity we have an low pass filter and we get back a pure differentiator.

So, I have done two things here I have made this guy to be causal or a realisable transfer function still comes with its own set of you know it still not done and what we also saw is that I can place the poles further away from the origin. So, in one of the earlier lectures we saw what is the affect of adding a pole to a transfer function what is the affect of adding a 0 to transfer function if I add poles to further to the left then the affect becomes small and small right. So, I can smartly choose the location of poles such that the affect is like a derivative controller it is causal and it also eliminates this the issues related to amplification of noise right.

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Now this brings us to some kind of approximation of this PID controllers which are in text referred to as lead and lag compensation.

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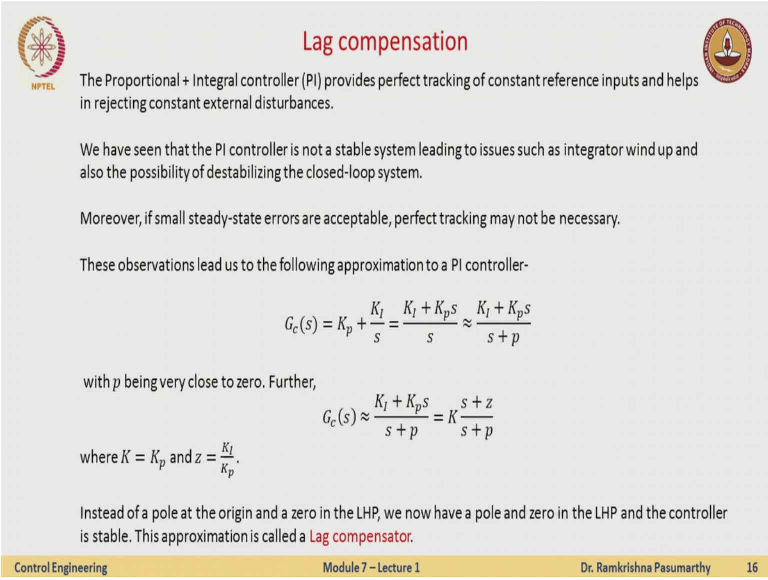


Well just start with a lag compensation which essentially is an approximation of a proportional plus integral controller a PI controller well I know provides perfect tracking for constant reference inputs and helps in rejecting constant external disturbances we said this repeatedly will keep saying this over and over right. Of course, this is not by itself stable it has a pole at ground at the origin and so therefore, well it is not a stable system and we also saw we have this integrator wind up property and also in some cases we had seen for higher order plants that it can actually destabilise the closed loop system. I add a integrator and I do the math it says steady state error is 0, but say hold on you know the system is actually unstable you cannot apply the final value theorem blindly.

So, now let us say let me say take a condition where small steady state errors are acceptable right. In that case a perfect tracking may not be necessary right if I look at the cricket ball I do this and it goes to I do this right it goes to the d r s its right even if the you know ball is hitting the stamps and if the margin of error is say point 0 one millimetre I am I am with that right it will still be the umpires call 3 reds or 3 greens and all those drama which adds up while we are watching while we are watching cricket these days, in that case a some small steady state errors are acceptable right. Now these things leads us to the following approximation of a PI controller.

So, we start with a ideal one which is K p plus K I over s that is K I over K p plus s, but I just add not a pole at the origin, but very close to the origin could be as much as 0.10

with p being very close to 0. Now the design procedure we will see how close to 0 it should be right. So, and further well this now looks like a gain K a 0 and a pole in such a way that the pole is very close to the origin and some 0 right this we will see how to design this right where K is K p and z is K I over K p right. So, the approximation of PI which is a lag compensator what does it do - instead of having a pole at the origin and see this was the pole at the origin a 0 on the left of plain. Now we have a pole and 0 both in the left of plain right and it is a stable controller right z is less than 0 p is less than 0 it is always stable and this approximation is called a lag compensator.

Now, we may ask ourselves from when you know actually you are actually you know playing around too much with a system right you are teaching us something and, but then you are approximating now we will see is the affect of these approximation disastrous or is this good enough.

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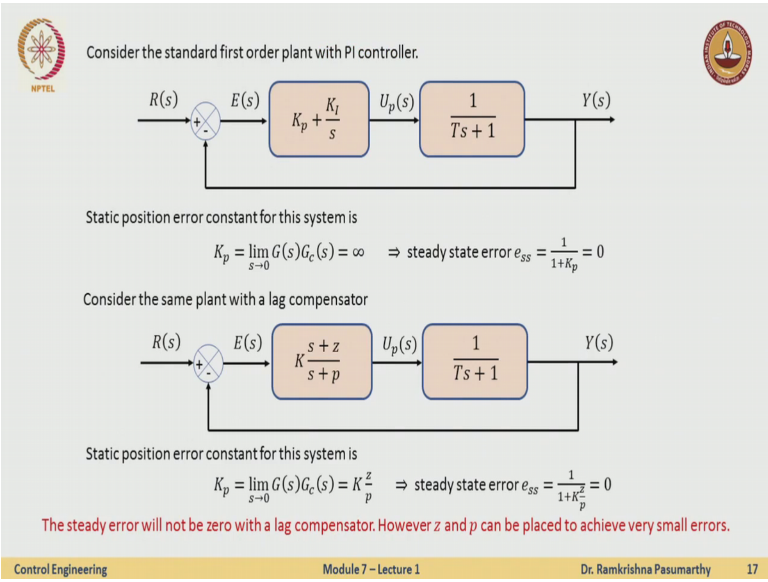


So, again we will start with the first order plant I have this PI controller and the static position error you know all these I do the error as say well because of this integral action the steady state error goes to 0. Now consider the same plant with a lag compensator this was the ideal one right which we said has very good properties in addition to some not very good properties too, now if I do this K s plus z s plus p and I pass it through my plant I do all the error analysis and what I get is that the steady state error is 1 over 1 plus K z over p this p is very small 0.001 or 0.01 and say this you know so based on these choices I can make these error as close to 0 as possible right. So, this is not equal to 0, but this is well as close to 0 as possible. So, that would be the objective right.

The steady state error will not be 0 with a lag compensator right. However what is in my in my control I can have K I have z and I have p and I can make this as large as possible to achieve as smaller errors as possible. Now what is the affect of adding the lag compensator on the root locus?

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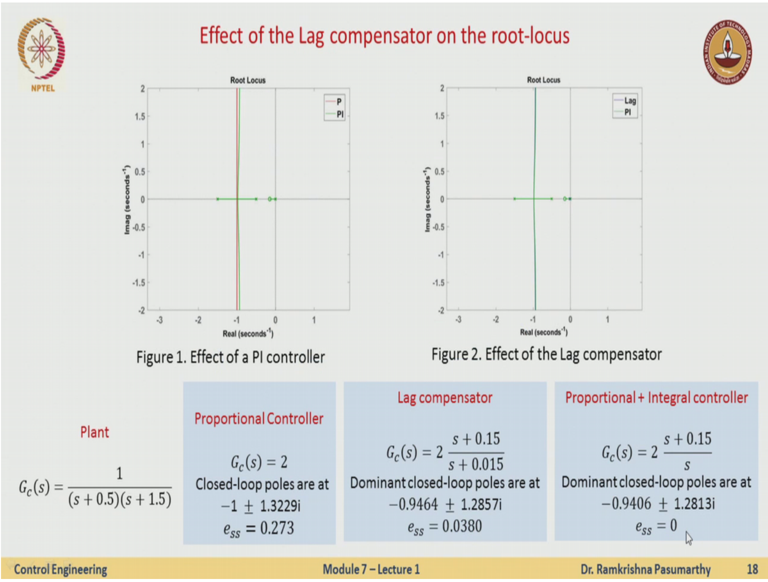


So, let me start with a plant well it just kind of fairly good looking I have two poles and there is no 0, so everybody goes to infinity and just by adding a proportional controller well I see you know this the red line right. So, the poles are always at an this complex conjugate poles and then I have a certain steady state error sorry this I could compute by hand.

Now if add a lag compensator first let us see what is the ideal case right the proportional plus integral controller I do all these things and get a steady state error to be 0 which theoretically I learn I have been learning since a very long time now. Now if I instead of this I do something like this I still have s plus 0.15 and I have a pole very very close to the origin just like 0.015 right and then the dominant closed loop poles are at this location and some somewhere here right. And then the steady state error instead of 0 is now 0.03 which is you know a kind of fairly acceptable right and if you look at the root locus it does not really change much. So, the qualitative behaviour of the system with respect to gain has not changed much if not little it will change via very small margin, but we still be in a very safe zone right.

So, I will start with 0 and 0.05 this is right. So, what these plots tell us right, so I have a plot here with the p and a PI. So, and all the time well we talking of p I we were talking of perfect tracking right disturbance rejection. So, all these are steady state behaviour these are all important add the steady state, now while I am playing around with a steady state behaviour or I am trying to improve the steady state behaviour does it have any bad influence on the transient behaviour. So, the first plot which is just adding a proportional controller and comparing it with a PI controller you see that this section is fairly close enough. So, there is not too much of compromise on my transient behaviour it will be very small right. However, now this is the ideal PI controller now I say well this is actually not very good and I need to actually have go to something called as lag compensator which is an approximation of a PI controller and here I say well they are more or less they are very they are very much the same right this show so much seem that with this plot I cannot really find out any difference if I zoom in like a hundred times possibly something would happen here right.

So, first thing is the PI controller has a good affect on the steady state performance without compromising too much on the transient behaviour and the PI is close to the lag compensator and therefore, I can conclude that the lag compensator if I implemented this way it improves my steady state performance without compromising too much on the transient behaviour. Let us say that we will keep in mind whenever we are doing a steady state error design for a or a lag compensator design.

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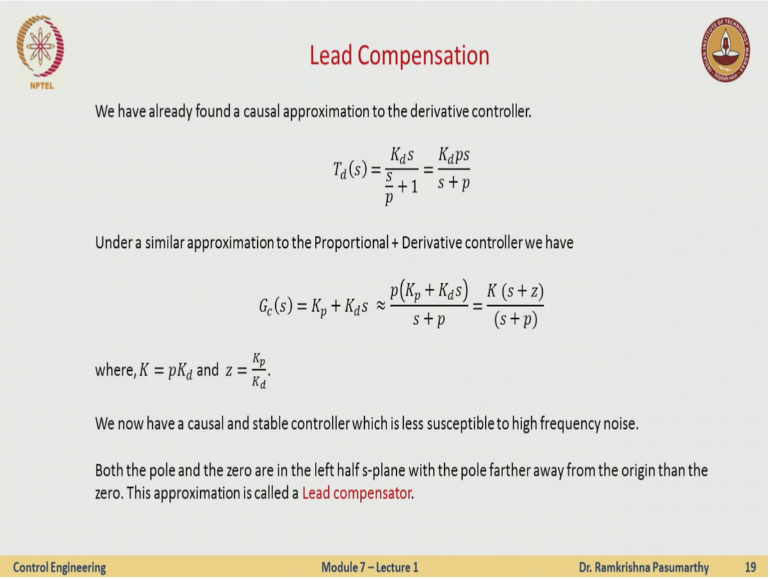


Now, having found a causal approximation for a derivative controller and we would again like to do something similar as we did for the PI controller right. So, under a single approximation, we will again K s plus z and s plus p where again K here I can find what are the constants and z and so on. Now again this is causal, this is stable and again this is less susceptible to high frequency noise right because it is not a pure derivative term right. So, both the poles and 0 are in the left of plain with a pole further away from the origin than the 0 then we will really see why this is true right. So, here so even though both the structures wore the same K s plus z K s plus p here I also had the same structure right it really depends then on the choice of the poles and 0s to decide which one is the lead compensator which one is the lag compensator and I will shortly tell you that.

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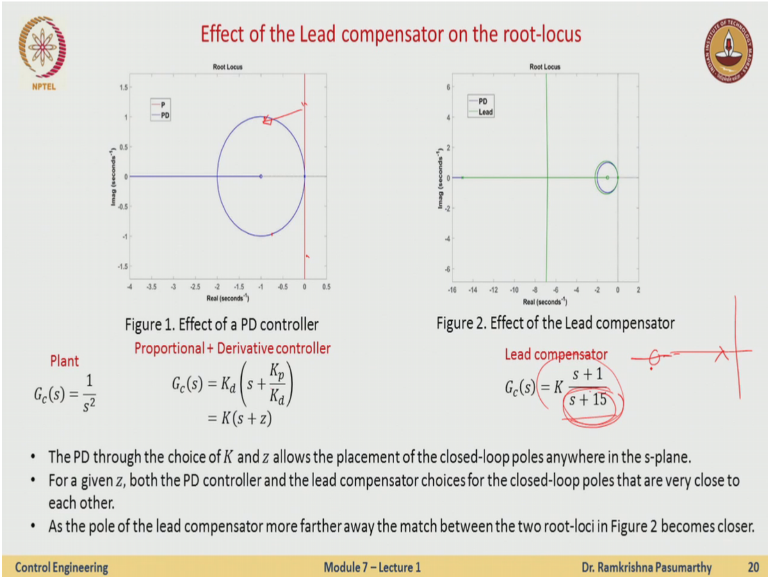


But before that let us again analyse what is the affect of adding a lead compensator on the root locus. So, what we saw earlier was adding a derivative component significantly improved my transient response without possibly much change in the steady state response.

So, start with 1 over s square for which I saw that all the time that my poles how much ever I change the gain they are on the imaginary axis. Now adding a proportional derivative control which is I am just adding 0 t just has the effect, it just pulls the root locus to the left right. So, the poles for a certain gain which were here sorry which were at this location have now becomes somewhere here right approximately right. So, through this PD control and a smart choice of K and z I can place my poles anywhere on this circle or the circle could be different for different kind of stuff right.

Now look at this thing right. So, this is desirable right that the root locus experiences a significant shift to the left and shift to the left is required because I am interested in improving the transient performance. Now this PD controller how does it compare to when I just look at a lead compensator which is an approximation, well say I have a say a 0 at minus 1 here and a pole significantly further right, now when I do this you will only see that there is not much difference right the green and the blue are fairly close enough to each other and again with some little adjustments here I can get my close loop here we to perform in a certain manner right.

So, what we conclude here is that the ideal PD controller it affects or it improves the transient performance via this kind of a behaviour that it pulls the root locus to the left it also helps in increasing the relative stability, but this approximation also does more or less same kind of a job and this is realisable this possibly is not this is never realisable right and you can say well then what does this pole do right. So, the only significant difference here is I have a s plus z, but I have a pole. So, the reason I had a pole very further away is that the affect of pole as it goes further and further to the left it reduces right as we have seen in the first lecture.

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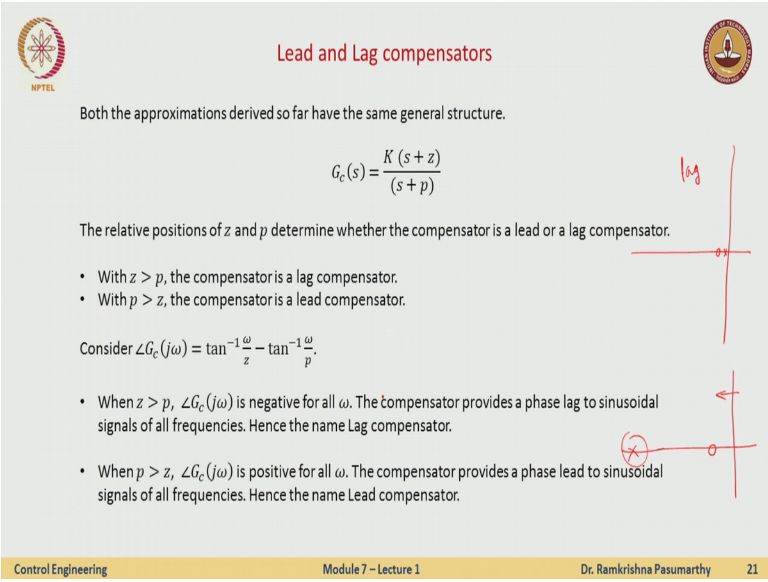


So, I will add a pole to make it a proper transfer function, but I will keep it far enough in such a way that it does not really affect my behaviour of the system right. So, both compensators now have this structure s plus z it has a pole it has a 0 it has a gain right. So, when z is greater than p then it is a lag compensator. If I were just to write down in a very you know very nice way. So, the lag compensator should have an integral action I just cannot have a pole at the origin. So, I place it somewhere very close by and then the 0 would be somewhere here right. So, that the 0 is to the left of the pole for a lag compensation what did we see for the lead compensation is that if I add a 0 here it pulls root locus to the left which is nice which is I want right it improves the transient response. But to make it realisable I have to add a pole. So, I add a pole further to the left which means the affect of this on the transients will be will be as minimal as possible right and the of course, when we will see why these are called lead and lag right. So, I will skip this discussion when we actually draw the bode plots of this compensators.

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So, having had an approximation for both PD and PI well will there exist what about PID controllers right. So, this problem is solved by using lead and lag compensators in cascade to each other. So, as to approximate a PID control action right.

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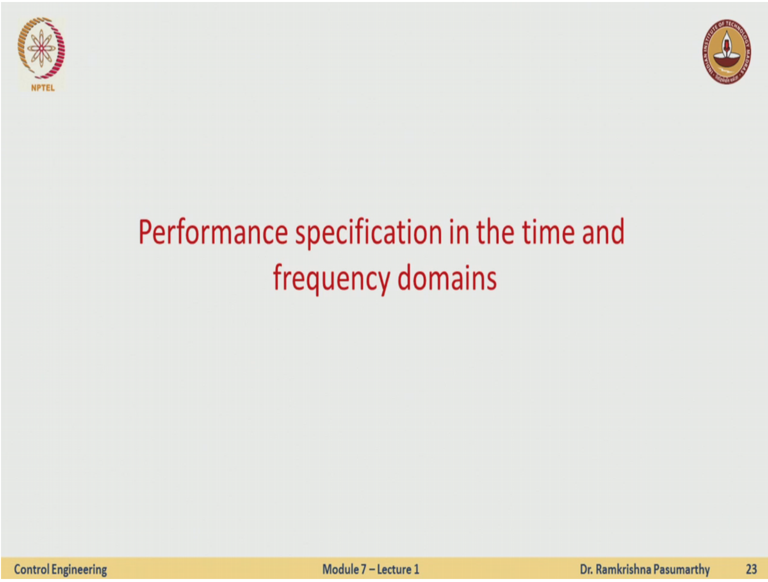


So, after all these design approximations the lead and the lag and the PID being a cascade of a lead and a lag how do we relate this to performance specifications. So, while we started our time response analysis we said that the performance was based on the overshoot t was based on the rise time, settling time, steady state error and so on, but we also had an equivalent frequency domain analysis where we talked of resonance frequency we talked of bandwidth and how are all these related to each other and how are all these related to the design specifications and how could we do this via a lead and a lag. So, before we do that we will just compare the performance specifications both in the time and the frequency domain and see if they are actually related to each other. So, so what we have done so far is from the description and specification of the close loop in terms of the overshoot as I said the rise time, settling time, steady state error and these are very nice to visualise right.

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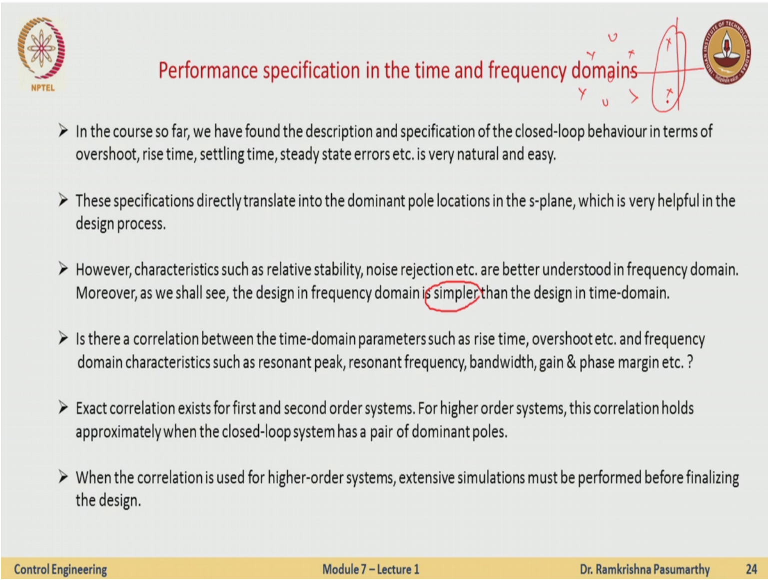


And this actually came across as finding all these, all these were like second order system analysis right. So, all these were nice when we say there are several poles in the system, but if there is dominant second order pole maybe all other guys lying further to the left and there will some 0s here 0s here. So, this is what detected mu behaviour of the closed loop system right. So, all those analysis could be just made based on this dominant closed loop systems right.

So, however, we did not really talk of talk much of relative stability noise rejection and this were nicer to understand in the frequency domain in terms of what we saw when I actually add a integral component or a pole in my what was said the lead compensator it had a good affect two good affects - one was it made that derivative controller look a little more realisable it also had the affect of attenuation of higher frequency signals which are essentially the noise signals right. So, and these are in some sense better to understand in the frequency domain and therefore, we see sometimes this parameters or the design specifications are well to loosely speaking simpler than the time domain. Simpler not in terms of that the problems which I gave in frequency domain are simpler than the time domain it will actually turn out to be the reverse that you know this might need a little more you know (Refer Time: 39:38) or some approximations than the design data domain. So, simplicity comes from this from one or two parameters I can actually get lots of information right.

So, before we do that we just find out is there a correlation between these parameters such as the rise time overshoot to gets to the frequency domain parameters which were the resonant peak the resonant frequency bandwidth the gain margin phase margin and so on. Again all over analysis for obvious reasons of the dominant pull behaviour was or we tried to find this correlation for second order systems right this correlation holds very nicely when the closed loop systems has a pair of dominant poles right. And for higher order systems where there is possibly know concept of this domination poles we may have to do some better simulations to achieve (Refer Time: 40:31), but for our analysis purpose we just restrict ourselves to the dominant poles and that also gives us lots of rich information. So, starting with a second order system I know all these formulas now, well possibly by heart right. So, peak overshoot and the settling time, the rise time and I have this error constants K p K v K a and so on right.

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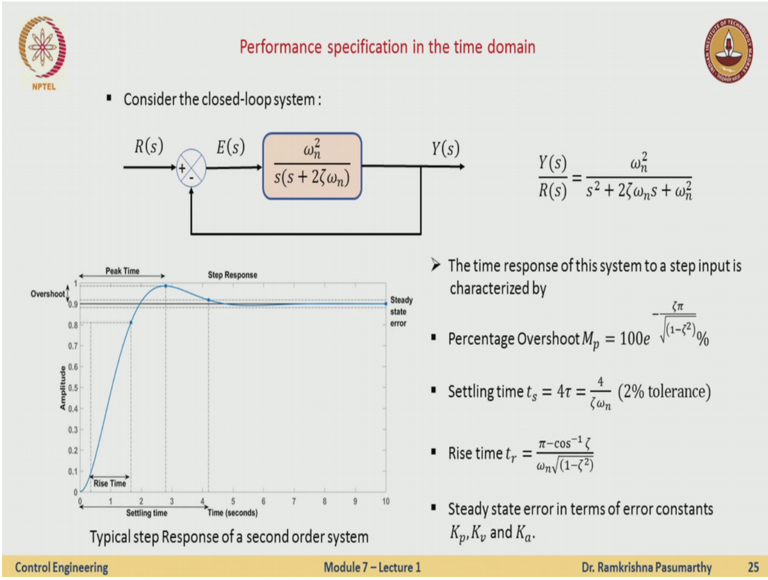


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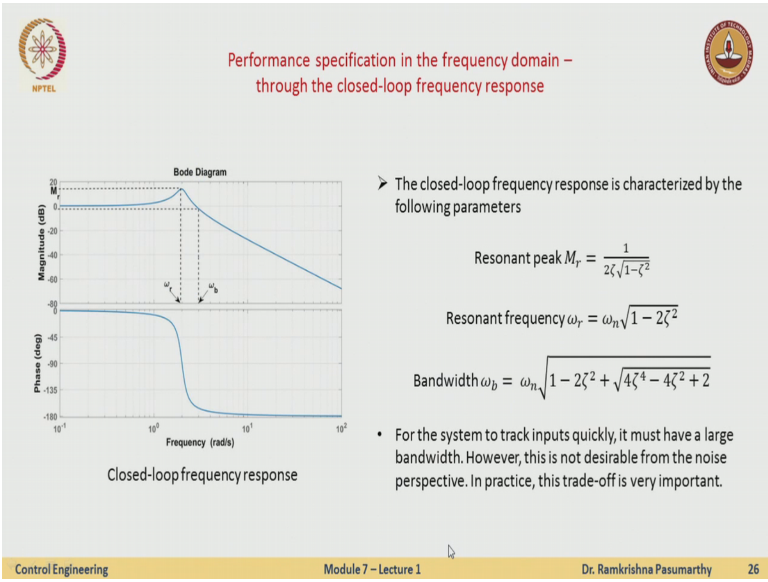


And at the same time I also have if I have a frequency response like this, this is Bode diagram of a typical second order transfer function I will have a resonant peak. So, this is the resonant peak occurring at some resonant frequency and these formulas we derived earlier. It will also have a bandwidth right this one of this is a ugly looking expression, but I will just quickly tell you how to do it by hand.

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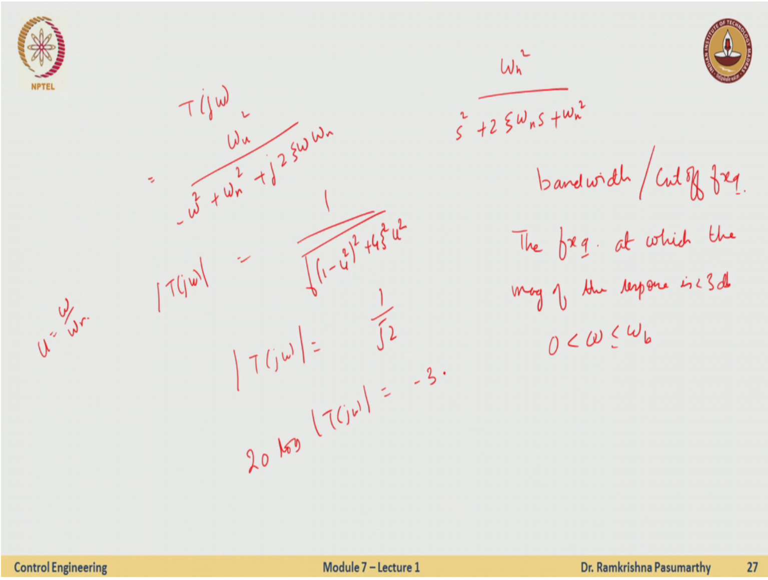


So, I start with a second order system which is omega n square by s square plus twice zeta omega n s plus omega n square then I do the sinusoidal version of it the T of j omega is omega n square over minus omega square omega n square plus 2 j two zeta omega and then this is should be j omega. So, I will have another omega n. So, when I define the bandwidth I look at this thing right. So, at which the magnitude is about point seven times the magnitude at a here. So, the magnitude of this guy t of j omega this 1 over square root 1 minus U square whole square plus 4 zeta square U square where U I had defined earlier it also to be omega over omega n right. So, first is how do you define the bandwidth or even the cut off frequency right. So, this is the frequency at which the magnitude of the response now in terms of the frequency response not the time response is three d b or below right and the bandwidth is not define as this 0 less than omega less omega b right.

So, how do we find this we just looked at take the transfer function and evaluate it to 1 over square root 3 in the absolute scale or in the in the log scale we say 20 log of t j omega should be minus 3 and I will just keep the computations because it is just kind of the business very manual right you can just do it by yourself. And then I can say that the bandwidth is related to zeta and omega n via this you know kind of ugly looking expressions square root inside the square root and so on.

So, I will not I ask you to differentiate this or anything like that, but we will just try to not even remember this formula only to show that there exists a relationship between the bandwidth which is essentially a frequency domain concept which relates directly to omega n and zeta which came from the time domain specifications right. For a system to track inputs quickly it must have a large bandwidth this again comes from here right what is bandwidth is depending upon omega n and zeta right. And however, sometimes it will be it is desirable from the noise perspective right because if I have a higher bandwidth as I saw when I were just having a pure derivative controller it was an all pass filter and it had the drawback of not dealing very well with high frequency noise signals right. So the impact is we will actually look at tradeoffs between good and bad right.

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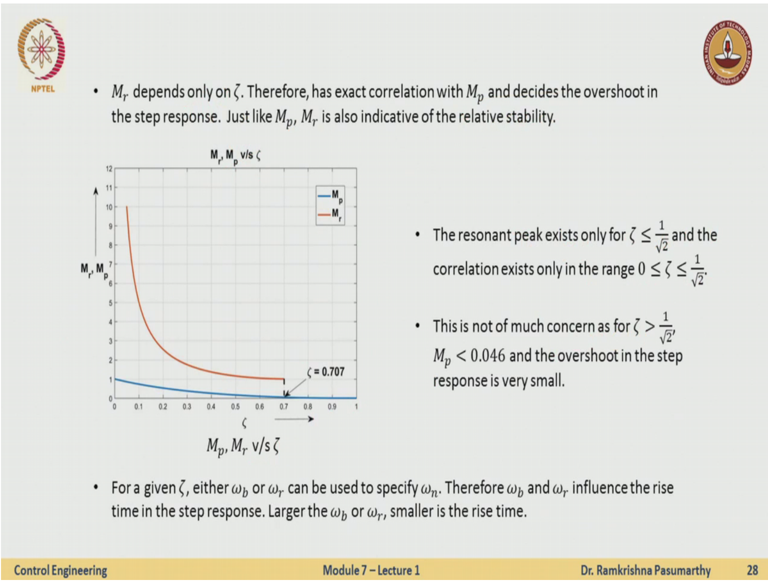


Now the M R or the resonant peak it depends only on zeta. So, somewhere here and then we also have this have the formula for the resonant frequency. So, in this case well it has an exact correlation with M p if I just draw M r. So, look at these two formulas right they are they are kind of nice. So, M R is directly depending on sorry M p the peak overshoot in the time domain specifications is directly depending on zeta nothing else similarly the M R also has a relation. So, if I find an M R here with use me a value I could correspondingly find a value for M p right.

Now can I do it all the time well let us just see that a the resonant peak only exists for these frequencies right and therefore, whenever I look at the correlation between M p and just from here till here from points point till the it is this point seven or seven right if you just because of this guy over here. So, for these guys well I can just see that you know there is a direct correlation between M p and M R of course, this is not defined for zeta equal to 0 right. And then this is this is really too much of concern not really because the peak overshoot for zeta being greater than 1 over square root 2 is very small right. So, for a given zeta either the bandwidth or the resonant frequency can be used to specify omega n and all these guys.

Given this guy you know I can compute these things right and therefore, the bandwidth and the resonance frequency influence the rise time in the step response and of course, I can just you know based on the formula for the rise time I can compute when will how is this related to the bandwidth or w r larger the bandwidth smaller is the rise time and so on could just be computed by just looking at this expressions here right. So, this is my bandwidth and this one.

So, the aim is not to introduce you to complex formulas which are difficult to derive and even worse to by heart, but just to show that there actually exists a good relation between the time and the frequency domain things and in your ends an exam whenever you take or if you take this I will give you all the formulas which are required for you. So, you do not really need to need to memorise formulas, you just need to know which how is approximately each of the formula looks like.

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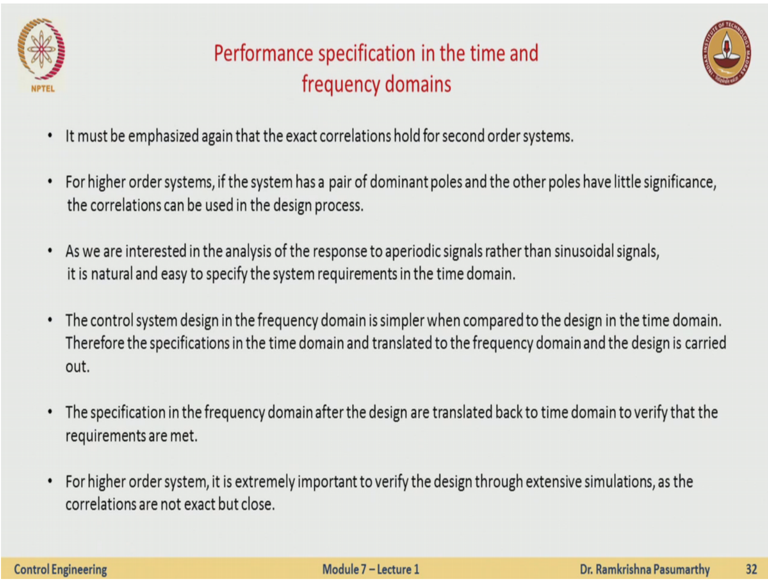

So, what are what can we conclude or based on all these things right. That exact correlation holds for second order system between the time and the frequency domain specifications, for higher order systems if there is a pair of dominant poles then we can still use this correlation right and as said earlier well the frequency domain control design gives us little more information than in the time domain I will still very carefully use the word simpler right. And of course, the specification in the frequency domain after design are translated back to time domain to verify if the time domain requirements are met or not. And again for higher order systems we may need to do designs through extensive simulations, but then these are not these are not very important words at the moment right. So, basic understanding of the second order system will give us good amount of intuition while we do this an analysis for higher order systems just based on some lots of experiments and simulations.

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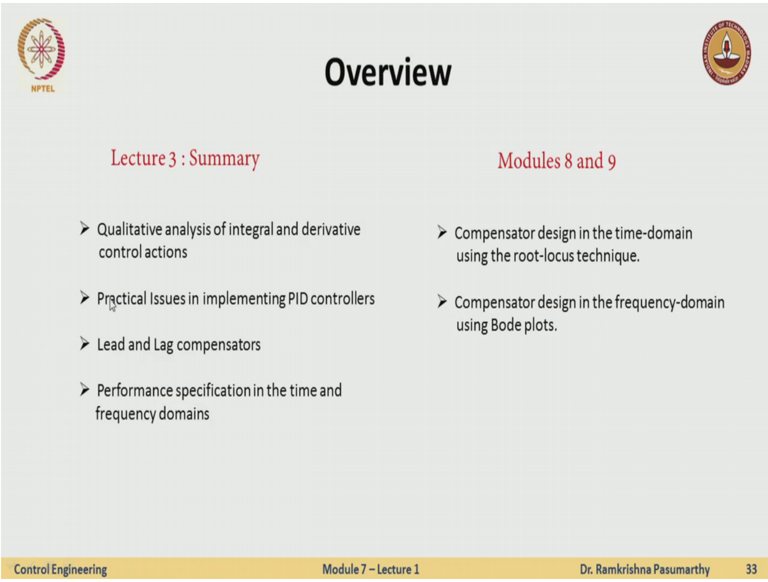


So, what we have done so far is to have a qualitative analysis of both the integral and derivative components. We have also saw what were the practical issues of realizability well we saw accurate saturation and so on, and we had some good approximation of this components called lead and lag compensators. And we saw also how the performance specifications in time and frequency domain are nicely interchangeable.

So, next we will see how to implement these or how to design these lead lag and lag compensators especially find the value of K find the value of z find the value of p. So, this were the important things right when we were realising those lead and lag compensators. We will do that using the time domain or essentially the root locus technique and also the Bode plot technique which is essentially a frequency domain technique. So, and that we will we will try and use lots of MATLAB, we will plot lots of things for ourselves, we will ask lots of questions before we come to a conclusion and hopefully that would be a little more little more intuitive while using MATLAB.

Thank you.