Control Engineering

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Module – 01
Lecture – 03
Elements of Modelling

In today's class we will talk about some basic elements of modeling. And this is not something which is very new, you might have learnt some of these concepts in your basic course on networks and circuits something coming from mechanics or even as early as your high school physics. I will see how all these basic elements which we will define can be used to model physical systems and we also find out if there is any analogy between different kinds of sub subsystems.

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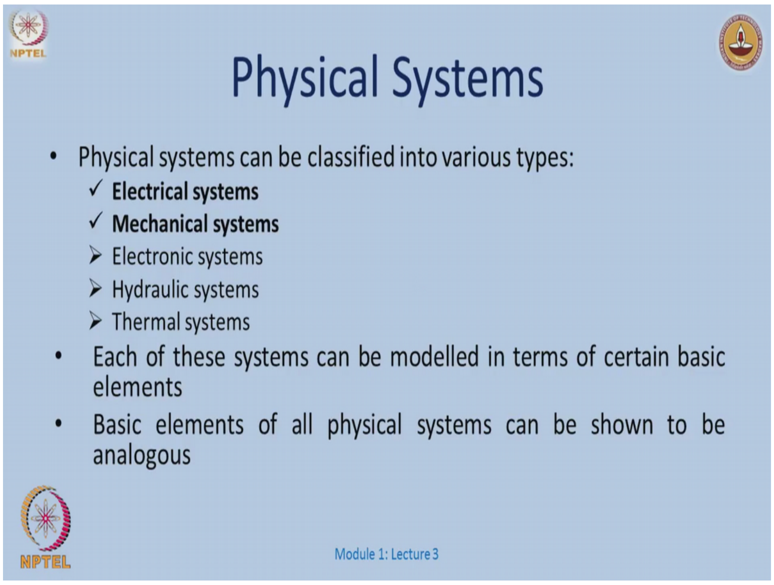

So, we start by classifying things into several types: it could be a electrical system a mechanical, electronic, hydraulic thermal and several other systems. And each of these models as we will see or each of these systems can be modelled in terms of certain very basic elements. And then we will see if there is any analogy between you know cost domain between electrical and mechanical systems.

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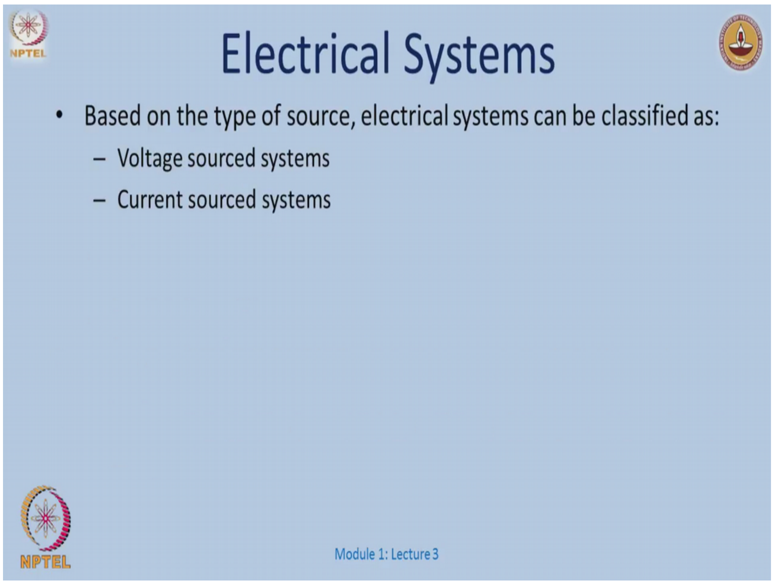

And through this lecture we will concentrate mainly on electrical and mechanical systems. So, systems in electrical systems based on the type of source we could classify systems either as being voltage controlled or current controlled source systems.

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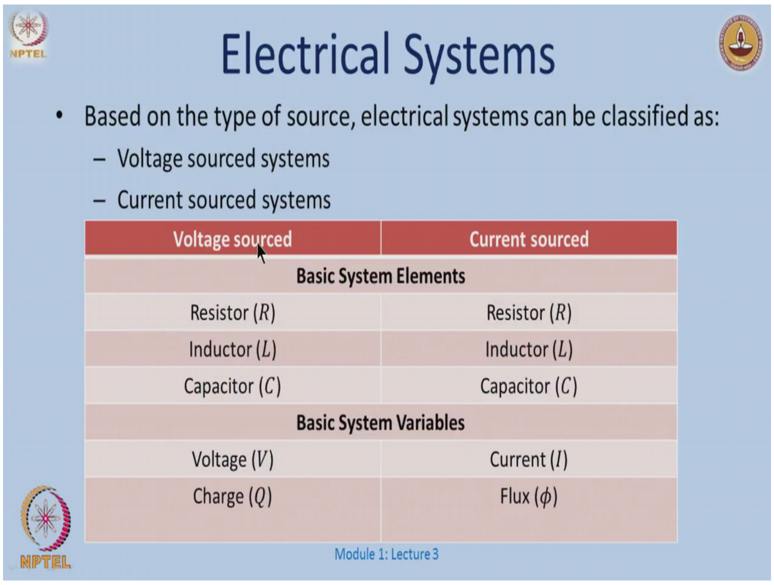

So, in if I talk of a voltage controlled system or a circuit my basic system variables are the voltage and the charge. We will see why this is true why it is a voltage and the charge why have voltage and directly the current or why not something else. And of course, the big three basic elements which we all know from circuits R a resistor, inductor, and the capacitor; and throughout this class we will restrict ourselves to linear elements.

Similarly, for a current sourced circuit or a system, my basic building blocks or the basic system variable are the system current and the corresponding flux, and also see why this is true of course, the basic elements that remain the same the resistor inductor and capacitor.

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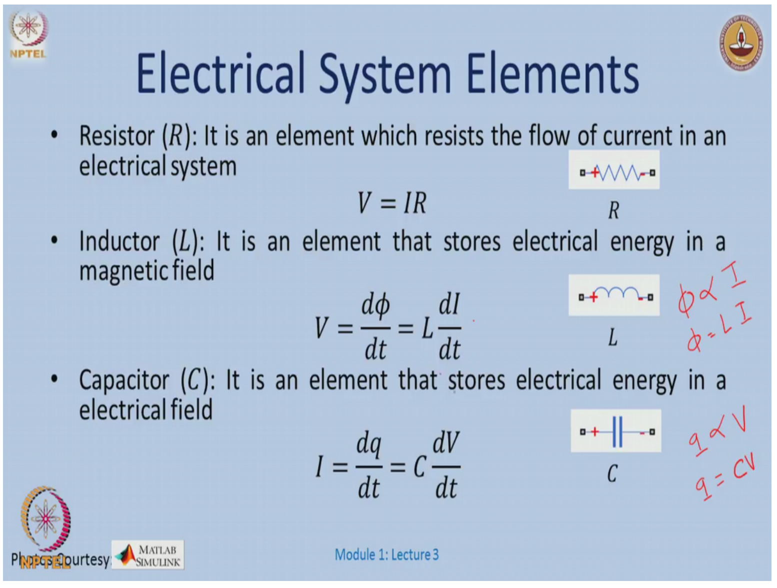

So, how are these things defined well we all know what are resistor rays it is an element which resist the flow of current in a system, and we all know this ohms law V equals to IR, modells simple linear resistor. If I go to an inductor I know that the voltage across an inductor is given by d phi by d t, or in the case when the flux is a linearly related to the current we will have V equal to L dI by dt say if I just write down some basic laws which we learn in electromagnetic. So, phi is proportional to the amount of current and in the linear scale phi equal to L times I where L is a inductors and then you have this relation between or this relation V equal to l d I by dt describing a linear inductor.

Similarly, for the capacitance, I know that the charge similarly for the capacitance we know that the charge is proportional to the amount of voltage and in case of linear elements q would be C times V and I have a relation that the current which is nothing, but the rate of charge is simply C dV by dt.

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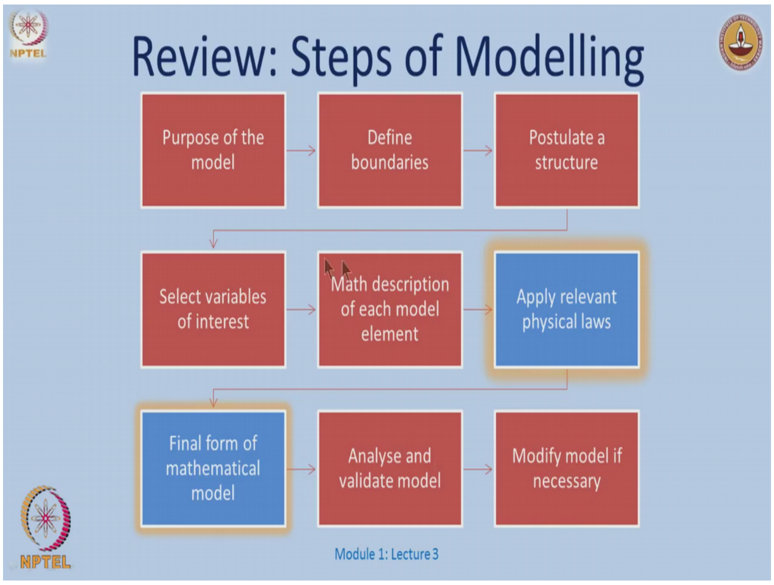

So, what we will concentrate now is on these two things highlighted in the blue, how do we apply relevant physical laws and get some final form of the mathematical model?

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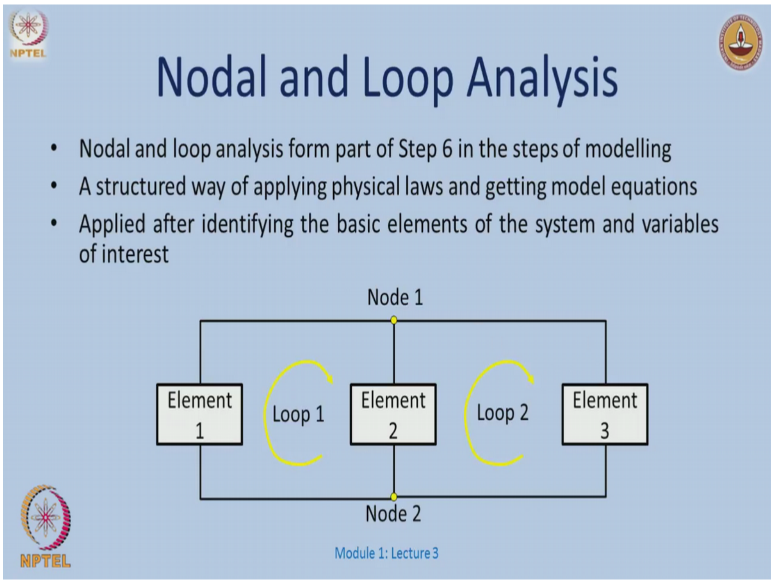

So, again coming from basic circuits we all know what are the nodal and the loop analysis, and these are essentially coming from the part six right. So, whenever we do nodal and the loop analysis we apply in the context of either the Kirchhoff's voltage laws or the current laws and so on.

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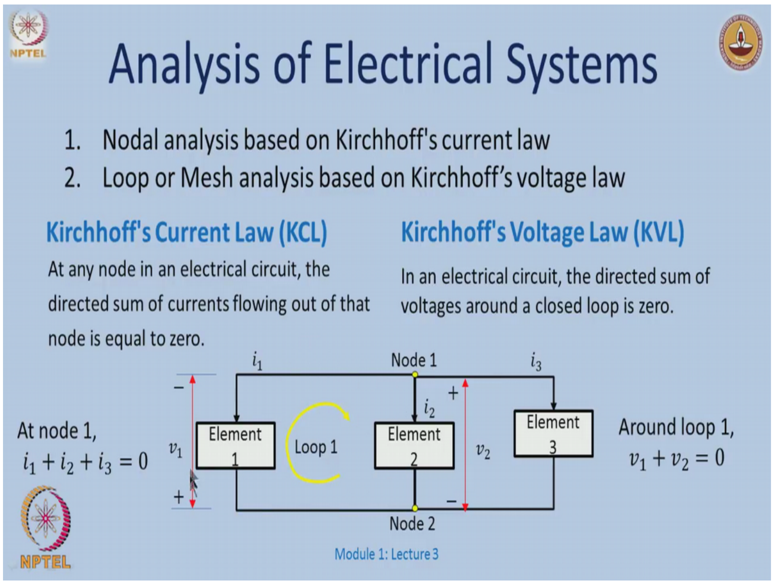

So, once we identify basic elements we inter connect them in a certain way which defines a circuit, and then we write down on the relevant equations. So, the nodal analysis as we know is based on the current laws and the loop or the mesh analysis we write down the voltage laws and this is again not not very alien to us, and the Kirchhoff's Current Laws says at any node the directed sum of currents flowing a flowing out of that node is equal to 0.

Similarly the sum of voltages across closed loop is always 0 that is what the Kirchhoff's voltage law tells us right. So, if I will just very simply look at this little circuit kind of thing here, I would know that i 1 plus i 2 plus i 3 is 0 and if I just look at this as loop I can say that v 1 plus v 2 equal to 0. So, just because of the sign side here I go from plus to a minus plus v 0 is equal to 0 or in other words v one is just minus v 2.

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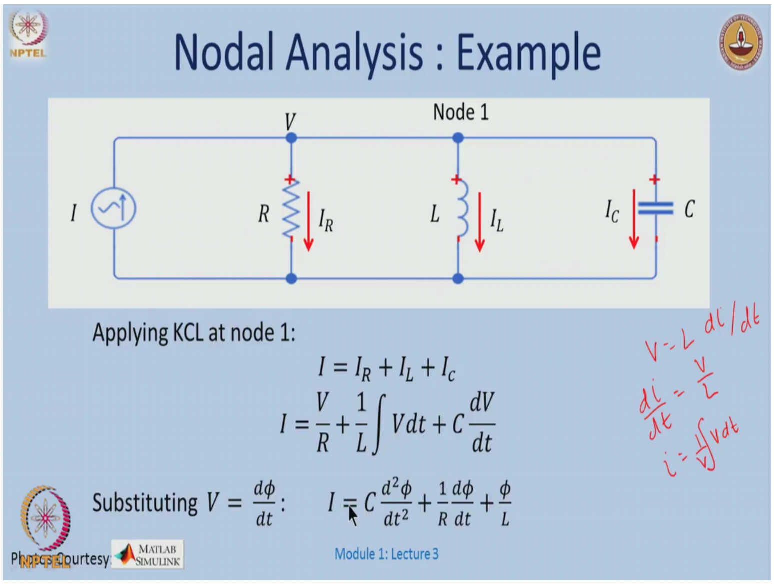

Let us start with the simple example I can we are start with very very basic examples which we have already know and then try to build up on those things. So, I have a current source, resistor, inductor capacitor all linear elements; and if I apply the Kirchhoff's Current Law at node 1 when the circuit is the single node that the input current which is I is equal to the current which goes through the resistor through the inductor and so the capacitor.

So, what is I through the resistor given V, that is simply I can I write it as V over R, across the inductor it will just be 1 over L integral V d t is just this all this equations again come from the basic laws which we have written earlier right V equal to L di by dt I can re write this as that the current is equal to 1 over L integral V dt. So, we had earlier that V is L d i by d t or di by dt is V over L and i simply becomes 1over L Vdt. So, just re writing things which we already know and finally, the current across or current through the capacitor is C dV by dt.

In what else do I know? I always know I already know that V is simply d phi by dt which means a current now can be written as C d square phi by dt square this term and across the inductor or across the register I V over R, what is V? V is d phi by dt. So, I have 1 over R d phi by dt and across the inductor that will simplify be phi over L right; and therefore, in the earlier side when I said let us just revisit that slide for a for while that the basic system elements are the current and the flux this is what I mean. So, I have current here and the flux and of course, these are the basic elements which build my circuit the capacitor inductor sorry the inductor here and the resistance here right.

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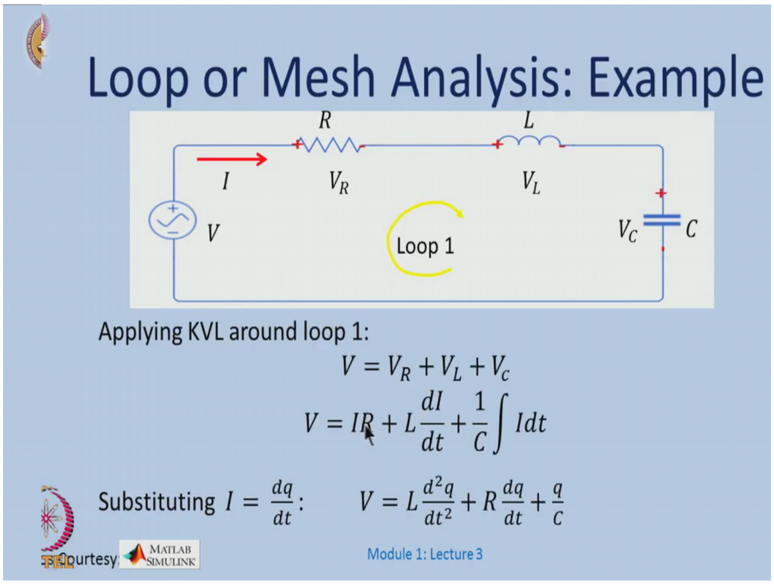

Now, if I do a loop or a mesh analysis you also called something like a series RCL circuit with a voltage source again all elements are linear, I have a voltage source V then if I just apply my voltage laws that with the total voltage V would be the voltage across the resistor plus the voltage across the inductor, plus the voltage across the capacitor whether current flowing in this direction.

Again I know these things again then V the voltage across a resister given I is simply I times R, and V across an inductor I already know from a previous slide that V is L dI by dt and voltage across the capacitor can be written in terms of this 1 over C integral Idt in the similar way as we have written the current expression for an inductor. So, where does this current come from well current is simply the rate of charge or I equal to d q by dt, which means that V can be written as L d 2 I by d t square just put this guy over here I is d q by dt. So, we will have over d 2 q by dt square R times I.

So, again I is dq by dt. So, I have R times dq by dt plus q over C, again here the basic things are the voltage and the charge we will quickly go back and check if this is true or not. So, when I said that for the voltage sourced system or a circuit the basic system variables that define my dynamics of the equations are the voltage and the charge right and we where somewhere in the steps we applied the relevant physical laws and got some final mathematical model.

We will see you again verify that we started with the basic physical model or writing down the basic conservation laws to arrive at this dynamics of the system. Similarly over here a voltage control circuit or a voltage source circuit I start with my basic laws and arrive at this differential equation which is the mathematical model of the system.

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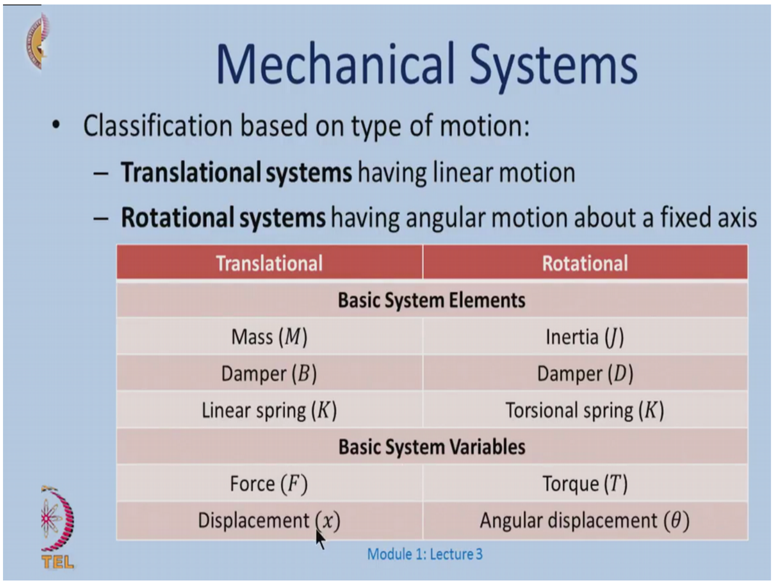

Translational	Rotational
Basic System Elements	
Mass (M)	Inertia (J)
Damper (B)	Damper (D)
Linear spring (K)	Torsional spring (K)
Basic System Variables	
Force (F)	Torque (T)
Displacement (x)	Angular displacement (θ)

Now, I go to mechanical systems there is something which we learn much earlier then we do a electrical circuit side when we do things in high school physics. So, I classify motions either translational motion or a linear motion as I called or rotational system or something which has an angular motion about a fixed axis. So, what are the basic things here? If I talk of translational motion my basic system variables would be a force which causes a certain displacement; in the rotational motion or the angular motion I have torque which produces some kind of an angular displacement.

And what are the building elements I will have a mass which is essentially like a kinetic energy element, I have a spring which is essentially like a potential energy element, and damper which is which represents the losses or fiction in the system or it could also be like a as we will see we can also have a external damper to the system. Rotational motion I would have the moment of inertia, I will have the torsion spring and again the damp corresponding damping or the resistance element in rotational motion.

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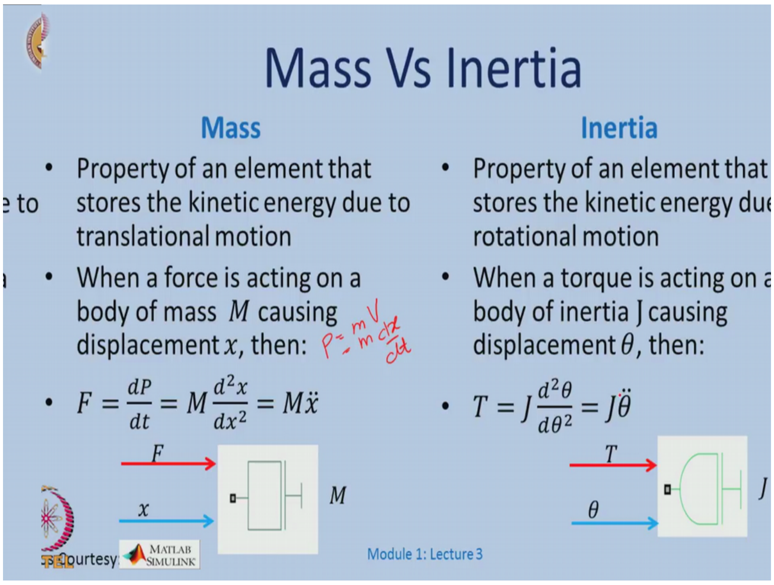

So, what is mass? So, mass is a property of an element which stores kinetic energy right and what is this mass. So, when a force is acting on a body of mass m it causes a certain displacement x, and what is Newton's second law tell me that the force is just the rate of change of momentum right this is say this is the statement of the second law F is dP by dt and if I use the relation between momentum and velocity. So, momentum is related to velocity as P is mass times the velocity, where velocity if I could write in terms of displacement I will have dx over dt and then I will have this final relation that F equal to Mx double dot or you usually called as also refer to as forces m times a or mass times the acceleration.

Similarly, in case of the rotational motion, I have the moment of inertia the property of an element that stores kinetic energy in the rotational motion, in such a way that when a torque is acting on a body of inertia J causing a displacement theta then torque is j times theta double dot with a same analogy over here this is a direct one to one relationship between F equal to M x double dot and T equal to J times theta double dot.

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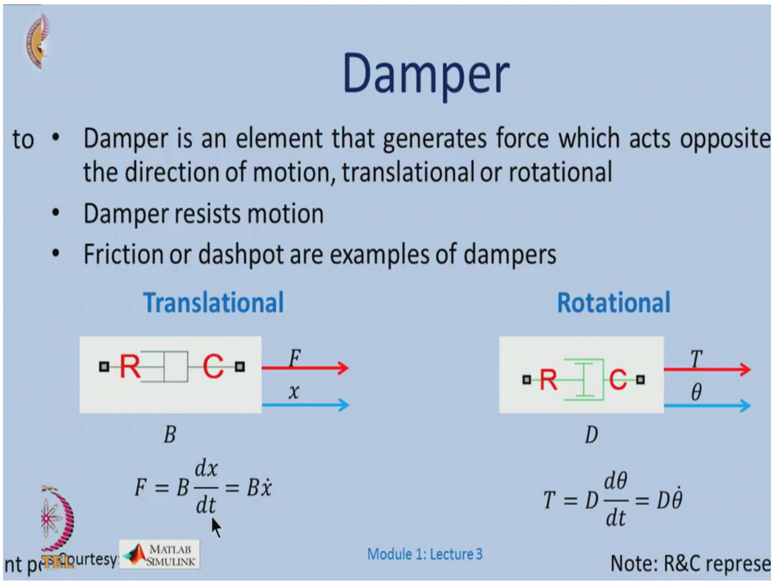

So, the second element is the damper well what is this guy? The damper is an element which is essentially which generates a force acting opposite to the direction of motion right it is essentially blocks your motion or resist the motion. What are the examples a natural friction which happens when we walk or when we drive a car or dash spot. So, these are the two examples and how are they modelled. So, if I have a force again causing a certain displacement x, the damper is modelled as F is B which is a constant which is like the damping coefficient dx by dt, a relationship linear relationship between force and velocity.

Similarly in the rotation motion I have the rotational damper or the fraction as a linear relationship between the torque and angular velocity which is theta dot written as T equal to T D times theta dot.

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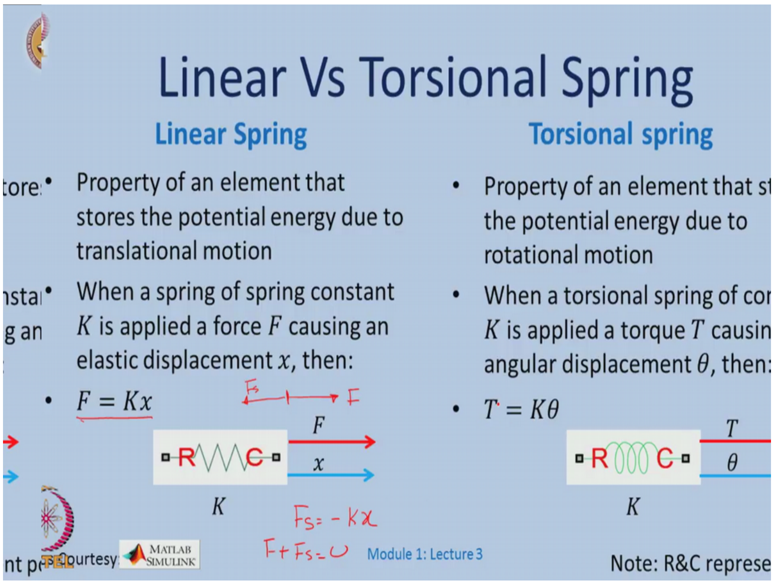

The linear spring as said earlier linear spring is an element that is stores potential energy in a way that when the spring when you have a force causing a certain displacement, then the force the spring as we know we will have a restoring force which will act on the opposite direction. So, if this is my force here F and. So, this is nothing, but the spring and then the force of the spring would be somewhere here I will call it F s right in such a way that F. So, if I write down the conservation of the force here that is the sum of all forces is 0, I will have F plus Fs equal to 0 and what we learn in school is V model this F as minus K times x right and then I put this here to I substitute Fx Fs equal to 0 resulting then in this equation right ok.

Similar thing happens in the rotational domain also right that the torsional spring is a property of an element it shows potential energy in the rotational motion, in similarly when I have a torque that causes a angular displacement theta, then the torque and theta are related linearly as T equal to K times theta.

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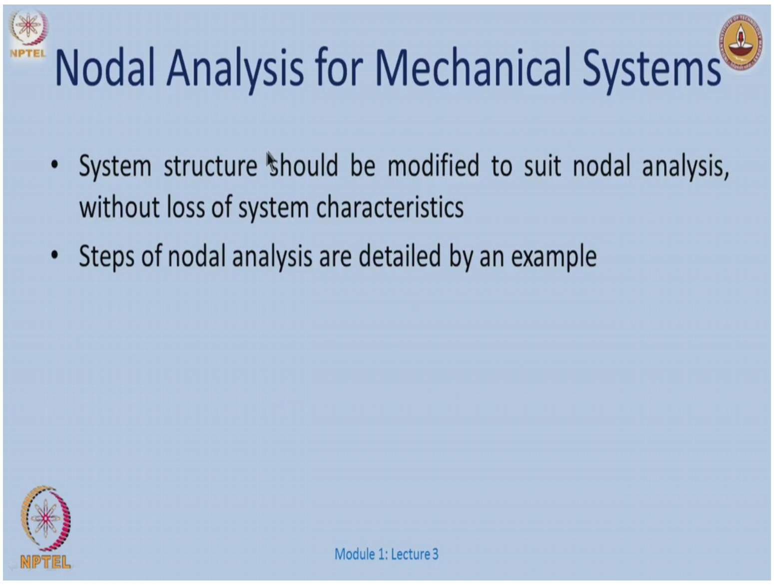

Now, how do we do equal in nodal analysis for mechanical systems? In the electrical systems we had direct laws right we have a circuit interconnected then you write the revenant identify the loops or identify the nodes, write down the voltage laws write down the current laws and you have set of beautiful equations, which describe for you the entire dynamics of a system. Let see the analogous of those kinds of equations or those kind of conservation laws in mechanical domain. So, we should first have our system structure that would suit the nodal analysis and let us see with help of an example.

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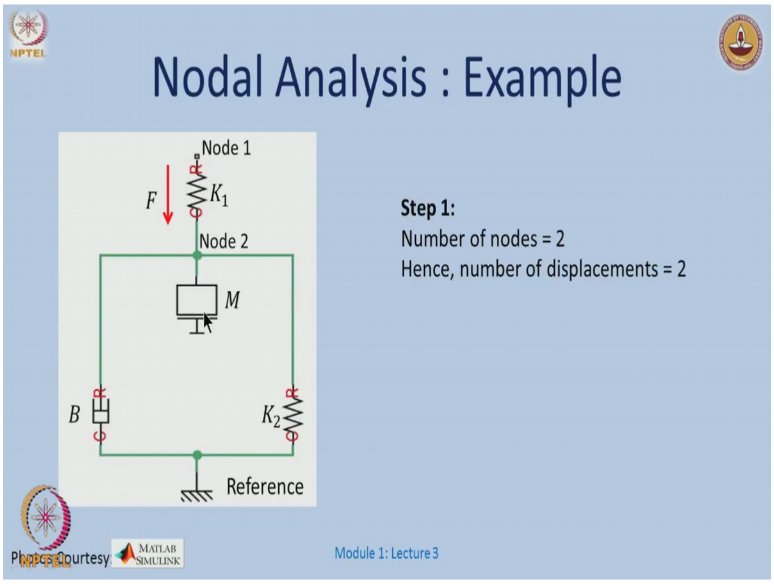

So, I have a system composed of a spring have a mass element, I have another spring and I have a damper or dash board and this is my reference node and with some external or some force over here. The step 1 would be to identify the number of nodes and these are identified usually by the number of displacements right. So, I will have one corresponding to this guy here and the other corresponding to this guy here. So, these are my two nodes right node 1 and node 2, which will give me these two displacements.

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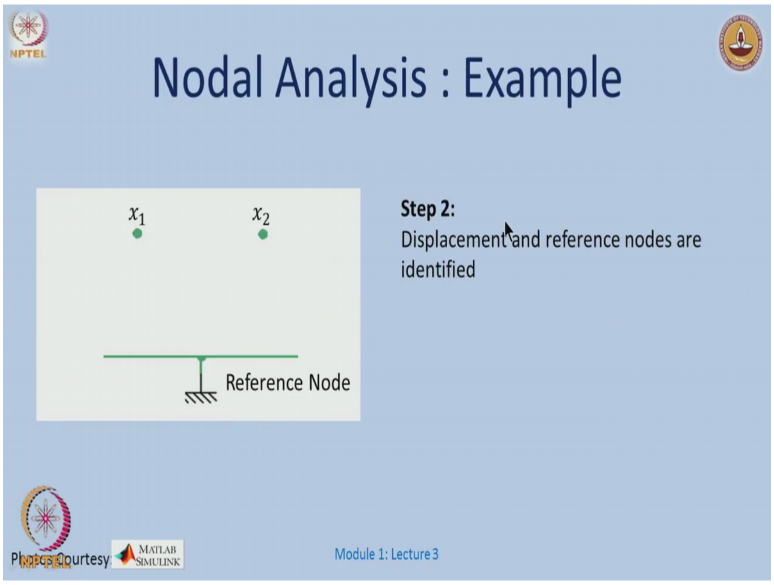

So, once I identify these two displacements or these two nodes, I just also fix identify these two nodes x 1 x 2 I mark them here and then I put I have a reference node step 3 right we just go back to how these guys for connected between node 1 and node and then with this means the reference I had the mechanical the mass between node 2 and the reference K 2 between node 2 and the reference B between the node 2 and the difference between node 1 and node 2 I have the spin K and the force was kind of acting on the node 1. So, we just draw some kind of a circuit diagram kind of a thing over there. So, I have a force x 1 x 2 or my two nodes.

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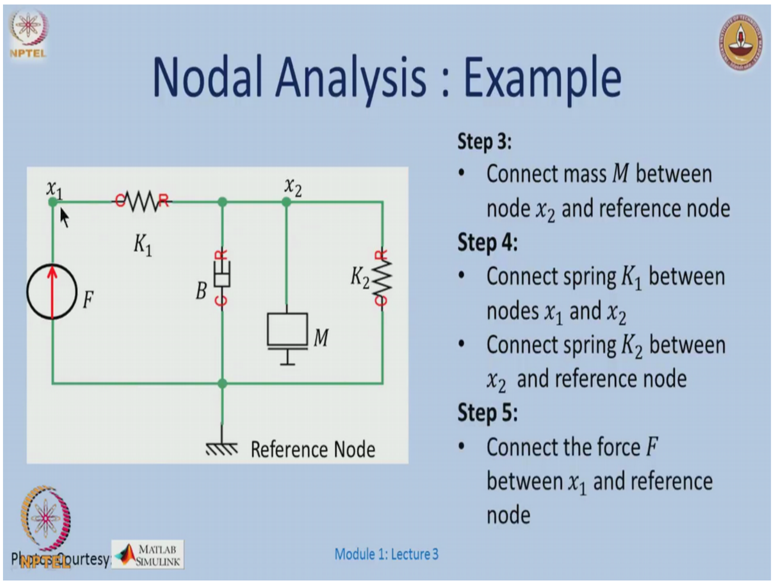

So, all these are the same this is also x 2 this is also x 2 this is my reference node. So, is this one this guy also a reference node? So, I have the force I have a spring K 1 the mass element second spring and the damping element right. So, once I do these connections as stated or as in the in the previous picture I have my some something like a circuit diagram right, and the force f acts on this node 1.

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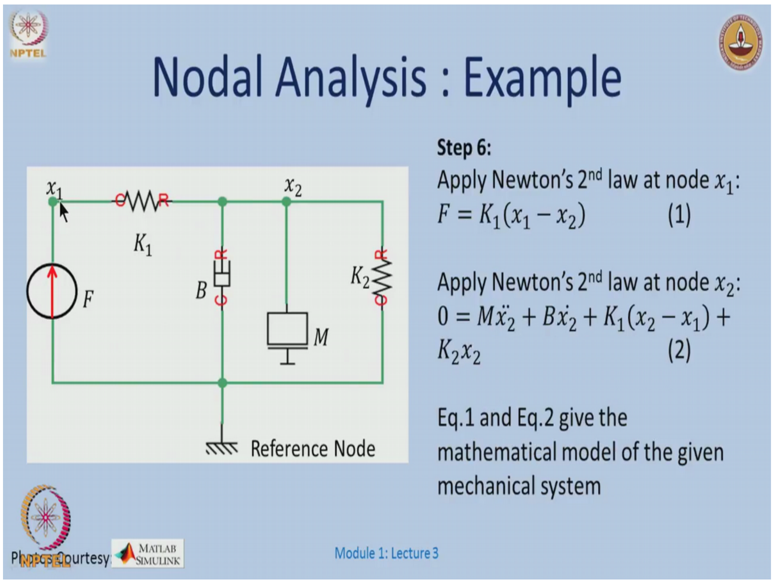

Then I apply the Newton's laws at node 1 that I have a force and I have a spring. So, F would be K times a displacement and since I am dealing with two displacements x 1 and x 2 as simply I have F is K 1, x 1 minus x 2.

Similarly, at the second node I say that sum of all the forces is 0, there is external force coming from here therefore, I will have the force corresponding to the inertia element as Mx double dot the force corresponding to K 2 would displacement x 2 would be K 2 times x 2, here would be B x x 2 dot and K 1 the displacement or the force corresponding to the spring K 1 would be K x 2, K 1 x 2 minus x 1 and I have these two balance laws for the for node 1 and node 2 which will give me the overall mathematical model of the system.

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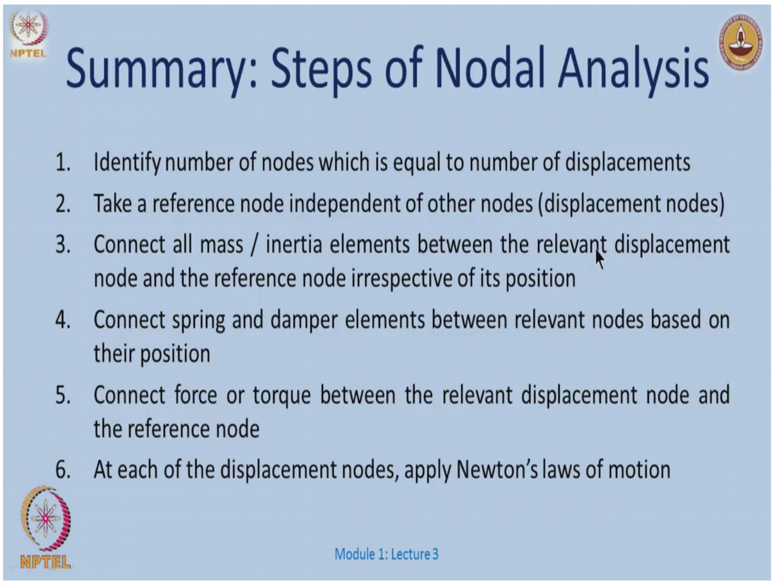

So, in summary what are the how do we identify the steps of a nodal analysis how do we come about getting our models.

So, first is identify the number of nodes which is equal to the number of displacements, select a reference nodes and then connect all the basic elements, which are the mass elements or the spring elements and so on according their positions, similarly do the for the spring and the damper if there is an external source which generates or which produces a force or a torque also add that to your system. And then finally, apply the Newton's laws of motion to arrive at your desired equations. Now if you see go back and you see that there are these kind of equations look similar to what we had even for electrical systems, and we then see where is there analogy because both systems are written a set of conservation laws right.

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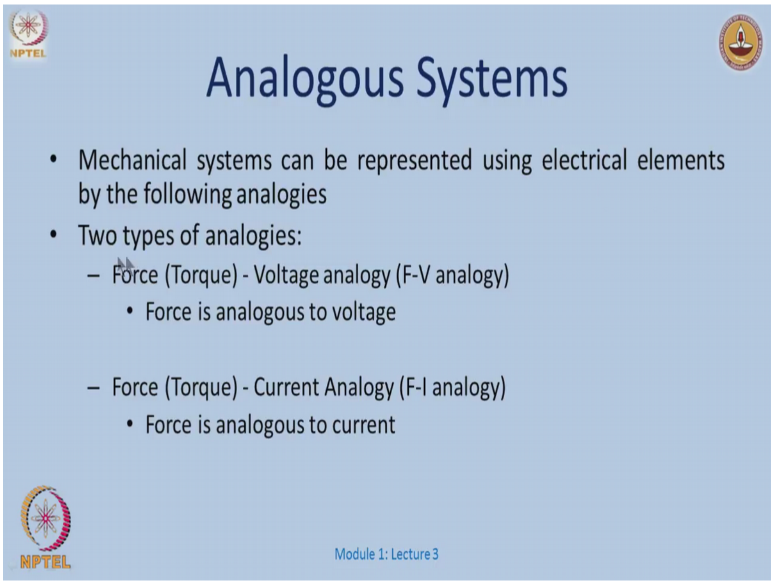

If I look at the electrical systems I have the voltage conservation or the summation of currents equal to 0 in the Kirchhoff's Current Laws, here I also I have some kind of conservation laws that summation of all forces is equal to 0. And based on these conservations laws we will just try to investigate is there a good analogy between these two these two systems mechanical systems analytical systems.

So, let us investigate that one. So, that we could typically arrive at two kinds of analogies one would be where you can say that the force or the input torque is equivalent to the voltage where we have the if we called the force voltage analogy or F V analogy, or we could also have situations where the force is analogous to current or what we called as the F I analogy how to establish this?

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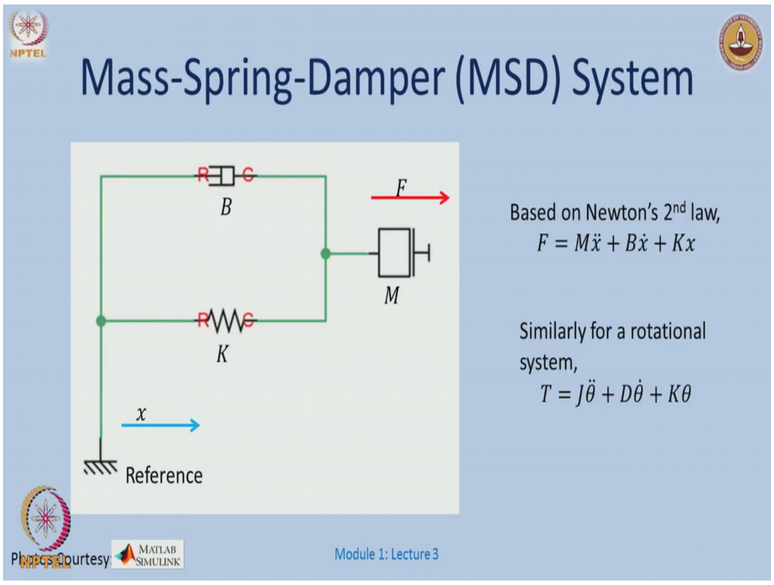

Let us start with the circuit right. So, again based on the previous steps which were listed I can easily write down that the force that the system which is exerted on this mass spring damper is Mx double dot plus bx dot plus Kx which has a mass.

A dissipative element and a spring with a constant K similarly for rotational motion I can write down the equivalent of it.

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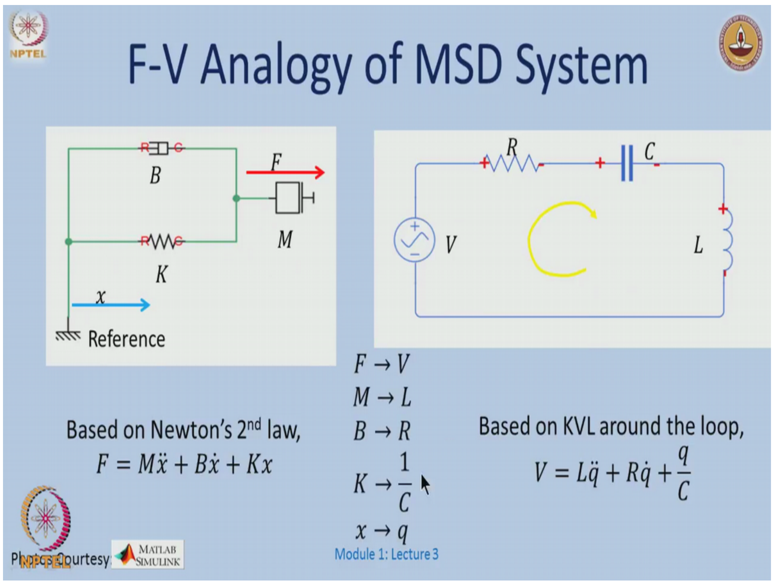

So, let us see I have the equations for this guy and see what does the how do the equations on the right hand side look like. Based on KVL we derived earlier also that L q double dot plus R q dot plus q by C is the voltage that is supplied. Now this looks very similar right. So, I have a force as a input here as a voltage as the input force produced a certain displacement, here voltage resulted in some charge and q dot was the current and so on. So, therefore, I could say that force here is equivalent to the voltage, mass would in this case correspond to an inductive element, B the dashboard or the damper would correspond to a resistance.

K the spring would correspond to a capacitor and the basic system variable the displacement now corresponds to the charge.

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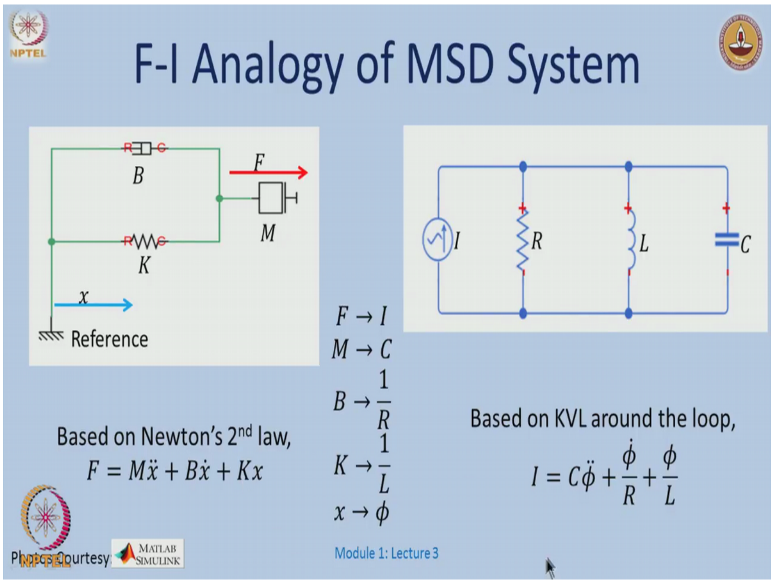

Let us go to the force current analogy. So, instead of having a voltage sourced circuit, now I have a current source circuit now which has elements in parallel which I called as a parallel RCL circuit. This thing remains the same F is Mx double dot plus Bx dot plus Kx and if I write the equations further for the circuit on the right hand side what I see that based on KVL I get I is C phi double dot phi is a flux, plus phi dot over r plus phi over L, the sum of all the three currents through the resistance the inductance and the capacitor.

There is some analogy now force could be analogously seen as the current, the mass could correspond to the capacitor b would be the conductance or the inverse of the resistance, the spring then would correspond to the inductance neck with the relation K goes to 1 over L and finally, the basic system variable the displacement here now corresponds to the flux here, and that is a reason we call it like the force to the current analogy. So, in my previous slide the mass element corresponded to a inductor now it is just corresponds to a capacitor and so on.

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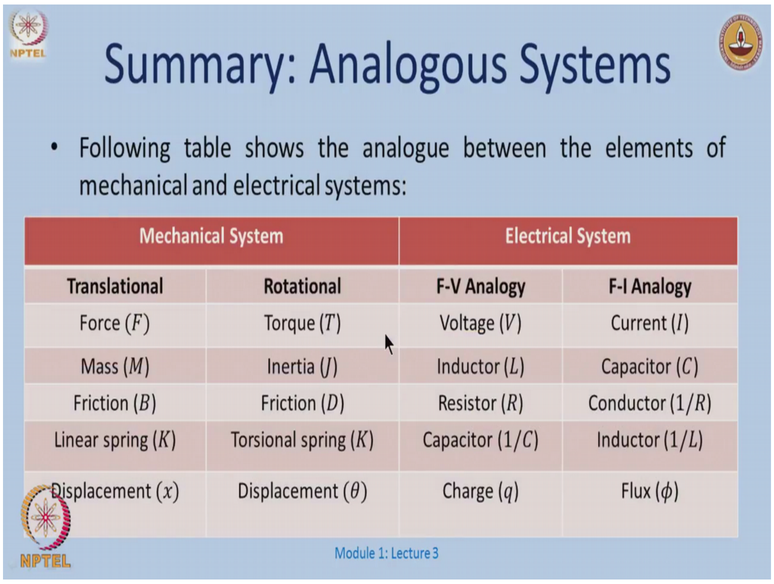

To summarize what we have done so far is to find or to establish an analogue between the mechanical systems and the electrical systems, the force to voltage analogy or the force to current analogy. Similar analogy would be even for the rotational domain where I start from torque to the voltage or torque to the current and this table summarizes the analogy between various elements between the mass and the inductor or the mass and the capacitor or the basic system variable which is the displacement, x here or a displacement theta to the charge or the flux.

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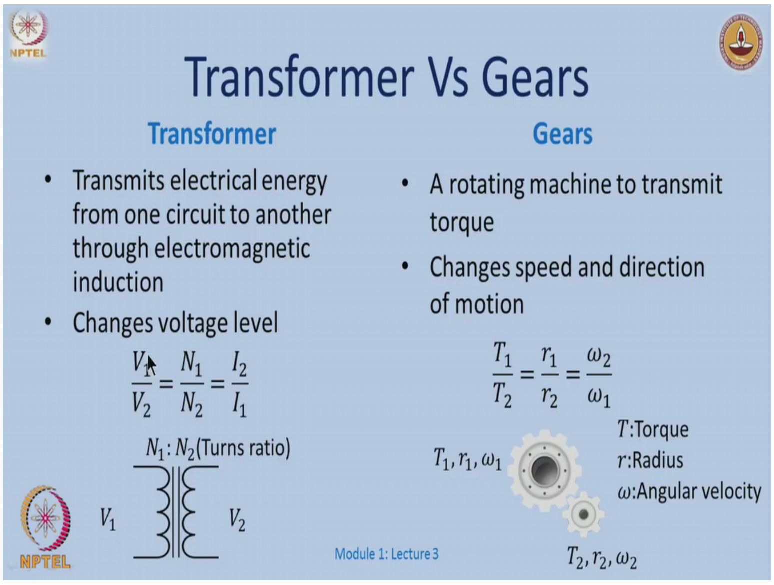

So, other kind of basic elements which we learn are systems which just transform energy from one system to other. In the electrical domain we all know the transformer which transmits electrical energy from one circuit to another through some electromagnetic induction properties. Analogously in the mechanical domain we have gears right which is right again it just transforms mechanical energy from one mechanical system to other via gears.

Here, well, I can say I have I can measure that the transformer as either a step down step up or a step down transformer which would change the voltage level, here it could change the speed and the direction of motion and they just look like this right. So, if I have a transformer it turns ratio N 1 to N 2 V 1 by V 2 is N 1 by N 2 and the current would be the reverse of it; it will be I 2 by I 1 right it will be a step up or step down based on if n one is greater than N 1 or N 2 is greater than N 1.

Similarly in the mechanical domain I have the torques on the both side related by this relation torque torque T 1 is equal to T 2 via the radius or inversely related to the angular velocity. So, these are again equivalent elements in the electrical domain to the transformer to and to equivalently a gear in the mechanical domain.

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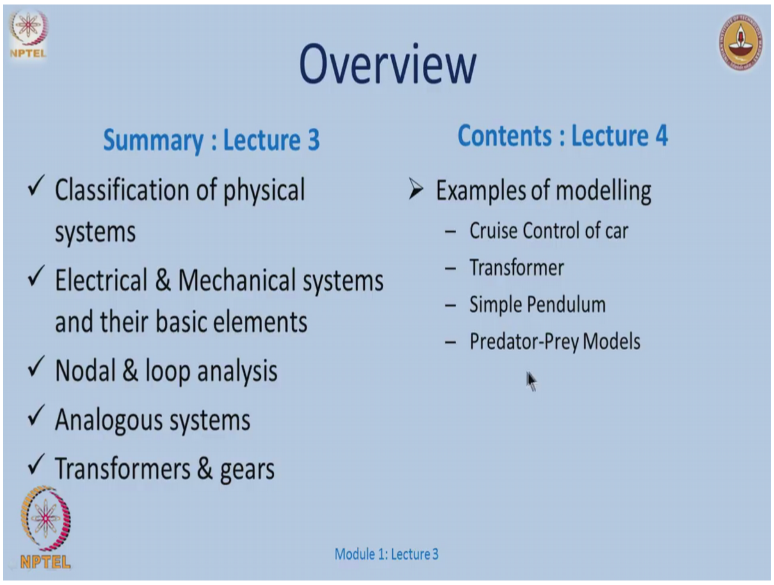

So, what have we learn so far? We have classified physical systems and as specifically concentrated on mechanical and electrical systems and their basic elements, we saw how by using basic conservation laws where the nodal or the loop analysis helps us get the dynamics of the system, we also established a beautiful analogy between electrical and mechanical systems and towards the end also learnt about energy transformation devices like this the transformers and the gears.

So, what we will do next is to also learn some more physical examples right. So, starting from cruise control of a car, we also look at some more details of the transformer of why what we saw in the previous slide is not really fit for analysis, the simple pendulum and something which is which might be a little new or call the predator prey models which I will explain you while we reach there.