Control Engineering
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Module - 07
Lecture - 01
Basic of control design – Proportional, Integral, Derivative actions
Part – 01

So, in this module we will start to familiarise ourselves with some basics of design. Now so far what we have done is modelling and lots of analysis of the system, analysis in terms of stability starting from the Routh criterion, root locus gave us lots of other information about how my closed loop systems change as I vary the gain from 0 to infinity. This is also parallel of what to do you know for gains from 0 to minus we will say for certain we will keep that for a little later that is not important at the at the moment. We also saw analysis in the frequency domain, we characterised relative stability in terms of the gain and the phase margins. So, what we will do now is to slowly learn concepts of designs.

Now, what are typical design problem. So, while we were doing time response analysis or even the frequency response analysis, we had some basic parameters or basic things which we observed like the rise time the settling time the peak overshoot and we saw how all these were related to the natural frequency and the damping. In similarly the frequency domain concepts like the bandwidth the resonance frequency were also in a way related to this system parameters. Now any design problem would be based on setting this values like the peak overshoot or the rise time or the settling time the steady state error, to some desirable numbers. Now that is what we will focus on how do we go about doing this. So, there are three basic elements in this control design. So, we start with a proportional controller, the integral and then the last would be the derivative actions.

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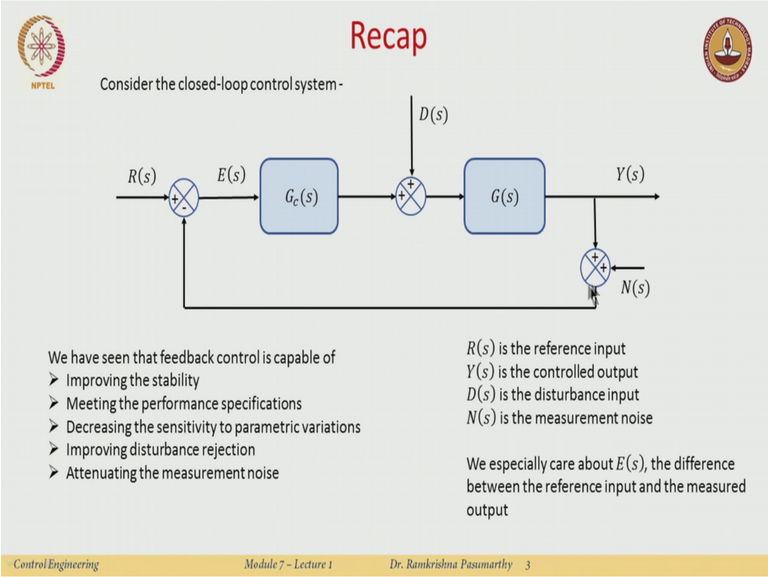

So, let us slowly go through these thing right. So, we started defining in our earlier lectures the closed loop system of the form of this form where you know, I had a plant to be controlled via this controller, I have a reference signal R of s which I need to track here there was disturbances in the systems, there was noise and so on right. So, this feedback configuration we saw is capable or it does improve the stability, it also in some sense helped in meeting the performance specifications we will elaborate on this a little more shortly, we also saw how feedback helps decreasing the sensitivity to certain parameter variations of the system, this step is ejection was also very obvious while we were doing the feedback control and of course, attenuation of the measurement noise and so on right.

So, of course, what was the defining factor even when we looked at the error constants was you know this signal E of s right.

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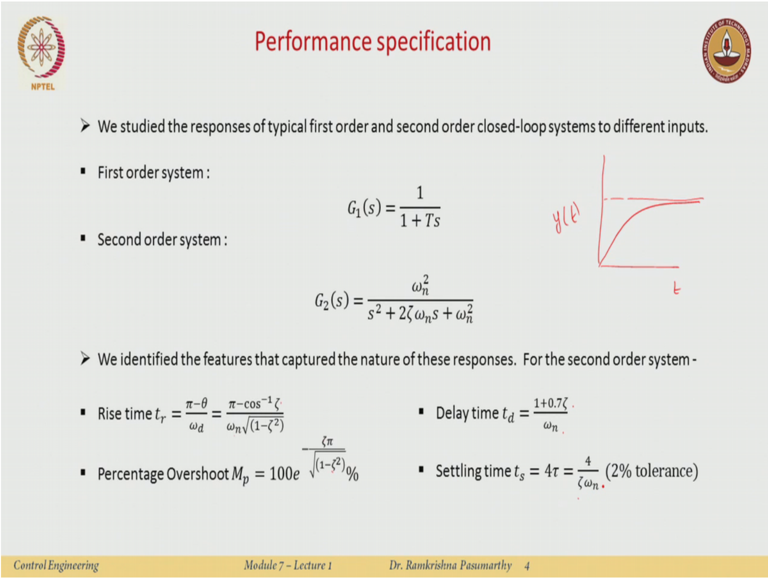

So, a typical first order system with some time constant T looks assembling of this G 1 is 1 over 1 plus T s right and then it is response was supposed to become like this depending on the time system, how fast or slow it was and it had. So, so inheritably some steady state error. So, this u of t this is we call this y of t or c of t depending on whatever terminology or rotation we are using. The second order system was of this form I had the natural frequency omega n square, I had the damping ratio zeta and so on right and for a second order system we defined some basic properties like the rise time again it was depending on zeta and omega n, the delay time was also depending on zeta and omega n similarly the peak overshoot was only depending on the zeta and settling time again both zeta and omega n ok.

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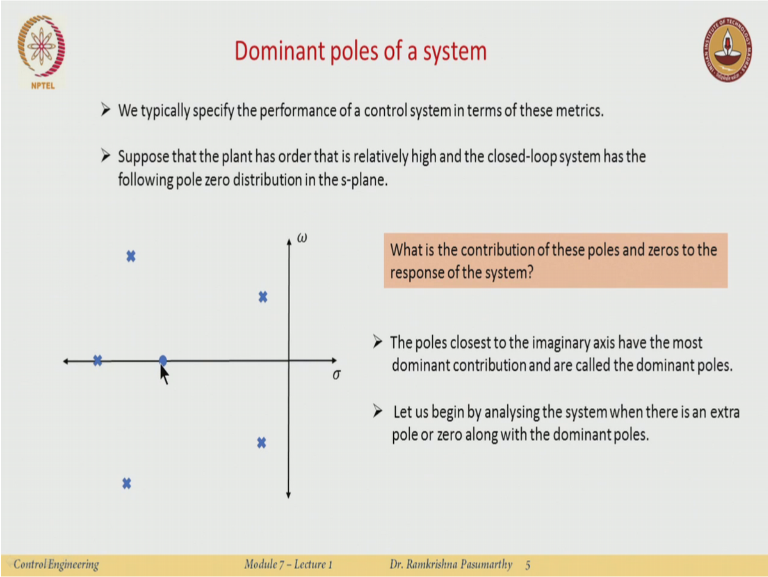

So, typically any performance matrix are defined in terms of these 4 or 5 things which we defined here rise time, delay time settling time, overshoot and sometimes also the steady state error. Now if you see that you know all these were defined only for second order systems and typically you may say well all not all these terms we come across are second order systems, it might be we actually saw third order systems we also did examples where we had a system with seven poles and so on. So, how do we deal how do we translate these situations or these design parameters are performance matrix two cases, where we have say several poles in the system. So, let us say I have a configuration like this I have these two poles herem I have these two conjugate complex poles here a pole here and a 0 here right.

So, this is something which has like 1, 2, 3, 4, 5 poles and a 0 right. So, say well what is the contribution of the poles and zeros to the response of the system. So, you can say that these two poles right I call them as the dominant poles of the system, because this as time increases the response of this now it dominates more than these 3 or 4 guys over here right. So, we will we will see now actually some plots and then we will do some analysis on this right. So, let us first start by analysing the system where there is an extra pole or 0 along with this dominant poles, and I think over the next few slides I think this concept of dominant pole should be should be more or less clear ok.

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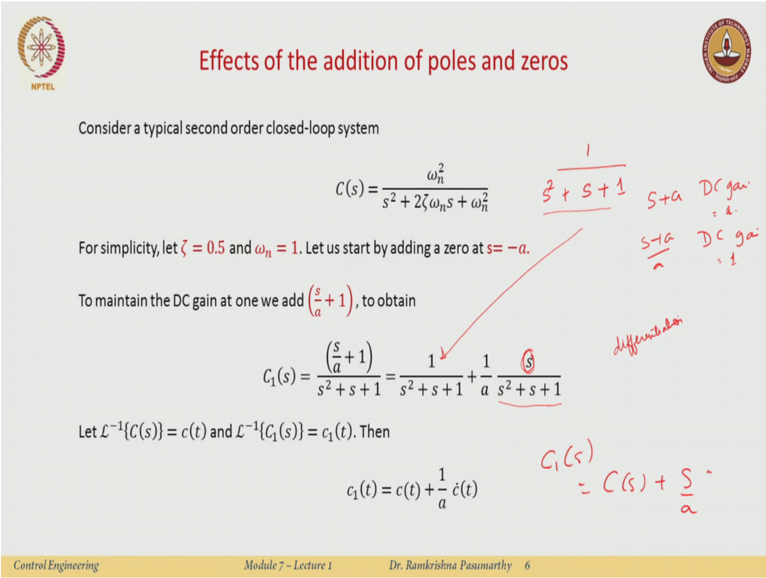

So, we start with a typical second order closed loop system and we know that this is always stable through we do not need to worry about stability here, and as far you know computation purpose of a simplicity I say well I say the damping ratio 2.5 and omega n is 1. Now I will start by investigating what is the affect of adding a 0. So, we do add a 0 let us say I add a 0 at s equal to minus a again in the left of plane. I just do a little manipulation here just to maintain the DC gain at one nothing really changes, but just for things to look a little neat, I just add you know divide this s, so the way I get this is as follows. So, if I just use the 0 as S plus a my DC gain would be a right and I just want to make it 1. So, I just do s plus a by a for which the DC gain is 1ok.

So, the response would now well how will my transfer function now look like C 1 of s is this guy the 0 at minus a, in such a way that the d c gain is 1 divided by this guy and I will get something like this right. So, I have the original thing. So, when zeta equal to 0.5, omega n equal to 1 this guy just becomes 1 over s square, plus 2 times 0.5 times 1. So, that is s plus 1. So, now, look at the something interesting here right. So, this is y original transfer function which is already here and then what happens with transfer function if I add this s over a plus 1. Well as you have this guy in the denominator and a numerator at s and we know what is the idea of this s right in the in the Laplacian domain; it has just to do with differentiation loosely speaking multiplying by s would mean differentiating that particular signal right and therefore, if I look write it in terms of in the time domain C 1 of t is c of t like the inverse of this plus 1 over a c dot of t.

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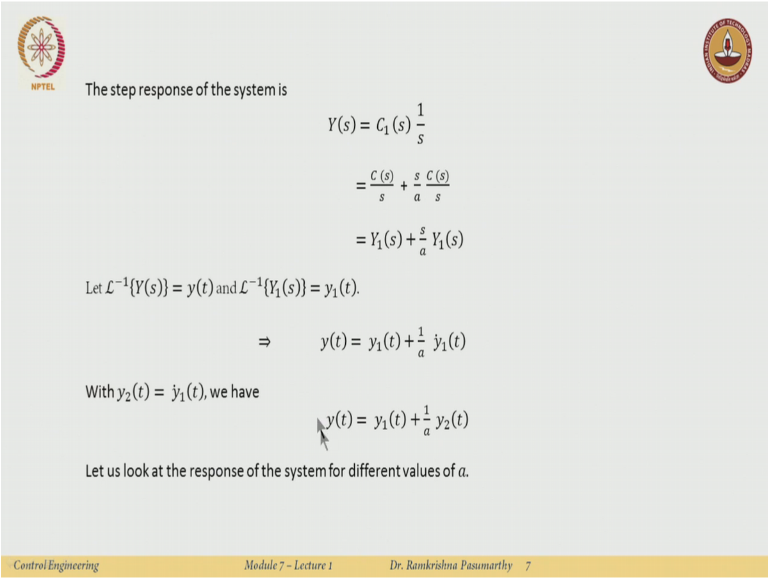

Now, what happens to their response when this for simplicity take the step response. So, Y of s is C 1 of s right over you know C 1 of s with a step unit step has a Laplace of 1 over s. So, this is C s over s plus s over a C s by s right. So, whereas, this come from this something here right. So, this is if I were just to write the properly. So, this would be in this C 1 of s is C of s plus s over a, c of s and that is what happens here. So, this is y 1 of s plus s over a y 1 of s where right.

So, I will just define the output corresponding to the original plant C 1 as s by 1. So, now, I go back to the inverse do the inverse of the Laplace go back to the time domain what I have is y of t is y 1 of t the original response without a 0 plus 1 over a y 1 dot t. So, when, so I just denote y 1 dot s by 2 and the total response is y of t is y 1 plus y 2. So, let us I mean let us just you know remember this for a while y 1 is the original response without the 0 y 2 is just the response of the 0 of adding the 0 and then y of t is the total response c ok.

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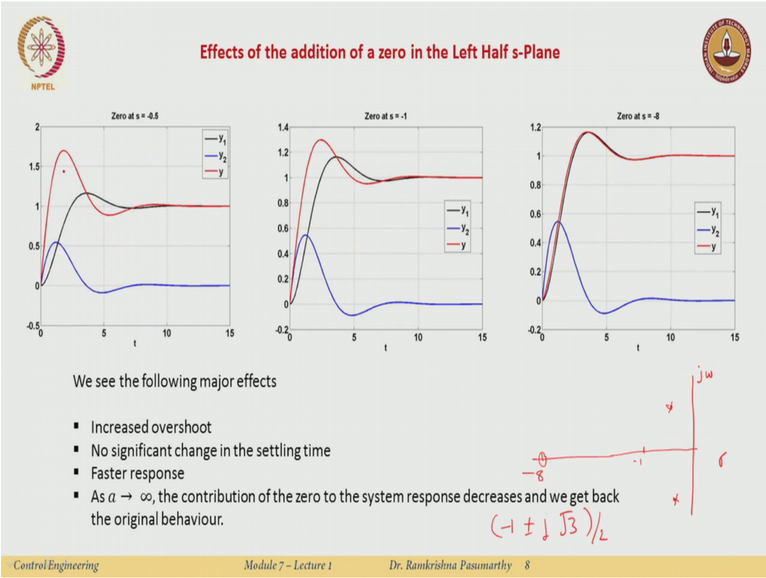

So, f I just take this system and I say I add a 0 at s equal to minus 0.5. So, what was my original response I am calling this as y 1 of t. So, this is the signal in the black right this is 1 of t, now I add a derivative term and what does sorry I add a 0 and what the 0 does is adds this extra term into the response 1 over a y 2 of t what is y 2 of t? Y 2 of t is y 1 dot of t. So, the total response y which was y 1 plus 1 over a y 2 is just y this is red line here. So, this, the black line or the black curve was the original response, I add a 0 at s equal minus a and then I get this one again by maintaining the 0 the maintain the d c gain to p 1. So, this is at equal to minus 0.5. So, wherever my original poles of the system, the original poles of the system were at minus 1 plus minus j square root of 3 this entire thing divided by 2. So, I have something here. So, this is my minus 1, I will have minus half plus minus root 3 by 2 somewhere here and here right. So, these two are the poles of the system and I am just adding the 0 over here and this is again the sigma and the j omega axis ok.

Now, say I add a 0 I remove this guy and I add a 0 over here, at s equal to minus 1 all I remove this guy right this kind of longer axis. So, what I have is the open the second order system plus a pole at sorry plus a 0 at minus 1. So, again we follow the same thing and see what happens to the response, the response in the black remains as it is the scales are different and therefore, this looks a little bigger right ok.

Now, y 2 looks like this, this 1 over a y 1 dot, and the overall response now is something like this the red line. Now I do what do I do I just remove this 0 over here and I say I add a 0 much further away right. So, is somewhere here and I say this is a 0 at minus 8, now what happens well y 1 just still remains the same, y 2 well the blue line there is something like this, and then I have the total response is something like this. Now again let us compare these three plots. So, as I as the 0 is closer to the origin you see here difference is quite big here right between the original response in the black curve and the red and the and the response in the red which is the affect of adding a 0, at s equal to minus 0.5. If I add a 0 at s equal to minus 1 whether is still some affect with the curves get much closer, add a 0 further to the left right at very far at s equal to minus 8, say that the response is more or less the same in a way I can just say will (Refer Time: 14:16) at 0 right and nothing changes ok.

So, apart from this distance in the curves here what else do we observe, what is the difference between this black curve, and the red curve here well one thing is obvious that this has an increased overshoot now here. So, the steady state value is one. So, I am my overshoot is like almost between 60 and 70 percent, this is at 0 at minus 5. Now here the pole is the 0 is slightly to the further to the left. So, the overshoot still increases, but not by too much well here well it is more or less the same well first is an increased overshoot depending on the location of the 0, but what happens here right there is no too much change in the settling time and it is much easier they more or less raised to 98 percent of the original value in fairly the same time right, all 3 guys including you know the original 1 and then well what is good here is that I have a faster response in a way that I reach the peak time much faster here, I was taking about two and half seconds here I reach in less than less than one second.

Similarly, here also the response is faster right now I reach this time you know when obviously, is like two and half seconds here, I reach about little over one second and so on. So, my response is faster and as I keep shifting or pulling my 0 further to the left and I keep doing this, what I observe is that as a goes further to the left and furthest the contribution to the 0 or the contribution of the 0 to the system response decreases, and we get by the origin behaviour say this is minus a t I do not really need to plot this curve also it would be almost exactly be the same right. So, there is some affect of adding the 0 and it is based on where we add the 0. So, we want them somewhere close to this these guys to see the maximum affect of course, there are also drawbacks here at about that in a way that the that the overshoot actually increases, but nothing changes in the settling time ok.

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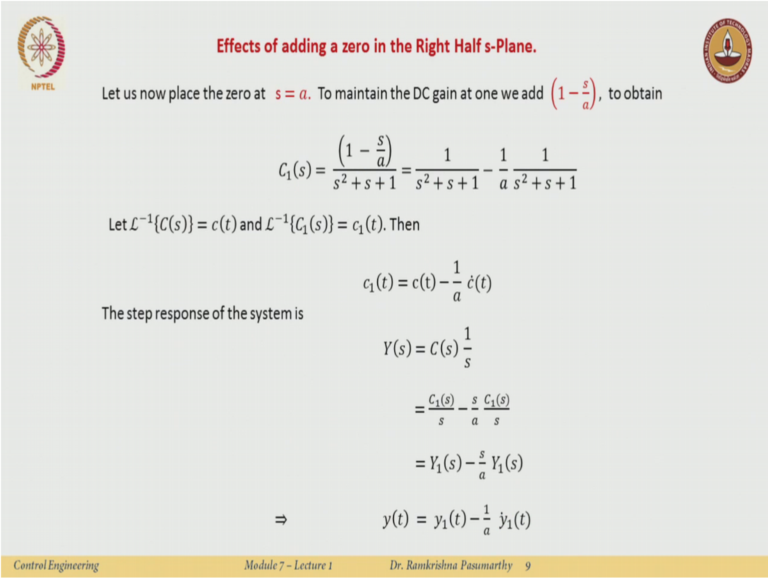

Now, what if I add a 0 in the right half plane right, again we do the same thing that to maintain the DC gain at 1 which has divide the entire thing by a. So, my transfer function of the system is now 1 minus s over a is again looks fairly the same, I have 1 over s square plus s plus 1 I just have a minus here, minus 1 over a one s square plus s plus 1 or in other words C 1 of t is c t minus 1 over s c dot is at a plus earlier.

So, here similarly the response here also translate in this same way now the total response, is the response y 1 and then in addition we have this response due to the 0 that is placed at s equal to a ok.

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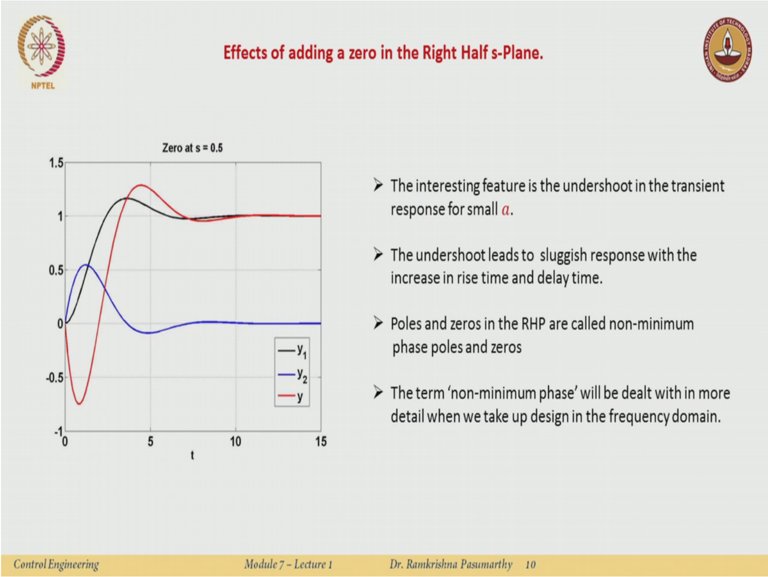

So, what is happening here well again the black curve is as same as in the previous case, the blue curve is the affect of adding a 0 on that half plane, now the red curve which is the combination of these two looks something strange now right. So, initially if you look at this plot you might see that well what I what do I want to reach I want to reach the point plus 1 right well with less number of overshoots and you know setting time should be fairly small and also no steady state error.

But what I am doing is initially if I just switch this on, and I see that well I am actually I am actually going away from 0 which quite mean well you know I just want to stop it over here because I think well this is something very weird right, but then if I just let it go for a little while, it reaches you know the minimum here and then it actually starts going up. Going up and it does something same as a previous one, but there is some increase overshoot of course, you know there is nothing much in the response is a little slower the overshoot has increased and of course, the settling time does not do much things ok.

So, what is the first observation is that well so, far we have only analysed overshoots right how much so I start from 0 my desired value is 1, how much do I overshoot before settling down at 1 right. So, that is over here right this is this was the overshoot, now addition to that we also experience something called an undershoot I go down from minus 5 to less almost minus 0.8 and then come back here right and then this undershoot what is does is that it actually increases my delay time the rise time and the peak time and so on right which means that the my response is quite sluggish now.

So, when poles and zeros are in the right half plane, these are called non minimum phase poles and zeros. We defined this last time and we will still stick to that definition that if all the poles and zeros are to the left then it is a minimum phase system, and if some guy sit to the right it is a non minimum phase system. We will do we will deal the we will deal with this when we you know deal when we talk about design in the frequency domain or maybe some special module or just dealing with a right half 0.

So, at the moment we will just restrict ourselves to this definition that will be enough for our analysis. Just that we will just remembered that if I have a right half 0, I experience an undershoot and this will always happen and I will show you a little proof for that little later ok.

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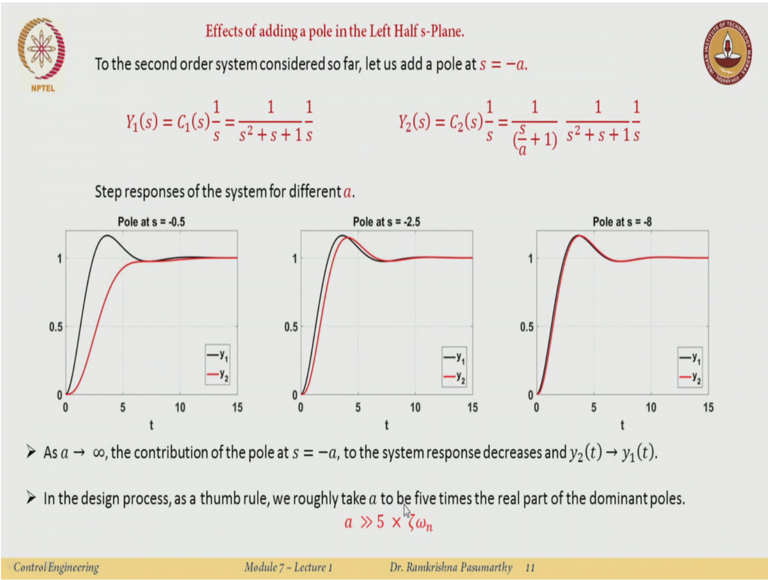

So, here what we will see is the affect of adding a pole to a standard step response of a second order system. So, well this is a standard response is this one, 1 over s square s and 1 and 1 over s. Now I add a pole right I add a pole at different locations a. So, first I see that if I just look at y 1 this is a natural response or the response without adding a pole, if I add a pole very close to the origin and you see there is a significant change in the in the response also good in a way because you know I kill the overshoots and other stuff. Similarly if I had a pole slightly further may be little little 5 times to the to the left and I see well the response is almost look the same right there is a very little margin here and here and so on.

Now, if I go keep going further I see that the red line actually touches the black line right now that is not surprising right. So, as a tends to infinity the contribution of the pole, it decreases and as you know as this pole it keeps on till infinity there is no affect at all. So, whenever we do in a design process where we have to add a pole such that it does not have too much effect on the transient response, we take it to the 5 times the real part of the dominant pole as in this in the middle case when we have a pole at s equal to minus 2.5. So, we will keep this in mind well we were well we will explicitly talk about design problems while when we are looking at you know improving the transient performance and steady state performance and so on ok.

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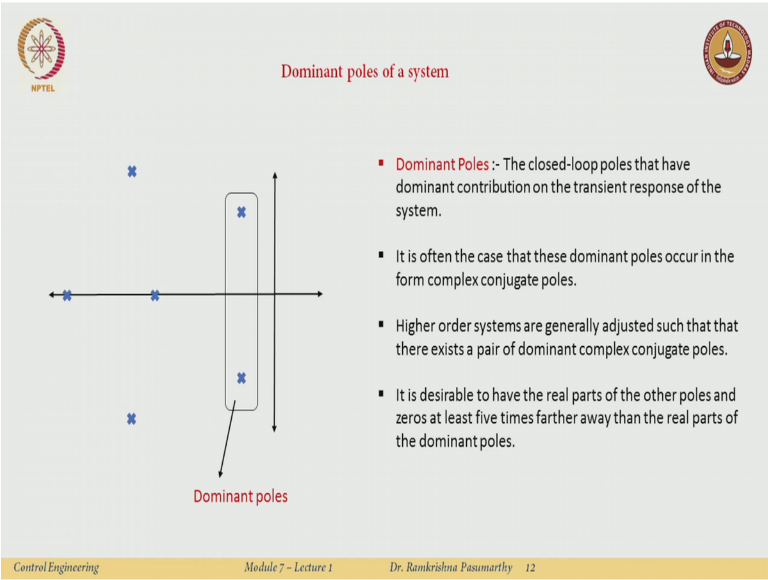

So, this leads to the definition of our characterisation of the dominant poles of the system. So, the dominant poles are the closed loop poles that have dominant contribution on the transient response of the system, and it is quite possible that this occur in conjugate pairs, they could also be just sitting here right on top of each other say at s equal to minus 1, and higher order systems are generally adjusted such that there exists a pair of dominant complex conjugate poles, which in that case we would just restrict our analysis to these two poles and we could the entire analysis and design right.

But for that well it is desirable to have the real parts of other poles and zeros at least 5 times further away then the real parts of the dominant poles. So, if this is in minus 1, this guy should be at least at minus 5 and further away. This is at minus 1 and there is a say a dominant not really dominant, but say some other pole pair at this one then I cannot really call this the blue guys as the dominant poles. So, these are like typically because the response of these guys over here they die down faster, the response of these guys is even like a slower than this, but still fast enough that this also dies on fast and what is remaining is just this one right and the limiting cases when the poles here right the response never dies down right. So, this if this plus this is the case then these are the dominant poles because this will still go to die down to 0 after a may be a fairly last time, but this will remain as it is.

So, if I you know look at this in terms of a classroom, the guys who are on the you know sitting on the last bench furthest away from me go to sleep much faster, then these guys, then these guys and possibly you know the dominant guys are the ones who are on the on the first row or the second row right. So, this is just like the attention panels or so right. So, these guys loose attention faster these guys little slower, well these guys possibly are a little more attentive than these other guys. So, if these guys are here well this will you know go to 0, but it little after possibly after 45 minutes after the class or something. And this is this guy is me here right the teacher n the inertia the axis he cannot go to sleep right he will always be talking talking talking.

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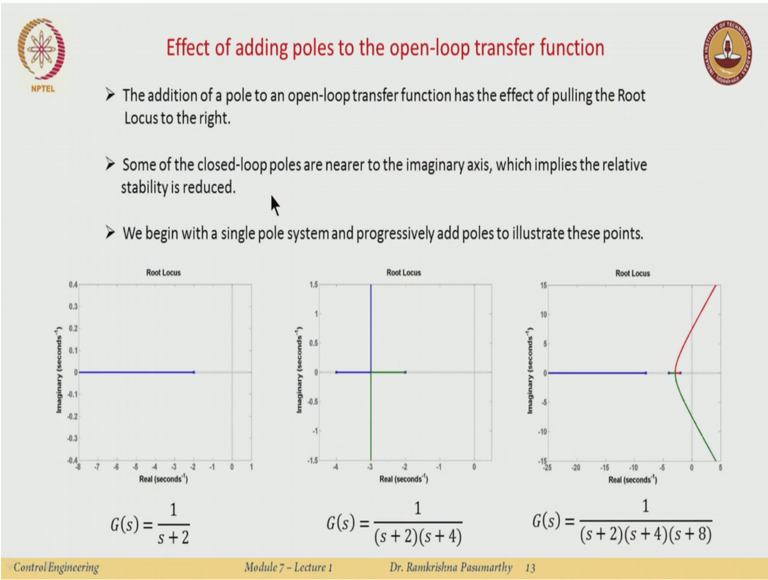

So, we will see what is affect of adding poles to the open loop transfer function. So, far we were just doing the closed loop alright s square plus 2 c tau omega n, s plus omega n square right. Now we will see, what is the affect of adding poles to the open loop transfer function well, we will do this in terms of the root locus. Now the step response sometimes may not give too much information, but you know root locus actually is a little is here to interpret and things are much much straight forward some in some sense to see from the root locus. So, we will switch to the root locus plot right.

So, let us say I just have G of s is 1 over s plus 2 is just a pole here, how the root locus look like as the gain k increases it just goes further and further to the left hand sits it is stable all the time. Now to this guy I add another pole at s plus 4. So, this is my original pole, I add another pole here and what I know from the first class of root (Refer Time: 25:46) root locus is that well this guy will move towards the right this guy will move towards the left, they meet each other and then they go their own ways ok.

Now, when I say well I will just do some more experiment I will just add at s plus 8. So, I have s at minus 2, minus 4 and minus 8. So, I have 3 guys all three would go to infinity because there are no zeros this guy will choose the lazy path go here, this guys have to move this guy has to move to the right this guy has to move to the left because something tells me that point on the root locus lies on the real axis only if the number of poles plus zeros to the right is an odd number. Therefore, this is not allowed to be on the root locus this guys well this entire line is not allowed to be on the root locus he just take the easy path and just goes runs away. Now these two guys meet here I calculate the angles of the asymptote that will be 60, then this would be 180 and minus 60.

So, these guys will go this way. So, what is happening right? So, first well the root locus was which were just going to infinity then the first case is now pulled now a little bit to the right. So, the root locus exists just in this domain it was earlier existing in this entire domain here right and I add a one more pole and it shifted further to the right, in such a way that it can also go to the verge of instability after I attain a particular value of the gain k right. So, these are the very simple things right there is nothing to actually remember as a formula or anything like that, but just that we are just plotting various things just kind of just doing some initial experiments to then you know evolve towards a general theory.

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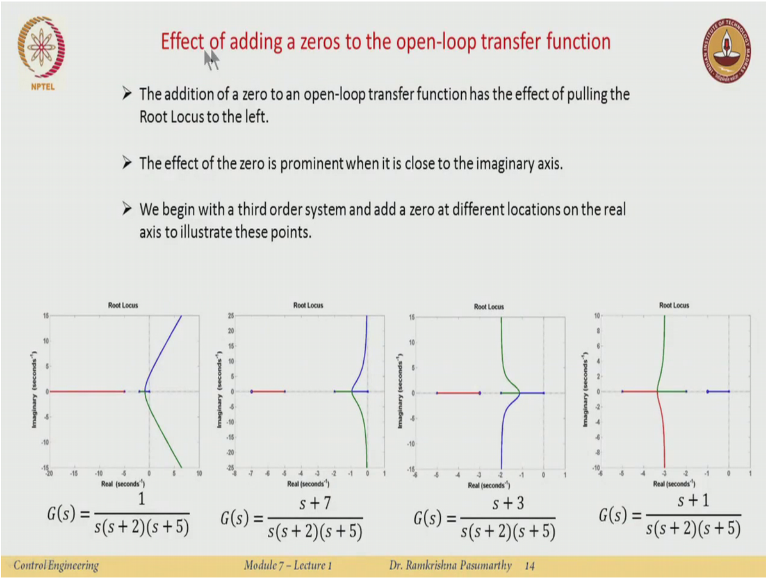

Now, what happens if I add a 0 now to the open loop transfer function? So, let me say I start with this open loop configurations. So, I have c a pole at the origin, a pole at minus 2, a pole at minus 5 and of ours you know I just plot a root locus this guy will go away to infinity these two guys will meet up each other and then they having go to infinity, and after a particular value of k at the Routh Hurwitz will tell me that I am going towards the unstable region. So, let me just add a 0 just slightly towards the left of minus 5 right. So, this was minus 5 and this is a 0.

So, this guy which was now running away is now trapped by this 0 here, now do not need this right. So, this guy also does something nicer not only (Refer Time: 28:33) of this guy here, he also does things in a way that the root locus now is well more or less shifted to the negative region right, there is no point of instability here because my asymptotes are just this is the imaginary axis. Now I add pole somewhere not really you know very far at minus 7, but somewhere between these two poles right this and this. So, well this guy again little just move to the right and stop here.

And now I see that the root locus has improved much because if I were to deal with this gain here you see that I am close to the verge of instability right this and this right. So, this this this kind of adding a 0 it makes my system stable because it is pulling this blue and the red lines to the left, but it is well it may not be too good in terms of relative stability. Now I add a 0 at minus 3 and say well it is actually fair, fairly stable for all values of gain k right, and if I add it further add minus 1 then I get a configuration which looks like this right. So, what does your observation tell us? The observation tells us that the addition of a 0 to an open loop transfer function it pulls the root locus to the left right and the affect of 0 is prominent when it is close to the imaginary axis. So, in these plots we just start with a third order system and then study several affects right ok.

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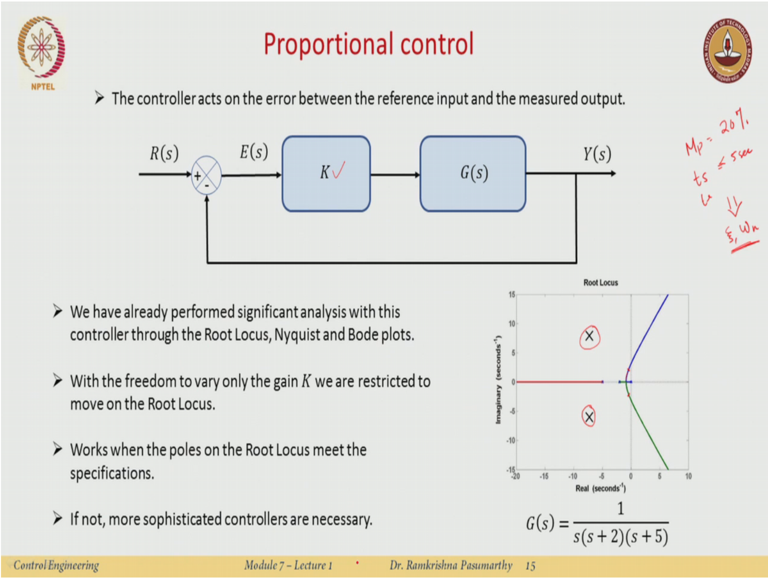

Now, based on these observations we will just start with the basic element of control. One is the first one is the proportional control is just means what happens if I just keep on adjusting the gain, does adjusting the gain achieve my control objectives or not right you either increase the gain decrease the gain and so on and we will see under what conditions just a proportional controller or just by adjusting the gain k we meet our performance specifications, we will also see is there any drawbacks right just by adjusting the gain even the system is stable does it always help us achieve good results that is what we will do here right. So, again I have an R of s the G of s is a plant, and my controller is just a proportional controller. So, as usual this is my reference signal this is my output this is certain error and then and then the controller is supposed to adjust according to itself based on the error and so on.

So, the first question is when will a proportional controller work. So, the steps we usually here are at the following let us say I am given that well that my peak overshoot should not be more than 20 percent, I am also given a specification that the settling time should be well say 5 seconds or less correct and maybe some other things on t r and so on. So, what this will translate to is some zeta some omega n, now I just keep on moving my gain from 0 and see at any point of the root locus, if this specifications are met if these are my domain impulse if it is met at this point and this point then I say yes I can just adjust the gain and then that value of gain K would sit in the controller and do my objectives.

But if this zeta and omega n translate to this pair of dominant poles, then no matter whatever I do with my gain I am never here right. So, when does this work well of course, we have the freedom to vary the gain K, which means we are recited to move on the root locus and therefore, this worse when the poles of the root locus eet this specification right. So, in just translate the given specifications into zeta and omega n this zeta and omega n translate to appropriate locations on the root locus or all or on the complex plane, if by adjusting the gain I could arrive at this complex planes then I did I just say this is my controller without the value of gain k if not well we need to something else right.

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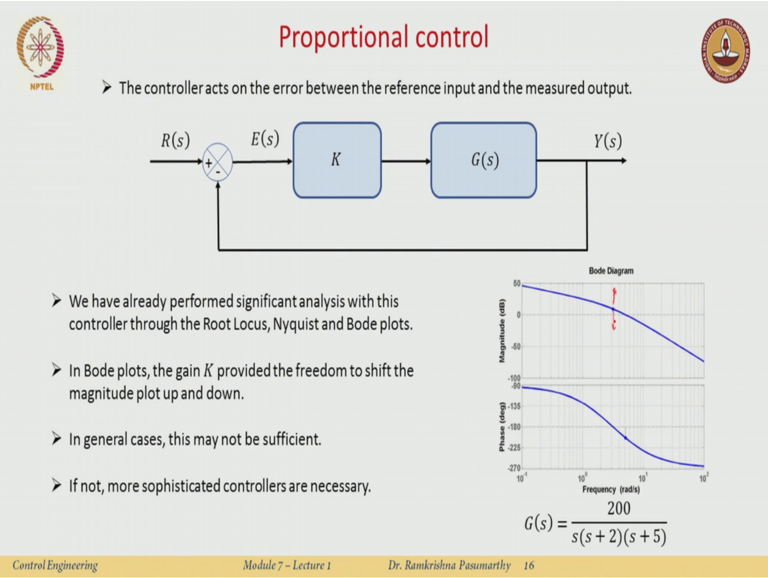

So what else is the proportional control do right. So, again it just depends on the controller adjust according to the error and so on, what happens in the bode plot right. In bode plot the gain K would just shift the magnitude plot, this does not contribute to the phase the phase remains the same the magnitude could go like up and down depending on whatever is the value of gain K again. So, this, the case of a unstable system could be easily seen here and we could see that I know by this in the gain I could actually push the bode plot or the system to the verge of stability.