Control Engineering
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Module - 05
Lecture - 04
Root Locus Plots

So, continuing again on the root locus thing. So, we just left with the couple of rules.

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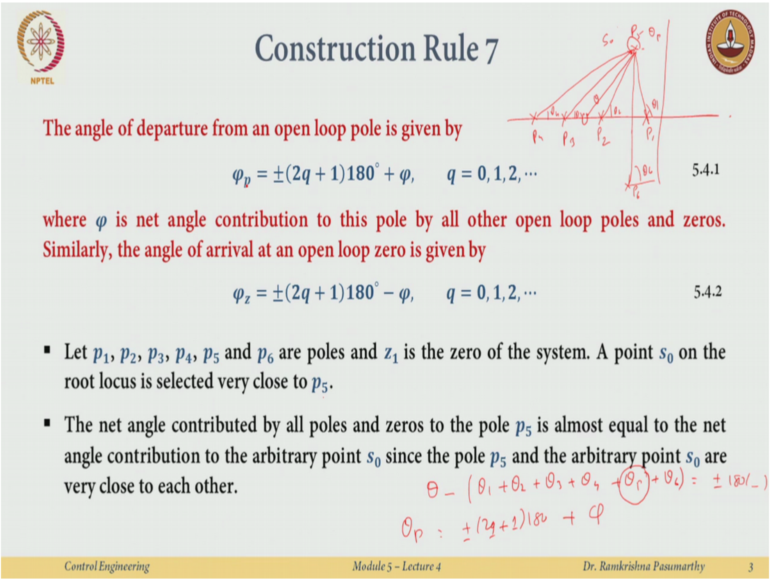

So, the first one was angle of departure from an open loop pole, and why is this important to compute. So, in one of our earlier examples, where we had a pole at origin then we had something at j 2 omega minus j 2 omega, one guy was going to infinity through the dual line and then we are asymptotes this way and these guys for going to like this way right like this way infinity this guy was going like this to infinity.

So, who decides right of how this guy goes why does it take a steeper angle and go like this, or why does not it go you know wander here and then I can come back and say oh this guys here I am not in this. So, we say who decides or what is there any anything that will tell us which direction I should be moving right. So, this is decided by something called the angle of departure or which direction I should go. So, the angle of departure the formula says is the following it phi of p from an open loop pole. Let say as I am just looking at one pole under consideration is again it should satisfy the angle criterion right.

So, all the time we are just looking at this one at 1 plus K times G s times H of s is 0. So, all this criterion all we do is just try to satisfy this criteria right not nothing else or just that you know the magnitude criterion and the angle criterion as simple as that. So, the angle of departure is given by again this angle criterion. So, little modification of that plus phi what is what is phi? Say I am interested in some pole and this phi is the net angle contribution to this pole by all other open loop poles and zeros.

Now this is for pole right I go away from a pole and then I end up at a 0. So, I arrive at a 0. So, which direction to wherever I arrival on the axis or do I just do some circular behavior whatever how do I arrive that is also important right. So, the angle of arrival is again defined similarly. So, let us try to understand what this means and say draw a little picture here. So, let us say I have 6 poles and one 0, and I am interested of you know how does p 5 move. So, let us get a little scenario here is this. So, this is my plane and say well I have what we call this p 1, p 2, p 3, p 4 and say I have a p 5 and p 6 b here right p 1, p 2, p 3, p 4, p 5 and p 6 and then there is a 0.

So, let us say that the 0 is sitting somewhere here. Now I want to see which direction this guy should start moving should it start moving vertically downwards or somewhere here or somewhere here for any arbitrary scenario, and let us say I just take a point very close by here this one and I call that point as s naught. Now for this s naught to lie on the root locus it must for sure satisfy the angle criterion now and this so, which this very close by right. So, let us see. So, I just draw this angles this will be theta 4. So, theta 3, I have theta 2, I have theta 1 and the angle here would be this guy right; so some drawing this line from here. So, that would be some theta 6, and then say angle of this of the 0, I just call this theta.

And then this guy is somewhere here right. So, the angle here would be. So, this guy is actually moving in this direction right this is the angle of theta p, the pole which I am interested in. So, here the phi s and thetas, but does not matter right you can just use whatever. So, for s 0 to lie on the pole what should happen? At all the sum of all this angle should again be multiples of 180 degrees. So, what is the how is angle criteria look like the angle of the 0 is just theta, and then I have angle of all the poles that is theta 1, theta 2, theta 3, theta 4, I have this little theta p, and theta 6, this again plus minus 180 and 2 q plus 1 blah blah blah.

Now I am interested in this angle right this guy this rigid angle do not raise to tell me where I will go. Therefore, I just you know substitute this and this theta p or even phi p whatever is simply plus minus 2 q plus 1 180, plus all the other demeaning angles they will just go here and I just call the sum from theta prime. So, where this guy or this theta prime here or I just call this phi just would be consistent with what is the notation over there, for all the others you guys. So, this phi is the net down angle contribution to this pole by all other open loop poles and zeros. So that is how I compute this theta p.

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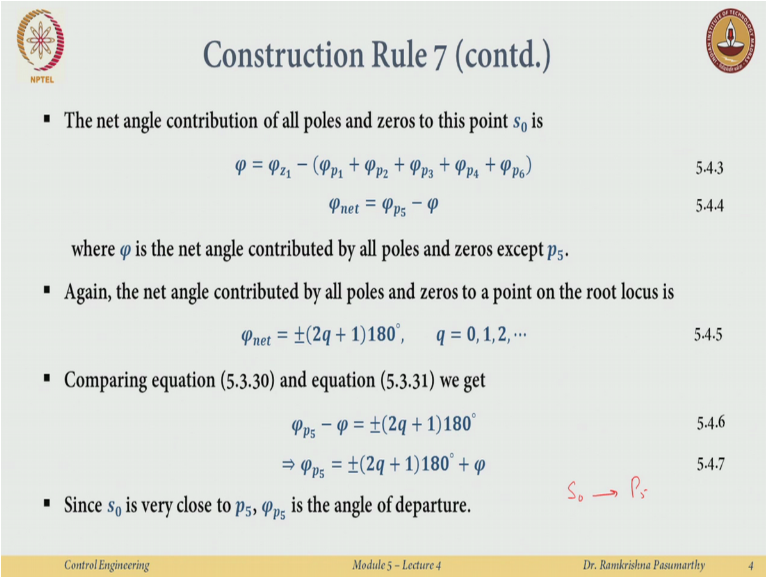

So, this is this how it would look like. So, this phi is the angle of the zeros minus all the angle of the poles and this is our guy get it, and it is a kind of straightforward to look at it right and again because you know s 0 is very close to the departure point or the pole this will be a departure point and as S naught tends to the pole P 5 well that particular angle contribution will come 0 and then you know this and then the other poles or the angle contribution by the other pull would satisfy the angle criterion.

Where I am not moving I am just at that pole and this is the limiting condition as s naught goes to s 5, but s naught being very close to s 5 how do I compute the angle? Well, that is just by this 5 wrote here this guy similarly I could even do for a 0 right its exactly the same procedure what I would do for a 0 take this 0 and you can just compute take a point just to the left or right or in a top or bottom of the 0 and then the angle could be computed kind of quite easily.

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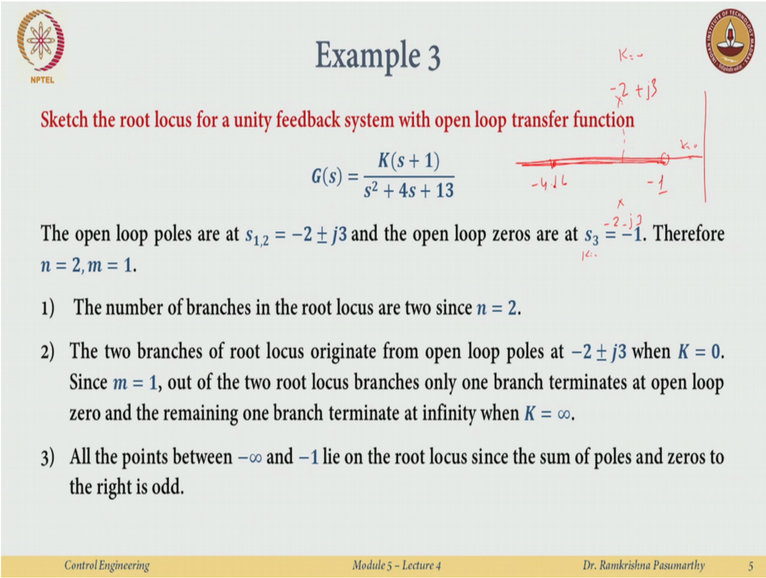

So, we do an example its case the root locus or a unity feedback system with open loop transfer function again. We have complex conjugate poles open loop this is 0 at minus 1, n equal to 2, m equal to 1 number of branches is 2 1 and there is a 1 0. So, one pole will go to 0 and other will go to infinity and the 2 branches originate from here when K equal to 0, and m equal to 1, one of the branches will go to this guy another will go to infinity. So, let us see how the plot looks like.

So, in this broad this is a rough sketch of it before. So, this is a good space here. So, I have a 0 at minus 1 and my poles are here, minus 2 plus j 3 and minus 2 minus j 3, a pole here and a pole here. So, I just want to see you know which direction this move, but before that will be do the first rules right which point is on the real axis I know that when I start from K equal to 0 and here K equal to 0 I am here, K equal to 0 I am here now just look at this is nothing here. So, this point would not this region would not be on the root locus.

There you can here then look at the point here right. So, from here I go to the left of minus 1 there is one 0 odd number, then I go again here there is again odd number one 2 and 3 sometimes we can even neglect this complex pose for counting because they always are in conjugate pairs. So, there is even before this guy they will always be an even after this guy if it is an odd there will be naught. So, the at least I know that the entire this guy is on the real axis you. So, sorry is on the on the root locus correct I will not even draw the arrow at the moment is to be a little safe. So, this entire line is on the root locus starting from the left of minus 1.

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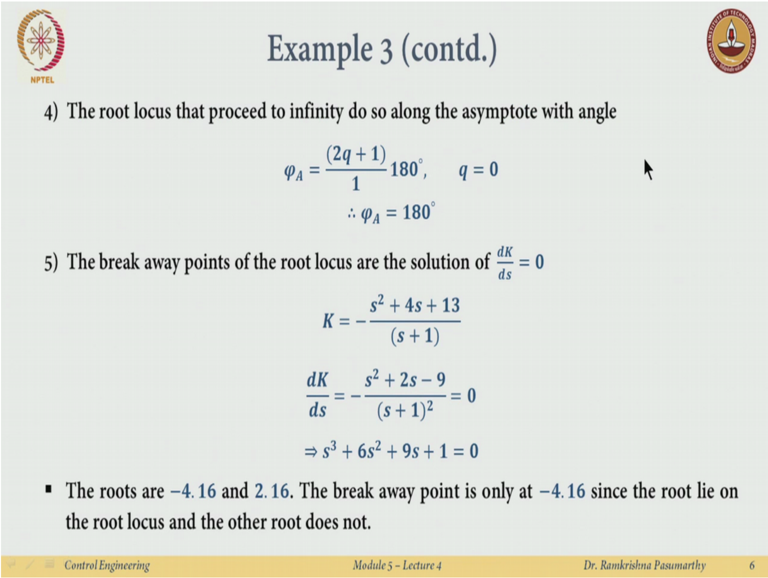

So, again I do all the basic computations that well there is one guy which goes to infinity what is the angle which that guy goes to infinity. Well, that angle is 180 degrees and is just very straightforward come n minus m is 1. So, this phi will be 180, well and these guys will do they really meet at some point and then go away I can just find by d K by d s equal to 0 and the solutions would be now this one minus 4 and 2. I do not really I am not really interested in this guy right this is on this point does not really lie on the root locus I am not ignoring this because it is an unstable region at I am just ignoring because the root locus still tells me that I just cannot go here, because I cannot be on any of this line because there are no poles or zeros to the right of this. So, I am looking at minus 4.16. So, this is this real part it is minus 2 and say this is this point is minus 4.16.

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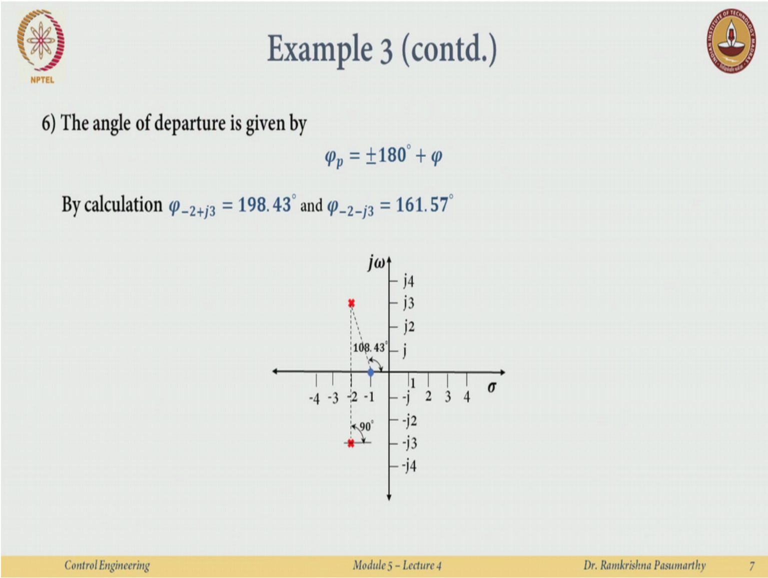

So, I just go now to compute the angle of departure, departure from this side and departure from this side right and then this formula phi of p if I am looking at this guy would be plus minus 180 plus the contribution of the other 2 guys, and if I am computing phi for this guy it would be this phi would be the angle contribution of these other 2 guys.

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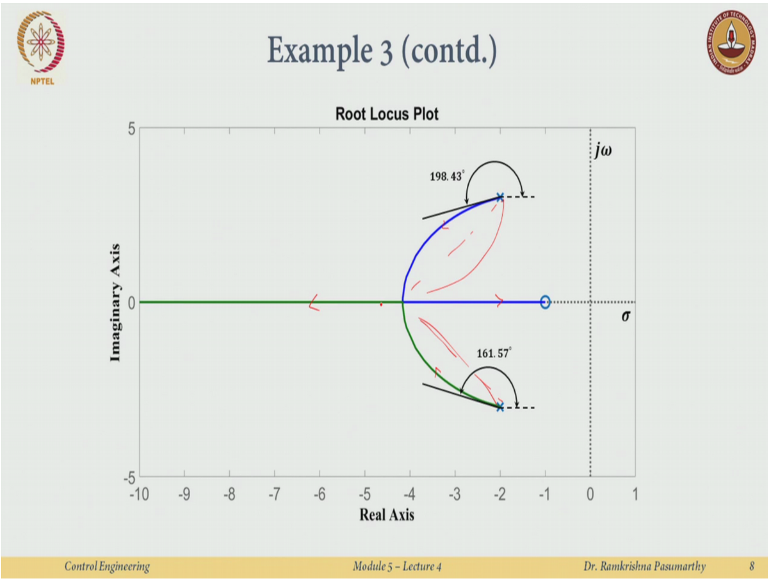

I can just compute this is kind of very straightforward to check that phi for this guy would be 198 this way. And phi for this guy minus 2 minus j 3 would be 165. So, I just know that I am just from here sorry I just move in this direction, here I just move in this direction I also know where I am meeting right that minus 4.16.

So, the locus will start from here, then will come here, then they come here and therefore, the directions would something maybe something like this. This guy moves here, this guy moves here, this guy goes here and this guy goes here right. So, now, I know why is why is it this way why is it not like something like this or any other thing why is it not like this right. So, this I just this angle tells me why how it behaves.

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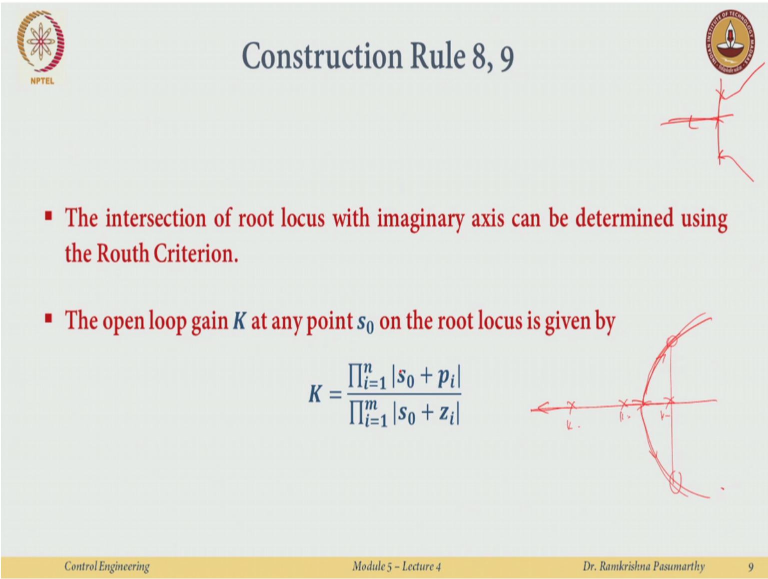

So, this is something not very surprising this rule, how do I know if my root locus intersects with the imaginary axis well when I hit the imaginary axis say I have a root locus which is like this let us say I have 3 guys here and I say well this guy is just like an approximate plot right. So, this guy there could be something like this right there could be possibilities of this thing and these guys are going here.

So, these are all K equal to 0, K equal to 0, K equal to 0, and I see that after some values you are actually going this side, they are going in the unstable region. Can I really compute this value when it becomes unstable, there is another example where my poles were here, here and here and then the root locus one guy was going this side and other rather 2 guys went here. So, for any value of K greater than 0 it was always unstable right that was not difficult to compute now how do I handle these conditions.

Well, I know something all the Routh Hurwitz criterion right, the intersection of the root locus with the imaginary axis can easily be determined by the Routh criterion just to see how can. So, we had done some problems where we were computing stability with respect to a parameter. If K was between this number and that number system is stable of if K is greater than this number system is stable and so on. So, and then the open loop gain K at any point on the root locus is just given by this, I think this we had even discussed in one of our very earlier slides what given a point K how do I find the sorry given a point on the root locus how do I find the gain K at that point well this is just simply like this is nothing special here.

Of course, now if you go do and mod matlab you just move the cursor it will actually tell you the values, but where does where does it come from that comes from something here.

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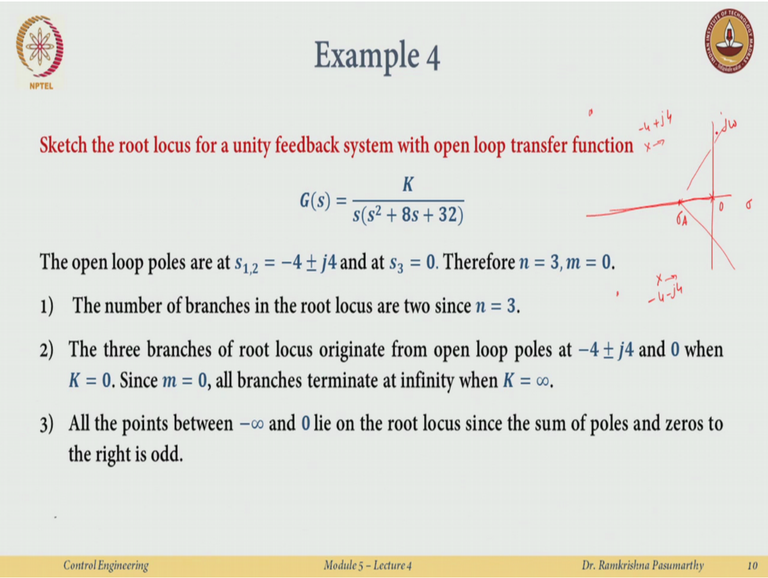

So, just to do another example: so we have G of s is K I have a pole at the origin and 2 other poles which are complex conjugate minus 4 plus j 4, and another pole at the origin. So, I start from here. So, I have a pole at the origin and I have 2 poles here. So, this is minus 4 plus j 4, minus 4 minus j 4 and this is the origin in the sigma and j omega axis. So, there are 3 branches all of 3 all of these things will go to infinity, because there are no zeros; so points of the real line the root locus. So, this is fairly obvious because at any point if I go to the left of 0, I look at the right there always be odd number of poles. So, one until I reach here and after that there will be 3.

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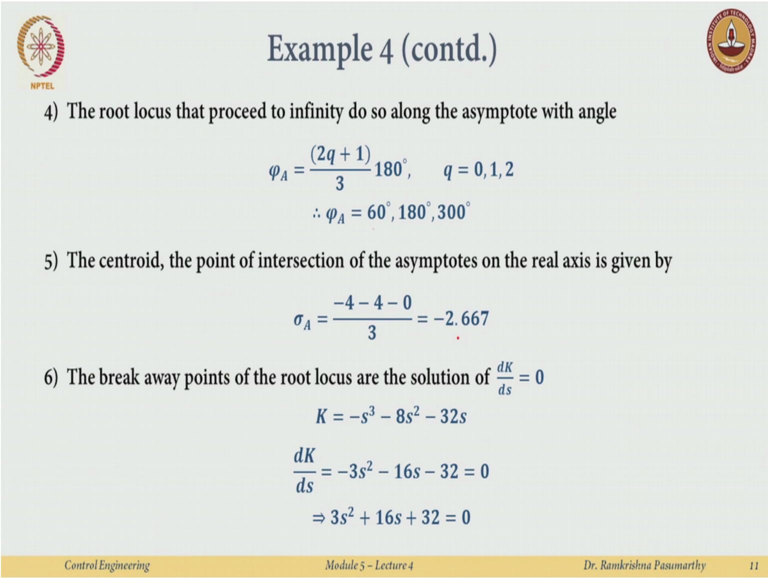

In the usual process of finding how the root locus or how the roots proceed to infinity they will be at angles of asymptotes at 60, 180 and 300 with the center formula and then the centroid point of these asymptotes would be minus 2.66: so somewhere over here. So, this is my sigma A. So, the breakaway points where I just look at d K by d s equal to 0 I find k.

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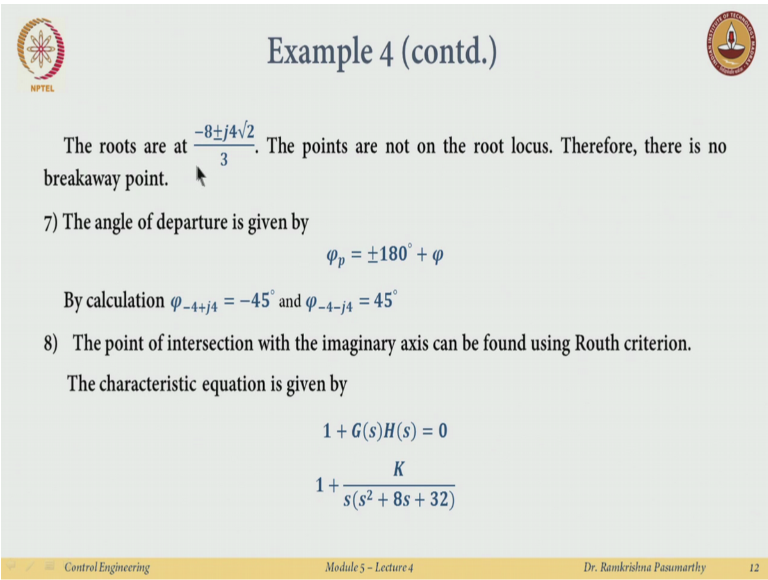

And then solve for d K by d s equal to 0 which gives me the breakaway roots at this point. And I could actually see from here that these points are very unlikely to be on the root locus right. So, look at this minus 8 plus minus j. So, I am this somewhere here somewhere here right there what do my asymptotes suggest is that well I am at 60, 180 and 300. So, which is at an angles like over here; so this is where.

So, I it shows that I should actually be moving towards the asymptotes somewhere in this direction somewhere in this direction right. So, there is no way that I could reach these points based on the location of the poles the centroid point and the asymptotes. So, for sure that even though d K by d s equal to 0 gives me a solution these points will not lie on the root locus you would also test this by the angle and the magnitude criterion. Now I am looking at the angle of departure at these guys. So, angle of departure would be computed by you know so, this angle and this angle.

So, that should be straightforward to compute. So, this angle would be the angle at minus 4 plus j 4 would be 45, and minus 4 and say 4 would be plus 45. Now since these are moving in this direction and the asymptotes are towards the right of the imaginary axis, there could be chances of these guys meeting the imaginary axis. So, let us see if that is true. So, I write down the characteristic equation and I have 1 plus K s s square plus 8 s plus 32 and I draw the Routh table.

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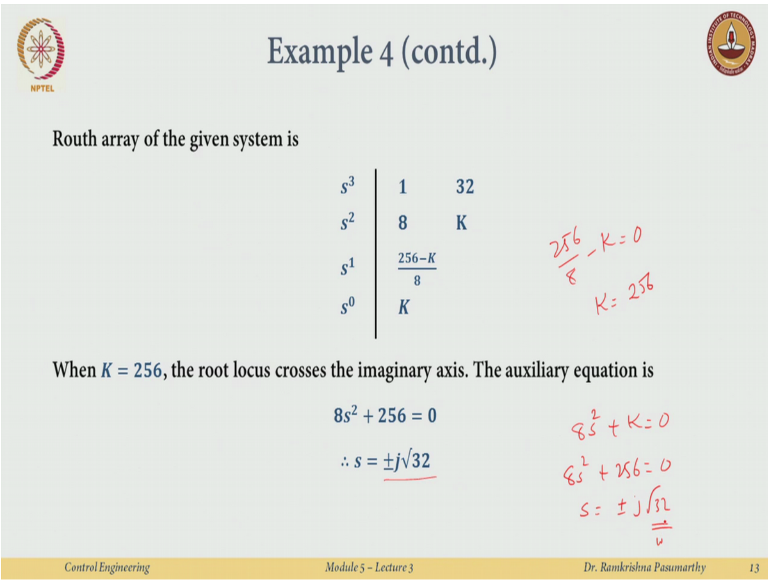

So, 132 8 K this would be 8 minus 8, 8 times 32 minus K by 8. So, what happens when say what does this guy go to 0 this guy will go to 0 when 256 by 8 minus K equal to 0 or for value of gain K equal to 256, I have this entire row equal to 0. Now let us see how the auxiliary equation looks like I have 8 s square plus K equal to 0 or 8 s square plus 256 equal to 0 which gives me s equal to plus minus j square root of 32 and this is the value of omega, where it will cross the imaginary axis.

So, this will be at somewhere here square root of 32 in a positive direction, square root of 32 in the negative direction right. So, this gives me a lot of information of that there are no breakaway points the root locus will actually cross the imaginary axis and it would actually do it this way.

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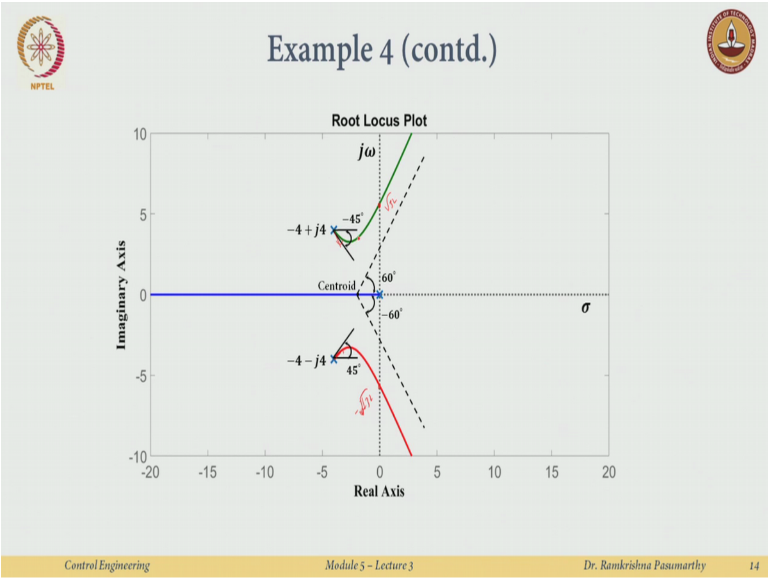

So, the angle of departure was minus 45. So, it will initially tend to move this way and this is square root of 32 in the j axis and then it will just asymptotes this way.

Similarly, this guy has angle of departure of plus 45. So, it will move this direction and it will cross the imaginary axis at square root of minus 30 sorry minus of square root of 32 and I will just do this one right. So, we saw few possibilities here right that there are no breakaway points even though d K by d s gives me a solution, second is to explicitly compute the angle of departure right and also to see exactly where the root locus meets my imaginary axis and this way the Routh table was very useful for me.

So, if I now want to have a problem where I said: well, what are the values of gain K for which the system is stable, I will say well for values of gain K between 0 and 256 my system is stable and for all other values it might go to the unstable region. So, I have a bit better understanding now of the relative stability in terms of the gain k.

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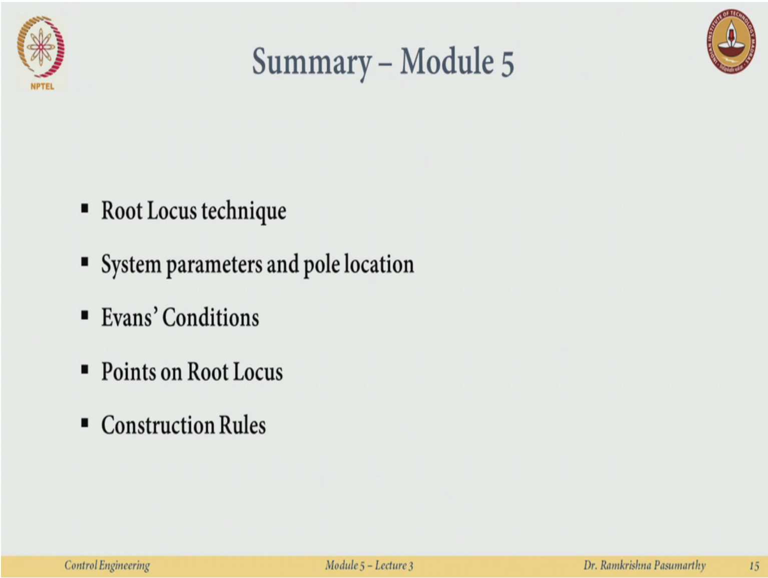

So, that is this about it and this guy is this guy is kind of straight forwarded you just keep on moving to the left until it reaches infinity right very straightforward route for this guy. To summarize module 5, we have learned construction route for the root locus techniques, we also knew know does the system go to instability.

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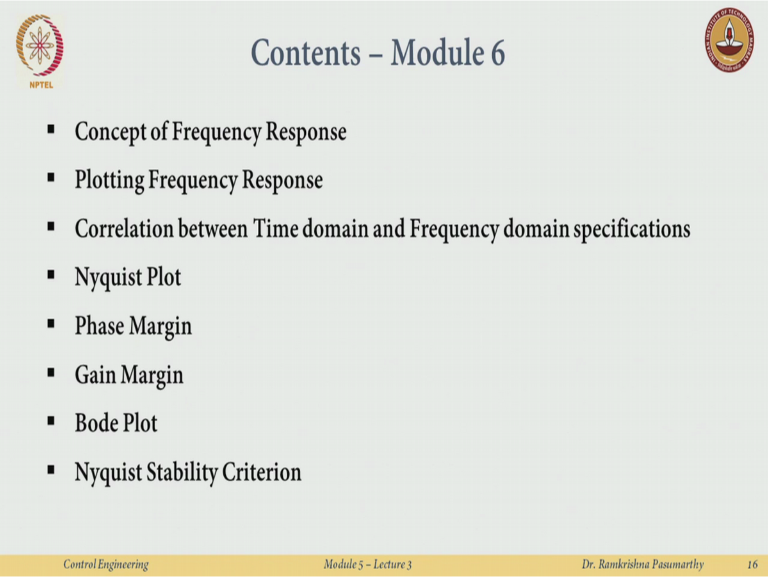

And next when we meet we will do lots of things on the frequency response of the system. Now we were interested in what happens when time goes from 0 to infinity or a gain goes from 0 to infinity. We will see what happens with changing frequency, so that will do in the in the next module.

Thank you.