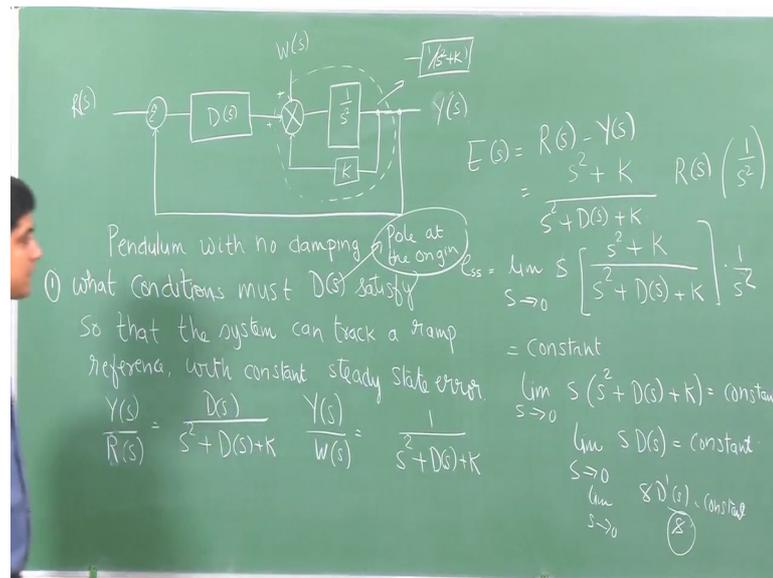


Control Engineering
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Module - 07
Lecture – 30
Tutorial

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So last time we had learnt the basics of PID controller or what was essentially like a proportional component, an integral component and a derivative component right. So, before we go into further analysis let us do some problems to build up a case for each of these components is are they useful are they not always useful is it sometimes not very you know wise enough widely add a component. So, that it reduces my steady state error and so on. So, we will slowly build that up with some with some motivating problems and then go back to analysis of that. So, I start with a simple looking example, which is like a pendulum with no damping and I will tell you why this represents a pendulum with no damping and then you have a controller and we expect this controller to do a set of chains for us right.

So, you have a reference input and then you the output the disturbance and the standard feedback configuration right. So, this little block here, if I do the block reduction this block would just be a series block of the form 1 over S square plus K right and this has the imaginary roots plus minus square root of k with a you know with a with a J right and

that response is oscillatory, and the response is always oscillatory and therefore, I call it is a system with no damping.

And these are the dynamics of the of a simple pendulum right we did in one of our very first lectures. So, first question which we would like to answer is what conditions D must satisfy so that the system can track a ramp reference. So, what should be are there specifications on the steady state error. Well the specification is my tracks are ramp reference with constant steady state error ok. So, first thing we would like to do is to find out transfer function right I will not spend time of this on this because we know how to do this out. So, this would be D of s over s square plus D s plus K .

So, this should be y over r and similarly Y of s or the affect of disturbance on the output is another transfer function of the form 1 over s square plus D of s plus K . Now what does the problem require me to do well it requires me to find what must what are the conditions on D if for the exact nature of D so that the system tracks the ramp with a constant steady state error. This is very similar to what we did while finding out the error constants right at type one system can track a velocity ramp signal with certain steady state error and so on where a little (Refer Time: 04:18) for that brand ok.

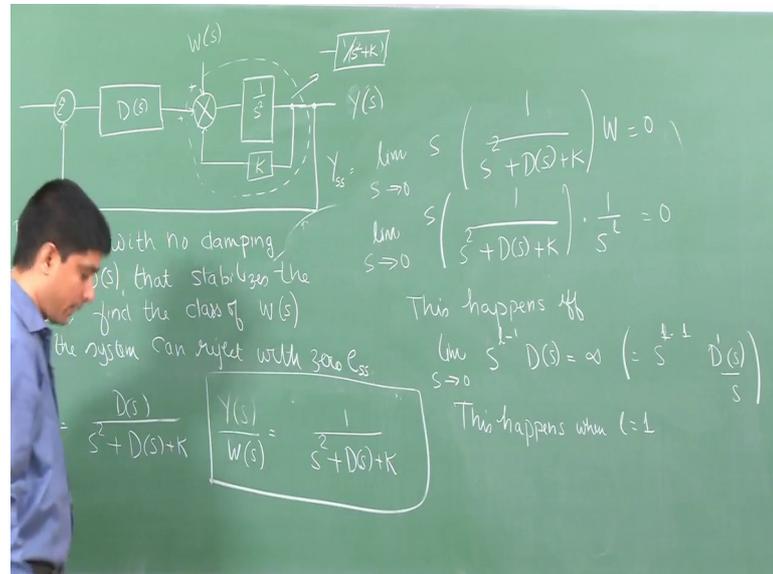
So, first thing is what is the error the error is the reference minus the actual output. So, in this case the error would simply turn out to be s square plus k over s square plus D of s plus K right times R of s now for ramp this would be one over s square ok.

So, what do we do need that the steady state error which is $\lim_{s \rightarrow 0} s$ times e of s is the entire guy here that is s square plus K , s square plus D of s plus K times 1 over s square, this thing should be a constant which means well. So, this some usual cancellations.

So, this means that limit. So, here I put a s equal to 0 there is constant here anyways. So, if I just look at this term here the limit s times s quarrel plus D of s plus K should be a constant, now sorry limit s going to 0 here. So, the first guy goes to 0 , third guy well it should also got 0 because I cannot change K , now let us see what it is. So, this means that limit s going to 0 , s times D of s should be a constant. So, what should be the nature of D such that s times D s is a constant well D s should be such that it should be some D prime s over s right in this case is this cancel out and then I just take the limit s going to 0 this will be some constant ok.

So, the condition on d such that these things are satisfied is that D must have a pole at the origin right. So, this guy must have a pole at the origin ok.

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This answers the first part; now in the second part for the D s which we obtained in part one which has pole at the origin, now for that he has that stabilises the system, right find the class of disturbance w t or w s that the system can reject with 0 steady state error. So, I need to find well for this D s which had a pole at the origin, I also assume that this it stabilises the system what are the class of the disturbance signal that I can reject. So, I will use this guy here right. So, y s over w s is this guy ok.

So, what I would want is limit s going to 0 s times 1 over s square plus D s plus K (Refer Time: 08:54) within a Y s set or Y s s if I would call right. At this steady state there should be no affect of the disturbance on the output which means the w should not affect this y that steady state and that is only because of w , I am not really considering what is happening with r at the moment. So, this would mean that since limit s going to 0, s times 1 over s square plus D s plus K let me assume that w is of some form s power l right could be a step could be a ramp or parabolic kind of thing and so on. So, I want this to be 0, now when is this 0 this happens if and if limit s going to 0, $s^{l-1} D$ of s is infinity right. So, why do I say this, well look at the denominator here.

So, this would cancel out this will be s^{l-1} , this $l-1$ would go here that term would become 0, k times s^{l-1} would also become 0 when I substitute s to 0. So,

for this got to infinity this guy should sorry this for this affect to go to 0 there should be infinity in the denominator and that is possible when limit s tends to 0 s power l minus 1 D s is infinity and this happens when l is equal to one because of let us see what happens when l equal to 1. So, when l equal to 1 this guy is s l is 1 minus 1 what is my D s, D s is D prime of s over s ok.

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The image shows handwritten notes on a green chalkboard. On the left, there is a block diagram of a control system. The reference signal $W(s)$ enters a summing junction. The output of the summing junction goes to a block labeled $D(s)$. The output of $D(s)$ goes to another summing junction. The output of this second summing junction goes to a block labeled $\frac{1}{s}$. The output of $\frac{1}{s}$ goes to a block labeled K . The output of K goes to a summing junction where it is subtracted from the output of $D(s)$. The output of this final summing junction is $Y(s)$. There is also a feedback path from $Y(s)$ through a block labeled $\frac{1}{s+K}$ back to the first summing junction.

Below the diagram, there is a note: "n with no damping at although a PI controller condition of 0 it will NOT yield a stable closed loop system."

To the right of the diagram, there are mathematical derivations:

$$D(s) = K_p + \frac{K_i}{s} = \frac{sK_p + K_i}{s}$$

$$\frac{Y(s)}{R(s)} = \frac{D(s)}{s^2 + D(s) + K} = \frac{\left(\frac{sK_p + K_i}{s} \right)}{\left(s^2 + K_p + \frac{K_i}{s} + K \right)}$$

$$= \frac{K_p s + K_i}{s^3 + s(K_p + K) + K_i}$$

Below these derivations, there is a boxed equation:

$$\frac{Y(s)}{W(s)} = \frac{1}{s^2 + D(s) + K}$$

So, this guy just goes away and limit s going to 0, I know this d as a pole at the origin and this guy goes to infinity right. So, in the third part show that although a P I controller satisfies conditions of part one of the problem, it will not yield a stable closed loop system. So, D s is a P I controller which means it looks something like this K p plus some constant K I over s or I can just write it as s times K p plus K I over s right. So, this satisfies the condition of the problem one that d should have a pole at the origin, and if d as a pole at the origin then I. So, the condition one the part one of the of the problem said find the conditions on d such that the error, that the reference is a ramp is a constant this satisfied right d satisfies the conditions of part a ok.

Now, with this d let us find out what is Y of s or the transfer function of the system. Well this becomes D s over s square plus D of s plus K, what is my D of s? My D of s is K p plus K I over s here and again I have the denominator s square plus K p plus K I over s plus K. So, this would I just do on the partitions K p S plus K I here I will have s cube

plus $s K_p$ plus K plus K_I . Now is this system stable how do I find out I find out by the roots of the characteristic equation or the poles of the system.

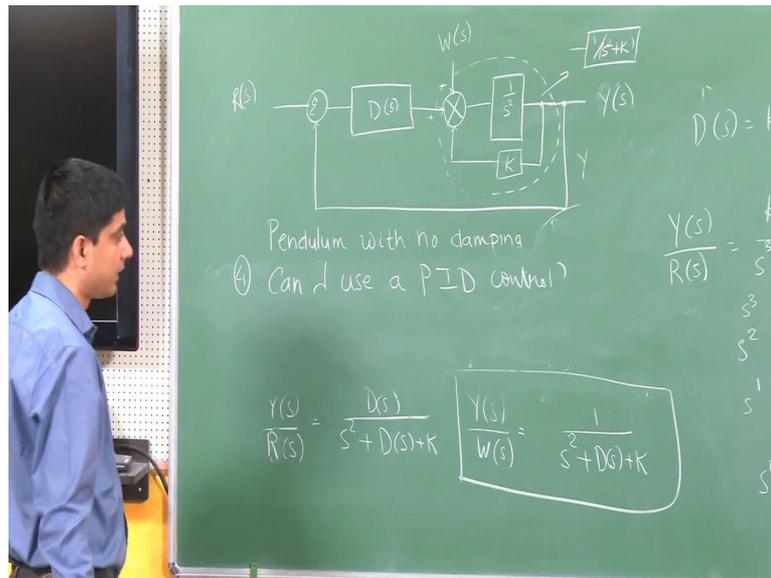
Now, look at this guy right. So, s^3 plus $s K_p$ plus K_I times s sorry s is already here plus sorry K_p plus K plus K_I is 0, and I do not even need to see the Routh table right to say that this is always unstable right for whatever values of K , you may just want to do it by the Routh table, but you see that there is no s^2 term here s^2 term is missing and as long as that is that is missing you will never have a closed loop system that is stable as a little exercise you can just convince yourself by drawing the Routh table.

So, the conclusion here is that although $D(s)$ satisfies the conditions of problem number one with a P I controller or which has a pole at the origin, it leads to a closed loop system which is unstable and therefore, even though a P I controller as the affect of reducing the steady state error, if this was not true in problem number one this would never be able to track a ramp right. So, with by putting a pole at the origin I see that well I track the ramp, but with some error right, but then the closed loop turns to be unstable.

So, these are little things which we need to be careful while we design P I controllers servicing badly. So, the conclusion is even though this theoretically satisfies it will be meaningless because my system is unstable. So, this saying that $D(s)$ has a pole at the origin is nothing because the system is unstable. For any of these analysis you need the closed loop system first to be stable it is not stable therefore, this is not true; it could be true for some other $D(s)$ which has a pole here and it could be some $s + 1$ over $s + 2$ that could be true, that could lead to a stable $D(s)$ right and here I am not really talking of a stabilising $D(s)$ right I am just saying condition one. The second part of the problem assume $D(s)$ to be stabilising if $D(s)$ was not stabilising there I do not need to really care about disturbances also right only assumption in the second part was $D(s)$ has a pole at the origin and it has a utilising affect.

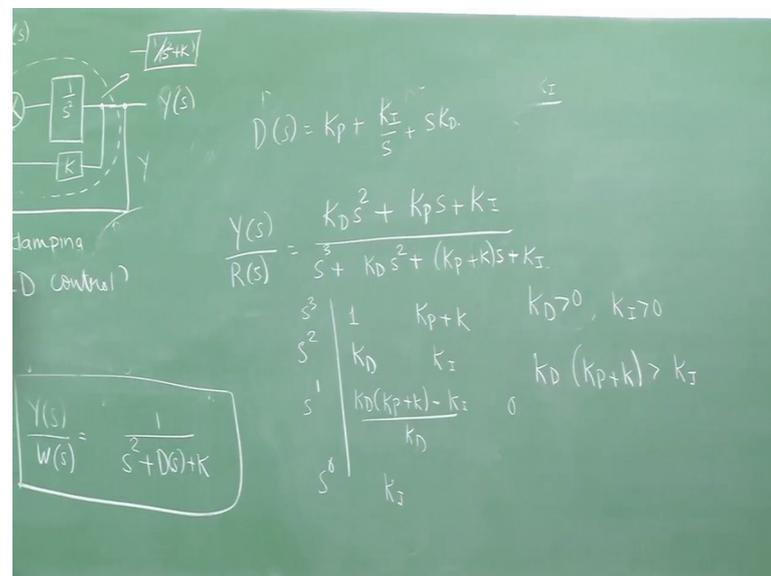
So, you see that just adding a K_p plus K_I by s or a P I control does not have a stabilising affect. In this case and it could turn out that is in any higher order cases this could be true. So, we need to be careful by before looking at the problem statement of improving the steady state performance through a P I controller is always comes with a bit of warning, that you should look at the closed loop system to be stable ok.

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Next is now part 3 told me that P I was not a very wise decision or a wise choice now can I use a PID control. So, a general structure of the PID control would be ok.

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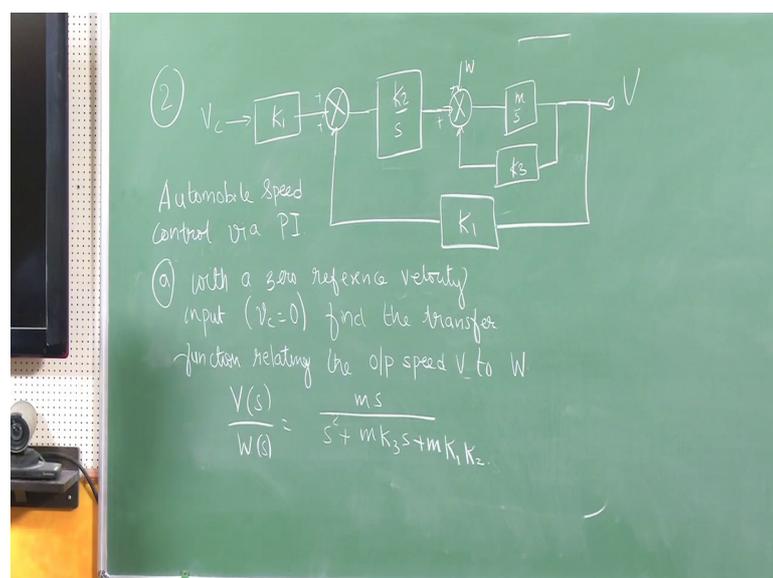


You have K p proportional term the integral term and the derivative term, in which case the closed loop transfer function Y s over R s becomes I cannot just skip the computations I just write down the as it are KDS square K p S plus K I over S cube plus KDS square plus K p plus K S plus KI. So, this gives me a little bit of hope because there the s square term is not zero and D s still has a pole at the origin ok.

So, the part one is satisfied now let us see what happens with this one. So, I can just draw the Routh table I have s^3 terms as $1 \cdot K_p$ plus K_d , s^2 terms K_d and K_i then I have I could compute the s^1 terms accordingly as K_d , K_p plus K_i over K_d and a 0 here, and s^0 would simply be K_i . Now this, this is positive this is positive this is positive and I can choose my K_p and K_d such that this is true. So, just find out. So, this is table for K_d greater than 0 and K_i greater than 0 right of course, K_p is always greater than 0 ok.

So, this is very very straight forward to find out. So, K_i should be greater than 0 K_d should be greater than 0 if these two are true you plug it in here, and you will find this also to be greater than 0 right. So, this the additional condition we need to impose is $K_d K_p + K_i$ is greater than K_i right for the system to be stable right the second one would mean K_d should be greater than 0 K_i should be greater than 0 and this is third one. So, as state earlier, this guy has an effect on the steady state performance of the system and the derivative component has an effect on the transient performance of the system right. So, this is a very basic illustration of when to use P I when to use PID. Again not a very general answer it is a just special case of it right of why we should be careful, why while we are using P I controllers we will see a little more different types of problems.

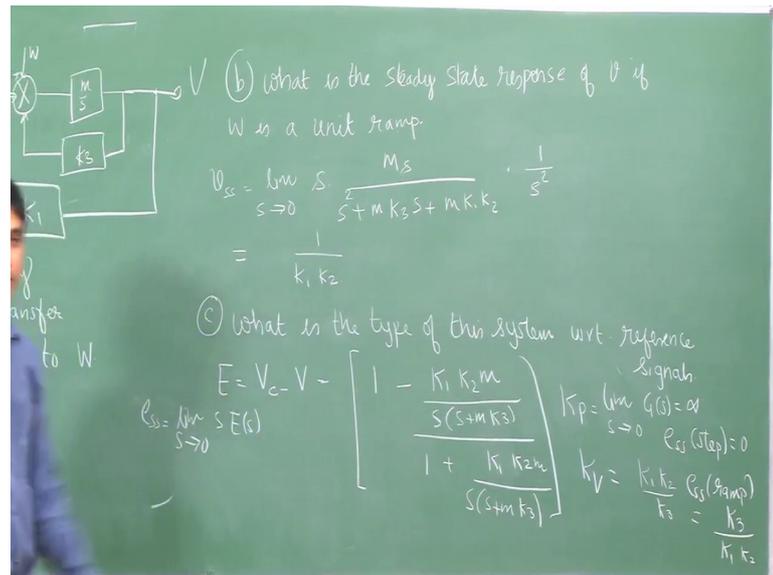
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In the second problem well let us say it is a very simpler simple model of automobile speed control using a P I control when there is disturbance also right. So, and we will see

what all we could do with this right. So, first I would say part a we will do is with zero reference velocity input is means this guy we see is 0, find the transfer function relating the output speed let me call this V relating the output speed V to the wind disturbance w here well. I can it is not really a big deal V s over W s assuming V to be 0 I can easily compute this as m s over I have s square plus m K 3 s plus m K 1 K 2 ok.

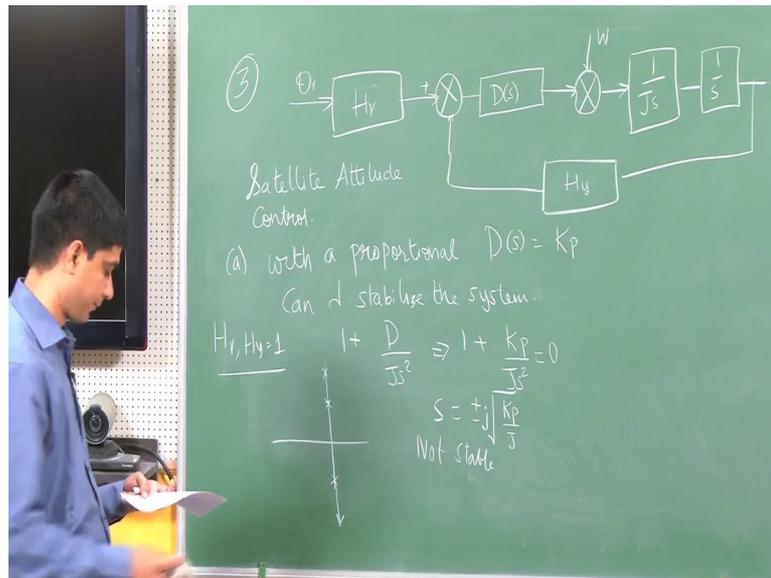
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So, there this straight forward I will not go into the details of doing that the second part, what is a steady state response of p if w is a unit ramp again still keeping we see to be could be 0. So, I will typical formula v s s is limit s going to 0, s times v of s what is v of s this is M s over s square plus m K 3 s plus m K 1 K 2 over I multiplied by 1 over s square ok.

So, this will turn out to be. So, this well this will go away I am just left with 1 over K 1 K 2 third part what is the type of this system may be with respect to some reference signals. So, we look at in terms of E is V c, minus V is 1 minus and usual the way we compute this steady state error is through the final value theorem they meet s going to 0 s times E of s. So, here the position error I skip the computations s trending to 0, G of s is infinity. So, the steady state error with a step input is 0 right similarly K v turns out to be K 1 K 2 over K 3 or e s s with a ramp is K 3 over K 1 K 2 right which means just a type one system that could be easily computed.

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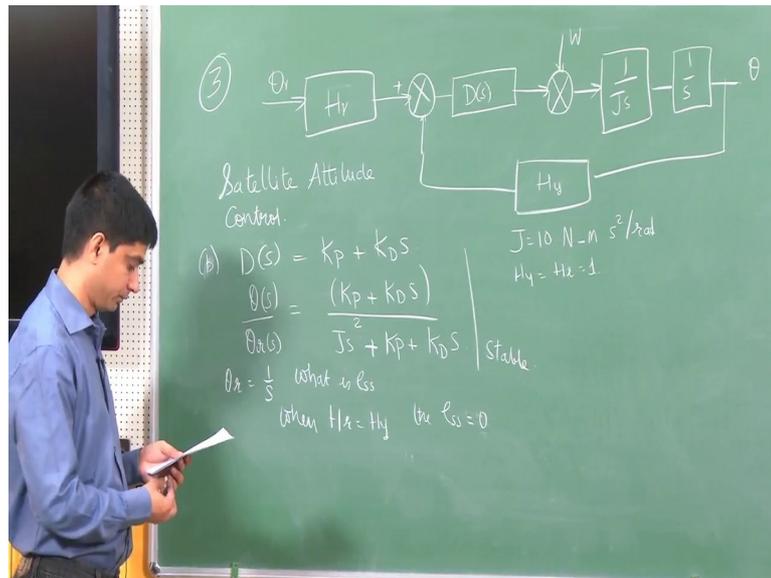


So, in the third and the last problem we deal with again a very simple model of a satellite attitude control. So, with J being the momentum of inertia. So, we have a certain reference attitude and then you have the actual theta right ok.

So, and then as usual w is my disturbance, disturbance stroke; in the first part with a proportional controller, this only sees the affects of p P I p d PID and c I d what they mean with respect to reference tracking and even disturbance rejection. So, first thing is to start with the proportional controller, which means my $D s$ is simple a constant $K p$ can I stabilise this system right. So, can I stabilise. So, let us say so for some of this purpose I would sometimes assume that this $H r$ and $H y$ are equal to 1 even though nothing will change even if they are non unity. So, when $D s$ is just $k K p$ then my characteristic equation is $1 \text{ over } D J s \text{ square}$ assuming this to be true which means one plus $K p \text{ over } J s \text{ square}$ is 0 or s is plus minus square root $K p \text{ over } J$ yes with the since this will be in the imaginary axis as $K p$ and J are always greater than 0 ok.

So, this system will always have poles no matter whatever I do with the $K p$ the poles will just lie on the imaginary axis right as an therefore, this system is not stable at best it can be called as marginally stable. So, with just a proportional controller I cannot stabilise the system this is a first thing ok.

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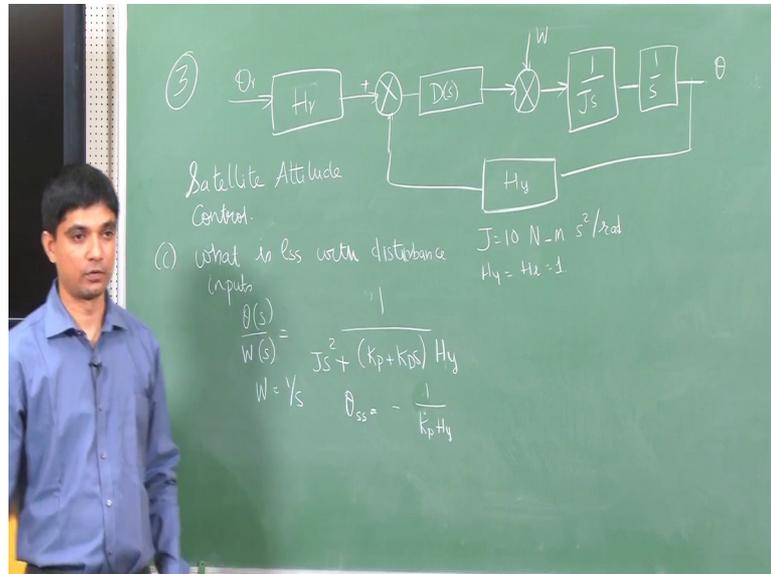
What could I do now with a p d controller, in that case well the transfer function theta over theta r of s is $K p$ plus $K D S$ over $J s$ square plus $K p$ plus $K D$ times S , now you look at. So, earlier we just had roots which were imaginary this had this term $J s$ square plus $K p$ now have this additional term $K D$ which introduces damping to a system and this is a second order system with all constants being greater than 0, I know that this system is actually stable. Now and by appropriate choice of $K D$ and $K P$ I can play around with the peak overshoot or the settling time the rise time and so on ok.

So, first thing with a proportional controller, I could not stabilise the system I could just move the roots of the system along the imaginary axis, I put a derivative term I add damping to the system right in the first case we just $D s$ being the proportional controller I did not have this damping term this term corresponds to $\zeta \omega_n$ which means ζ equal to 0 if $K D$ is equal to 0. There is no damping so, the system will always be oscillatory the second case I introduce a damping and stabilise the system.

So, if my reference is $1/s$ what is the steady state error. So, for this I will just use some constants to compute these values, that J is 10 since it is in the moment of inertia thing. So, it is being Newton meter second square per radians then H_y H_r is equal to 1. So, then I just used that theta r is equal to $1/s$ or theta r of s is $1/s$ then when H_r equal to H_y the steady state error is 0, I will leave that as our exercise to you because

you can see lots of things happening here right you have well if I just consider this I have two poles at the origin ok.

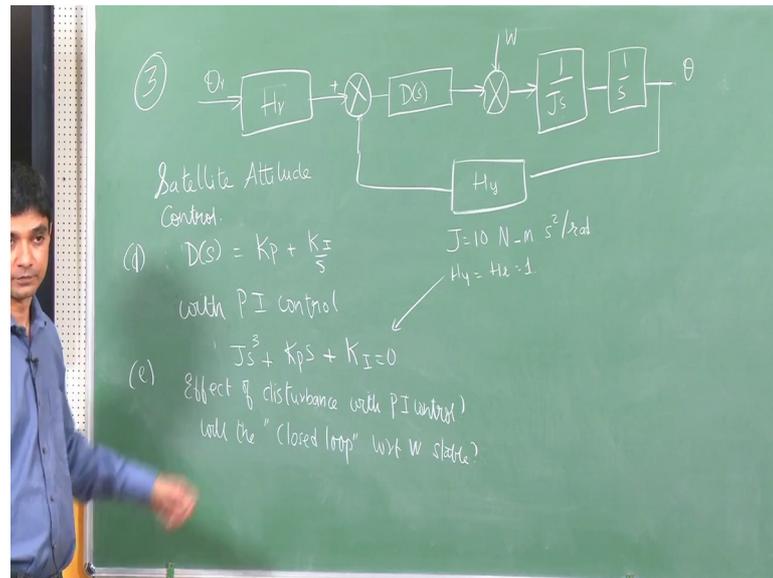
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So, therefore, I can use any track or step signal same case what is steady state error with disturbance inputs well for that I need to find out what is a transfer function, theta s over w s or the affect of w on theta then I could compute again this straight forward computation $K_p + K_D s$ times H_y right and then well if the disturbance is $1/s$ then theta s s and steady state value of theta where w being equal to s is minus 1 sorry over K_p times H_y .

So, this kind of controller does not eliminate disturbances of this form you will get more complex when you have a square and a cube and so on. So, what do we observe from here right. So, again go back to the plant, the plant has two poles at the origin and these two poles were to eliminate the steady state error when I was tracking a step reference, but these guys are not useful enough to reject disturbances, that is seen from here and therefore, a pole at the origin helps in the reference tracking, but I were to reduce the affect of disturbances I may need an integral component here or a pole at the origin for the controller ok.

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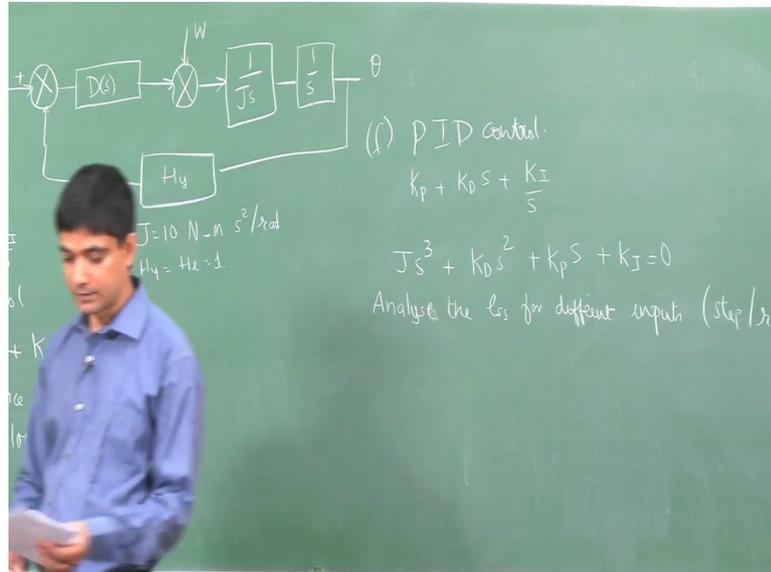
So, in the next we will just see what happens when I use a P I kind of controller. So, what does this controller do. It is again not very very different then what we did earlier right. So, in this with P I control the characteristic equation becomes $J s^3 + K_p s + K_I = 0$ this is again assuming this to be true. Now this is unstable right I do not really need to analyze this any further. So, this with a P I controller I know is unstable now it does not really help me to compute the steady state error anymore because whatever I do the plant the system is unstable therefore, just by doing a P I controller and doing a steady state analysis is really meaningless right.

So, first we always have to check for stability before we do any other analysis even though the math of that you know of $e(s) = \lim_{s \rightarrow 0} s E(s)$ will give me a number, but this was valid only if this is a valid transfer function, or $E(s)$ is a stable single if this is unstable then nothing can be done right. So, this we even did while we were discussing about the final value theorem right the system must be stable so, as to apply the final value theorem right, so that we need to be careful of all the time.

So, similarly we could also do the affect of you know disturbances right. So, the affect of this I will leave as an exercise affect of disturbance with a P I control. Just check this will the well I will just use the word close loop even though it is not the actual close loop the close loop with respect to w stable now with a P I control, and just try to solve this it

should be once you know this should be very obvious. So, the last part now is to check again what happens with a PID control.

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So, PID control again I will have K_p plus K_D times S plus K_I over S , I skip all the all the steps and you know you just have the transfer function of the form $J s^3$ plus $K_D S^2$ plus $K_p S$, plus K_I equal to 0. To similar to what we had in one of our earlier examples right and now I can really find out conditions under which the close loop system is stable right.

Now, again, so again I just leave this for you as an exercise. So, analyse the steady state error for different inputs with the different step and just for a ramp right. So, I will just give this as an exercise because it is just a very very manual thing. So, what we have learnt right just to summarise is, we need to be careful when we are adding an integral control. It might seem a little misleading that it will improve my steady state performance, but it might lead to a unstable close loop system right and then of course, when I see this PID well I just have to then we should choose the constants K_p K_I and K_D such that the close loop system is stable this can come from the Routh's table right this we are now very well familiar of how to analyse stability of this right. So, what we will do next is to take this as bit of background and analyse these kind of things a little more can I directly do this practically are there any difficulties right.

So, can we relate to what we learnt in terms of the root locus plots, can we relate these things to what we learn in terms of the Nyquist or the bode plots, can do this guys talk anything about relate to stability at the moment we just have no doubt anything about relative stability right. So, all those things we will analyse once we have understood what adding of these controllers mean, are they good, are they bad, are they sometimes, good and sometimes bad ok.

Thank you.