Control Engineering
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Module - 05
Tutorial
Lecture - 23
Root Locus Plots

Hey guys. Seen this, but will just quickly run through few root locus plots; I will just run you through three or four problem just to tell you how to use those steps

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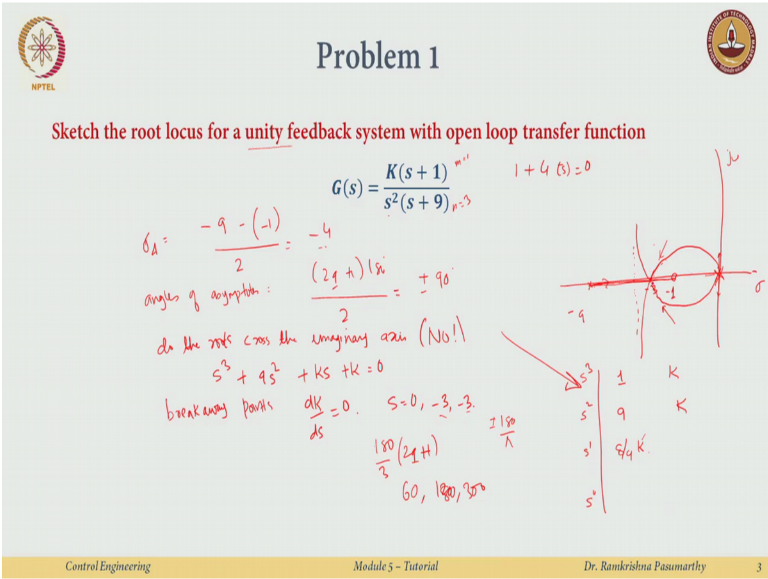

So this is the first problem: G of s is, all through again I assume that everything is a unity feedback system so do not really worry about what is happened to the h and then I just get the k in here as well. So, let us try plotting this. So, I have m equal to 1 n equal to 3. So, the n equal to 3, so I will have 3 branches right so 2 starting from the origin here I have a pole at minus 9 and I have a 0 at minus 1 open this is not drawn to scale so this should be. And this is again the sigma and j omega just for one; these are the two roots at the origin.

So, first thing is where the three poles 1 0 the root locus will have 3 branches it has 1 0, so one of these poles will go to a 0 and two of them will go to infinity. We need to find out how these two guys go to infinity. So, first is we need to step four we need to find out what is sigma A or the centroid point of the asymptotes so that is just like minus 9 and the zeroes are here are minus of minus 1 n minus m is 2, so this will be minus 4. So, somewhere here is my central point again drawn to scale is over here.

And then what are the angles of these asymptotes? Angles of asymptotes we compute them as 2 q plus 1 times 180 degrees over n minus m that is 2, so that will again give you plus minus 90 degrees; so somewhere in the direction, vertically above and vertically downwards.

Now, other thing which we need to look at is do the roots cross the imaginary axis. So, I just write down the characteristic equation of 1 plus G of s equal to 0 and this gives me that s cubed plus 9 s square plus k times s plus k equal to 0. Now see when does this actually meet the imaginary axis, while I quickly draw the route table. So, I have s cube s square s power 1 s power 0. So, s cube I have a 1 and k I have a 9 I have a k and what comes here is 9 times k minus k by 9. So, that would be 8 by 9 times k. So, for what values of k this goes to you know 0 k equal to 0; this guy goes to 0, but if I substitute k equal to 0 over here I see that well there are no real there is no other point apart from the origin that it meets that the root locus actually meets this j omega axis. So, the roots do the roots cross the imagery axis the answer is no; it is just obvious from this table here.

Now, the breakaway points: so what do we do, we just take dk by ds we equate it to 0. I will not do is I think that should be easy to compute so will have s equal to 0 s equal to minus 3 and minus 3. So, one of course you can see here, there already a breakaway point because there is multiple root here and then we have two guys sitting here at minus 3. So, this is how it looks like.

Now, we have to decide how do these guys move from here, which direction they go do both of them go here both of them go here or they just go around no wander here for a while and come back, I know that they never go to the to the right half plane because of this Routh criterion. So, you just have to be somewhere in the left half plane. And how do they go away? Well, they go away from here at an angle plus minus 180 by r where r here is a number of the routes. So, they will be plus minus 180, so the initial moving point will be sorry will be like the tangent would be here plus minus 90 degrees. And then this these two guys go here and then they have to arrive here, arrive here so some at this point.

Look at this guy so this guy has: ok so even before that I think what we forgot is to see what point of or which part of the real line is on the root locus. So, I start from here what about the area between 0 and minus 1, I just look at my right I have two poles, but this number should be odd. So, this area is ruled out I go from minus 1 to minus 9 I just look at the right I have 1 2 and 3. So, this entire part is on the root locus: the part from minus 1 till minus 9. So, I know that these two guys will move away from each other and they will arrive at this point minus 3.

Now where does this guy go? This guy moves to the right, right because it is on the root locus and this guy also moves and it also arrives at this point because you have 2 s minus 3 coming in. Now these guys go away at plus or minus 180. And I now look at how. So, there are three roots coming in and three roots will go away. So, I just use the angle criterion I have 180 by 3 2 q plus 1. So, that will give me 60 and then 180 and then 300. So, this is 60, this is 180 and its 300 and 11 minus 60, this is 60, minus 60.

So, here the roots will arrive here this way, this way after they come here they just go away to infinity like this along the asymptotes which are given by this and minus plus minus 90 degrees. So, I just try to join the dots here. So, I will have a root locus which looks like this from here from here and so on. So, all the steps are clear right.

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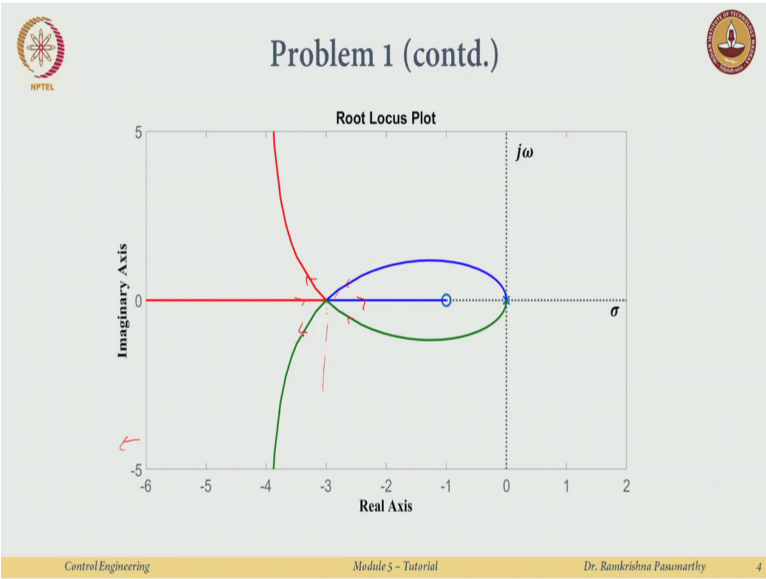

So, this is how it goes. So, I start: so this is a just do not draw a further to the left because there is nothing happening there. So, there are two roots, here there you go they meet at minus 3 which is the braking and breakaway point. So, we just would break in here and they also there. So, go away from here and then the three guys which come in 1 2 3 and then they go away here, here, and here. One pole is going to the 0 here; the remaining two poles are going to infinity.

So, that is our just is following all this all the steps as before.

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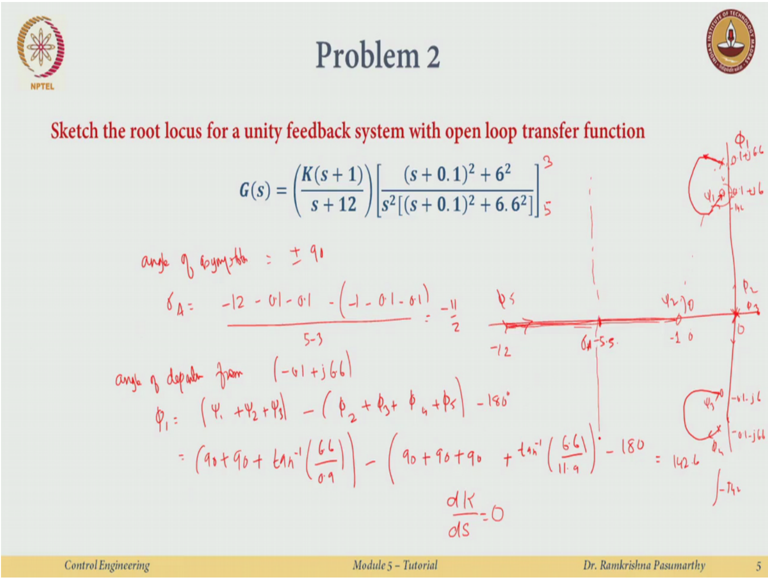

So, the second guy: so here I have 2 and 1 three zeros and I have 1, 2, 3, 4, 5 poles. So, five poles would mean 5 branches. So, let us just locate these poles and zeros. So, I have a 0 at minus 1 and then this is the 0 at 0.1 plus j 6 another 0 at minus 0.1 minus j 6. Now these two will tell me that there is a pole, again I am not drawing to scale, but just to make this a little more visible 0.1 plus j 6.6 and here is will be minus 0.1 minus 0.1 minus 0.1 minus 0.1 minus j 6.6 and somewhere here is my origin; sorry, this is my origin and this are my poles. So, the distance is 0.1.

And then there additionally there are two poles here at the origin, so we have 1, 2, 3, 4 and one guy is sitting here at minus 12; so five poles and three zeros. So, the root locus will have 5 branches 3 branches will go to the zeros: 1 0 here 1 0 here and 1 0 here. And then there would be two of them going to infinity, we will find out how they would go to infinity.

The first question we would like to ask is well which part of the real line lies on the root locus. So, I look at the region between 0 and minus 1, I do not really look at these guys because they do not really change the even or odd numbers. So, if it is even before then after them would say no if it is even before them it will be when after them and so on. So, I do not really need to count this. So, I start from the origin here between 0 and minus 1 I just see two guys to the right. So, this is not allowed. Then I come here and I say minus 1 I just three see between minus 1 and minus or I see three guys to the right so this entire region is on my root locus. So, one thing I know that this guy will move in this direction.

So, then I can just look at it and see well may be this is the pole goes to 0 so may be just goes this way right; that is a lazy way to look at it from this front from here and this one to here this one is going to infinity and these two guys I do not know just here they just do not know what to do and then you go to infinity from here that may not be true, because there are some other rules which we need to follow as well. So, I just remove this in the not about this.

So let us see the; well, you see the angle of the asymptotes. So, you could easily compute them to be plus minus 90 like two guys which are going to infinity. The centroid of the asymptotes sigma A that would be again of all the poles is minus 12 minus; I can I just take all the real parts if you remember how we derived these things minus the 0 here which is minus 1 and then the real parts of the poles 0.1 minus 0.1 everything divided by n minus m that is 5 minus 3 and this will give me a centroid point of 12 and minus 11 by 2. So, somewhere here 5.5; this is my sigma A 9 minus 5.5.

The next thing I would like to do is well the angle of departures. Departure from let me say this pole minus 0.1 plus j 6.6. So, I need to look at the angles which these two guys make here and then I need to look at the angles which these guys make. So, let me call this zeros as let me call this is psi 1 the angle made by this 0, let me call psi 2 by this 0, and then psi 3 by this 0. And then I have five pole psi let me call this phi 2, phi 3, phi 4 this is the zeros and in 4.

So, the angles of departure and I just need to compute first what is phi 1, and then phi 1 so is all these angles sides are just angle by all the zeros minus all this phi 2, because there is no angle contribution from phi 1 to itself phi 2 plus phi 3 plus phi 4 plus phi 5.

And then two just get the angle of departure what I do is I just add an angle of minus 180 degrees here.

So, let us compute each of these angles. So, I am just looking at the 0 starting from psi 1. And from here till here if I measure the angle that will just be a plus 90; so I have a 90 plus what is psi 2 here psi 3 would also be 90 and then this angle would be the angle over here. So, that is the inverse tangent of this distance that is 6.6 divided by the horizontal listen that is 0.1 or 1 minus 0.9. So, that is 0.9 or 0.1; so this 1. And then minus we have the angles from phi 2 that is very very nearby so I could as well I approximate this by 90 degrees. So, phi 2 would be 90 phi 3 would also be 90, phi 4 would also be 90, and then I have this guy that is inverse tan of from this 0 again so that is 6.6 by the horizontal distance 11.9; and all of this minus 180, so that will give me an angle of sorry; of 142.6.

So now, I know that from here I am actually going this direction. Similarly, I could also compute what is the angle of arrival here: the angle of arrival here would just again depend on well, this angle I can always have all the other contributions. And that we could easily compute again just by doing all the same things to be similarly about minus 1 minus of 142, so it is arriving from here. And then angle of minus 142 approximately x because there will be some little change because of this 0.6 number.

So, I know that this guy is not directly falling on to this directly here which is taking a little detour and coming here and doing this. Similarly this guy also will do the same right this is also should be easy to compute this and this. Now, what happens to these guys; so this guy I know goes and sits up here. So, we need to compute what are the breakaway points. So, break away points could be computed by just looking again at dk by ds is equal to 0. So, again it is a little complex expression here, but it will give us some points and let us see how the final plot looks like.

So, what we would expect is of course, there will be one breakaway point here, these guys should meet at some point and then go to infinity along as inputs given by this one.

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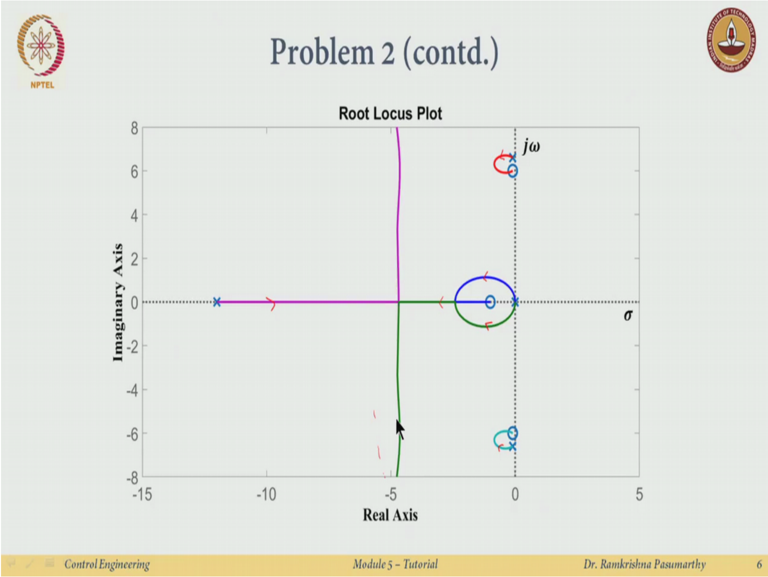

So the plot would say well there is a breakaway point here, there is a break in point here. And then let us look at each of these roots. This is easy to find out this guy goes here, this guy goes here, these two poles well this guy goes here, this guy goes here, this guy travels here and then they come here and then they just go away to infinity this is a point minus 5.5. So, I am just avoiding computing this one because it will just be a little too ugly to write down these expressions, but we could compute.

So, the idea here is to just that the complexity here is not to blindly say that this guy will just directly follow on to this guy and this guy you just directly fall out of this guy, right. So, they will actually follow a different path like this and like this and then the rest all is just a computational procedure you have to dk by ds and then equal to 0 and then you get this plot which looks like this, right. This is a little ugly here, but then we have to draw it by hand so it looks a little distorted. But this is more or less how the pot actually looks like.

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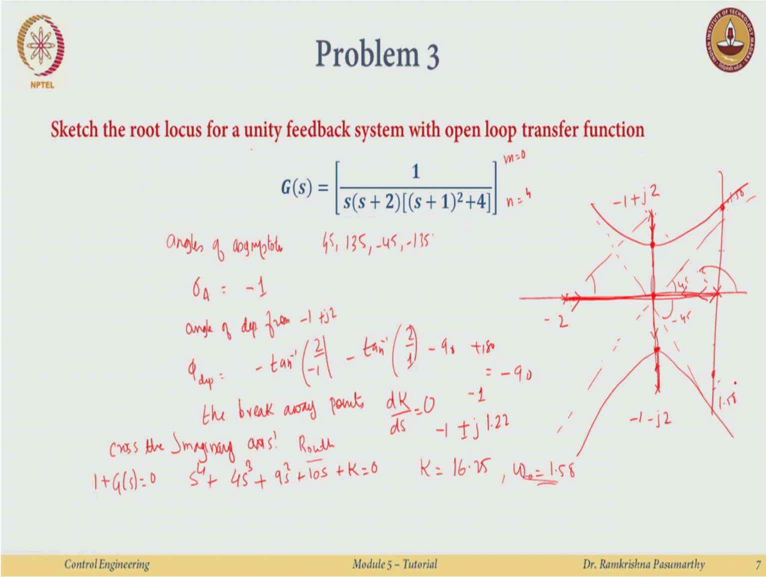

The other example: so again I have all and again a unity feedback system let us see where my poles and zeros are I start by drawing this diagram here. So, I have again a pole at origin a pole at minus 2 and then I have a pole here at minus 1 plus j 2 minus 1 minus j 2. So, m equal to 0, n equal to 4. So, which part of the root locus lies on the real axis? There is one pole at the origin so I look here from the right. So, then this entire line lies on the root locus; this entire thing. So, this guys will gives this mark as these guys will travel this direction. So, what is the next thing over here?

Then you have to compute the angles of asymptotes. And also asymptotes again make it four of them. So, will have 180 by 4 then you keep adding 45, then you have 135 and you have minus 45 and minus 135. And then you are also looking at the center of the asymptotes sigma A is you can just check that this is a minus 1; that I am skipping all these computations. So, if this is your center of the asymptotes right and then they go with ways this way and this way this is 45 this is minus 45 and so on; 135 minus 135

So let us see where this guy goes angle of departure from minus 1 plus j 2. So, what are the angles I am interested in? I am interested in this angle, I am interested in this angle, and I am interested in this angle. So, that phi angle of departure is just the negative with all our poles will just be negative 2 over minus 1 for the first guy, then you will have minus tan inverse 2 over 1, this guys at 90 degrees, and then I add 180 degrees of the formula. And I get all this will just add up to minus 90. So, this guy will just manage by itself because it is you can also feel symmetry or so I just vanish.

So, one thing I know is from here I am moving here. Similarly, I can compute here that I just go here and then I just going this way. So, if I am moving this way I just keep moving this way, and I see that well something like this happens right that the entire line between minus 1 plus j 2 and minus 1 minus j 2 is on the root locus. So, this is the angle of departure for minus 1 plus j 2 here I can compute is rewrite it is just b minus 1 from minus 1 minus j to just be plus 90.

So, there is something interesting happening right. So, everybody is meeting here, well that is what it seems like right and then what do they do after they meet here. Well, now let me compute the breakaway points again by setting dk by ds equal to 0. So, this means I will have of course, one breakaway point at minus 1 and I will also have breakaway points at minus 1 plus minus j 1.22. So, this is 2, this is 1, so I am having breakaway points it will here; something more interesting happening. This is 2, this is 1 here and I am angled 1.22. So, guys come here they even go away. So, this guy comes here and then maybe then it says it seems that this guy will go and these two poles will meet here at this guy goes here from here till here and they meet here this guy goes from here till here and then this guy they meet here. So, something strange is expected to happen here.

Now, look at even the asymptotes right: they look as if this might actually come here and then cross the imaginary axis. So, does the root locus cross the imaginary axis? Well, I just look at the characteristic equation and then I am looking at the Routh criterion. So, my characters equation turns out to be 4 ok. So, just computing quietus equation again from 1 plus G times s equal to 0 I have a characteristic equation which looks like this s 4 plus 4 s cube plus 9 s square plus 10 s plus k equal to 0.

And I do again the Routh table; and what it gives me is that, well there is again k at a number 16.25 I will skip all the details. By now you should be experts in computing these guys. And also at an omega 0 of 1.58. So, they cross the imaginary axis at this point z is running approximately again not to scale this is 1.5 a to minus 1 and this is 1.58.

So, now I know that it seems two of the roots will cross the imaginary axis. So, let us again try to analyze what is this; this guy is going here and well it is a breakaway point.

So, he is just waiting for somebody else to come and meet him here so that he could just say hello and go by. Similarly this guy, this guy starts here there is again a breakaway point. So, these are complex breakaway points. So, these guys come here and this guy also comes here they meet and I say well we have to go away because it is a breakaway point. So, this guy comes here, he goes and then they meet here. After this well they decide where should they go they should go asymptotically along this line right; this is this very this guy will go, this guy will go this way right. Again at angles 45 minus 45 and so on.

So, to the asymptotes, again this will be like this and this will be like this; sorry just messing up the scale here, but the point where it meets the imaginary axis is at this omega naught which we computed from the Routh table this is 1.58.

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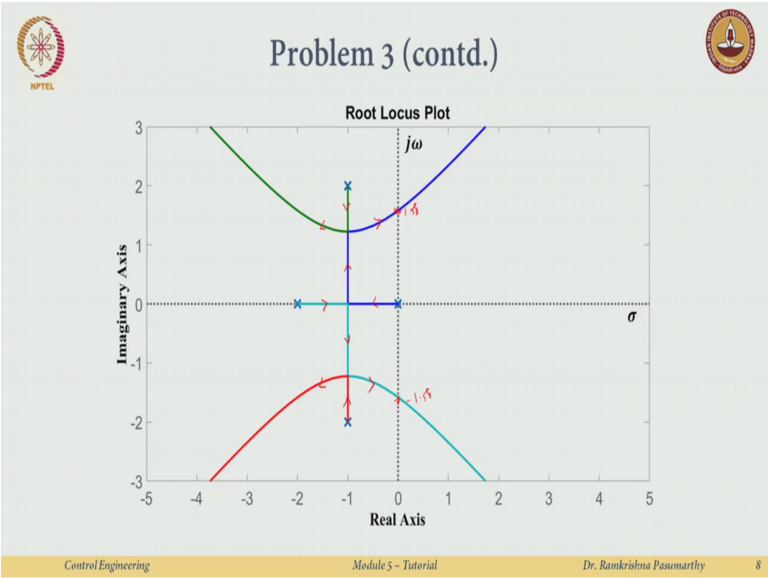

So, I will just show you how the plot-turn out looks likes. So, this guy comes here goes here, this guy goes over here, this guy goes over here, this is this point is 1.58, this point is minus 1.58, again this guy here, here, this, this, and this. So, just we just using all those steps to just see the entire plot and then see this intersection with the imaginary axis. So, I know that after some point if I keep the increasing the gain the system might actually go unstable it is not mind it; it will actually go unstable after my gain increases to beyond 16.25.

You could just do this on MATLAB; I will shortly post a little tutorial for yourself to see how to use MATLAB to get these basic plots. Other plots like the step response how to give a command for the transfer function, find the poles, and zeroes and so on. So, I will just post a little little a tutorial on that. I will not do it in the class over here, but that tutorial would be self-sufficient for you to learn by yourself.

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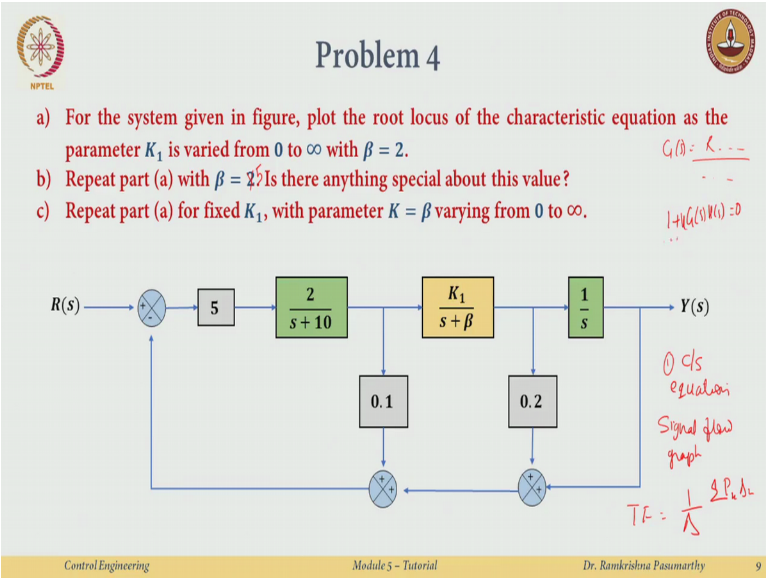

So, far things were nice. If I give you a block diagram which looks like this and I say for the system given plot the root locus as this guy K 1 is varied from 0 to infinity with this parameter beta being equal to 2, and then this should be 5 here; repeat with beta equal to 5. Not only that then I get keep K 1 fixed and then I vary beta from 0 to infinity. So, how do I handle this? So far everything was given to me that a good G of s was k in the numerator some zeroes here and then denominator. So, how will I do this?

So, first is I need to get the characteristic equation. Now given this guy do I know anything which I learned in the past; that would directly give me the characteristic equation. So for that I will do the signal flow graph analysis, the transfer function had 1 over delta plus I think summation p k delta k or something that. I do not really need to do the entire transfer function; what I just need to know is what is in the denominator of the transfer function, the characteristic equation what was it 1 plus G s H s if I had a k outside this was 0 this is just a denominator of the closed loop transfer function.

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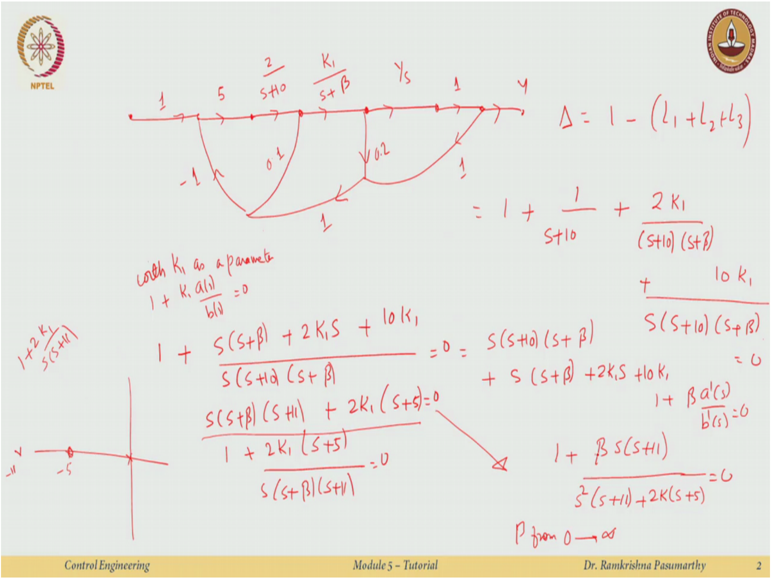

So, first let us draw the signal flow graph of this. So, the signal flow graph would look something like this. So, 1, 5 and 2 over s plus 10 then I had K 1 over s plus lambda and then I had 1 over s beta 1 and a y. So, there are five 2 over s plus 10 K 1 over s plus beta; sorry I called it a lambda here. So, let us erase that and put a beta. So, what happens after all this? So, I have after this guy I have a loop of minus 1, from here till here I have a gain of 0.1, this is plus or minus it is plus. Then later I had s plus s over beta, I had a 0.2 here, and here I have 1. And here I have one and now these are all the directions of the arrows.

So, I am just interested in finding the overall delta. So not really the transfer functions, because I know that I am only interested in the denominator which I call the characteristic equation. Why is that? Because, I am just interested in half my poles or how the poles of my close loop system vary as I change a parameter from 0 to infinity- which is K 1 in this case.

So, the delta here is 1 minus, well you look at all the individual loops there are three loops here right: one loop this arrow is here, one loop is this, one second loop is this one, and the third loop is the outermost loop. So, the delta turns out to be 1 plus; I just look at this 1 so I have 1 over s plus 10 plus look at the other guys one. So, I have 10 times K 1 over 0.2 sorry. So, 2 times K 1 over s plus 10 s plus beta and this take the five and point to s becomes 1 so that this goes away from here.

And in the last term: so I go the entire since here I have 5 and 2 10 times K 1 over again. So, I have s in the denominator s s plus 10 s plus beta. So, this is this equal to 0 is my characteristic equation. So, if I want to draw the root locus in terms as with K 1 as a parameter I need to get the equations in the form 1 plus K 1 times something in the numerator polynomial a of s something in the denominator polynomial b of s equal to 0. So, it should look something like this.

Now can I rewrite this equation that way I just do some do some manipulations and what I get is. So, first this will become 1 plus s s plus beta plus twice K 1 times s plus 10 K 1 over the entire denominator s s plus 10 s plus theta is 0; which I could also write as this is equal to s s plus 10 s plus beta plus s. Again say that the s plus beta plus twice K 1 s plus 10 K 1. So, this will further simplify in to s s plus beta s plus 11 plus twice K 1 s plus 5 equal to 0, which I could also equivalently write as 1 plus twice K 1 s plus 5 over s s plus beta s plus level equal to 0.

Now I have, well I can just do this right so I have assuming beta is always greater than 0 I have a 0 at minus 5, I have a pole here, and a pole at minus 11. And then depending on wherever the beta is I will have the location of the other pole I just do the simple one for myself so beta equal to 5. So, I will have a pole also here. So, this guy will just go this side will just be nothing here. So, I will just be left to the transfer function which looks like 1 plus twice K 1, this guy will cancel out s s plus 11. And I see know how to draw the root locus for this right I am not going to do in to the details of that.

Now, when k is fixed and lambda is varying; I just again look at this entire expression and now I want to write it in the terms of a transfer function of where 1, I have 1 plus lambda some other polynomial a prime is divided by some other polynomial b prime s. Again I can write it down in a fairly straightforward way again, so I just start from here from here. So, I just skip a bit of the computation, but I will just s s plus 1 over s square s plus 11 plus 2 k where k is a fixed value now according to the problem statement well you can just say any just K 1 equal to 1. And now I can look at what happens to the root locus when now lambda varies from 0 to infinity keeping k in to fixed, this being keeping k fixed. So, the idea is to just write down this characteristic equation in this form 1 plus lambda a prime by b prime being equal to 0. I will skip this steps because now again the looking at the steps of the root locus should be kind of very straightforward now. So, those steps sorry its it a beta; beta n beta

We now see how to draw the root locus with beta as a parameter or in other words I want to write the characteristic equation as 1 plus beta a prime s over b prime is equal to 0. So, I just go from here, I just skip the computations here so I just have 1 plus beta s s plus 11 over s square s plus 11 plus 2 k s plus 5 equal to 0. So, for a fixed k I know what will be the poles here I know what are the open loop zeros and I could just say as beta going from 0 to infinity. I will skip the steps because this are just the routine step.

Just to give you an idea of how to formulate a problem in a way that I could use the root locus techniques directly. It was not very obvious from here because so far all the problems are very nice that I was just given G and H and the K and I just had to do. If it is a little complicated there is nothing ready to worry I can just look at my signal flow graphs and then just compute the delta, which is a denominator I do not even really need to look at what the numerator is like because I am just interested in the characteristic equation which is essentially the denominator; and then the steps will follow.

So I just stop here, and I will leave these things for you as an exercise to do.

Thank you.