Control Engineering
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Module - 06
Lecture - 03
Relative Stability

Welcome to this new lecture again we will continue today with the Nyquist plots. So, what we saw yesterday was or in the previous lecture was how to analyze stability in the frequency domain. So, we answered several questions based on the Nyquist plot, the first question was what Nyquist would tell me is how many zeroes are or how many poles of the characteristic equation lie on the right half plane. So, that would be determined by the number of encirclements of the point minus 1 plus j 0 and we also see saw how we shift from the origin to minus 1 plus j 0 and how minus 1 plus j 0 analyzing this point will just help us getting the closed loop behavior by just looking at the open loop transfer function right and so what we will do today is to see or just handle some special cases right.

So, what we said when we were defining the conformal maps from the s plane to the q s plane is that this map should not pass through any single point or through any poles, but that is well it is mostly possible, but sometimes you could see that there might be a pole on the imaginary axis and lets for simplicity say I just start with the pole at the origin ok.

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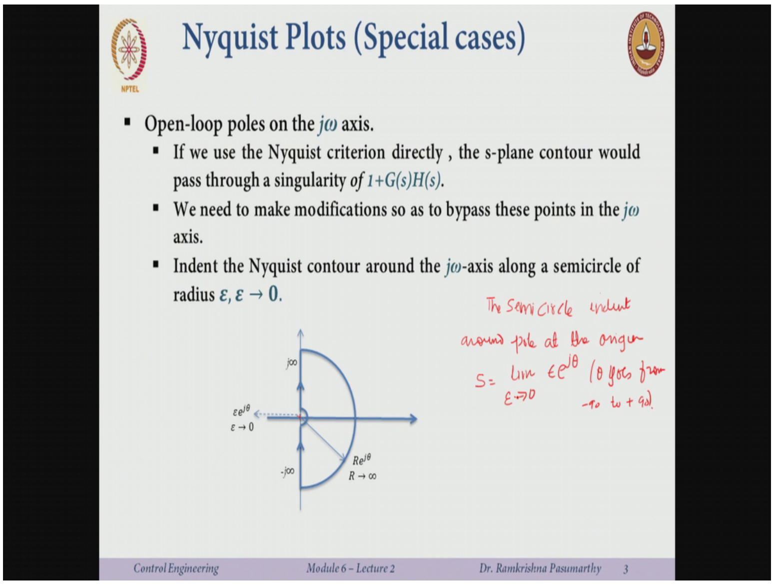

So, the question we will ask is how do we handle these conformal maps or the Nyquist contour when the open loop poles are on the j omega axis.

So, if we use the Nyquist criterion directly the s plane contour would pass through a singular point of 1 plus G of s this would just drop. So, this could be similar in a way to see you know when we were doing the Routh Hurwitz table of when you get an epsil, when you get a 0 right in one of your columns one of your entries becomes zero. So, what do you do you just substitute with a very small number called epsilon, you assume it to be greater than 0 and take the limit as epsilon goes to 0 and then do the rest of the computations. So, we will do very very similar here it is a little tricky here than then what we did last time. So, let us see how we will do this. So, we need to make modifications, so as to bypass these points in the j omega axis and therefore, what we will do is if there is a pole here I just take a detour here, and this detour I will just say well I will just take a little semi circle of radius epsilon.

So, this is how the semi circle looks like epsilon e power j theta and take the limit as epsilon goes to 0 right, if I just write down a little more more formally. So, what we would have is take the take the semi circle indent round say there is a pole at the origin. So, if I say this guy this is pole at the origin I just draw a little semi circle and S would now look limit epsilon going to 0 epsilon e power j theta. So, as theta goes from minus 90 to plus 90 and then we will just do this.

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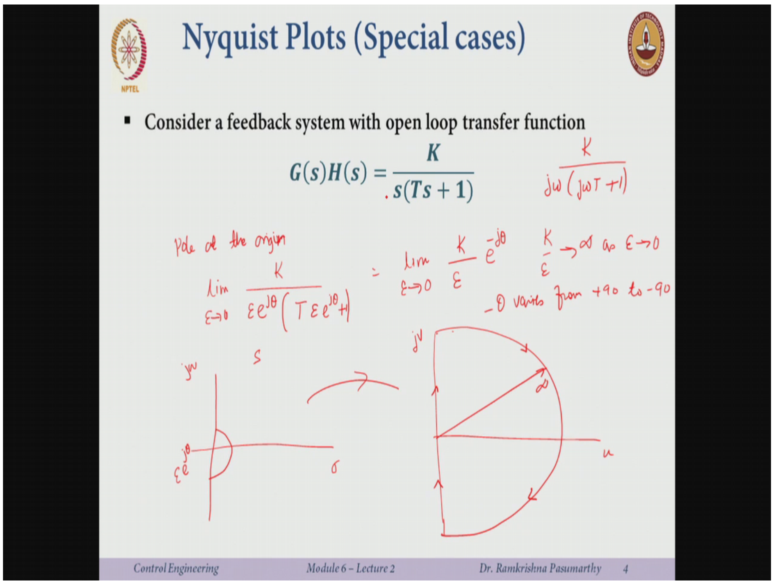

And then put this back here and see how the how the plot plot goes like. So, to just get a little understanding we will start with the help of an example right we have feedback system I have a pole at the origin and another pole. So, sinusoidal transfer function we just substitute s with omega and we know already why we know why we do this right why we just substitute s with j omega for the sinusoidal transfer function ok.

So, let us start by dealing with the pole at the origin right the standard thing that I learnt in the last class will not work because there is a singularity when I substitute x as equal to 0. So, I take this little thing limit. So, this guy limit S or limit epsilon going to 0 this guy this is the value of S. Now we will see how this value of S defined by this little semicircle given by this thing epsilon e power j theta epsilon going to 0 and theta going from minus 90 to plus 90. So, how this little region will map to the q s plane, so, I take limit epsilon going to 0 K what is s now? S is epsilon e power j theta now I will have t again epsilon e power j theta plus 1. So, this guy becomes limit epsilon going to 0 I have K in the numerator epsilon this guy goes to 0 and I have e power minus j theta ok.

So, this guy K over infinity K over epsilon this goes to infinity as epsilon goes to 0 and what does theta do? Theta varies now from or this is minus theta right minus theta varies from plus 90 to minus 90 or plus pi by 2 to minus pi by 2. So, I just first want to see how this little region here or I will draw a little better do this, this is line straight right this little region. So, this is epsilon e power j theta. So, how will this map into the q s plane right and s sigma j omega u j v this is how we defined it last time right. So, what happens how does this guy map here. So, what I have is as epsilon tends to 0, the magnitude goes to infinity and the angle goes from plus 90 to minus 90. So, I will just have arc like this right something like this is of radius infinity and it will travel in this direction here here here and here. So, this is just to do with the pole at 0 region and the rest of the Nyquist plot follows the same procedure as before ok.

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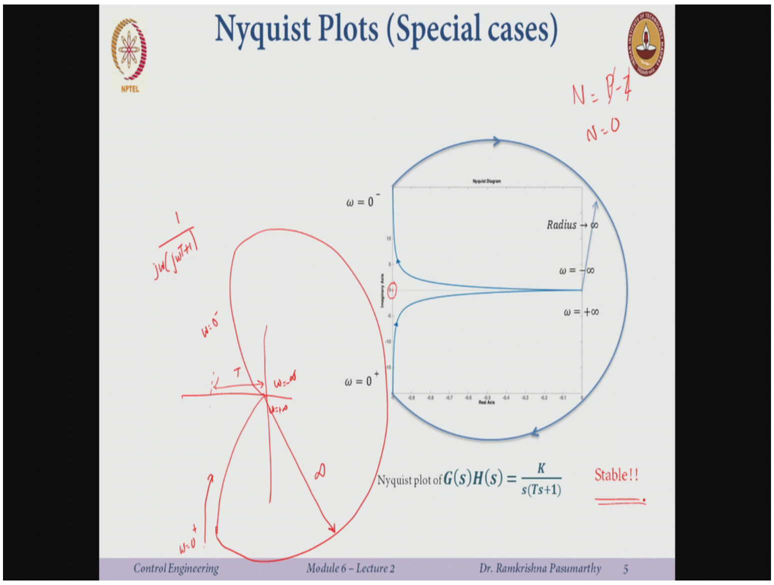

So, if we remember the polar plot of this guy it just looks something like this right. So, the polar plot of 1 over j omega j omega t plus 1 was something like this, it will asymptote here this guy is T omega equal to 0 I start from infinity. So, I start from somewhere here I go here and all the way I come back to omega s plus infinity right and if I want to do the Nyquist then I just take the even the other frequency right something like this right. So, this is how the plot will look like this is what more MATLAB will tell you right when you when you draw these plots and this we derived last time, so how do I really look at it right. So, I start at omega equal to 0 or let us call it 0 plus right. When omega equal to plus infinity I will be here when omega is minus infinity I will be here omega equal to 0 plus I am going here omega equal to 0 minus for negative frequency I will just be here right.

Now, what happens at omega equal to 0 right. At omega equal to 0 no sorry omega is 0 plus omega equal to 0 minus and then I have to really complete this, this thing right where. So, this thing will be the region of infinite radius. So, just, this region is what I have to map now right this is what when epsilon going to 0 this little region right this also I have to map it here and therefore,.

So, the plot is complete by starting from here I started omega equal to 0 minus all the way come to omega equal to 0 plus and then go to omega equal to minus infinity sorry plus infinity. Minus infinity it is 0 plus 0 sorry I will say it again omega is minus infinity

0 minus 0 and 0 plus and omega equal to plus infinity. So, if we draw on MATLAB it will give you this line these 2 lines like this one and this is the polar plots it would not really give you this line because it would not be able to plot, but this is like this is just the infinite radius from here drawing this very approximately, but you know we could look at it this way. Now the question is the system stable well what was stability I just looked at encirclements of the minus 1 plus j 0 point right where is the minus 1 plus j 0 it is somewhere here right this is the point of interest. Now what was the number of encirclements was n equal to p minus z p is 0, z should be 0 therefore, the number of encirclements should be 0 right. So, since the number of encirclements is 0 we can say that the system is stable ok.

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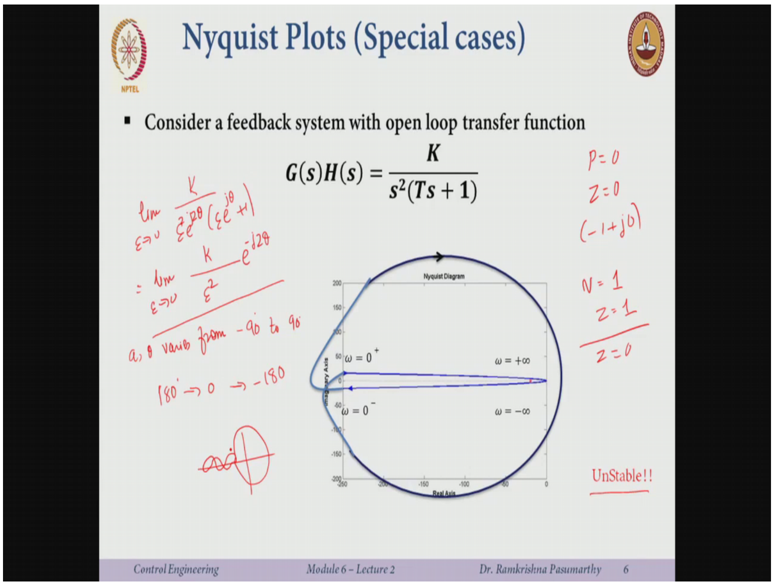

Now, what if there are 2 of these poles at the origin. So, we just do again the same kind of analysis. So, I am just dealing with this term alright. So, I have limit epsilon going to 0, 1 over epsilon square e power j 2 theta then epsilon e power j theta plus 1. So, this I am just looking this like this. So, the K in the numerator limit epsilon going to 0 K over epsilon square e power minus j 2 theta right. So, of course, the magnitude goes again goes to infinity, and as theta varies from minus 90 to plus 90 in the s plane, the q s plane it varies from angle of 180 degrees to 0 to minus 180; this is what the then the plot would actually look like.

So, what MATLAB will give you is this, this kind of blue line here right. So, I just go here this I terminate here at omega equal to minus infinity and this sorry I terminate here at omega equal to plus infinity, I start from here at omega equal to minus infinity and up here at omega equal to 0 plus. So, what happens later on is again and just looking at this of infinite radius going from angle minus 180 to plus 180. So, the plot is completed in this way minus infinity 0 minus I go here go here and do all the way. So, there is some kind of an encirclement it is little tricky, but it is you can just understand it how to complete these lines. So, what MATLAB would show me is these 2 things right, starting from omega equal to 0 plus omega infinity, minus infinity and 0 minus and these guys have to meet somewhere right and they will meet again via this expression. So, it will go all the way till infinity with this 180 to minus 180 and then come back here ok.

So, what about the stability? So, P equal to 0, the system to be stable Z equal to 0. So, I am again interested in the encirclement of the point minus 1 plus j 0. If I look at the axis here minus 1 plus j 0 is somewhere sitting here right not really to scale, but somewhere here. So, if you see closely that I go here here and I am actually encircling the point minus 1 plus j 0 one times, and there is no pole right. So, therefore, z equal to 1 and for stability what I need that z should be equal to 0 and therefore, the stable is this system is unstable right you would also just do an equivalent root locus analysis of it or just look at the characteristic equation do the Routh Hurwitz and so on, we will still get the same result ok.

So, this is all about special cases of Nyquist we will not really go into plotting complex plots that is not the aim of the course of you know given a you know something you know something of s power 4 and s power 6 and so on that is not important right. So, we just try to understand what this guy tries to tell us. So, there is something also called as relative stability right. So, most of the times I will say well I am interested in stability; the system is just about stable.

So, then the next question I will ask is how stable it is right. If it is stable will it remain there for all time or will a certain change in parameter push it to the verge of instability, certain change in gain will push it to the verge of instability. So, I want a reasonably good, good measure of how stable my system is ok.

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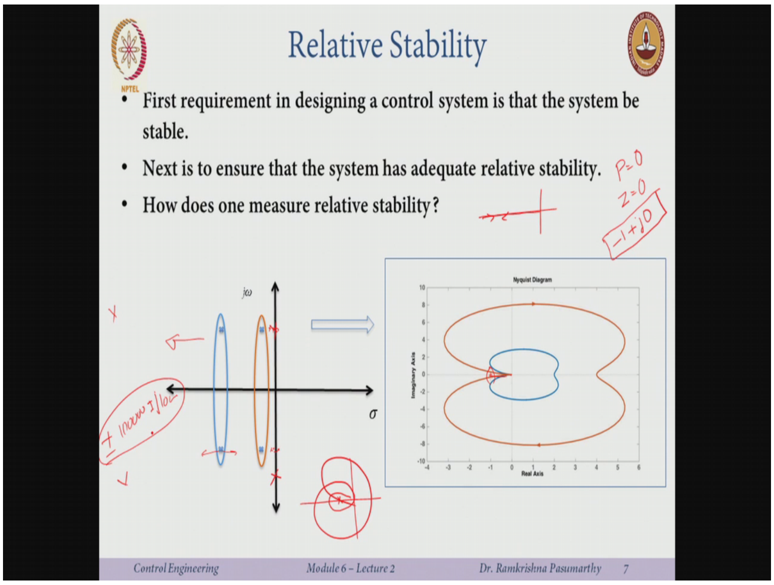

So, what we know after we say well the first requirement is that the system should be stable, and next is to ensure that it is adequately relatively stable, now how does one measure relative stability sorry. So, look at these 2 things this brownish kind of color I do not know what it is called. So, this is a set of poles here open loop poles and these are also a set of poles right now I go. So, this is my s plane sigma j omega these are the poles or also called the dominant poles I will tell you later why I use the word dominant poles, but let us say the system has just 2 poles and I plot the Nyquist for this right. So, this guy blue guy goes here and this brownish guy goes here.

Now, here I look at stability in terms of the distance from the imaginary axis right. So, if I am closer here get closer until here, and there could be a point where I just tip off to the right side it is here and here. Even if am just very slightly to the right I am still I am still you know unstable even if I lose by one run in a cricket match or 100 runs I still lose right. So, it is the same thing here. So, how does the Nyquist thing now look like here I just cannot directly measure relative stability I can say well this is further from the origin. So, it will take you know it will still be safe if there are some change in parameters or things like that it may just come slightly to the right or even move to the left and so on.

But I can say that this looks little more hopeful the blue line than the brownish one this paired poles. Now let us come back here right. So, what is the point of interest right? So, as long as P equal to 0 say the open loop system is stable I just want Z to be 0 right and if Z is 0 Nyquist tells me that the system is stable now who decides this. So, I just. So, this is my guard right who says well you cross this line you trespass and you become unstable. So, the idea would be [FL] well I just stay as far as possible from this line this this number minus 1 plus j 0 right.

So, look at the blue line here it starts here and goes here the brownish line is still closer here right this is the this guy inside is the minus 1 plus j 0 thing right and I want to stay away from here, say something changes and then you know if I could just redraw plot over here like this. So, this is my point minus 1 plus j 0 and if I do I know something like this then I am gone right because I am actually encircling the point minus 1 plus j 0 I did not really draw a smart picture here, but something like this. So, I want to you could see this one if something keeps changing you may just stick to over to the to up, and then the what happens when I am here that is poles on the imaginary axis I am just here. So, the Nyquist here I will just draw it here, it will just be on the horizontal line this and this depending on the omega increasing or decreasing right ok.

So, based on this can I quantify something as a measure of relative stability. Again the idea is stay as far as possible from the point minus 1 plus j 0 or further to the left of course, you cannot really say that I will have poles at plus minus 1 million plus minus j whatever something like say j 10,000 this is also not practically feasible right. So, we have to make some reasonable assumptions there or you get some reasonable reasonable estimate of things ok.

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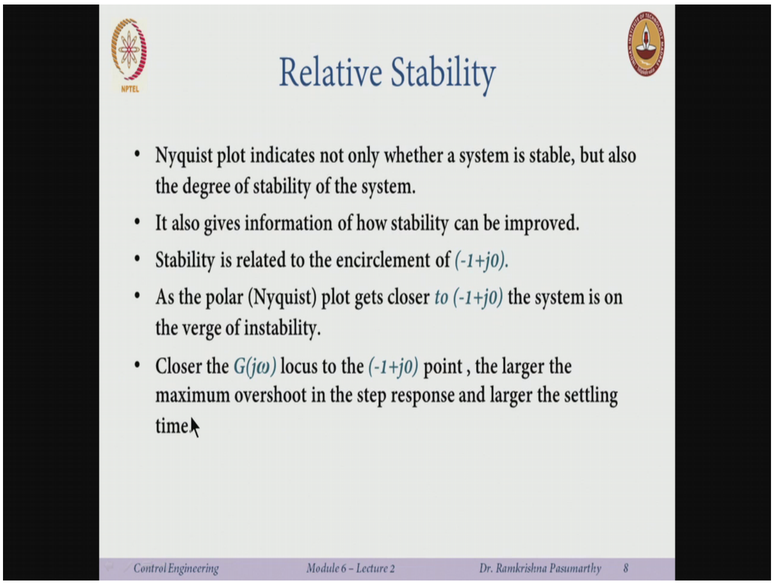

Now, what is then the beauty of the Nyquist plot? So, Nyquist plot indicates not only whether the system is stable, but also the degree of stability.

So, we will also see how it will give some information if the system stability can be improved right and then since the deciding factor is this guy minus 1 plus j 0 and this entire stability criterion is depending on the encirclement of the point minus 1 plus j 0. So, what does the plot tell us either the polar or the Nyquist that as the plot gets closer and closer to minus 1 plus j 0 the system is on the verge of instability closer it is to minus 1 plus j 0 larger is also the maximum overshoot in the step response and so, is the settling time closer I am to the minus 1 plus j 0 which means closer I am to the origin here which means the damping is very small and things like that now ok.

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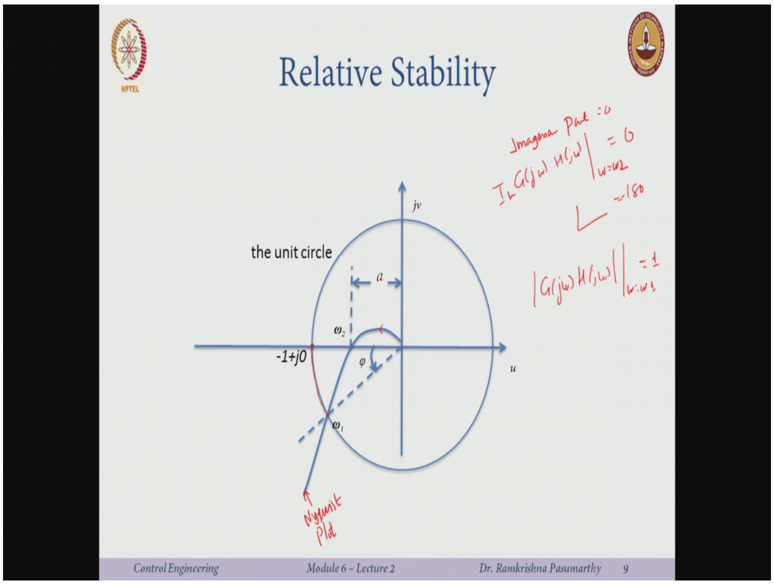

So, let us do it a little graphically, let us say I am not interested really I am not interested really in you know in the Nyquist in the exact transfer function, but say something looks like this right this is the Nyquist plot this guy or even the polar plot and this is the point minus 1 plus j 0 and I just draw a circle around here and say this is the distance from here till here is minus 1, the distance from here till here is a. So, what is this distance this distance is when the imaginary part goes to 0. So, let me call this G of j omega H of j omega at omega equal to omega 2 sorry the imaginary part of this guy goes to 0 or in other words you can even say that the angle would be 180 minus 180 degrees. So, this is this is the thing here right at omega 2 ok.

So, what could happen that if this a is getting closer and closer here, then I can you know I am on the verge of instability say I have something like this here and say I have something changes I increase the gain and I go here. So, once I go here I am actually encircling this point right. So, I will just go some do some detour like this and I can say that the system is going to be on the verge of instability here is this thing. So, this is not important. So, there is something which this a can tell us the distance or how far this guy is from this point not only that now look at this circle, and say well it intersects this guy here right at frequency omega 1 the magnitude of G omega H j omega at omega is omega 1 this guy is 1. Now keeping this, this constant and if I keep on moving along this line which means I am adding some kind of angle to the system right I am just moving from here till here then if I add say some angle of phi in this direction then I hit the point minus 1 plus j 0. Once I hit the point well then I am on the verge of instability.

So, there are 2 things the distance how far is this a from here and second also how far is this angle from being minus 180 degrees right. So, these 2 will quantify this a and phi will give us some measures of relative stability. So, far I just said just be further from the origin in the s plane when I would talk in terms of poles, that the poles they are on the imaginary axis the system is just marginally stable slightly to the left well it is stable, but it is on the verge of a stability right it is not it is not very. So, now, I can actually quantify these things ok.

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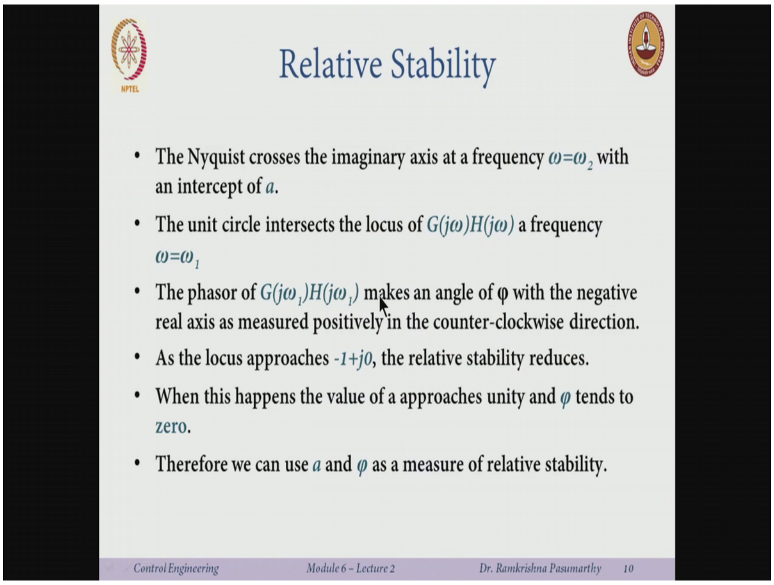

So, let us say these things a little more formally now. So, the Nyquist crosses the imaginary axis at a frequency omega is omega 2 with an intercept at a right here omega 2 and this intercept is called a. And then the unit circle intersects the locus of G H with the frequency omega equal to omega 1 here right and I just draw this unit circle because this is the point of my interest right minus 1 plus j 0. So, the phasor of this makes an angle of phi in the negative real axis, again the usual measurement is we measure positively in the counter clockwise direction. So, this angle phi from here till here and then well this is now kind of known to us right and we know why this is true that as locus approaches minus 1 plus j 0, the relative stability reduces and this happens in 2 cases right.

So, this happens in the value of a this should be a little bit highlighted; because it is the when the value of a approaches unity or phi approaches 0 or you know. So, for this this angle if I measure from here the angle of G j omega should be minus 180. So, a approaching one would be bad news for me, in the similar way phi approaching 0 will also be bad news for me. So, therefore, we use these guys a and phi as a measure of relative stability.

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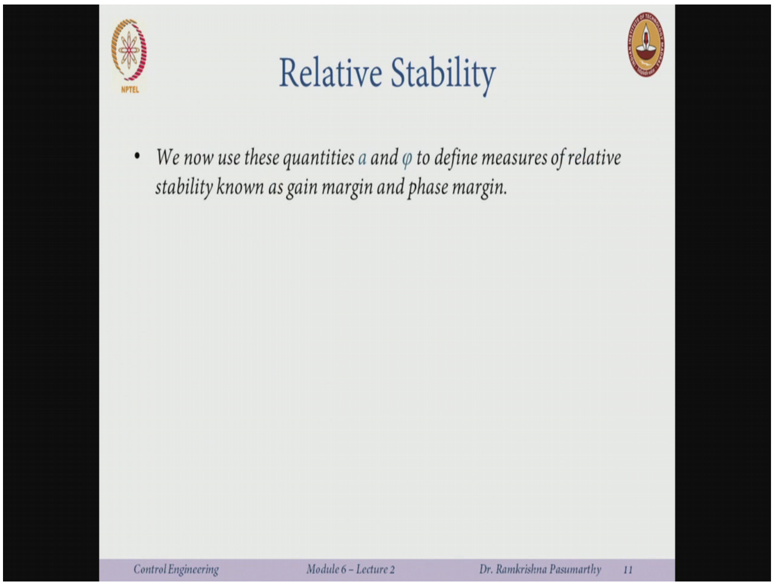

So, we will use 2 quantities a and phi to define the measures, now we will name each of them right. So, this guy related to a we will call the gain margin and this guy we will call related to phi we will call as the phase margin the names will already tell us what they mean.

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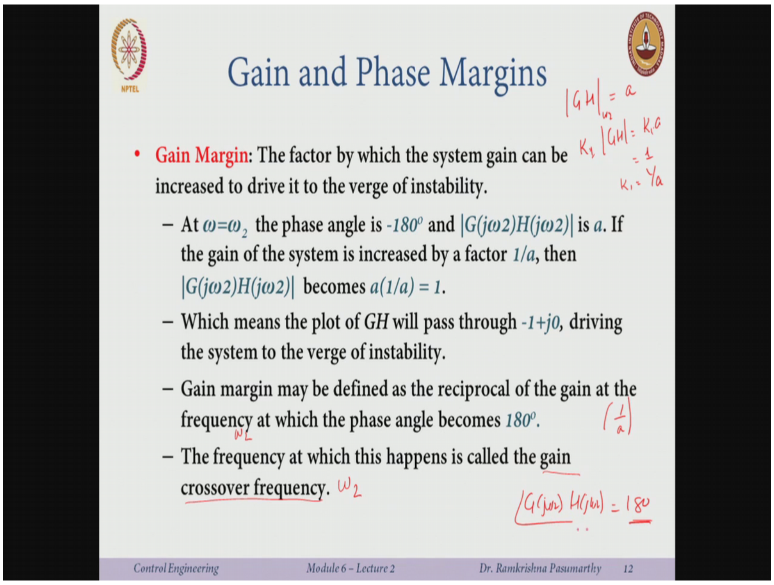

But let us understand that little more formally in little more details. Gain margin what is this guy this is the factor by which the system gain can be increased to drive it to the verge of instability. So, slowly we will try to understand. So, at omega equal to omega 2 this guy omega equal to what is the phase angle? The phase angle is minus 180 degrees and the magnitude of G times H is a, now I want to say well. So, say G times H now I will not use this omega 2. So, G times H is a I can have some gain right I can have some gain. So, it will be K 1 multiplied by G times H would be K times a I say well at what point does it reach one how much can I increase the gain factor well that will be K 1 is K 1 is 1 over a right then G times H becomes 1 right. So, it means that if the gain of the system is increased by a factor of 1 over a this is K 1 I am multiplying K right. So, do not confuse with that with addition here right.

So, some what is the gain margin will say this is one oh how far I am, I am just 1 minus a you are not 1 minus a because the gain comes as a multiplicative factor right here that is why that if I add more gain it will show up here. So, this multiplied by the actual magnitude a when does that goes to one that goes to 1 when this additional guy here K 1 is 1 over a. So, if the gain of the system is increased by a factor of 1 over a then the magnitude of G 1 becomes a this is the original magnitude, multiplied by the new magnitude right K 1 the additional gain which we put 1 over and this is one this means the plot of G H now again I am being a little lazy not writing the entire thing of G j omega H j omega, but that should be obvious which means the plot of G s will pass through this point minus 1 plus j 0, driving the system to the verge of instability ok.

So, therefore, now how do we define this what is this gain margin in terms of a number right. So, this gain margin may be defined as the reciprocal 1 over a right reciprocal of what reciprocal of the gain at the frequency at which the phase angle becomes 180 right. So, that is this one. So, this frequency we defined as omega 2 right at omega 2 angle G j omega 2, H j omega 2 was 180 right. So, at this omega 2 I calculate the gain. What is the gain at this omega 2? This the gain at this omega 2 where the angle is 180 is a now the gain margin is defined as the reciprocal of this a, what is this a? The gain at this frequency which is omega 2 at which the phase angle becomes 180 and the frequency at which this happens is called the gain crossover frequency omega 2 is the gain crossover frequency, the gain margin here is one divided by a ok.

So, by what factor should the gain be increased such that the system goes to the verge of instability that is 1 over a, again is defined as the value of G times H of omega 2 what is omega 2? Omega 2 is the frequency at which the angle becomes 180 degrees, similarly the phase margin ok.

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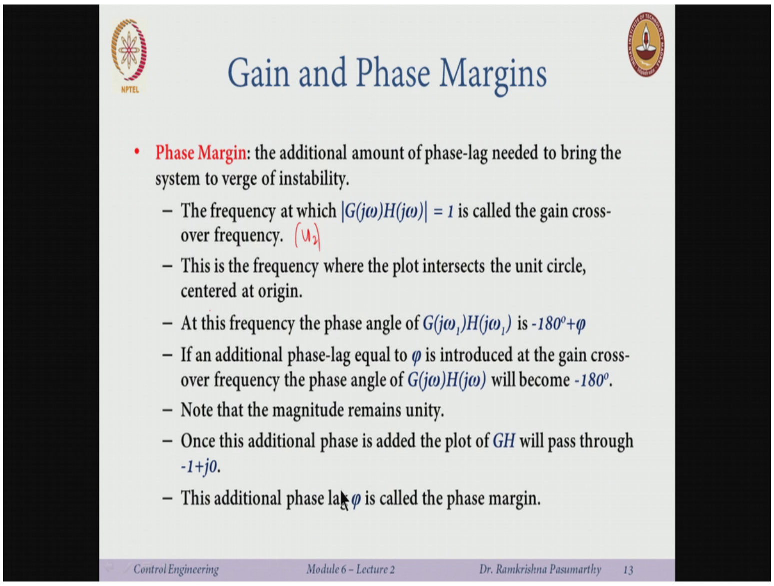

So, we could just you know say oh now I understand gain margin. So, phase margin should still be the same right some similar definition, you will say oh additional amount of phase that I needed to bring this system to the verge of instability well this is vaguely true, but we need to be little more careful in formulating the statement or getting the exact value. So, we will just see what are those steps. So, first is well we are interested let us revisit the picture again for a while.

So, what are we interested again is here right this intersection here, this phi this guy I call this omega 1 and what is this omega 1? The point at which you know the magnitude becomes one here this guy. So, the frequency at which G j omega H j omega is equal to 1 is called the gain crossover frequency omega right this is omega 2 the magnitude is 1 how did you find this? Just by the intersection of the plot with the unit circle. So, this is the frequency at which the plot intersects the unit circle of course, this unit circle is centered at the origin for obvious reasons because we are just interested in minus 1 plus j 0, I just want to pull it along that angle to that point minus 1 plus j 0 and at this frequency what is the phase angle? At this frequency the phase angle if I have to just measure is 180 or even minus 1 either wise 180 and this phi ok.

Now, if an additional phase lag of phi is introduced in the system, right if I just push it this guy little up with an angle phi the phase becomes minus 180 and while I am doing this along that unit circle the magnitude remains unity right, the magnitude remains one and once I reach that point where the angle is minus 180, what will be G H then? The G H or the plot of G H will pass through minus 1 plus j 0, once I have added that extra angle phi and just see this one right I am just pulling this omega one here along the circle right I am adding some 1 degree, 2 degrees and so on I add 5 degrees I end up here right what is the magnitude here? Minus 1 what is angle? 180 or minus 180 right. So, I am just adding this extra amount of angle phi. So, now, this additional phase lag phi is called the phase margin now where do how do I compute this phi well.

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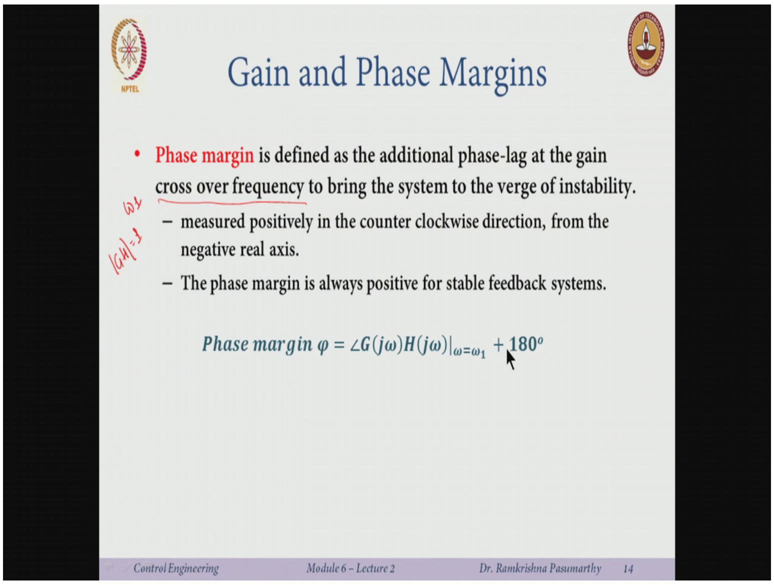

So, now formal definition is the phase margin is defined as the additional phase lag at the gain crossover frequency and this is the frequency omega one where the magnitude of G times H becomes 1. So, what we started off saying that well I need to add some phase now where do I add this phase? This phase additional phase I add at the gain crossover frequency again the standard terms of measurement will be in the counter clockwise direction from the negative axis and so on. For stable systems phase margin is always positive that is may be even true for the gain margin right. So, if I just want to formally write down what is phi from that which you think I had earlier here this guy phi is the angle of omega equal to omega 1 plus 180 degrees ok.

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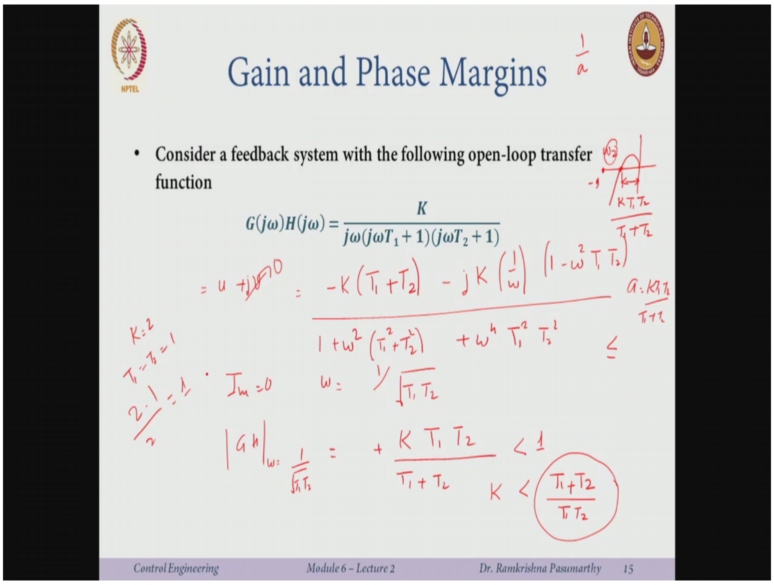

Now, let us see how this system looks like. So, now, I have a system of the form then I have pole at the origin and then j omega T 1 plus 1, and j omega T 1 T 2 plus 1. So, I can write this as I split this into the real and the imaginary part, I skip this computation I will just write down the final value minus K, T 1 plus T 2 minus j times K, 1 over omega one minus omega square T 1, T 2 over bunch of terms in the denominator 1 plus omega square, T 1 plus T 2 squares come here plus omega 4, T 1 square plus T 2 square just check this computation for yourself also. So, if I look at again the plot which I had shown and right and if I say I am interested in finding the gain margin so at this omega 2 which I defined earlier right. So, the imaginary part should go to 0 imaginary part goes to 0 when this term goes to 0 this term goes to 0 when omega is 1 over square root of T 1 times T 2 ok.

Now, at this, this is what this is omega 2 right at this? Frequency T 1 T 2 the magnitude is minus K T 1 T 2 over T 1 plus T 2 or just put the magnitudes this will be plus. So, this will go away. So, now, for ability what should what do I want if this is the point minus 1. So, this magnitude which is now this one right K, T 1, T 2 over T 1 plus T 2 this guy should be less than 1. If I put less than or equal to then there is some kind of marginal stability and therefore, this system is stable as long as the gain K is less than T 1, T 2 sorry T 1 plus T 2 over T 1, T 2. See it is quite straightforward.

So, I start. So, my thing is well I want to compute what the values of gain K is the close loop system stable well. So, I just see use this criteria right. So, this how far is this guy from the point minus 1 plus j 0 how do I compute this I just put the imaginary part of this guy to 0 and I compute the frequency the frequency is omega 2 or what we call as the phase crossover frequency what is the magnitude at this phase crossover frequency? It is K, T 1, T 2 over blah blah blah and the gain margin is this one right. So, T 1 plus T 2 by T 1 T 2 and what we have defined earlier? This was 1 over a and what is a here? A is K, T 1, T 2 by T 1 plus T 2. So, we just get the get the inverse of that.

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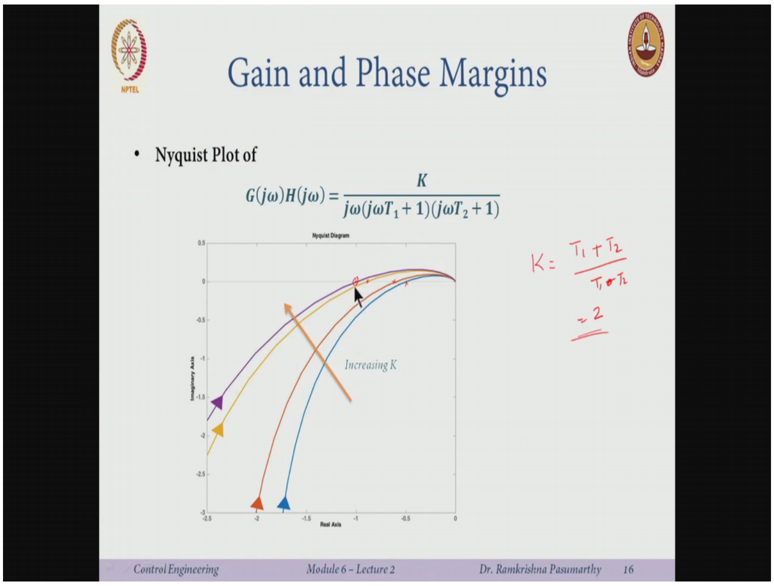

Just to see some plots here. So, I start. So, this is the same transfer function with T 1 and T 2 being equal to 1 and I keep on increasing the gain K right. So, this is some point here I reach the point minus 1 and this point I can now exactly compute right. So, K plus what T 1 plus T 2 over T 1 times T 2. So, at the gain K equal to 2 I just reach this point these are all K for 1 1.5 1.75 and so on. So, as gain in K increases my root loc sorry my Nyquist shifts to this side and if it is shifting to this side I am reaching closer and closer to minus 1 to this. So, this was my omega 2 there, omega 2 omega 2 and now I am here. So, this is we will just show you how the plot changes with varying gain how to quantify that well that is this one right this guy right.

So, what was this magnitude here K, T 1, T 2 over T 1 plus T 2, what happens when K equal to 2, T 1, T 2 equal to one this guy becomes 2 times 1 by 2. So, this is one. So, that is what is exactly happening here at K equal to 2, I can compute analytically and also check by the plots ok.

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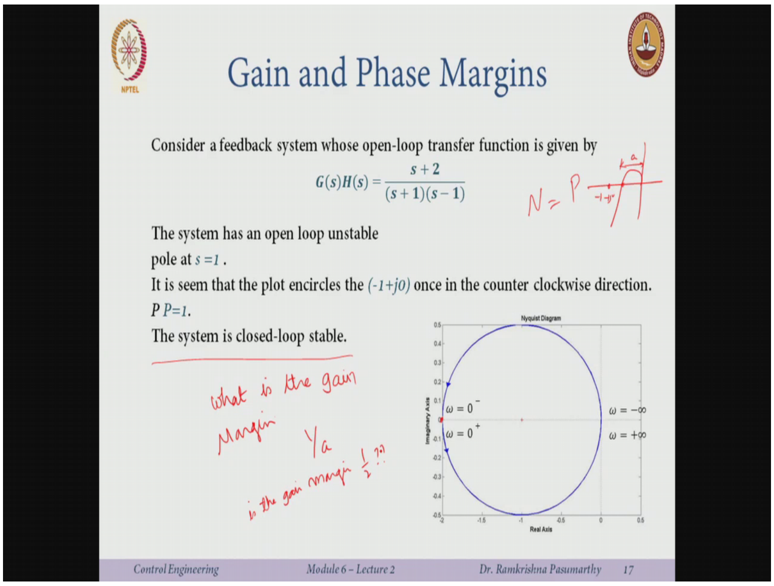

So, this thing we had check seen last time also right. So, I had a system with an unstable open loop pole at s equal to plus 1 I had a 0 and this 1. So, well the system has an open loop pole at s equal to 1 and the plot. So, this is the plot minus 1 plus j 0, and I see that the plot encircles minus 1 plus j 0 once in the counter clockwise direction right and p equal to 1. So, what should n be? N should be equal to P for stability P is one and n is 1 therefore; the close loop system is stable ok.

Now, what is the gain margin? Gain margin is we had said well the reciprocal of a, how did we define a was defined like this. So, this was a, this was the point minus 1 plus j 0 1 over a now which means I am looking at the intersection with the real axis now see where this intersects with real axis. So, it intersects here what is the magnitude well it is 2, now is the gain.

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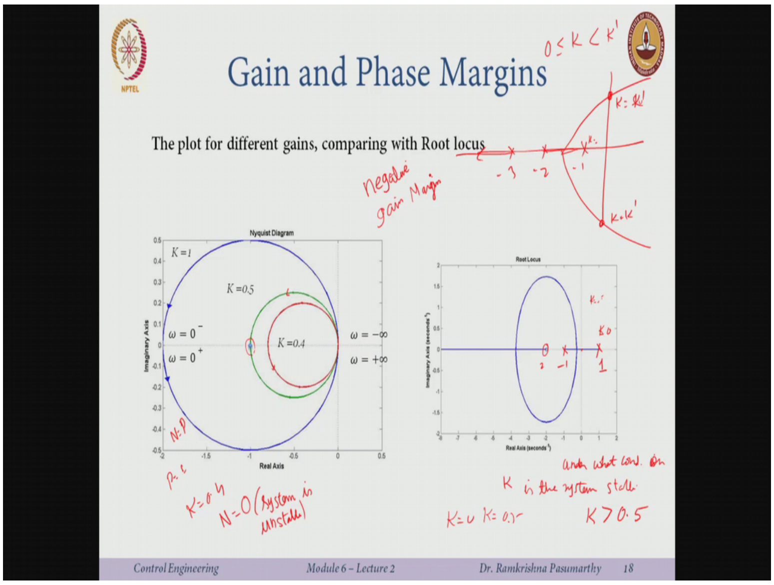

So, is the gain margin 1 over 2 well we will see what that means, right. So, if I start with gain of K equal to 1 sorry I am somewhere here right this is the original plot this is this plot right s plus 2, s plus 1, s minus 1 with K equal to 1. Now oh sorry this for K equal to one I am fine right. So, this this is K equal to 1, N equal to 1 or N equal to P where P was equal to 1 system is stable.

Because I am inserting the point minus 1 plus j 0 and I come down, come down, come down here I say well K equal to 0.5, K equal to 0.5 well I am just here just at the point minus 1 plus j 0. K equal to 0.4 what happens? The K equal to 0.4 what is the number of encirclements of minus 1 plus j 0 we take this direction, well there is no encirclement of minus 1 plus j 0 right this one. So, N equal to 0 therefore, system is unstable; what is happening here? Now let us compare the root locus right root locus will have a pole at one another pole at.

So, I have this is another pole at minus 1 I have a 0 at 2. So, one pole goes to the origin other goes. So, this is starting from K equal to 0, I reach some point here right say K equal to 0.5 and when am I stable. So, what under what conditions is conditions on K is this system stable usually let us then do something else right say I have a system with a pole at minus 1, pole at minus 2, pole at minus 3 and there are no zeroes and I will just do all the rules of the root locus. So, one guy will go to infinity this way these guys will meet here right. So, they do this way right. So, there is some value of gain K say some number say K prime right this is K equal to 0 right. So, what are the values of gain K for which the system is stable I will say for gain 0 to K less than K prime right if this is the value of the gain K when it intersects the imaginary axis ok.

What can I say over here? Well the system is stable what happens for K equal to 0? K equal to 0 I am unstable K equal to point one I am still unstable until I reach K equal to some kind of 0.5 here right where I am just at the point minus 1 plus j 0 here. So, this system is stable for K greater than 0.5 correct. So, I start with a open loop unstable system I have to reduce. So, the system is unstable until I have a certain gain. So, therefore, what we call these systems are as a system with a negative gain margin right. So, the earlier problems were formulated in such a way that how much can I increase the gain K, before it becomes unstable now it is the reverse I am starting K equal to 0 is already unstable K equal to 0.5 is already unstable.

But there comes a threshold where after a certain limit I pull the poles back to the stable region and unstable for all other gains. So, that is the concept of a negative gain margin that first I have to make a system stable starting from an unstable open loop configuration, I increase the gain K and I push it to the stable region. Now why is it negative we will quantify that it a little later while we do the bode plot analysis in terms of decibels. So, we will stop here with the Nyquist thing, again getting or really you know spending time to get beautiful plots by just writing the imaginary and the real parts may not be very helpful, but this is just we have to keep in mind for our basic understanding right.

Now, we will try to do an equivalent version of the plots called the bode plots and there the quantification will be much more clear, the plots would be much easier for example, here I really would not know how does this how does this cross here there are also plots where you may say encounter things like this right like I am doing this and this and this and so on and something like it is much more complicated than what your driving exam would ask you to do it just asks you to make one 8 figure, here it will be several combinations of those. So, we are not really going to get expertise on drawing these plots MATLAB will do it for us or there are several other tools also right. So, once we have the basic understanding of the concept of the stability and relative stability in the frequency domain via Nyquist.

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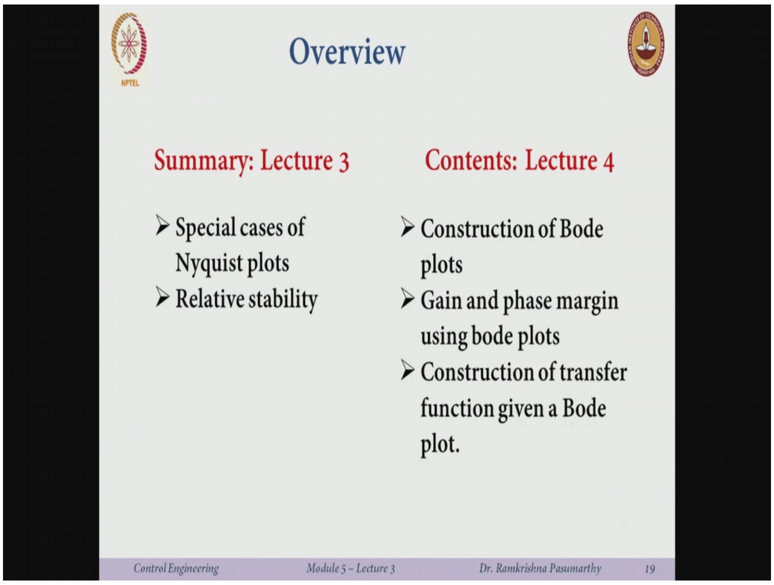

We will go in the next class to see the construction of bode plots. You might have seen this somewhere in your networks course if not then do not worry we will do it all over again.

So, I will just start doing this on the blackboard first right just to slowly explain you the construction and then we will go to the MATLAB and then I will show some slides on that then what are the equivalent of you know how do I get gain and phase margin using bode plots and then lastly some interesting thing we will also see is if I am just given a bode plot. Can I reconstruct back the transfer function of the system and that is also kind of pretty pretty exciting; and that will give you insight of I just start with a black box I just do not know what is in the system I gave several frequency signals, I construct the bode plot and I could kind of get an understanding what is sitting inside the system right. So, all these things we will do in the in the next class.

Thank you.