Control Engineering
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Module - 07
Lecture – 01
Part - 02
Basics of control design – Proportional, Integral, Derivative actions

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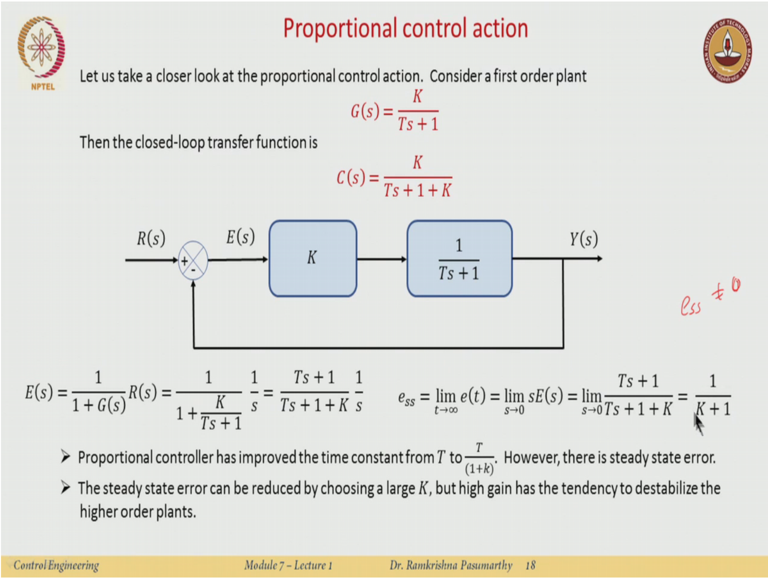

So, b proportional control action now right. So, let me start with a first order system, a proportional controller with the closed loop transfer function is K T s plus 1 plus K, what is the error? If the reference is a step function right so, the steady state error by to evolve this calculations we had done this before also is 1 over K plus 1 right the first observation is that the steady state error is non zero though I would ideally want it to be 0, but it is not zero in this case.

Now what does this proportional control do well the first observation is that the control or the proportional controller it results in improving the time constant, which means the response is faster on the flip side there is a steady state error right. Now what we could do is now just by observing this say that well my steady state error can be made smaller and smaller by increasing the value of the gain K, but this may not always be practically possible or even desirable because I might amplify unwanted signals and so on right.

In some case of cases of higher order plants which I will show you a example increasing the gain could also have a tendency to destabilise the system right for example, if I were to look at this root locus for I just keep on increasing the gain, but I just notice that I am now on the verge of stability right and that is that is not desirable ok.

So, the proportional controller it improves the response of the system, it also helps improve or lessen the steady state error, but only up to some factor right I just cannot get it to 0 ok.

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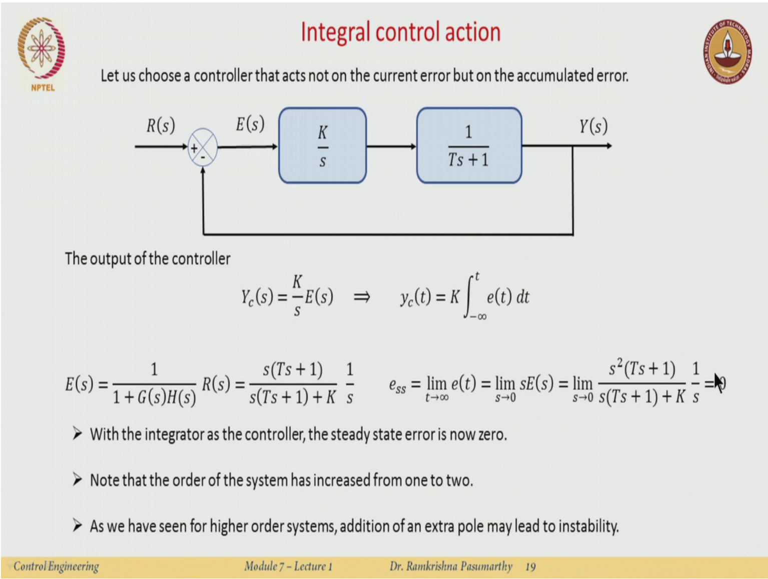

Now let us see what a pure integral control action do, and this actually go very well with the with the term side. So, integral control action is when I have a K, I divide by s one over s has an integral action right the Laplace transform of an integral of a signal just was translated to 1 over s of the original signal. So, the output is (Refer Time: 02:53) make something like this. So, now, what is the error? The error is right is something less happened here right. So, if I just go through all these computations the steady state error becomes 0 this we also can see in when we did the error constant analysis right type one system if I am tracking a step the steady state error is always 0.

So, with addition of the integrator as a controller when the steady state error is 0, but the system order has increased from 1 to 2 and if we keep on dealing with higher and higher order systems increasing a pole or adding an additional pole at the origin which is essentially an integrator might lead the system to the verge of instability. So, two things to observe here right first is proportional control, it improves my transient response it also helps in reducing the steady state error, but as an eliminate it completely. Integral action well I add an integrator to the system which means I add a pole at the origin it helps in reducing the steady state error to 0, in this particular example which we are talking about or in this particular case of what is now the type one system.

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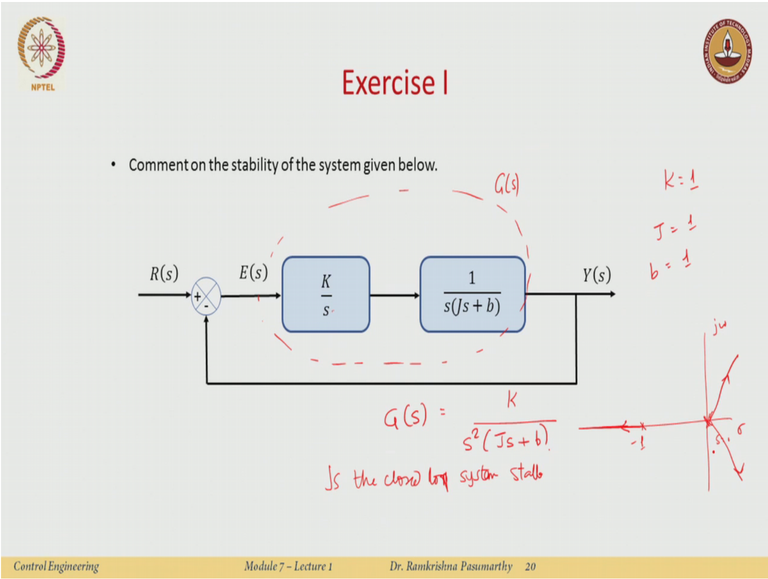

Higher order systems it may not always be desirable right that just keep on buying integrators and dumping into the system, it may not work and we say why it may not work. So, if I take this example here well, I need to see well how the system looks like now right. So, so total G of s if I call this as the open loop transfer function G of s here is K over s square J s plus b ok.

Now, let us may let me just say that K equal to 1, J equal to 1, b equal to 1 and let us see well what is the closed loop system stable well I can write if you do the Routh analysis, but I will do the root locus right because now we are fairly familiar with doing that. So, where are the poles where there is a pole here and a pole here and I write as equal to 0, in the sigma and J omega axis or the complex plane, but another pole at s equal to minus 1, now I think we now remember the rules of the root locus by heart. So, this guy will go here and we see that this guy is actually go this way closed at a this particular example while you were doing he root locus parts and you see that just adding an integrator here does not help me much because the system is now becoming unstable ok.

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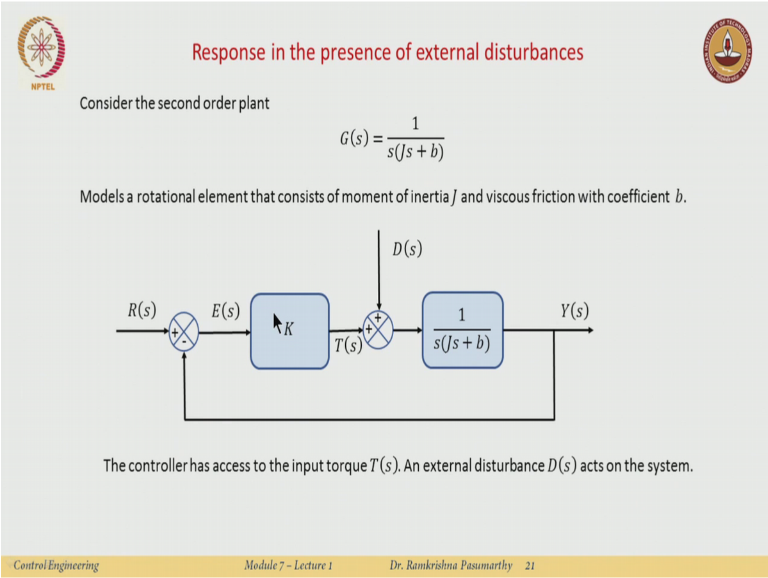

So, we have to be careful in that way, now what is the affect of disturbances right when we use these kind of controllers. So, again let me talk take this kind of plant here which is a rotational element with a movement of inertia J, and some frictional coefficient b. Now let us say there is some disturbance D of s and I will say what is the affect of this disturbance on the response of my system and can I get rid of those disturbances by adjusting the value of K ok.

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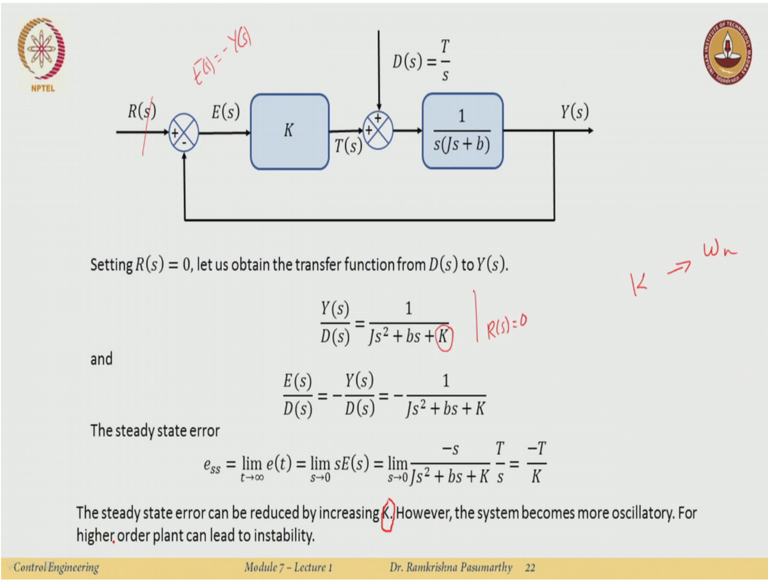

So, let us say well I am just interested at the moment in a step disturbance. So, I want to see what is Y s over s means the affect of the disturbance on the output Y, I can just do this by setting R equal to 0 within this how to derive this transfer function much much earlier n the course. So, now, what happens is well E s because of D s this becomes the negative of y. So, I am just. So, y goes here there is no R. So, E s is simple the negative of y.

So, I can also see what is the affect of the disturbance on the error signal I just stay forward to see because what we do all when we do this, we just compute this with R of s equal to 0, R of s equal to 0 that is matching here Y of s enters here what is E of s. E of s is 0 minus the signal what is coming here. So, E of s is the negative of Y of s ok. So, what is the steady state error because of this disturbance well the steady state error just do all the formulas se the final value theorem, and I can get that this is minus of T over K. Now I can take care of this steady state error I gain by increasing the value of gain K, now what is the affect of this gain K I am keeping on increasing gain k.

Now, where does the gain sit in my characteristic equation, the gain is sitting here. So, K now has some direct relation to omega n right s square plus 2 zeta omega n plus omega n s right if I just normalise it properly K. So, if K increases the system becomes more and more oscillatory right and or higher order plants as we saw earlier it can also go to the verge of instability ok.

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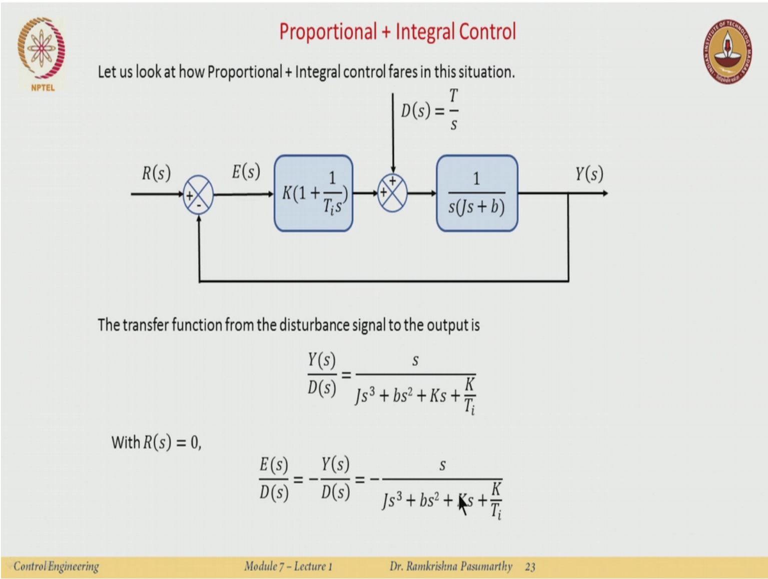

So, ext we see what is the affect of integral action, and we have this integral action addition to the proportional action and therefore, we call this a proportional plus integral control action. So, again Y s over D s I could be easily computed to e this way and I see that you know earlier we had stated that increase or introducing an integral action increases the order of the system. So, now, I have three poles with R s equal to 0 the affect of the disturbance on the error becomes something like this ok.

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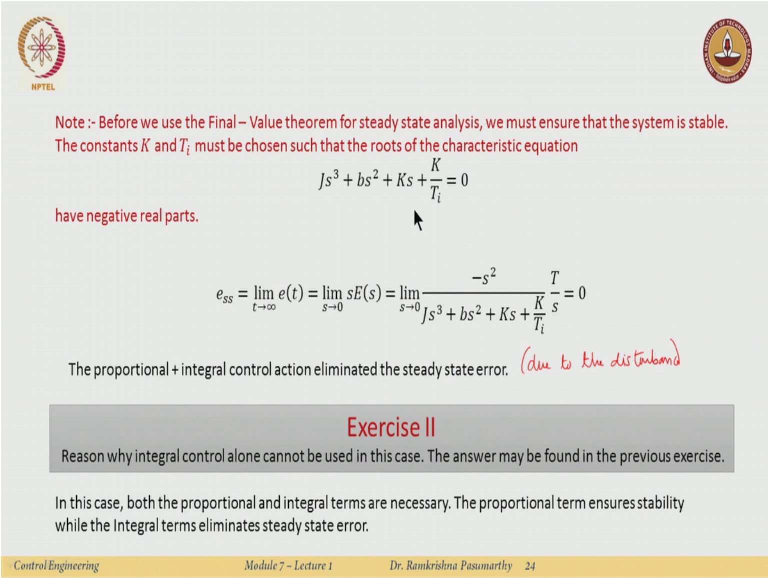

So, what are things which I have in my control from the control that is T i and I have a K here. Now I might be tempted directly to use the steady state error formula from the final value theorem to see what is a steady state error to yes stop disturbance. So, in order to compute what is the affect of the disturbance on the steady state error, while we are using a proportional plus integral control, we might be tempted to use the final value theorem directly, but only thing which we need to be careful is that when we use the final value theorem we must first ensure that the system is stable, which means I need to choose the K and T i such that the overall closed loop system is stable or in other words the routes of the characteristic equation have negative real parts, J and b are given to me. So, I am I can just play around with K and T i.

Now, under this assumption that the system is stable or that K and T i are chosen such that this system is stable, the steady state error I just use the formula for the final value theorem is 0. So, the observation here is that proportional plus integral control action helped us in eliminating the steady state error, now this is the steady state error due to the disturbance. I will just make it a little more explicit here which means the affect of disturbance can be nullified in the system. Now you may ask a question why did I not choose only an integral control action why do I need K plus 1 over T i s why not only this guy 1 over T i s or K y 1over T i s well the answer will be found in the previous exercise.

So, just be now just try to write down things and then well the answer is very obvious there right. So, to conclude the what we can say is that to eliminate the affect of disturbance, we need both the proportional and the integral terms. The proportional term was not enough as we saw earlier; adding an integral term alone gives us problems which we found in the previous exercise. So, the proportional term here ensures the stability while the integral terms eliminates the steady state error, and that we could compute from here now just look at a derivate control action.

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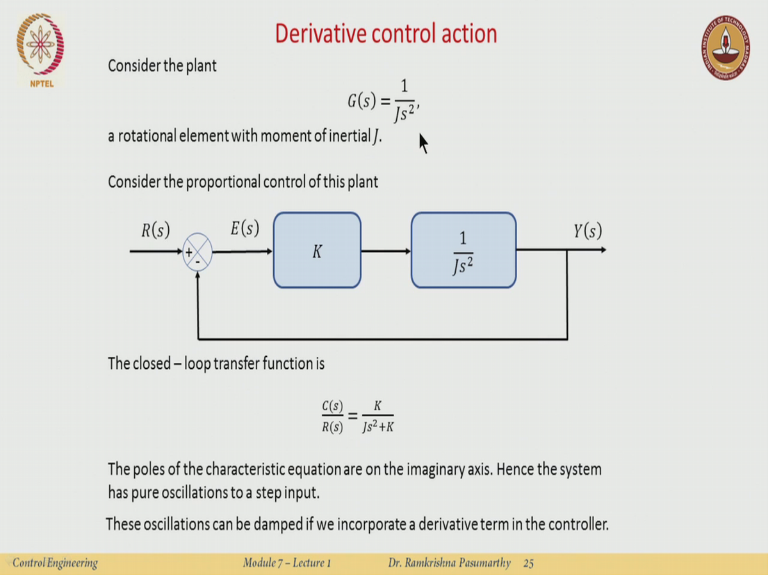

So, I just have my plant as a rotational element with a moment of inertia J. So, I will let me say I just start with a proportional control action. So, why do I even need to use a derivative action. So, I just say well the put a K here, the closed loop transfer function is K over J s square plus K, if I plot the root locus it will tell me that the poles are always on the imaginary axis as the value of the gain K increases from 0 to infinity right.

So, my response of the system will be purely oscillatory right and this oscillations would not die down. Now I would like these oscillations to be eliminated right and we see how these oscillations can be eliminated and when I say that the oscillations are to be eliminated it means I must introduce some damping into the system. So, this system here does not have any damping term, J s square plus K there is no term corresponding to the coefficient s right 2 zeta omega n s right.

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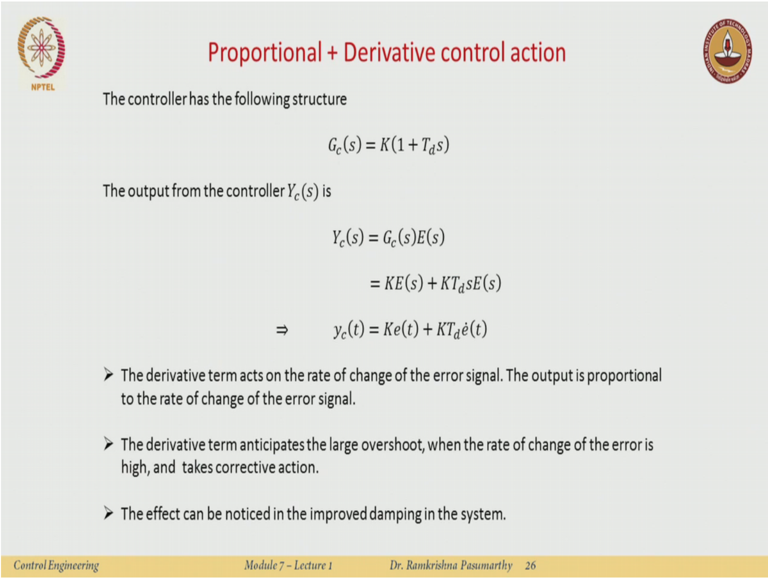

So, the oscillations can be damped if we incorporate a derivative term in the controller, and incorporation of a derivative term leads to introducing of the damping term in the system I will shortly show you how that looks like. So, can take a proportional for plus derivative controller, where the controller now looks like K 1 plus T d of s and the output from the controller Y c of s is all of these guys. So, I have Y c is K e of t plus K T d now e dot of t right we saw this when we were discussing the affect of adding a 0 right this is this is actually like adding a 0 to the system right.

So, you have the error and the derivative of the error also. So, the output now is which was earlier n the case of the proportional control only depending on the error is now also proportional to the rate of the error change e dot of t. So, what does this derivative term do? This derivative term anticipates a large overshoot with a e dot now how fast it is moving right e dot of t tells me how fast my error is moving. So, it anticipates the large overshoot based on the rate and the and when the rate of the change of the error is too high it takes a corrective action right. So, this e dot tells me how fast my error is growing right and this affect can be noticed in the improved damping in the system ok.

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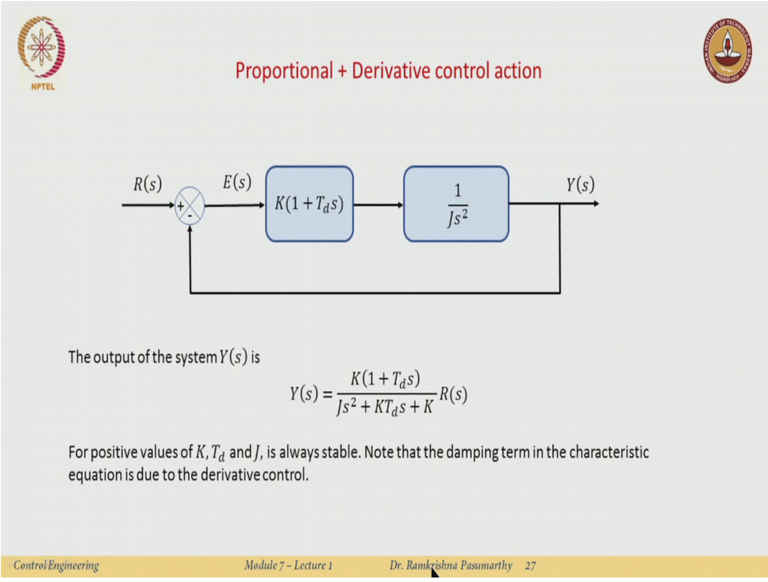

So, what is the affect of adding a derivative or a proportional plus derivative term to the plant 1 over J s square which does did not have any original damping. So, the output of the system is now well this K this entire thing will be numerator ,now look at the characteristic equation or the denominator J s square plus K times T d times s plus K.

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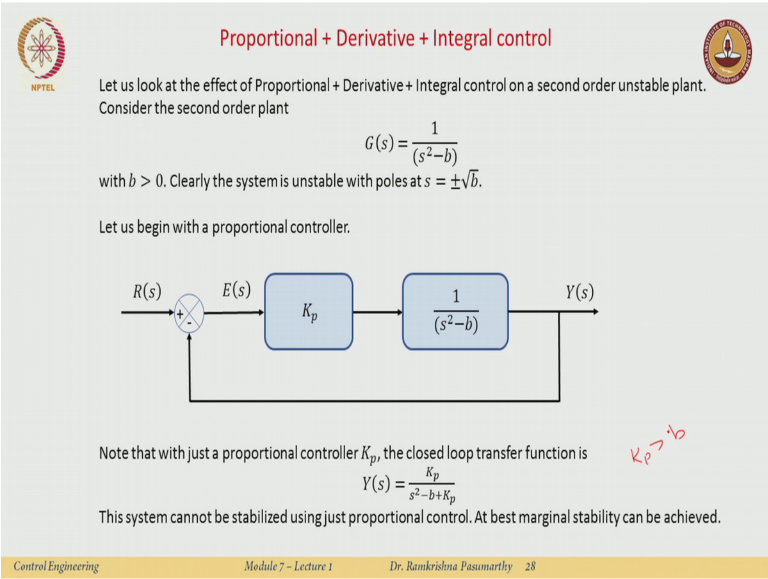

Now, this is just a second order system and we know that for positive values of J K and T d the system is always stable. Now what did the derivative control do, you just added this extra term here which was absent earlier right it added a damping term into my characteristic equation and this damping term is due to the derivative control action. Now let us combine all these things we started with proportional, we started with proportional plus integral control and now we later on saw proportional plus derivative control well, let us combine all these things together and see what is the affect of each of these things right. So, let us look at the affect of p plus d plus I on a second order unstable plant. So, let me say that I am starting with a plant which looks like this G s is s square minus b and for b greater than 0 the system is always unstable because my poles are plus minus square root of p ok.

So, the first thing right. So, in an earlier slide I said proportional control helps in you know improving the stability of the system, let us verify if it is true or not. I add a K p here and with this K p I am just calling this with you know subscript p, because I am just dealing with a proportional control action. With this proportional control the closed loop transfer function is now this one K p s square minus b plus K p. Now s is some stable well at this it could be marginally stable right because there is no damping term here right, but at least starting from this unstable system I could at least get it to a marginally stable configuration by appropriate choice of the value of K p.

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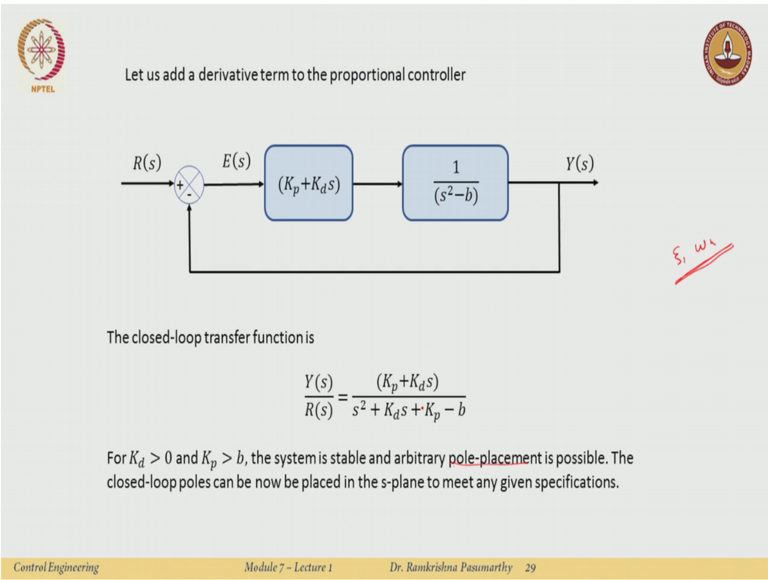

Now, let us add a derivative term the derivative term, well what we saw will add some kind of a damping to the system. So, the proportional control got an unstable system to become marginally stable, adding a derivative term will actually make it what we call also the asymptotically stable system right.

So, now the response Y s over R s has an additional damping term in the system, and for K p and K d K d greater than 0 and K p greater than b the system is always stable right and therefore, I can arbitrarily place the closed loop poles or give an any zeta and omega n I can design the system for any zeta and omega n this is what I mean by pole placement and because the poles are decided on the values of this I can place this where ever I want right based on appropriate choices of K d and K p. So, this K p greater than p is also necessarily here right when this K p is greater than b then I have marginal stability ok.

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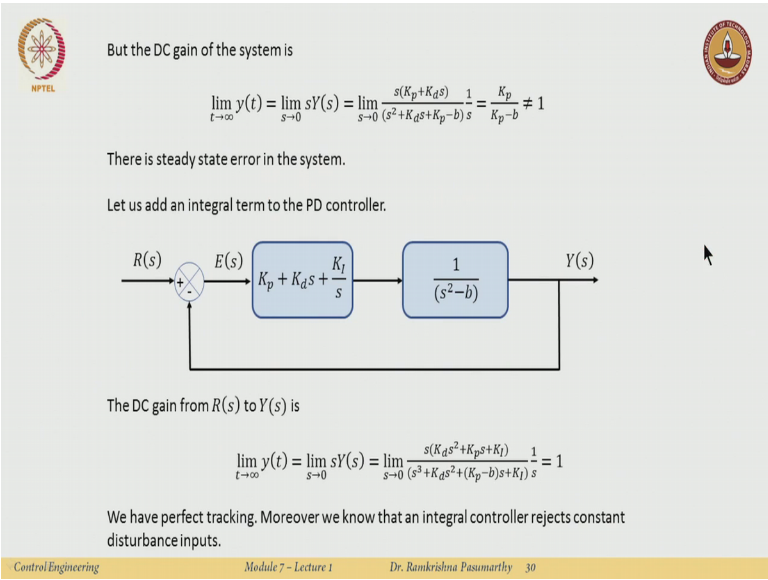

Now what happens to the steady state error? Well the steady state error I just look at the again the formula and I get that well it is I would like it to be one because I am tracking this reference signal well this is not one right therefore, there is a steady state error in the system now we will see what is the affect of adding an integral term in one of the earlier slides we said that adding an integral term eliminates the steady state error.

Now, let us add a integral term to this guy. So, I have a K p the proportional term the derivative term and now I have an integral term., the response through this and I compute the steady state error. So, limit t tends to infinity this is one right. So, what I am. So, my

R of s here is a unit step, but I am not tracking a unit step here right. So, my steady state value is K p minus b. So, I am not computing the steady state error here, but I am just computing the final value the final value should be one because I am I am tracking the step signal it is not one, but depending on K p and b. So, again appropriate choices of values of K p we saw earlier also that I can keep on increasing the K p to be very large value, but that may not help me all the time right. Now adding an integral controller gives me the output as one, which is what I am tracking right I am tracking a unit signal sorry I am tracking a unit step right. So, this adding of an integral term eliminates my steady state error or it means that I can actually track a unit step right and the.

So, the conclusion is that we have perfect tracking of this unit step moreover we also know that having an integral control action rejects constant disturbance input. So, we individually saw what are the affects of adding a p term the proportional controller a derivative term, useful in adding damping to the system or improving the transient performance and in integral controller which is useful in improving the steady state performance in terms of the steady state error.

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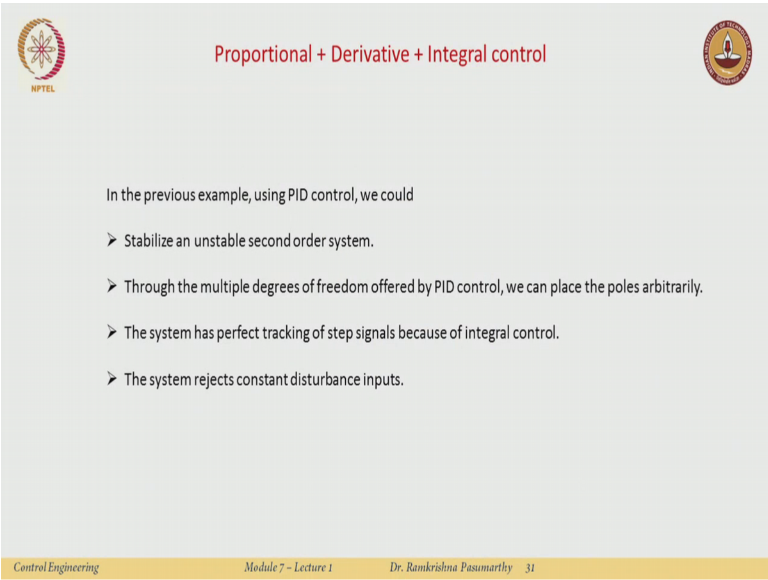

So, just to summarise what we have learnt in terms of the proportional derivative and integral control, is when we could start with an unstable system second order and we could stabilise it right we could also place the poles arbitrarily or in a way given any choice of zeta and omega n or any performance specifications, I could choose my gains

K p and K d appropriately so that I achieve those performance objectives the system has perfect tracking because of the integral control and what integral control also does it rejects constant disturbance inputs right.

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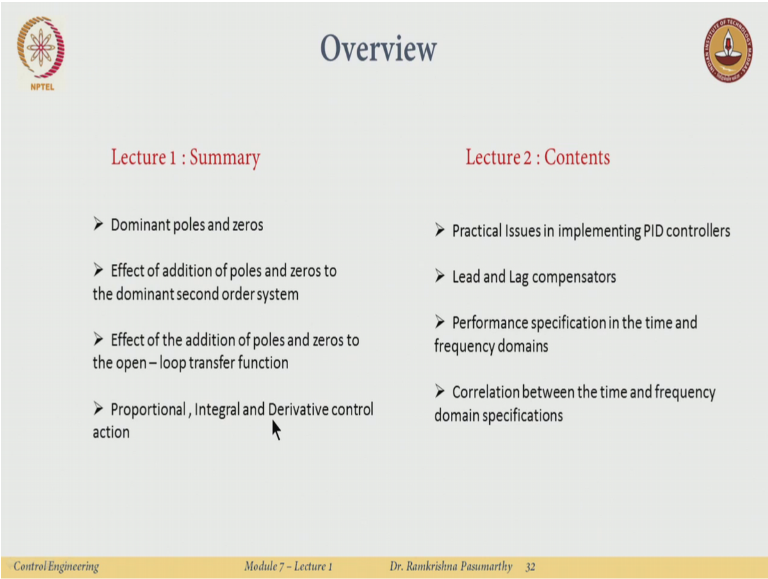

So, to summarise we define what are dominant poles and zeros, we by just you know using MATLAB plots we solve what were the affect of adding poles and zeros to the system right that was for second order systems we also saw the same thing when I add poles and zeros to an open loop system, what is a consequence on the closed loop system and we defined three basic control actions proportional, integral and derivative control action. So, in the next lecture we will see some issues in implementing PID controllers.

So, this stands for proportional integrative and derivative controllers and we will see slowly define what are lead and lag compensators, what could be the performance specifications in the time and frequency domains and the correlation between these two specifications if any ok.

Thank you.