Control Engineering

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Module - 06
Lecture – 04
Bode Plots

Hi guys. So, we are almost half way through the course. So, today we will compute to complete module 6 and let us see, let us see how it goes. So, I will start with what is called as the bode plots. So, last time we plotted Nyquist and that gave us lot of information. It gave us if we start with an open loop system which is unstable can the close loop system be stable. It also gave us accurately or with some good measures or some good quantification of the relative stability. And we defined 2 different terms called the gain margin in a way how much the system gain can be increased before it reaches the verge of instability. And otherwise phase margin, how much phase can be added to the system before it reaches the verge of instability. We also saw open loop system which was unstable and then we had to actually decrease the gain to get back the system into a stable configuration. So, what we will do today is to look at little easier way of doing this right. So, it is called the bode plot it is also called the logarithmic plot. So, I guess somewhere in circuit is you might have encountered this.

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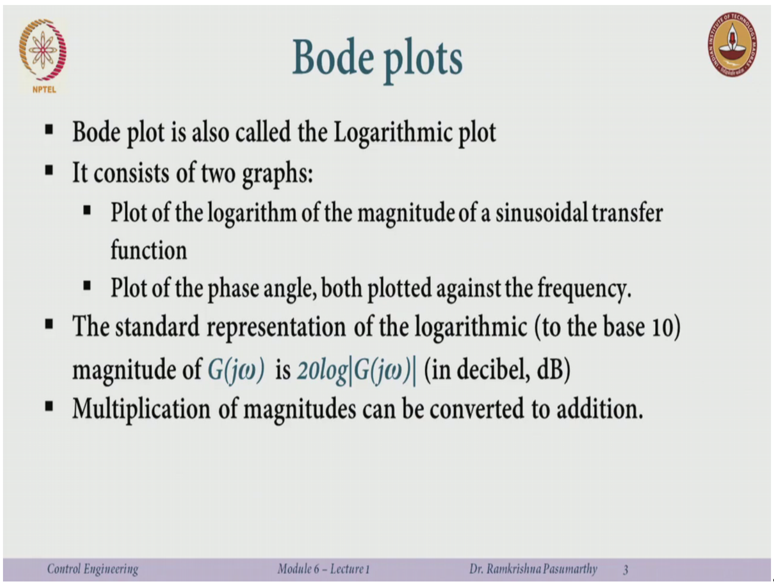

So, it consists of 2 graphs again we start with a sinusoidal transfer function. So, I first plot the logarithm of the magnitude and, So this magnitude it will be worsest of the frequency and also the phase angle against the frequency. So, some basic nomenclature or some basic notations will define. So, the standard way to represent this is in terms of log to the base 10 and multiply it by 20. Not really important of why this is 20 and why not logarithm to the base n we will not worry about that right. So, the advantage when I do this and we will be which will be fairly obvious in the next few minutes is that, multiplications of magnitudes can simply be converted to additions. So, we will see how these things actually translate to just or plotting the frequency response in the in the logarithmic scale just translates to just plotting straight lines, like y equal to m x plus c. And if I have 2 lines I know how the addition of those 2 looks like right. So, this kind of lots of lot of simplification whereas, if when I was in the Nyquist plot I had to be careful of lots of things, right. It wasn't really clear what happens at omega equal to 0 sometimes. So, there are poles on the imaginary axis and then, here are the bunch of things that there could be multiple intersections on the on the real line, how do I detect those all those were fairly tricky and then it required lots of computation by hand which could sometimes go wrong right.

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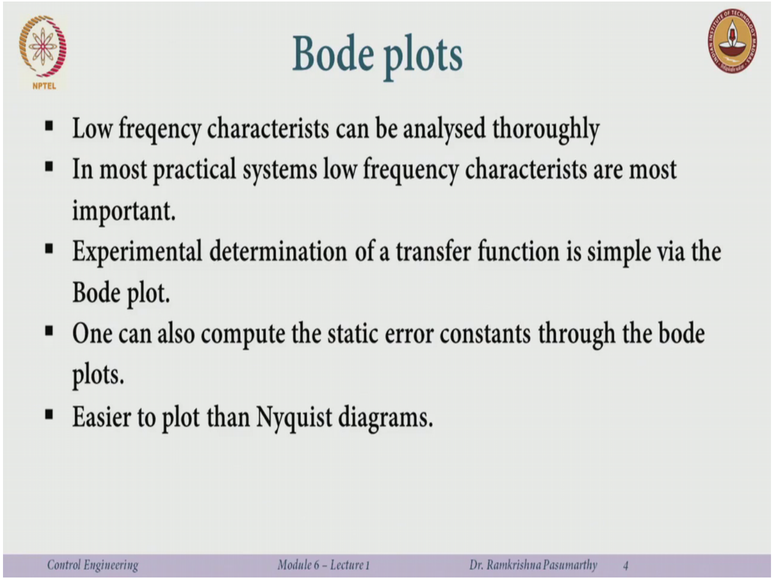

So, we will see here an easier method of doing that. So, what is the advantages of these? Again that low frequency characteristics, I can really see very well, right. And in most practical systems low frequency characteristics are very important in terms of disturbances, right. Disturbances always occur at low frequencies or their characteristic is always low frequency characteristic. Experimental determination of a transfer function is simple. So, I just plot how the system gain and the phase changes and I in a way that I just give the system lots of signals or varying frequencies of course, you know sinusoidal signals and I just I see what the output is. So, earlier I said that I will do this experimental determination of a transfer function as a part of this lecture. Why do a slide d 2 right? So, I will do this when in the module where we discuss non minimum phase systems. And it will be little more you know appropriate over there, because So far we haven’t really talked of the classification of minimum and non minimum phase. I will not even make the distinction of that because what we do from now on we know we will try to avoid that in keep that separately for an elaborate discussion a little later, right.

So, what we will also see interesting interestingly is that it is very easy to get this static error constants through the bode plots, right. Which we which was not very obvious in the Nyquist diagrams, right. And what we will see here these are just very easy to plot than Nyquister diagrams. So, I will leave the screen for a while and I will just go to the go to the black board to do some initial things and then I will come back to the screen again. So, there is a reason I am using the board because I think I can draw a little more freely I just wouldn’t directly want to show you the matlab process because that may not be very obvious of what I am doing right.

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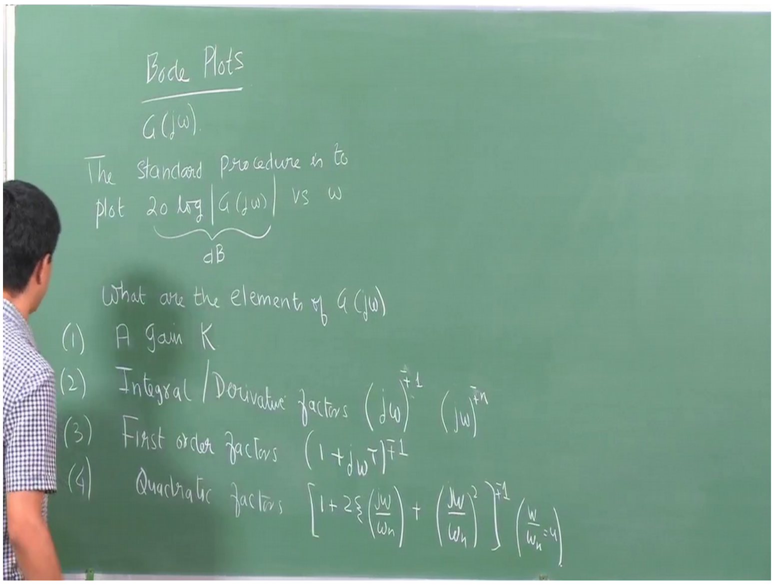

So, what do what would we do in bode plots. So, we are given a transfer function in the sinusoidal form G of j. Omega and the standard procedure is to plot 20 log of the magnitude of j omega versus the frequency, right. And the frequency will be in a in a in a log scale. We will see what is also the importance of doing this on a on a log scale ok.

So, this magnitude is usually in this term referred to as decibels. Of course, omega is in is in the standard things. Now if I look at a transfer function. So, of a sinusoidal transfer function G of j omega, first thing. That will be gain k, right. Then there could be integral or derivative factors integral factor would mean a pole at the origin given by j omega power minus 1 and a derivative factor would be a 0 at the origin would which should mean like plus omega. If I could write it respectively will be minus plus. Minus for the integral this way and if there are multiple poles it will just be j omega minus plus n. Then there are first order factors again both for the pole and the 0. And they would look like this 1 plus j omega T and remember I want to write the transfer function always in the time constant form. It will be minus 1 for a pole and plus 1 for a 0 and then lastly I will have a quadratic factors and this quadratic factor represents complex poles. How would they look like? Just pick 1 minus twice zeta S is j omega over omega n plus, constant will be a plus j omega over omega n square. This will also be minus or plus 1 depending on if the poles are complex, depending on if you are dealing with poles or if you are dealing with zeros. And earlier we had one ease of notation define omega over omega n as u and we sometimes use this notation for u, what for omega equal to omega n ok.

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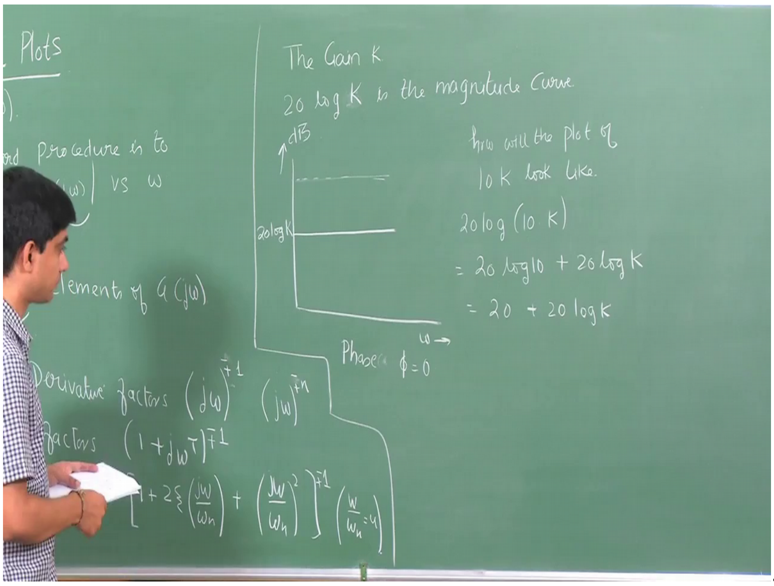

Let us start with a basic of all these things. Let us say my transfer function just has the gain term k. So, so what will we you should we do, right. We will say I take 20 log of the magnitude. So, 20 log of the magnitude which is simply k. This will represent the magnitude curve. So, if I would just plot it for increasing frequencies. So, I would just see that it is a horizontal line for all frequencies, right. To this horizontal line of magnitude K for all frequencies, right. And the phase contribution phi will always be equal to 0. So, imaginary part is 0. Another thing to just look at. So, if I have k, So how will the plot of 10 times K look like? The first phase still be 0. So, let me just 20 log 10 times k. So, this is 20 log of 10 plus 20 log of k, log of 10 is 1 right. So, this would be just 20 plus 20 log K you know. So, we will have a line somewhere here.

So, this is all in decibels, right. Similarly I can have 20, 40. So, it will just be again we are just adding constants right. So, the plot becomes quite straight forward.

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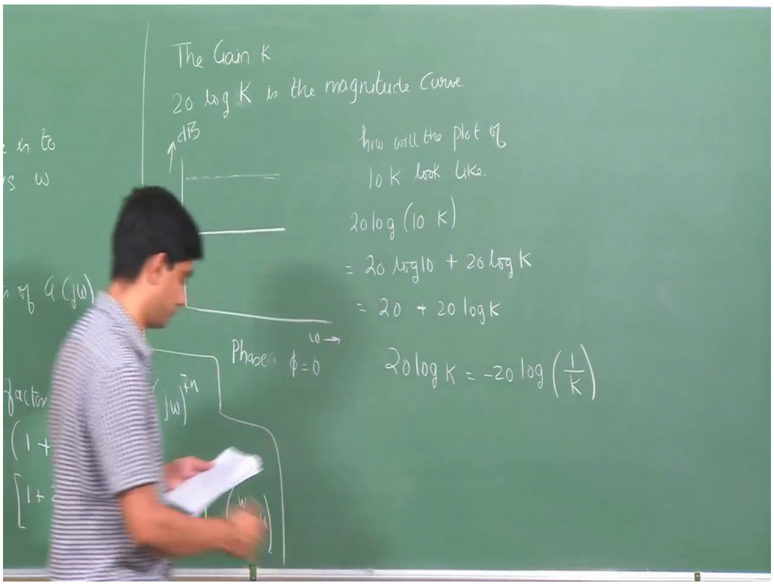

And lastly well this is 20 log K is minus 20 log of 1 minus K 1 over K. So, this is the basic one, right. How do we plot the gain K? It just a straight line horizontal line verse when plotted versus the frequency. Cos one for all frequencies, not surprising there is no frequency term here there is no j omega term in this sinusoidal transfer function. And the phase angle is always 0 ok.

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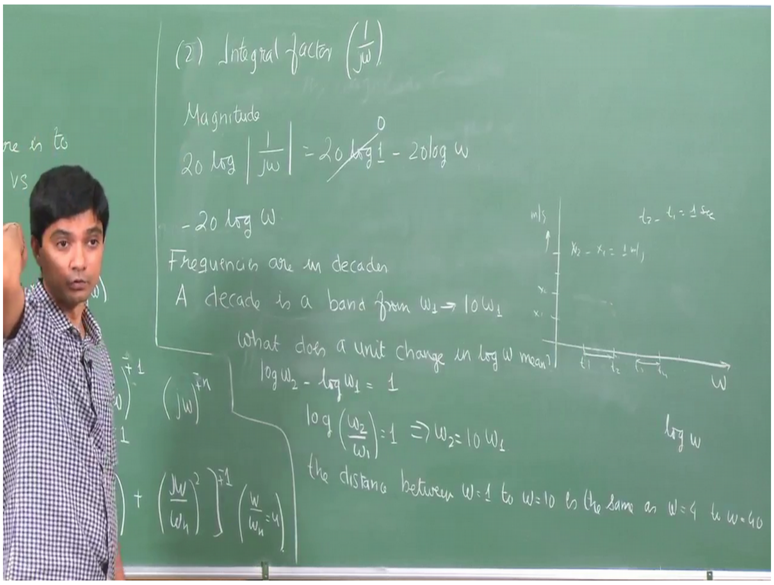

So, let me take the integral factor 1 over j omega. This is good for us because it helped in the steady state analysis or we tried adding a integrator always helps in reducing the steady state error as we saw. So, how will the magnitude look like? So, this is looking at 20 log 1 over j omega right. So, this is I expand this it will be 20 log of 1 minus 20 log of omega this guy is 0. So, what is left is minus 20 log of omega ok.

So, in this thing. So, what we will do is a on the on the omega scale how is the frequency defined right. So, this is omega and I use this on a log scale right. So, this is essentially log of omega. So, if I just you know plot it on the x 1 x 2. So, this is x 2 minus x 1. So, it will be like the unit is which we say sometimes 1, centimetre 1, meter, 1 millimetre and so on. Let us you solve onto a scale. So, here we will not do frequency from 1 2 3 4 5 six, but we will do in something called decades well, I will explain you what it what it is in decades. So, the frequency is are in decades. And there is a good advantage of why we do in decades. So, a decade is a band from omega 1 some frequency omega 1 till 10 times omega 1 ok. Now what does a unit change in not omega, we are looking at log omega, log omega mean? Again if we draw in the standard x y plain if we talk of distances and time. So, one unit change would mean 1 second 1 minute that is what we write here, right. Time in seconds distance in meters and so on.

So, what does a unit change in log omega mean? A unit change in log omega. So, if it is this by time I will say this is T 1 T 2 and then T 2 minus T 1 is 1 second, right. And if here if it is say some other thing distance, right. X 1 x 2 if I say this is a velocity in meters per second x 2 minus x 1 you say one meter per second, right. This was the unit. So, which we learn in high school how to draw the graphs, now what does the unit in log of omega here mean? So, this is log of omega 2 minus log of omega 1 is 1, a unit change. Now this is also written as log of omega 2 over omega 1 is 1. Which means that omega 2 is 10 times omega 1. So, which also means that here if I say well this is T 3 T 4 and so on. The distance between T 2 and T 1 which is 1 second is also same between T 4 and T 3, right. All the all the graph you turn to that scale. So, here the distance omega equal to 1 to omega equal to 10 is the same as from say omega equal to 4 to omega equal to 40. You can do this, right? Log of 10 over 1, right. You will get is a log of 10 here also log of 40 over 40 right. So, this is a log of 10 over 1 is 1 log of 40 over 4 is also 1 right.

So, the distance between 1 to 10 is a same as the distance between 4 to 40, 40 to 400 all in multiples of 10 400 to 4000 also, right. Now will come back to this one right. So, we will be interested in measuring the frequency decades. So, what is here is in decade.

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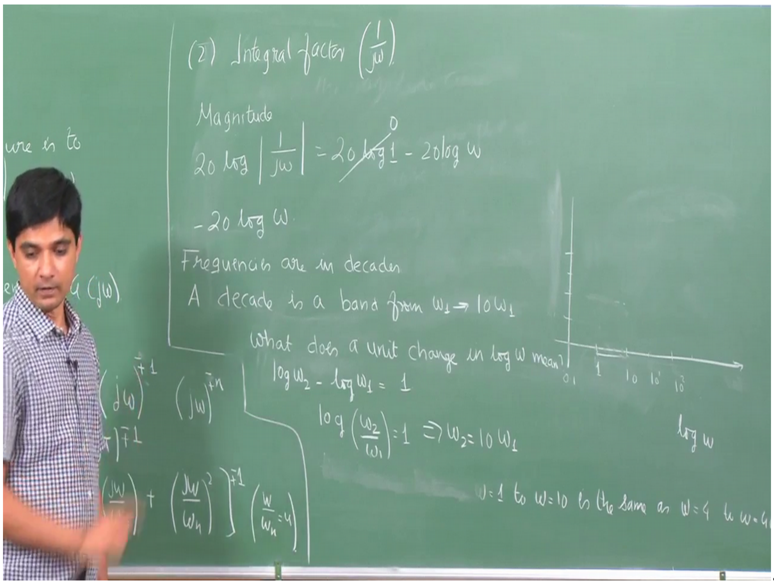

So, the frequency is usually will be on the log to the base 10 as multiples of 10, 0.1 1 10, 10 power 1, 10 power 2 and so on, right. And the distance between the omega 2 omega 1 this is unit distance unit distance unit distance all measured in the log scale this way, right. And this is what we need to be careful of this is the only important thing here. Now once we do this everything now translates to just writing down equations of a straight line.

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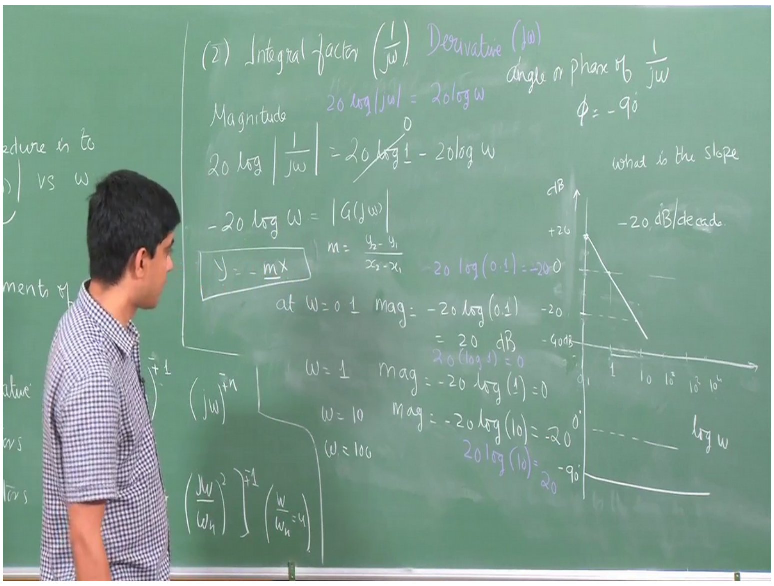

So, just say this is my y axis magnitude of G of j omega. So, I am plotting magnitude of G of j omega in decibels versus the horizontal line versus the horizontal axis that is log omega. And this is you can see is sorry, this of the form y equal to minus m x, right. Magnitude y x is a log of omega, this is a straight line right.

So, if I were to just plot this from, I will not plot at omega equal to 0 because the log value is not defined, but any value slightly if apart from 0 we could we could plot. So, at omega is 0.1 the magnitude is minus 20 log of 0.1 this will be 20. Similarly at omega equal to 1. So, this is again all in decibels at omega equal to 1, the magnitude is minus 20 log of 1 is 0. Omega equal to 1 of this omega equal to 10 the magnitude is minus 20 log of 10 this would be plus 20 sorry, minus 20. And similarly you could do for omega equal to 100 and so on. So, when we compare with this guy, we say well we are interested in the slop so give me the slop and I could plot y equal to m x, right. This is a minus m.

So, a negative slope. So, here you say unit change in y. So, what is m all right? This is like 11 standard geometry y 2 minus y 1 x 2 minus x 1, right. Unit change in y over unit change in x. So, here we will see, right what is the unit change? Unit change here is from 0.1 to 1 decade, right. And then here I measuring the y axis in decibels. So, let us start to plot this, right. Say let me say this is my minus 40 decibel, this is minus 20, 0 we will draw it a slightly better minus 40 minus 20 this is 0, this is plus 20. Now from point at omega equal to 0.1 the magnitude is twenty. So, I am here and at omega equal to 1 the magnitude is 0 on this here it will be line here. Now at omega equal to 10 I am here, right and so on. So, I just draw line just looks like this ok.

Now, what is a slope, right? See how much the y axis changes, from here till here when I am measured from frequency of 0.1 till 1 it changes 20 in the negative direction, again one till 10 it goes from 0 to minus 20 again negative 20 right. So, this has a slope of minus 20. Minus 20 what? What is in the horizontal axis? From 0.1 to 1 or 1 to 10 or 10 to 100 there will be gonna 10 power 2 10 power3. So, this falls at a rate of minus 20 per decade minus 20 decibels per decade. So, this is how we will we will measure the slope here again it is very similar to what is happening here, right. Y 2 minus y 1 that is 20 decibels x 2 minus x 1, right. That is a decade, right. And the advantage of doing this is that I will have a 10 power 3 10 power 4 and so on. Right. And frequencies are usually for very low frequencies of 1 hertz till 10 power 3 10 power 4. And if I were to plot that on a on a graph sheet in the market I would need to plot a simple transfer function a graph sheet as big as a football stadium, right. And I wouldn't want to do that right.

What instead I do I can just do that in a in a log scale, right. In the log scale my frequencies well from 0, 0.1 till 10 power 4 10 power 6 now I can really get very big frequencies in a small now a 4 size this much of paper, right. Whereas, I cannot do it on a graph sheet. That is another advantage why we do this in the log scale right. So, the magnitude of this looks like, like this it has a negative slope of minus 20 decibels per decade what about the phase phi is what I was looking at the magnitude of this I am sorry, the angle of this minus j omega. So, the phase the angle or just say the angle or phase of 1 over j omega that is phi is always minus 90 degrees write all the phase here if I say this is my 0 degree line the phase will always be constant at minus 90 degrees ok.

So, the integral factor will have a magnitude plot at minus 20 decibels per decade and a phase plot which is minus 90. Now how will the derivative factor look like? Let me use a different colour. So, the derivative factor this is j omega. Then I am just write down these steps again, So 20 log. So, I am not looking at 20 log 1 over j omega, but I am looking at 20 log j omega which is simply 20 log of omega is the is the magnitude, right. Now how will the plot of this look like? Let us say at omega equal to 0.1 I am looking at minus 20 log. So, it will not be minus anymore this is a plus here this one 0.1. So, this would be now plus 20. Now sorry, 20 log 0.1, 0.1 is minus 1. So, this will be minus 20 similarly at omega equal to 1 the magnitude of 20 log 1 is 0. Omega equal to 10 the magnitude of 20 log 10 is this becomes plus 20 and so on ok.

Now, let us start plotting this, at omega equal to 0.1 I am at minus 20. And at omega equal to 1 this one this line, of this colour omega equal to 20 I am at 0 sorry, omega equal to 1 I am at 0, omega equal to 10 I am at plus 20 ok.

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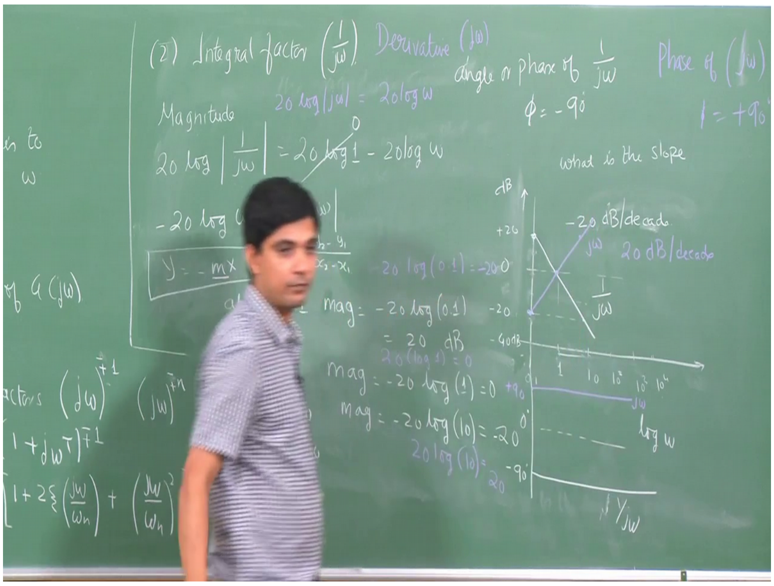

So, this is the plot of j omega, the white line is a plot of 1 over j omega, right. Just a mirror image which is not surprising. And of course, what is a phase of sorry, phase of j omega phi is always plus 90 degrees right. So, I will be all the time here just see this the phase of j omega at plus 90 and white guy is the phase of 1 over j omega at minus 90 all the time, right. Stay straightforward, right. You just need to remember something like this and then at the slope. So, this slope of the purple guy is positive 20 decibels per decade, right. Because I go from here till here I increase 20 again, I increase 20 and so on.

So, each decade my magnitude goes up by 20 decibels in a straight line why is it straight line because of this guy. Something slightly, what if I will I will erase this I would not make it anymore mess. What if this is 2? Ok.

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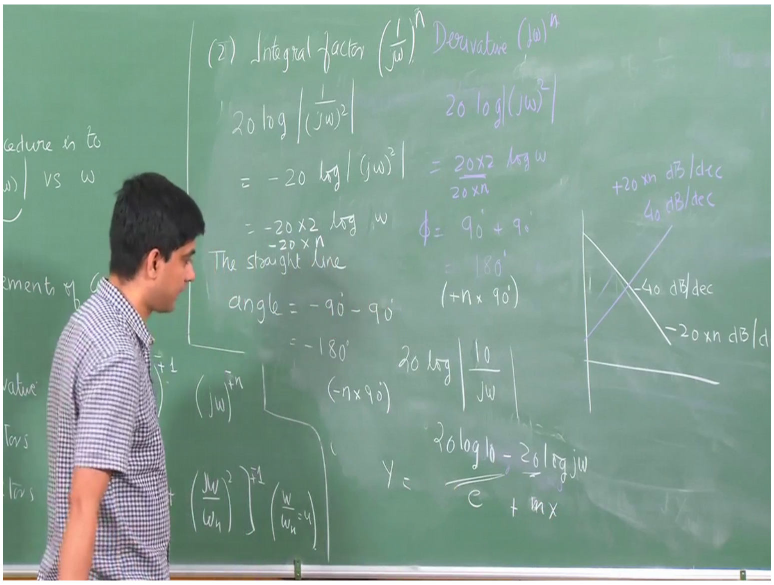

So, I am looking at 20 log of 1 over j omega square. Now this is minus 20 log of again j omega square or minus 20 times 2 log of j omega or say just omega if I just look at the magnitude. So, this 2 just comes and sit is here, right. Therefore, how will the straight line look like? It still be a straight line, what is this slope? Minus 20 times 2 that is minus 40 right. So, it will something like this minus 40 decibels per decade. And what will be the phase? Phase well, this is phase of 1 over j omega is minus 90 I have another of the same guy this will be minus 180 degrees all integrals. Just write straightforward I will not really now draw this scale and what is this and all that is a kind of now easy to compute. What happens to this guy? 20 log j omega square this is 20 times 2 log of omega, right. Look at this number now this will be something like this, 40 decibels per decade and a angle and an angle phi of 90 plus 90 is 180 degrees. So now, I can just say well generalise this 2 here it should be 2, right. For this analysis I can generalise this to n here and n here right. So, what will change here is minus 20 time’s n, right. Here also plus 20 times n. So, the slope would be minus 20 times n decibel per decade and here it would be plus 20 times decibel per decay right.

So, the angle would be minus of n time’s 90 degrees the angle here would be plus n time’s 90 degrees, straight forward a little exercise, right. I without, So if I have a transfer function now of say, say 10 over j omega, how will this go like? Well I am just looking at the magnitude 20 log. So, this will be 20 log of 10 minus 20 log of j omega. So, it is just be shifted by this number right. So, I will I will not draw this, right. You just tell that y equal to m x plus c kind of thing there is a slope still here this is y on the horizontal axis this is will my c is the intercept minus m x plus c, right. Very, very straight forward. One observation we could make here is that the plot or at least the magnitude plot of the 0 is just a mirror image of the pole right. So, there it was going at minus 20 and plus 20, right. We will see that still continues or not. We will see now not these things, but what was the third kind of thing, third kind of thing was first order factors.

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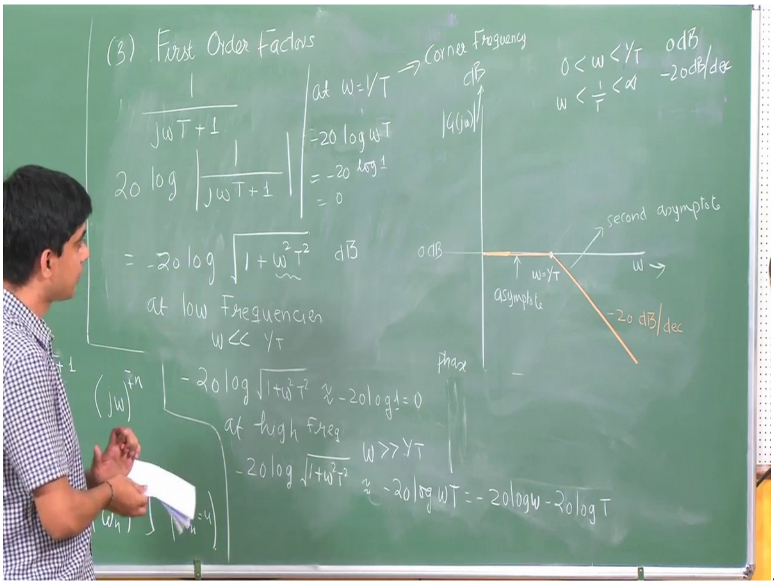

First order factors are this j omega T plus 1. Now how the magnitude look like? 20 log 1 over j omega T plus 1, right. Magnitude of this thing. So, what is the magnitude? Well, you can just get this 20 log I get this upstairs that would just be square root of 1 plus omega square T square, right all again in decibels. Now it is a little more interesting it is no longer like y equal to m x plus c kind of form. So, little more complicated, but we will try to simplify this. Let us try to draw this looks like, this is my omega this is my magnitude of G of j omega in decibels somewhere here would be the phase right angles. So, let us see what happens at low frequencies, low frequencies would mean omega is very small omega is very small this term can be ignored a bit and you can say that minus 20 log of this guy is approximately minus 20 log of 1 is 0. So, what can I say is that for low frequencies if this is my 0 dB line and I just be here, I will I will use a colour chalk for this one, here. Until what point?

Let us see let us see when this changes at high frequencies. High frequencies omega is difficulty large as compared to T. So, minus 20 log of square root of 1 plus omega square T square is minus 20 log of omega T. So, this one goes away and then omega square T square the square root will this be omega T. So, how does this look like? This is minus 20 log omega, minus 20 log of T. So, let us see at one particular interesting number here would be at, not here no here, this I write this here at omega equal to 1 over T in this the magnitude, right. Sorry like 20 log sorry 20 log of this entire guy 20 log omega T with a, what sign here? Minus 20 log of omega T would be minus 20 log of 1 and that will be 0. So, you see at some point of time or some frequencies is somewhere here which is omega is 1 by T this guy goes to 0. And after that well I am just moving downwards at this frequency minus 20 log omega. And how does the product just look like? Well, I know it looks like this, right. This is again at a slope of minus 20 decibels per decade and this is also well I can say where it just moves here these 2 guys meet here and this goes to.

So, well I can I am just making some approximations here I am just starting at very low frequency then I say I am just at the 0 dB line. I keep on continuing, continuing and at a higher frequency range I am just doing this going down at minus 20 decibels per decade. And this is you can say that the plot can be approximated by 2 lines right. So, this is one asymptote this is the second asymptote. And these 2 meet at this point. What is this point? Omega equal to 1 over T and this omega equal to 1 over T is the corner frequency. Now can see that this way that for frequencies, for frequencies between 0 to omega to 1 over T my plot is approximated by the 0 dB line, again for omega 1 over T infinity it is can stop at minus 20 decibels per decade. So, these are 2 approximations we get. Now what is the actual plot? The actual plot may have some errors, right. We just may at this omega equal to 1 over T it may not be a zero, but something somewhere close to 0. So, will see what that is ok.

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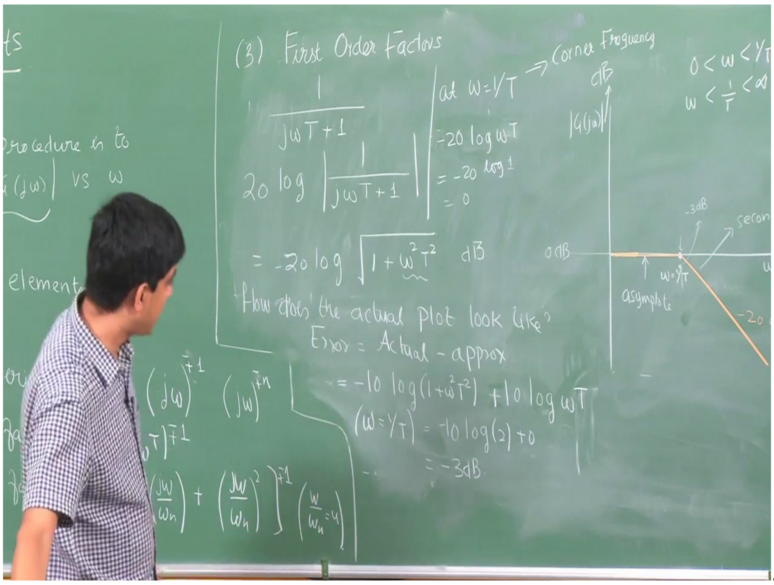

So, now how does the actual plot look like? We come to the phase a little later. We will see what does the this business for. So, the error is usually the actual minus the approximate. So, what is the actual? The actual is well I can write this square root can go here this is minus 10 log 1 plus omega square T square. This is actual thing the approximate is what 10 log of omega. So, the minus and the minus become plus approximate also had the minus 10 right. So, what happens at omega equal to 1 over T this becomes minus 10 log of 2. So the So, this will be omega T here. So, from here, right. There are the higher frequency things. So, minus 10 log 2 this will be 0. This would be minus 3 decibels no per decade right. So, at omega equal to 1 over T I am just having a error of minus 3 decibels. So, I am somewhere here where this length is minus 3 dB or similarly I could ask myself what happens at other points.

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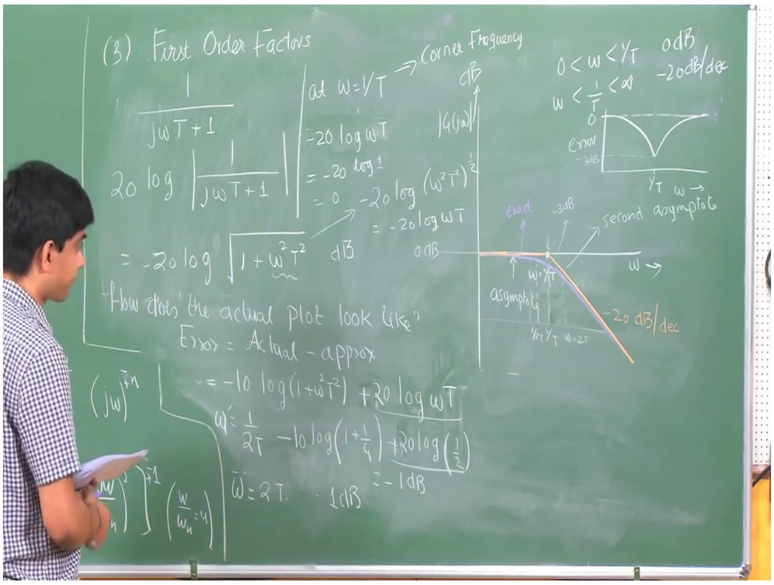

So, the same guy at omega equal to 1 over 2 T would like well minus 10 log 1 plus 1 over 4 plus 10 log omega is 1 over 2t 1 by 2 right. So, this you could compute to be minus 1 dB right. So, omega is 1 over 2 T. So, if I just add this is 1 over T somewhere here let us say let us say somewhere here is 1 over 2 T. So, here the error is 1 dB. Similarly I can compute for omega is 2 T the error is again minus 1 dB at here. So, this is say omega is 2 T. So, the actual plot would look something like this. If I just use this purple colour at the low frequency it is still start here and at some point it will start will go it will go and this will go down and then meet this guy asymptotically, right.

Student: sir you explained 20 log omega, T is approximate.

here?

Student: that one yeah.

This should be 20 square root disappears. So, the again that just recall the approximate thing that was 20 log with a minus sign omega square T square 1 over 2 that is 20 log omega T, sorry about that this will again be 20, right. Thanks for pointing that, just be I am sorry about this the approximate ones. This will have 20 this will go 10 because the square root just comes here. So, this is the approximate plot and the exact plot would look something like this. And you see at that the maximum error is just at the corner frequencies. So, if were to just you know where to plot how the error changes with frequency right. So, let us say this is 1 over T this is T 0 error. So, this will be some plot like this, right. And these are in this case this is minus 3 dB ok.

So, what about phase here? So, you can keep doing this for you know omega equal to 3 T and so on and just compute the error. That is again for a straightforward procedure ok.

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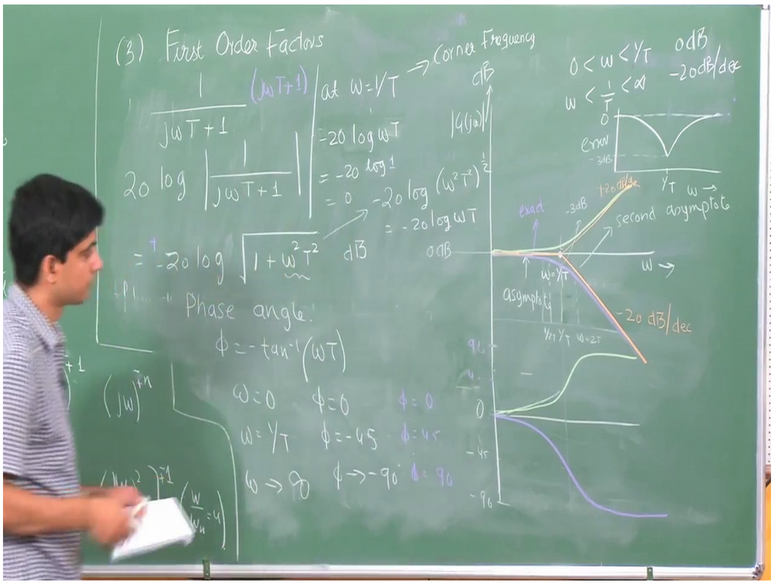

So, we will we will now come back to the phase the phase angle. Phase angle well, this is this computed as the inverse tan of omega T first one here. So, at phi and at omega equal to 0 we will have phi equal to 0, omega is 1 over T and phi is minus 45. So, it can be a minus here and as omega goes to infinity and the phi and the phase goes to minus 90 degrees. So, if I were to just plot it here. So, let us say this is the 0 degree line see somewhere here is minus 45 somewhere here is minus 90 right. So, the plot would look something like this at omega equal to 0 and asymptotically here omega equal to 45 sorry, omega equal to 1 over T and at 45. So, the phase would look something like this. Again for a exact values I could compute substitute for omega and T and this stuff.

Right, So this were first order factors of the form 1 over j omega T plus 1, 1 over j omega T plus 1 I could do this exactly in the same way for a 0 or first order factor j omega T plus 1 in the numerator. So, nothing would change just that is minus 20 will now become plus 20 everything will be the same just that in this case phi would be 0 phi would be 45 phi would be 90 well, these 3 cases right. So, this is plus 90, 45 then the plot would have I use the another chalk and so on right. Something like this and the magnitude plot would just be again the same, right. If I just draw the asymptotic magnitude plot using the same colour. So, I am assuming both are the same corner frequencies it will just be like this right. So, it will have a plus 20 decibel per decade and the exact plot would look something like this while computing the errors, right. This and then asymptotes here ok.

So, it is a very straightforward computation all these process would go to my minus S would go to plus and so on. And the way you compute for low frequency it will still be exciting the same for higher frequencies the approximation would still be a same for low frequencies I can just get rid of. So, low I can just get rid of this term for higher frequencies I can just get rid of this term and it will just be like a mirror image, right. Similarly even for this kind it gets down this guy goes up I will pause here for a while. And then we will we will continue to look at these guys quadratic factors these are all little interesting, right. To plot the errors. So, we will deal with this quadratic factors.

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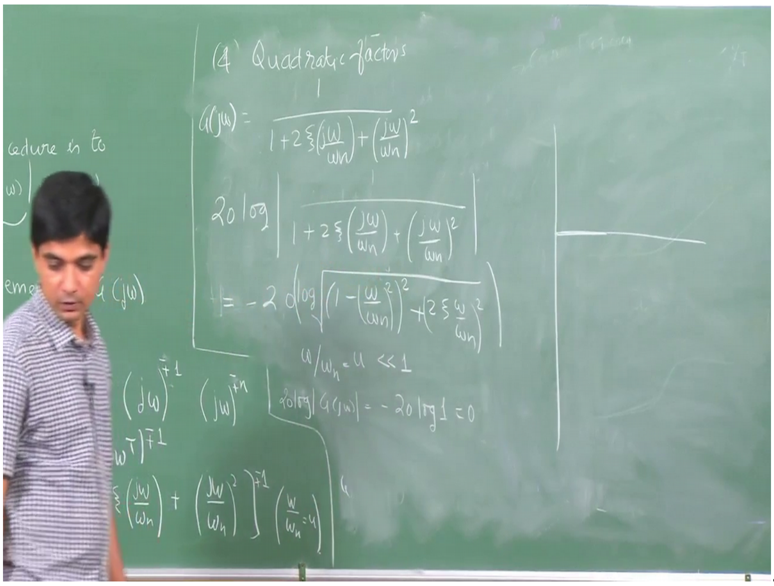

So, we will say they are of the form one over this is my G of j omega is 1 over 1 plus 2 zeta with a j plus j omega over omega n square right. So, again as you I will just do it first for the pole and the 0 is just again a mirror image ok.

So, how does a magnitude look like? So, I am just looking at 20 log of this entire guy, I write 1 over 1 plus 2 zeta j omega this is a, So minus 20 real part this is 1 over omega by omega n square whole square plus twice zeta omega over omega n square. Again as usual we will first look at how the asymptotic plots look like. So, here we will see well what happens when omega over omega n which we defined as u earlier is very small, right. If omega n over omega. So, this is sorry, this should be a log here. So, for lower frequencies So, the magnitude 20 log G of j omega is minus 20 log it is remain 1 this will be 0. So, well nothing but, nothing but changes here. So, for low frequencies I am still at the 0 dB line here ok.

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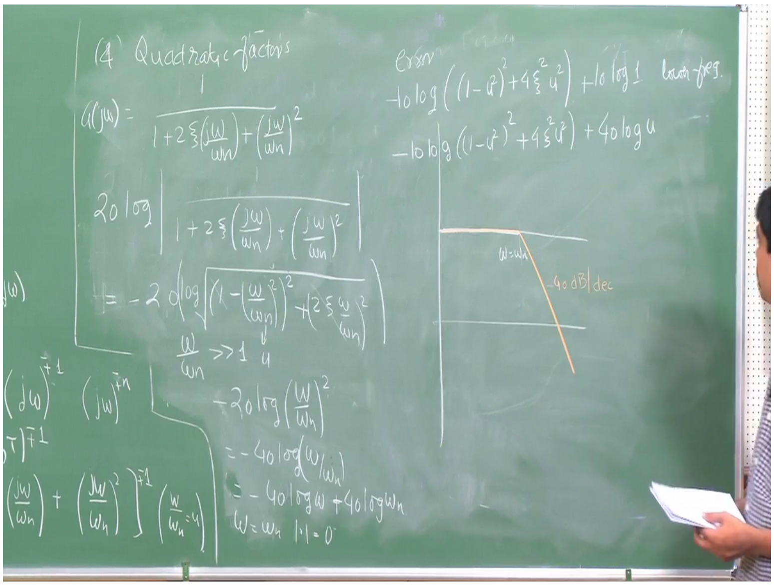

So, what is the other approximation? The other approximation comes when omega over omega n is much larger than 1. In that case my magnitude this becomes minus 20 log. So, all these guys will go away all these guys will go away. So, here just remains is just this guy omega over omega n whole square. So, we ignore this guy we ignore this guy. So, we will have omega over omega n whole square and this square root. So, this will this square root and this 2 will cancels. So, I am have something like this. So, this would be minus 40 log of omega over omega n or minus 40 log omega minus 40 log omega n correct. So, again at what point will these 2 asymptotes meet this is again a straight line of slope, now minus 40 decibels per decade answer may accept here, right. This can be computed because the omega 1 is already given to you, right. And when omega equal to equal to omega n when these 2 frequencies match then this sorry, this is minus 40 this will be a plus here and because it just jumps to the numerator, right. And then when omega equal to omega n the magnitude goes to 0. So, I am here I say I will be just be 0 0 0 until this point, what is this point? At this point, I will use the coloured chalk, this guy this till here this is omega is omega n and after this point I go down at a slope of minus 40 decibels per decade right.

So, again I can just compute the errors as it is. So, what is the actual one? The actual magnitude is, if I just take the square root here it is minus 10 log of 1 minus, let me again call this term as u I said (Refer Time: 54:59) So, I am here 1 minus u square whole square plus 4 zeta square u square plus 10 times log of 1. This is for lower frequencies. Then well again for higher frequencies the error would again be the same right. So, minus 10 log 1 minus u square whole square plus 4 zeta square u square is the actual value plus, I have this 40 this is higher frequency approximation, right. 40 log of u and this guy. Now what we see here is that is no longer just computed by substituting a value of omega n is given to us right. So, this is this comes from. So, omega the they are substituting I just cannot get a close some expression. So, what we see here it is depending on of course, omega n and also for different it will be different for different values of zeta. We will not really spent time in computing the simple I just show you how the plots would look like. Just before that how would the angle look like?

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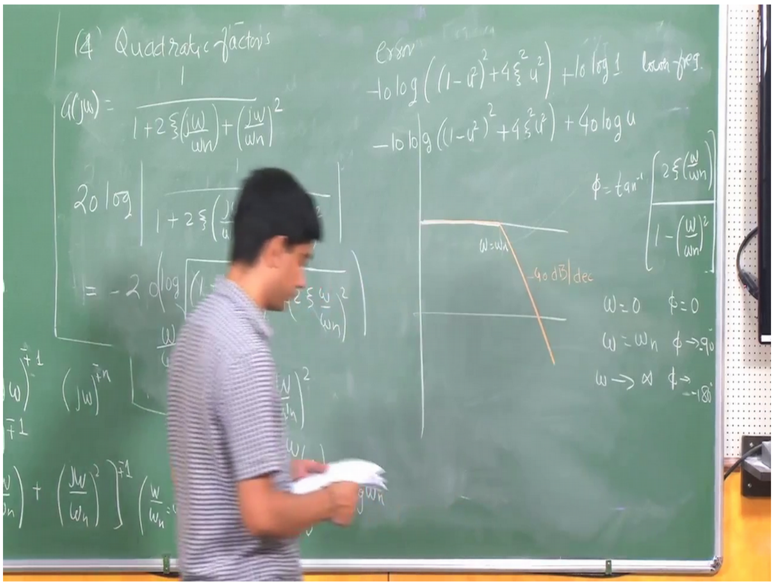

The phase angle phi is tan inverse of 2 zeta omega over omega n over 1 minus omega by omega n whole square ok.

So, when omega is 0 phi is 0, when omega is omega n phi is 90 degrees and when omega tends to infinity the phi which is to be minus 90. The phi would go to minus 180 degrees right. So, similar thing will happens when I have this guy in the numerator, will just be this one all other errors is will be the same right. So, we will we will go back to the screen and see how these things would look like right.

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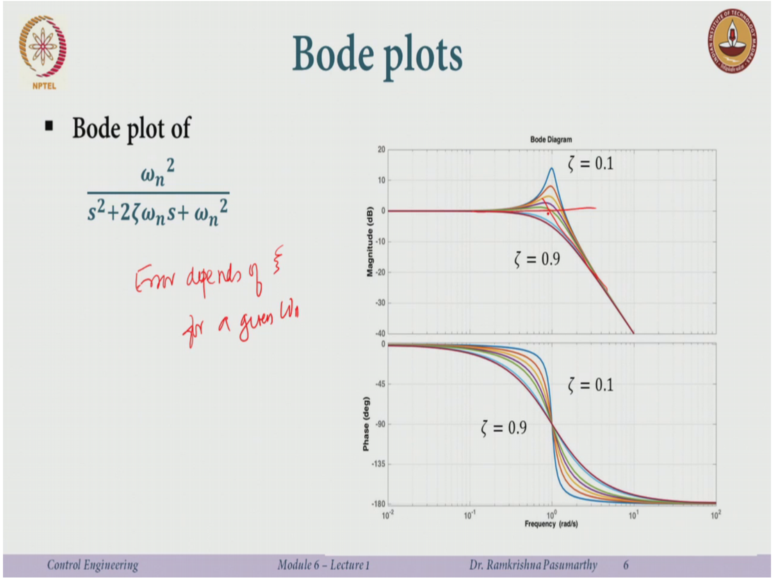

So we saw that the error was depending on on zeta for a given omega n. And these are my asymptotes right. So, just the 0 dB line till here and then this guy is somewhere here somewhere here. And you see that the error is maximum when zeta is very low I am just plotting for 0, 8, 0.1, 0.2, 0.3, 0.4 is be 0.9 and then as this guy increases when you see the error kind of kind of decreases, right. Similarly with the angle right. So, zeta equal to 0.1 I have this angle and zeta equal to 0.9 my phase plot looks something like this I just plotted this directly from matlab it should be straightforward to compute these guys given zeta and omega n, but this to avoid all that computational process I am just showing you this one I could not even draw this accurately on the board ok.

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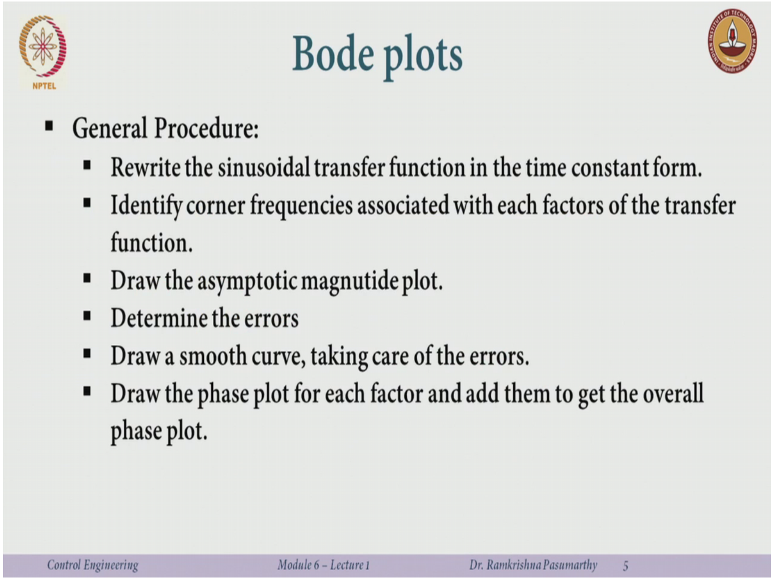

So, now given all these things how would be in general plot a bode diagram, given any general transfer function. So, the first thing is we rewrite the sinusoidal transfer function in the time constant form. We identify the corner frequencies associated with each other transfer function corner frequencies were the one where the asymptotes met what is sample here well this was the corner frequency omega equal to 1. Then you draw the asymptotic magnitude plot like these 2 straight lines. Then what we do is we determine the errors for lower frequencies for higher frequencies, how do we determine the error? The actual minus the approximated value again for both lower frequencies and higher frequencies. And we draw the phase plot well this is kind of straightforward you just look at the what is the formula for phi and then you add all these phases to get the overall phase plot ok.

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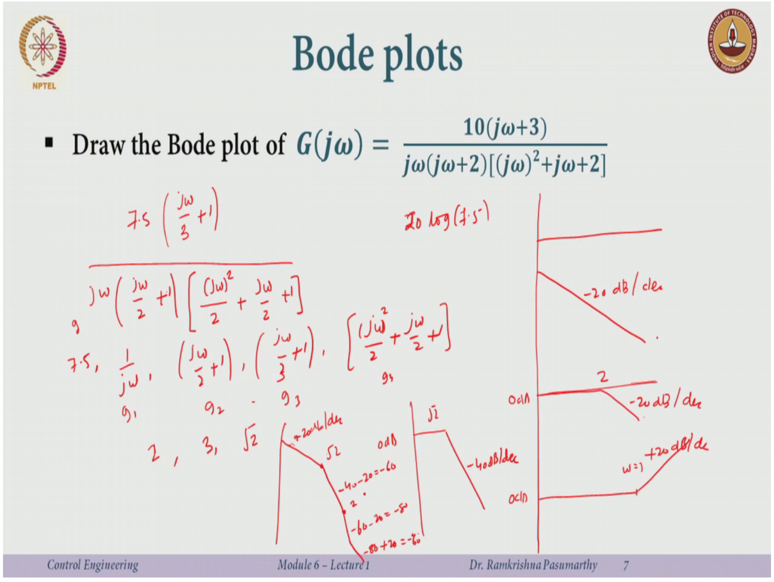

Ah just try to do a few examples on how to how to plot the bode, how to plot the bode diagram. So, first thing is write down this transfer function as a sinusoidal transfer function or in terms of the time constants.

So, I just rewrite the final expressions that will look like 7.5, I have j omega over 3 plus 1 over j omega and then I have j omega 2 j omega by 2 plus 1 and then the quadratic term should look like this. J omega square by 2 j omega by 2 plus 1. So, how many terms here we have we have the gain 7.5 and then we have this guy plot at the origin first order terms as j omega to j omega by 2 plus 1 and then we have the other first order term in the form of a 0 plus 1 and then quadratic term, j omega square by 2 j omega by 2 plus 1. So, well I will call this 7.5 the constant as G I will call this guy G 1, G 2, G 3 and G 4. So, well let us identify what are the corner frequencies there is no corner frequency associated with this guy, there is no corner frequency associated with this guy, well this guy has a corner frequency of well 2 then 3 and this guy has square root of 2. So, let us try to plot this individually right. So, the first guy the magnitude would 20 log 7.5 right.

So, it will be a straight line, right. At that in a proper magnitude and then I cannot read out scale by telling just then 1 over j omega would be something like this, right. With a slope of minus 20 decibel per decade. What about G 2? G 2 is the first order term it is a pole. So, I will have in the in the axis. So, this goes straight for a while and then go down. This is like again a minus 20 decibel per decade and the corner frequency here is

2. Then for the for this guy j omega over 3 plus 1. So, I go till 3 and I go up at plus 20 decibel per decade this is omega equal to 3. And then the last term well I have got a space let us I will just write here S it will start at omega equal to at the 0 dB line right. So, this is this is at 0 dB line for this guy this is 0 dB line for this guy and after a frequency of square root of 2 it will go at minus 40 decibels per decade ok.

So, let us let us see right. So, what how will the overall plot look like overall plot would be the combination of all these 1 2 3 4 and 5 lines. So, well this is like this just addition of 2 lines. So, so I start with this looks close to minus 20 disables per decade until this is a first corner frequency this is this is, So this line is just a combination of these 2, right. And after I did the first corner frequency my slope decreases further. So, this will be minus 40 until I reach 2 at 2 I have a pole So, I will go further down to minus 60 and after this is at root 2 this is 2 and at So, this will be 20 say minus 40 minus 20 this would be minus 60 it should be minus 60 minus 20 is minus 80 and then I hit a 0 right. So, that will add minus. So, I will have minus 80 plus 20 is minus 60 is a slope right. So, this it looks look something like this and I can actually compute what are the errors by the formula for the errors which we used ok.

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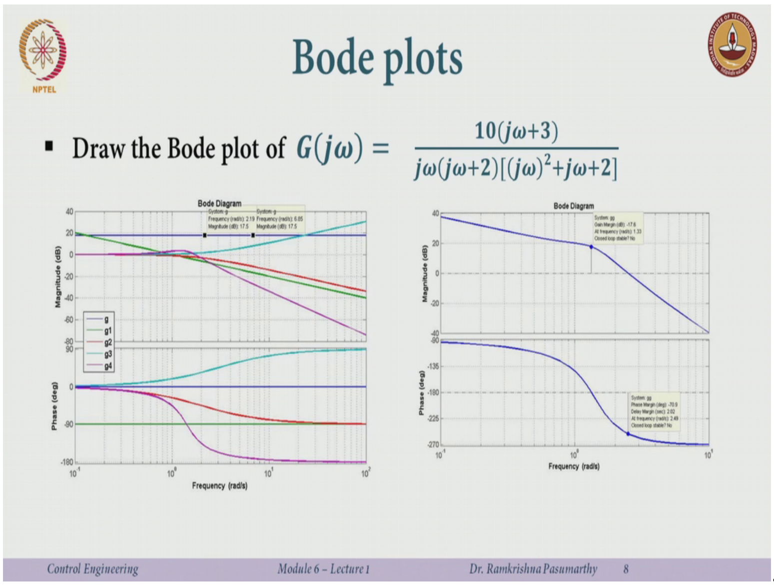

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So, just to to summarise. So, this is how the first guy would look like the constant then the green line would be the 1 over j omega term, right. The red line would be the first order factor which corresponds to the pole. The greenish line I do not know what this curve is called, but this would be the 0 the first order term which appears in the numerator and then the purple line would just be a second order term. I add all these lines and I get a plot which actually looks like this. So, well can we get something more some more information about what how does the close loop system look like. So, what I do on matlab is matlab actually shows me the stability margins. So, we quickly see what is happening here and will postpone the complexity of it to a little later. But what it shows me is that the closed loop system is it is stable well the answer is no similarly here, right. If I look at how did I do define gain margin, the gain margin was I look at the point at which you know the my phase becomes 180 and I look at how far it is from the point minus 1 plus j 0 in the in the Nyquist thing. Here it translates to decibels right. So, I am just looking at this line and I am also looking at this line for the phase margin where say I hit the minus 1 point, right. Or the unit where my magnitude becomes unity magnitude becoming unity in the log scale would be 0 dB line right.

I do all these things as well the closed loop is actually unstable, what does it mean? Right. So, I have a if I just look at. So, let us just try drawing the root locus for this. Like I will just slightly go to matlab and I will just draw it.

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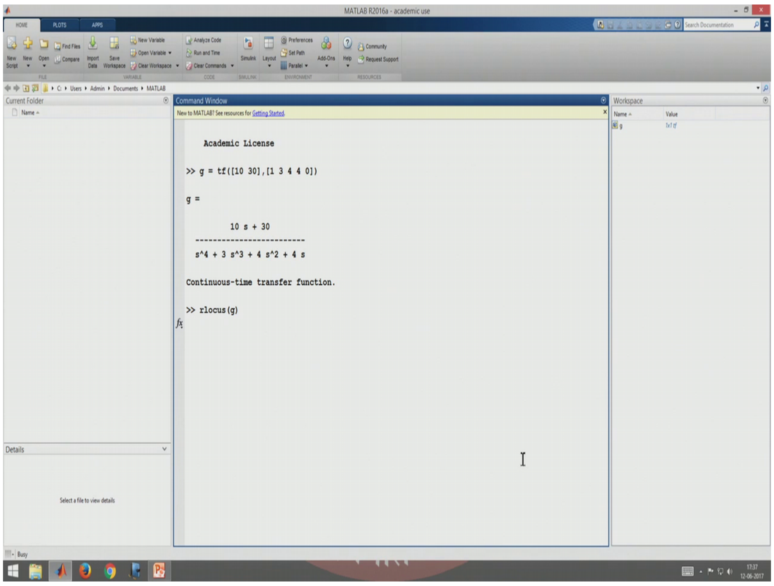

I hope I just remember the commands properly G I will call as a transfer function be the poles. So, I will have 10 and 30. And in the denominator I will have fourth orders one as 4 as cube S square S power 1 and S 0 I will just I will just write the earlier transfer function in this way and this is what I get. So, if I just look at the root locus of this guy G ok.

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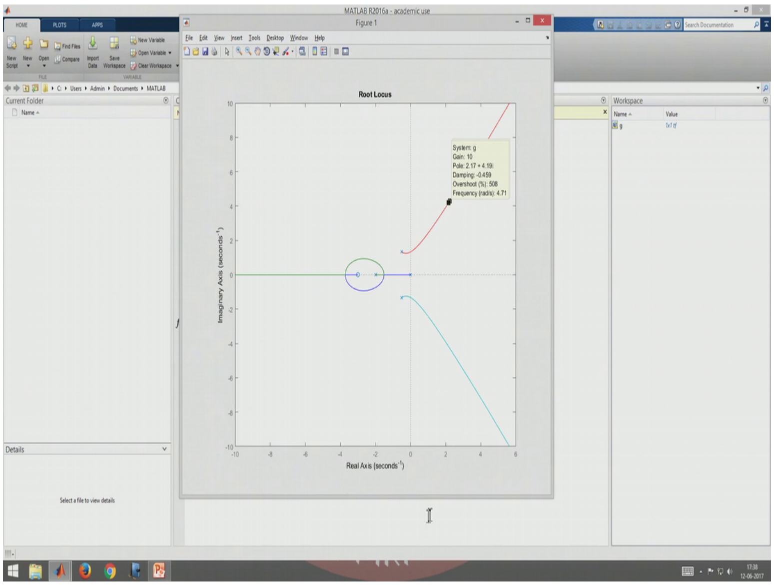

So for So, for this put my cursor here it will tell you what is the gain right. So, gain is 0 and if I look at well I have the system looks like the system here has a gain of 10 right. So, if I see if I go till here. Well, I am already in the in the unstable region, right. For all gains greater than this guy a 0.14 in absolute terms and stable. So, this all gains from 0 to 0.1 and if I am at the at the gain of the system which is 10 I will become unstable right. So, that is what at gain at 10 my poles are on the on the, right. Of plain and my system becomes unstable. And that is what this guy is trying to tell us here, right. Well say at the closed loop system is actually unstable ok.

Not to worry much about this we will we will keep on revisiting this several times, until we understand this exactly what it means right. So, we will kind of postpone this where we will try to understand at this gain my closed loop system is unstable, what is the closed loop system? The characteristic equation of 1 plus G times S equal to 0, right. Ok.

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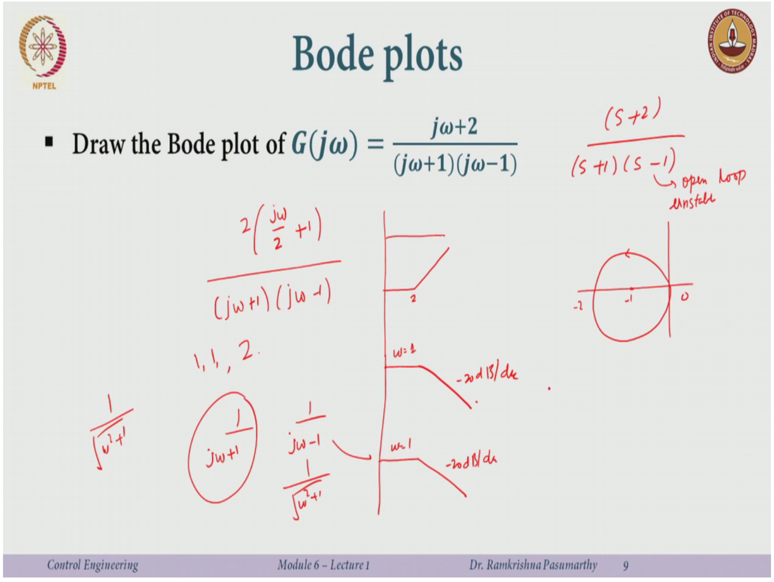

So, now we will got to this example, right. Which will look a little bit familiar if I say this is S plus 2 and S plus 1 and S minus 1. So, this is we are plotted the Nyquist for this, right. Which actually did something like this is 0 this is minus 2 I was encircling the point minus 1 plus j 0 ones and then the system was stable, right. We said open loop system is has an you know this is serious open loop unstable, but I know that the closed loop is stable. So, how will the bode plot look for this guy? So, I will just write this down as 2 j omega 2 plus 1 and again j omega plus 1 and j omega minus 1. And the cross over frequencies would be 1 1 and 2 from this omega is raised to the power 1 over T ok.

Now, few interesting things here. So, let us see well the plot of 2 would be just the straight line, let me this is plot of 2 would be straight line the plot of j omega over 2 plus 1 would be something like this, they meet at 2 and then they do this. Now look at these guys right So, 1 over j omega plus 1 1 over j omega minus 1. So, these guys will do something interesting. So, the magnitude plot for the first guy say I will take the stable guy first and it will just go to the omega equal to 1 sorry, this is not draw to scale, but you know and it goes here, right at minus 20 decibel per decade. Now the second guy again So, this is plot for the unstable guy this goes till here to the omega equal to 1 and still be minus 20 decibel per decade. This is because the magnitude is a same for this guy. So, what is the magnitude for the first guy that is 1 over square root of omega square plus 1 that is same here also magnitude is, the magnitude is 1 over square root omega square plus 1 also ok.

So, just by looking at the magnitude plot I may not be able to say this is semi stable or not. Because this will correspond to as plus 1 this is also correspond to plus minus 1 right. So, what will change is the phase right. So, this will have different phases and I will show you the exact plots now how they look like right.

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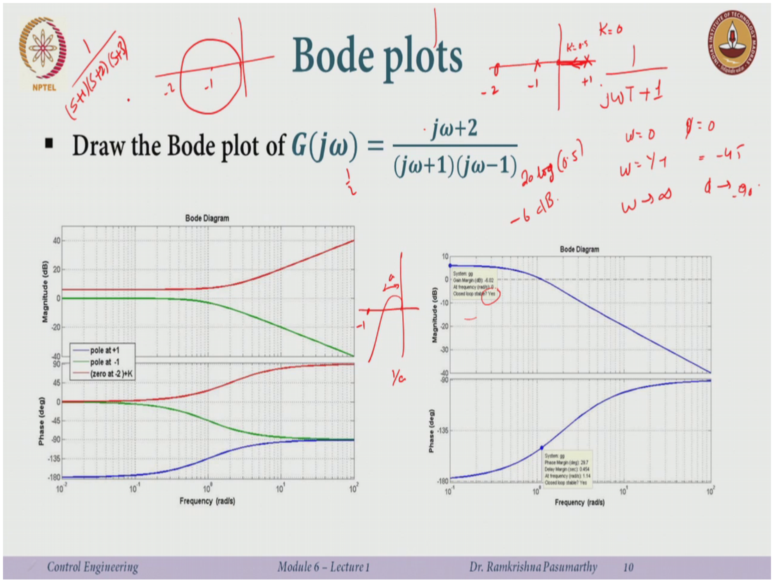

So, this red guy is the So, this red guy would be you know the 0 at minus 2 plus k, right. And then the green guy So, this plots actually over lap and this will be both the stable and the unstable pole, right. And then now look at the phase right. So, we see that you know if I were if you just remember how the first order terms look like. So, the angles started from 0 it went to minus 45 at the cross over frequency and then here. Similarly with this guy and this is how the overall plot actually looks like again.

So, couple of questions which we can try to try to answer here. So, can just by looking at the bode plot can I say if this is if it corresponds to a stable or an unstable pole? Well, just by looking at the magnitude plot well at least I can say that this is a pole, right. Because this 0 will go up. Now what was the how did the magnitude behave for a term like this one? Sorry, the phase 1 over j omega T plus 1, I start with at omega equal to 0 I start from phi equal to 0 then omega is 1 over T, I start at then I go at minus 45 and as omega tends to infinity my phi goes to minus 90 degrees right. So, that is that is happening in the green line, but the blue line does something strange, right. This starts at 180 and ends up at minus 90. So, this is one thing which you can distinguish just by looking at the magnitude plus the phase plots of if it corresponds to a stable pole or not. So, how does the overall system now look like? Overall system well the transfer function looks something like this and this ok.

So, what was a gain margin well the closed loop system is stable, right. If you just look at 1 plus you know this thing being equal to 0 the closed loop system is stable, but it has a negative gain margin, right. Minus6 So, which essentially means if I draw the root locus. So, I have a pole here a pole here and a 0 here this is plus 1 this is minus 1 this is minus 2 this is 0 here. So, for some values of So, at K equal to 0, I am unstable well, not I am not unstable the system is unstable. Then I keep on moving until at K equal to 0.5 I reach this point. So, my system is stable after this thing. So, the gain margin would be 20 log 0.5, right. And this will turn out to be negative or 6 decibels right. So, minus 6 dB is the thing. And if you remember last time I said these are systems which have negative gain margin, right. Even the Nyquist thing right. So, look at the how the Nyquist looks like. So, this was this was minus 1 this was minus 2 and for a system I am just really messing up this, but that is for a system when we started the Nyquist arguments we say if this was an a 1 over a was the gain margin right. So, if you increase the gain by a factor of 1 over a the system was stable. Here what I am doing? I am decreasing it by a factor of 1 over a, because this you know 1 over 2 in the negative direction and therefore, I have this negative gain margin minus 6.02 this I think the matlab will tell you directly.

Of course lots of things again we will come on this well we are dealing with stability and also minimum and non minimum phase system, at the moment you can just understand this from what is happening through, what we learnt through the Nyquist that I really have to decrease my gain for this for K decrease my gain. Because the open loop gain is one and also for the root locus it is obvious, right. That I start from an unstable system and I have to make it to a stable system. And the close loop is stable here unlike the previous case. And that is what you know Nyquist tells us. So, before this I will just do try to do one more one more example right. So, I will just let me say I am looking at. So, so far we have just been doing you know positive in a negative transfer functions. So, in the negative gain margins. So, let us say I take a system 1 S plus 1 S plus 2 and S plus 3 and it is still not ok.

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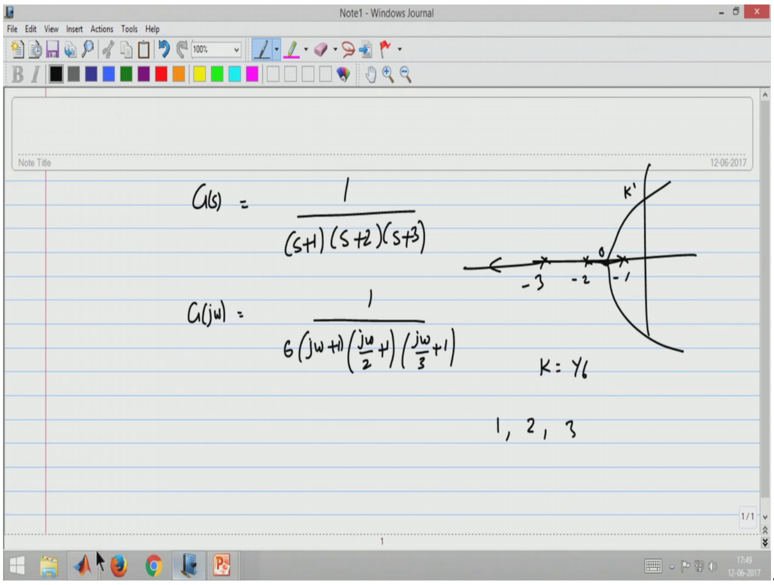

So, G of S is 1 over S plus 1 S plus 2 S plus 3. And I could see which is also plot and check at this is minus 1 minus 2 minus 3. And the root locus would look this guy will go here these 2 guys will meet up here. And they will go here at this is K equal to 0 and this is K equal to some K prime that is we will we will find out.

Now, let us try to write this in the sinusoidal form. So, this is 1 over j omega plus 1 this is j omega by 2 plus 1 this is j omega by 3 plus 1. And what will come here is 6 So, I have a thing of 1 over 6 and then I have cross over frequencies as 1 2 and3. So, let us draw the root locus and the Nyquist both for this I will check ok.

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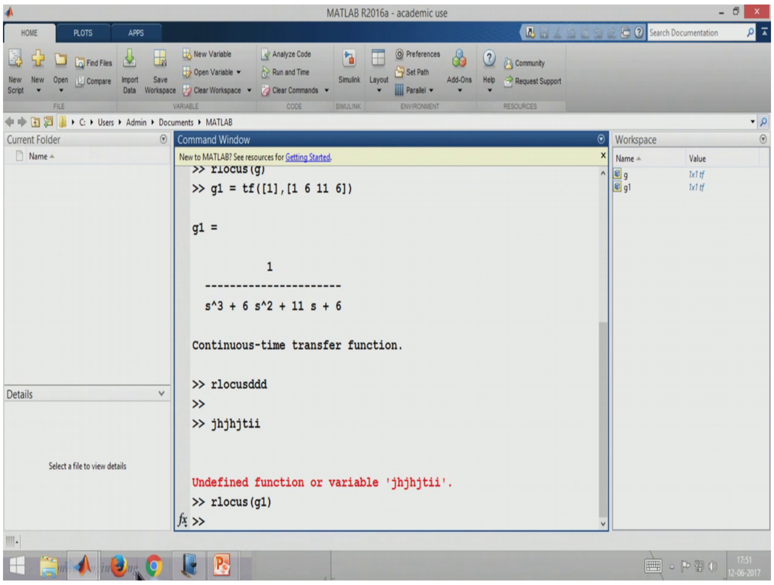

So, I go here I will call G 1 as my transfer function. So, on the numerator I will have 1 and then the denominator polynomial would be 1 S cube plus 6 S square plus 11 S plus 6 now perfect. So, I would want to draw the root locus. So, I will have root locus of G 1 and see how it looks like ok.

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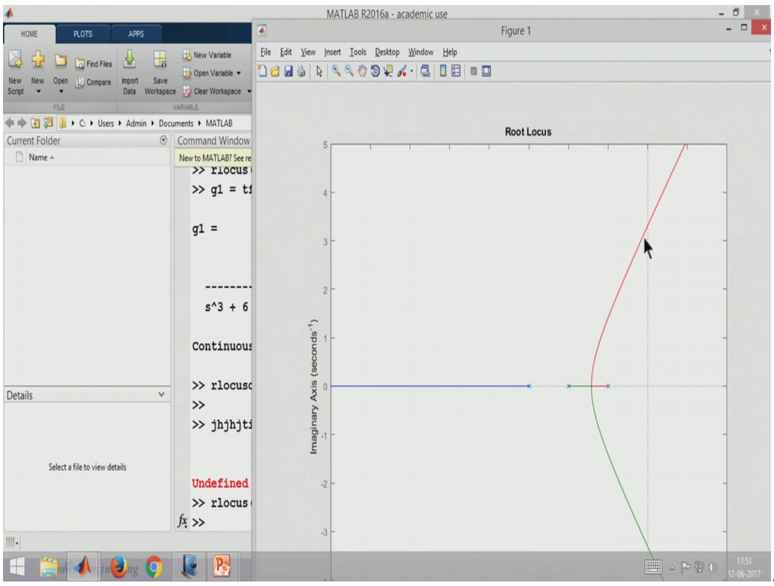

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So, this is almost like what I have drawn. So, you see at a gain of around 60 or slightly around right. So, I think if we have around gain of 60 my system is on the verge of instability. So, that is I was from 0 to 60 the system is stable otherwise the system is unstable.

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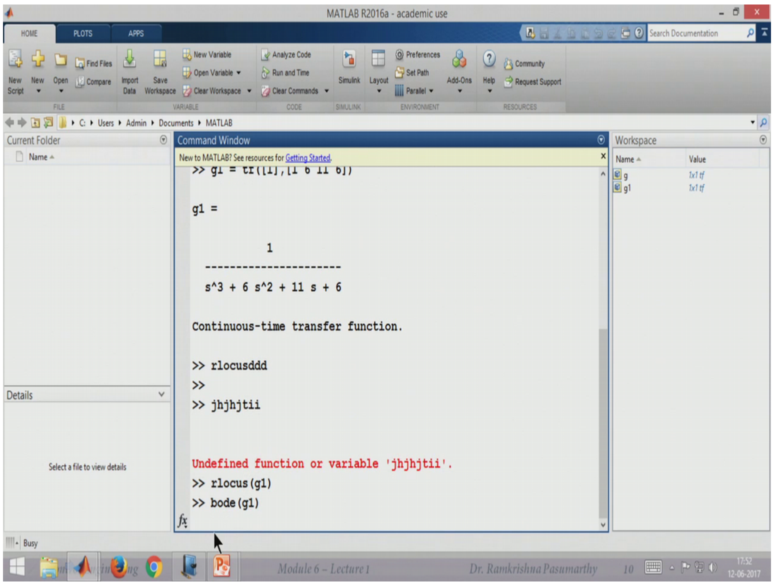

Let me say what the bode says, bode of the same guy G 1 ok.

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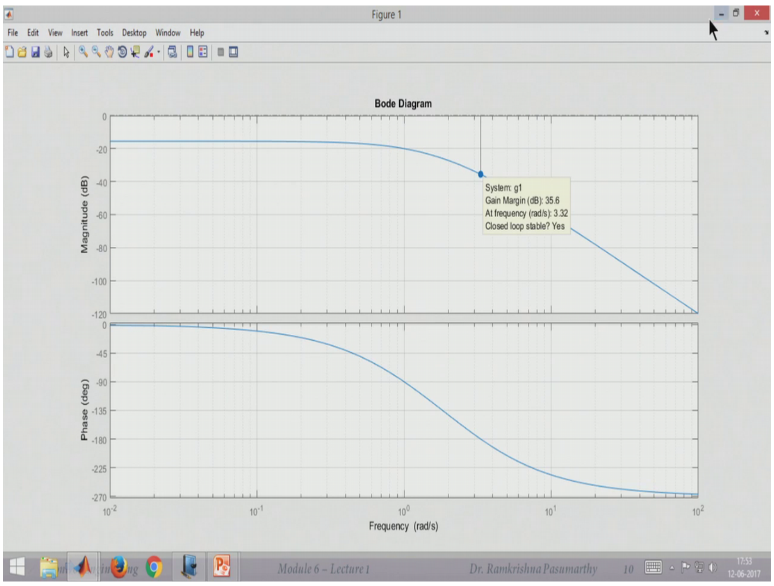

So, this is how the bode diagram looks like and if I ask the system to show all it is stability margins on the grid then this is something nice right.

So, you have gain margin of you know this is a positive 35.6 dB which will actually be 20 log of 60. And then similarly it will have well let me phase margin well it is stable for all figures. How do we compute this? Gain margin again we look at the phase cross over frequency at 180 degrees, right at 180 degrees some somewhere here. So, this is a gain margin the phase margin well I never actually touched the this 0 lines. So, there is no question of I computing the phase margin here, but this is system which is which starts with a stable configuration when K equal to 0 it stay stable until K equal to 60 in absolute terms or 35.6 dB and later on it just goes on to the verge of instability, right. There are also systems with let us say an infinite gain margin.

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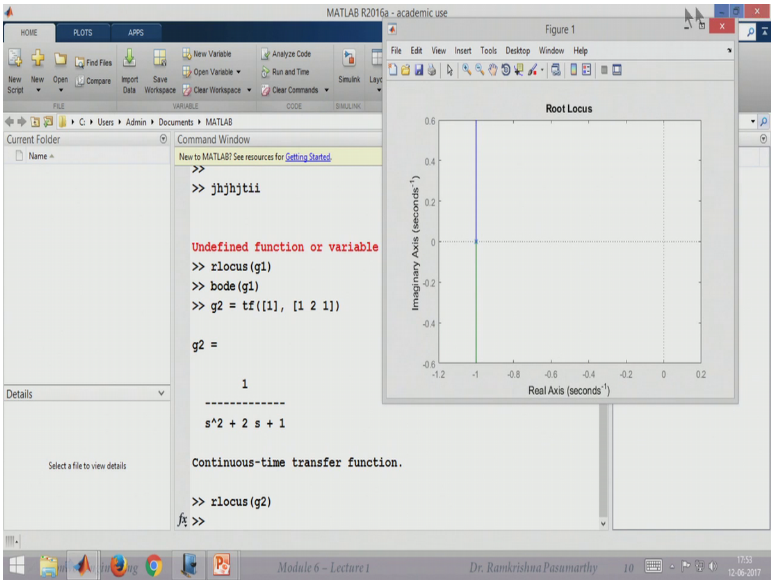

Let us say a standard second order system say G 2 is tf it is like this 1 this is a very simple second, order system and this is what the first example we had done when we are even doing root locus 1 2 and 1 and this looks good and let me just first draw the root locus of G 2 ok.

You see the root locus is exactly what we have drawn right. So, this is for all gains K it is stable.

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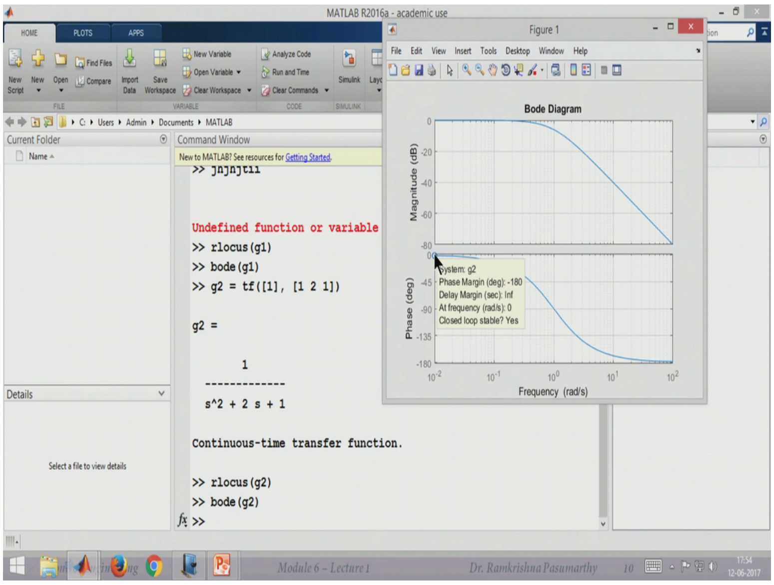

Then you look at how the bode of this looks like. Now I will go to the grid and I see how the stability margin look like right. So, you will have a phase margin of 180 and then there is actually the this actually never reaches well the 180 degrees which is at infinity. So, you will just have the gain margin to be infinite, you see it never reaches 180 degrees right. So, these are systems which have infinite gain margin and a phase margin of minus 180 means kind of kind of very good for us right. So, back to the slides. Now to just before we conclude what we will try to do is can I actually find out error constants from the bode plot ok.

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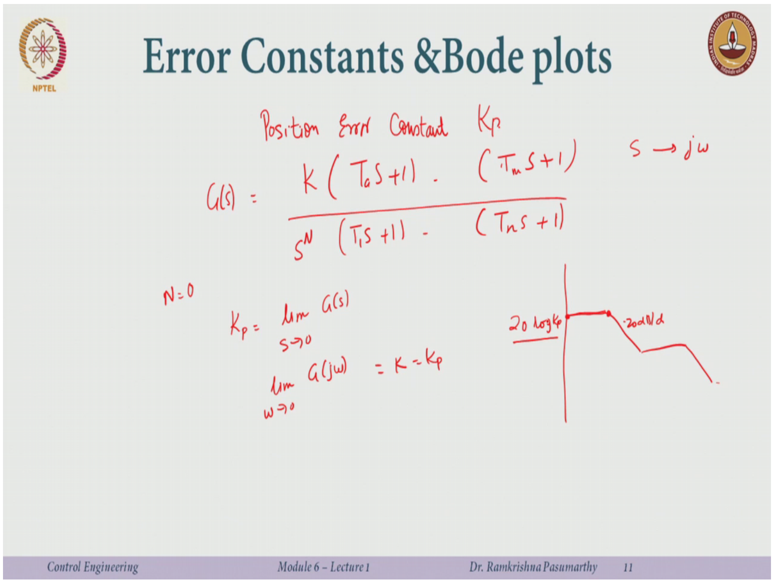

So, let us say first we look at the position error constant K p, right. How was at typical transfer function like when if I added in the sinusoidal form? I have K, I will all the set of zeros say I will call this T some number m over some poles at the origin and I will have t 1 S plus 1 and some T n S plus 1 small n. So, when I can I just write this in the sinusoidal transfer function just by substituting S with j omega.

So, let us say n equal to 0 and this is the only case where I am interested in the position error constant right. So, when S equal to 0 well how will my bode plot look like? So, my bode plot will initially have a certain constant magnitude. Because this guy does not does not appear right. So, this is just one. So, it will just be K times this or things whichever is this. So, all these guys at the lower frequencies are at the 0 dB line what else changes K right. So, this will be something like this and then they will go whatever if there is a here and then there is a 0 and again a pole and so on. Right. So, this is the minus 20 dB per decade and this is the first corner frequency. So, what is this magnitude? The magnitude here is So, how do we define a K? The position error constant that was a limit S going to 0 G of S. Now same thing if I say, what happens when limit? Omega going to 0 G of j omega is well that is simply k, right. This guy goes away I just put omega equal to 0 this is what I am left with.

Now, this thing is again at omega equal to 0 right So, 20 log of K p and this K is essentially K p, right not, So that I have directly written the time constant form. So, how do I find K p? Well I just see where does this intersect this, this what is what is this constant? And then this is a actually 20 log K p when I once I know 20 log K p I can find out what is K p. Then we look at the velocity error constant.

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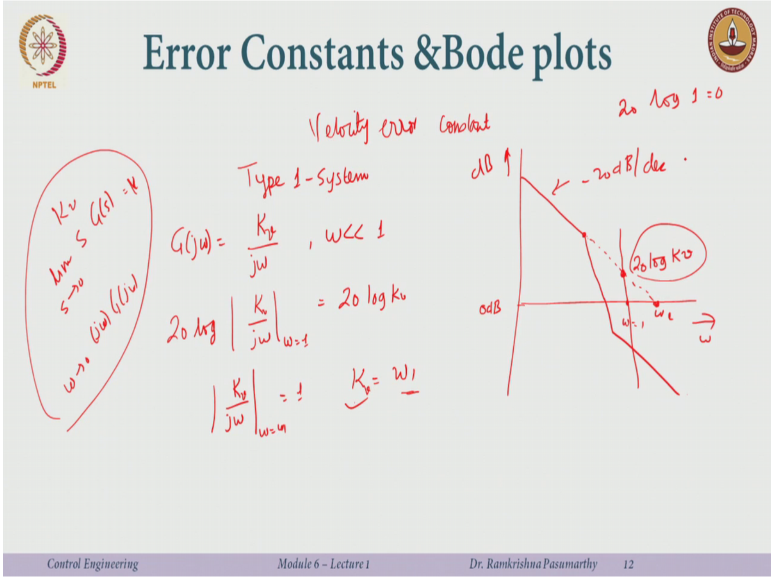

And this has typically will we are interested only in a type 1 system. So, which means the air for lower frequencies the plot would look something like this. Or I can say a plot would look something like this and then well may be after a corner frequency I just go down and you know do whatever. So, for lower frequencies G of j omega because the system is type 1 will look will look something like this. These are again for very small omegas ok.

So, 20 log of the magnitude of K over j omega. So, let us say that well there is some point. So, I just extend this line, right. Till where I do not really worry about what happens at the corner frequencies. And say what is this value at omega equal to 1? Well at omega equal to 1 the value is 20 log times K v. So, I take this say omega equal to 1 here extend this line and this value would be 20 log K v. So, this is my magnitude in decibels and this is frequency now well something else has happened right. So, I am just going down and somewhere I am in intersecting the somewhere here, right. The 0 dB line. 0 dB line essentially means at the magnitude K over j omega at some say omega equal to omega 1 is one let me call this frequency has omega 1 right. So, I just I just keep going down and this one in the inn in the decibels turns out be 0 dB I am just looking at 20 log 1 that would be a 0 dB line. So, I will ask myself what is the frequency at which the magnitude of K over j omega goes to one well that turns out to be K is omega 1, right. And what is this K this K is essentially my K v, right. All these are K v S where I can say what is K v limit S tends to 0 S times G of S or similarly omega going to 0 j omega G of j omega.

So, if I take a type 1 system this will be 1. So, what whatever remains after everything is substituted to 0 which is K and this K is my K v right. So, G of j omega for lower frequencies and this is K v, K v and. So, the frequency omega 1 at which this line intersects the 0 dB line this omega 1 is then my velocity of a constant. So, I can do it 2 ways see what is a magnitude at omega equal to 1, right. And that that will give something 20 log K v given 20 log K v I can always find out what is K v or other ways I can just extend this line until I get the 0 dB line. In absolute terms what does this mean 0 dB means a magnitude of K over j v at one and this K is essentially K v because of this guy this is K. And this is from the definition of K v this is k. So, K v can be found out by the intersection of the initial line with a 0 dB line. And the frequency at which this happens is my K v then this should be straightforward to visualise.

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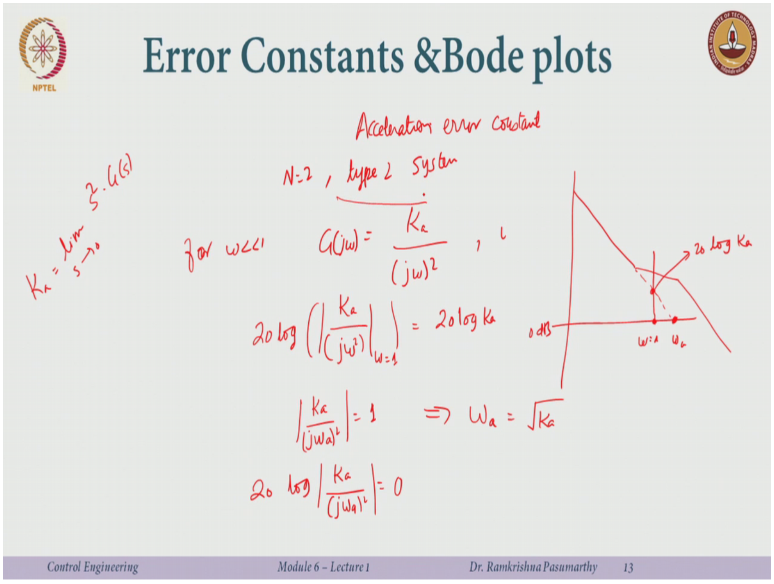

We look at the acceleration error constant, right. In this. So, so we will have n equal to 2 or a type 2 system right. So, here So this initial thing will be minus 20 decibel per decade line. And here well, what happens for low frequencies? I am now looking at G of j omega is K and this K a again turns out to be the acceleration error constant, right. By j omega square how do I compute K a is limit S tends to 0 S square G of s, right. And then due to the j omega and whatever K is left is actually the K a ok.

Again these are for a lower frequencies. So, if I just plot I will have a minus 40 decibel per decade line and then may be something else happens something else happens and so on. So, just extend this line. So, I am interested in log 20 log K a by j omega square, what happens to this guy at omega equal to 1? This guy at omega equal to 1 is simply 20 log K a. So, let us say this is a omega equal to 1, right. And their intersection is 20 log K a. Other way to also compute is to see at what point of time this touches this axis. So, I am looking at magnitude of K a by j omega a square is 1 sorry, or equivalently in decibels 20 log K a over j omega square is 0, 0 dB line, right. It is a 0 dB magnitude of one in absolute terms corresponds to this guy. So, what will this tell me is that omega a, So let me just call this at omega a, right. At omega equal to omega a, So omega a is square root of the acceleration constant. So, I just keep going this till I reach the point omega equal to till I reach the 0 dB line and the frequency at which this happens is the square root of the acceleration error constant ok.

So, again these are just occur only for type 2 systems, K v occurs only for type 1 system and K p occurs only for type 0 systems. And we know why this is true right. So, we saw we saw several advantages now of the bode plots, right. Much easier to compute then the Nyquist, right. We just have to deal just by adding straight lines, right. And well a bit of we could visualise how the gain margin and phase margins look like even though we will really extensively discuss that a little later. What we also knew towards the end is that given a bode plot I could actually compute what are the K p the position error constant velocity error constant and the acceleration error constant right. So, that is what we learnt today.

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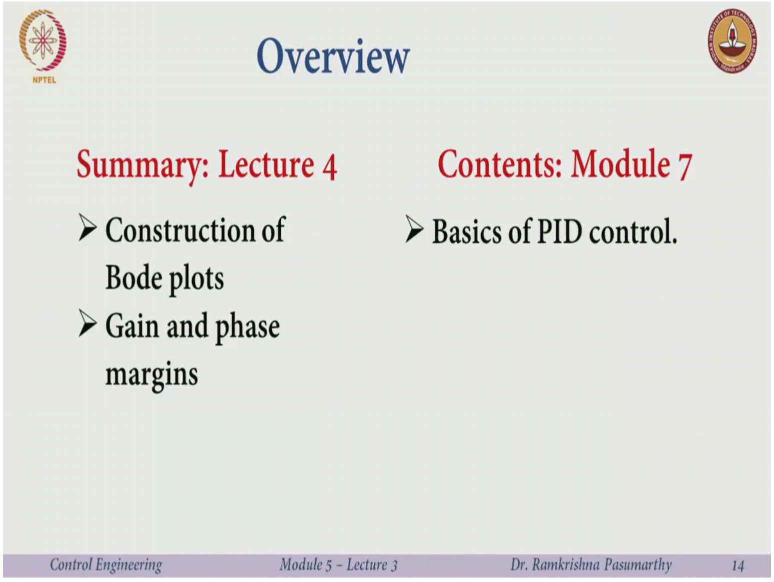

So, what we will postpone the experimental determination of transfer functions via of bode plot when we finish learning what are the minimum phase systems I will not really even attempt to define that at the moment, but let us assume there is something excess or just to please you I would say well minimum phase system are the ones for which all the poles and zeros are to the left of plain. For example, this system was a non minimum phase system because there is a pole on the right. So, that everything is into the into the left is a minimum phase system. So, just juts remember it that way for the moment, right. Ok.

So, next class what we will do is to slowly understand the concepts of designing a controller, right. And what are the basic elements. These are well what they call as a proportional integral and a derivative control. So, we will see how to smartly use these individual elements when to use this is a proportional controller enough for sometimes I need to add a integral control, whatever a derivative control can I actually realise a derivative control, if I cannot realise the derivative control what are the other methods I will do all these ways slowly try to build up the theory behind it, right. Ok.

Thank you.