

Control Engineering
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Module – 01
Lecture – 02
Modelling of systems

In this module we will talk about Modelling of Systems, and various aspects of modeling, what are the things we need to be careful of while modelling a system.

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What is a Model?

- An elemental or mathematical representation of a plant or system.
- Model helps in the analysis (input-output) of the system.
- Captures the dynamics of a system.
- Dynamics refers to evolution of system variables.
 - The room temperature when an AC is switched ON
 - The speed of car when accelerator is pushed by certain angle
 - The evolution of current flowing through an inductor when a voltage is applied

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graph LR; A[Plant or System] <--> B[Equivalent Mathematical Model]
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Module 1: Lecture 2

First, what is a model? By definition we could say that it is a elemental or even mathematical representation of a system. And this model which we derive will help in analyze the system. And more in control terms it would help us more analyze the input output behaviours of the system. This model which should be such that it captures the dynamics of the system; what are these dynamics? This refers to the evolution of system variables. For example, how the temperature behaves or how fast or how slow it goes to the desired temperature as soon as AC is switched on? How is the speed of my car changes before it reached or reaches the desired speed?

Similarly, if I look at an electrical circuit how a current or a voltage evolves. So, if I look at a model in terms of very basic, why do I need a model of a system? And modeling is something which we do in our everyday lives. For example, if I go to an interview and I

am being interviewed for the post of a control engineer how would I judge; if that person is fit enough for my job or not, I would essentially ask him a set of questions. First I would look at his background, his education qualifications, experience in some sense, and then I would question him with some of the basics of control possibly.

And over there it does not really make sense for me to ask who is your favourite football player or which IPL team do you support, because that answer would not help me judge if he is a control engineer or not or a good control engineer or not. And the model essentially is I will ask him a set of questions and based on his responses I should be able to judge if he his fit enough for the job or not. That is an essential part of modeling.

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The slide is titled "Types of Mathematical Models" and is presented in a light blue background with green and white rounded rectangular boxes. It lists three types of models with their characteristics:

- Differential equation model**
 - Dynamics of the system represented in terms of differential equations
 - Time domain representation of the system
- Transfer function model**
 - Dynamics represented in terms of a Laplace transform expression
 - Frequency domain representation of the system
- State space model**
 - State is a set of variables that describes the system behaviour in conjunction with the system inputs
 - Dynamics are represented by a set of first order differential equations using these state variables

Logos for NPTEL and a university are visible in the corners. The text "Module 1: Lecture 2" is at the bottom right.

So, in our course what we will look at is a couple of types of mathematical models. A basic model which be a differential equation model; this the where the dynamics would be represented in terms of some first order, second order differential equation sometimes even higher order differential equations. And this is essentially or usually also called as a time domain representation of the systems.

We also know that we could transform these differential equations via Laplace transforms to linear equations. And can derive also the transfer function models. And this also helps; this Laplace transform models also help us analyze the frequency domain response of the system or the frequency response of the system.

Lastly which we would like to investigate or something which helps us understand the system behaviour better in the control point of you or this state space models. What is the state? State is a set of variables which describes how a particular quantity in the system varies and these dynamics are represented as some set of finite set of linear differential equations. How do this look like.

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The slide, titled "Types of Mathematical Models", features an RLC circuit diagram at the top. The circuit consists of a voltage source V , a resistor R with voltage V_R , an inductor L with voltage V_L , and a capacitor C with voltage V_C . The current is denoted by I . Below the circuit, three mathematical models are presented in green boxes, with red arrows pointing from each model to the circuit diagram:

- Differential equation model:** $\frac{dV}{dt} = R \frac{dI}{dt} + L \frac{d^2I}{dt^2} + \frac{I}{C}$
- Transfer function model:** $\frac{I(s)}{V(s)} = \frac{1}{R + Ls + \frac{1}{Cs}}$
- State space model:** $\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V$

Background text on the slide includes: "Dynamics represented in terms of a second order differential equation", "Frequency domain representation of the system", and "State is a set of variables that describe the system behaviour in conjunction with its inputs". The slide is from NPTEL, Module 1: Lecture 2.

Let us take a take model or take an example of a very simple linear RLC circuit. So, if I just asked you to write down the Kirchhoff law what you will tell me is that dv by dt is di by dt and so on. I could do the Laplace transform of this and I can write it as a input output representation as- a voltage being the input gives me a certain current as an output and which is given the ratio of the output to the input is given by something called as a transfer function.

This is just rewriting my previous equations. The thing I could also do is to write down. So, this is a second order differential equation, and I could write it as a set of two first order differential equations which looks like this as a bit of matrix structure and this is what we will call as a state space model. And we will eventually learn each of this in detail and see which model is fit for word, which model gives us what kind of information and so on.

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Modelling a System

- Two methods of modelling systems:
 - Analytical modelling :**
 - Involves systematic application of basic physical laws to system components and their interconnections
 - Combination of physical modelling and mathematical modelling
 - Experimental modelling :**
 - Selection of mathematical relations which best fit the observed input-output data of a system
 - Also called modelling by synthesis

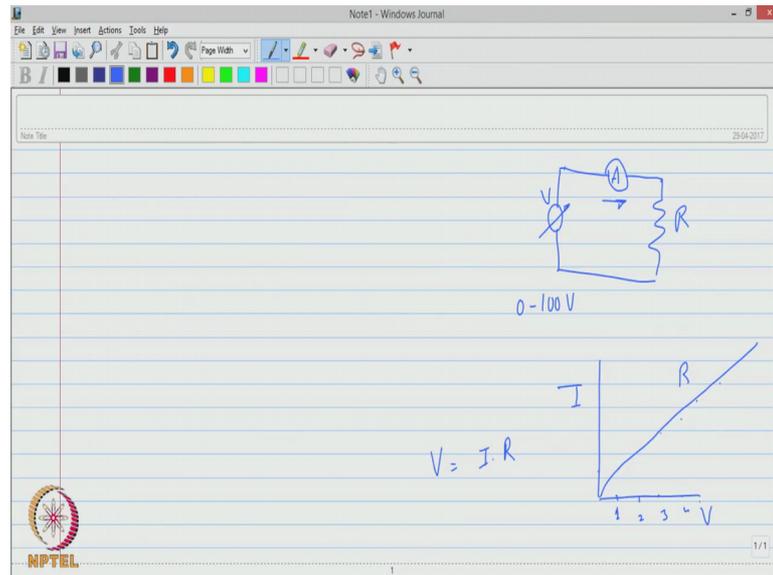


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The measures of modelling could be classified as either analytical modelling or experimental modeling. In analytical modelling we look at systems as application of some kind of basic laws; basic physical laws which are used to model the systems. And their interconnections and then we combine all these physical systems to evolve at some or to get to some mathematical model of the system.

Experimental modelling could be selection of a mathematical relation which best describes the observed input output data of a system. This also called modelling by synthesis. This is something which we would have done already in one of our electrical engineering labs.

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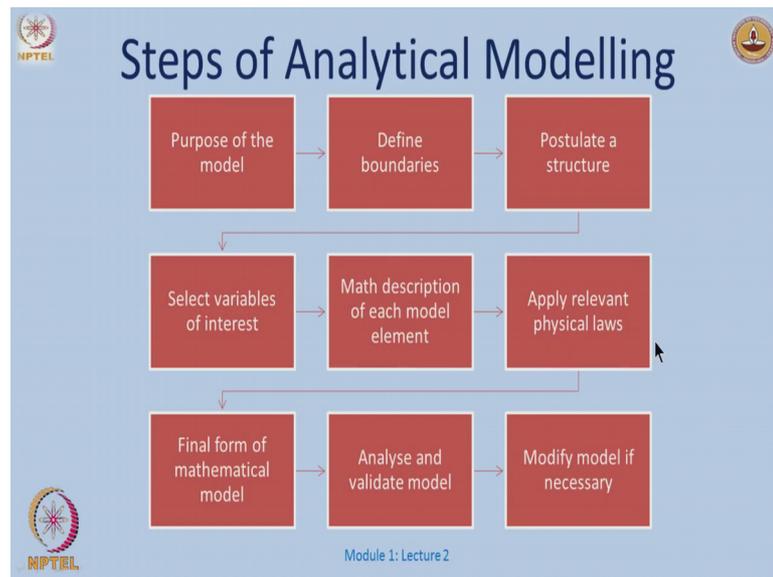


So, let say I want to I am given a resistor and I am given a variable voltage source; say which could vary from say 0 to 100 volts. And I would ask you a question; can you derive a model of this? So, what would I do? I would connect this, I would put current measuring a metre here, call the ammeter, I know what is the input voltage. So, I start from say applying 1 volts I analyze the behaviour here it gives me a point here, I do the repeat experiments for 2 volts it give it could be me a point here, I repeat it for a 3 volt could give me a point here, 4 volts and I keep on doing this when I generate a series of observations.

And I just see if this is a linear resistor that this is R which defines the relationship between V and I and based on this model or this experiments I could conclude that V is I times R . That what this is graph tells me and this is something which we derive in our basic circuit experiments, to when you observe the behavior of how the current changes through a resistor when we apply a certain voltage.

So, this is also one of the ways which we could model a resistor.

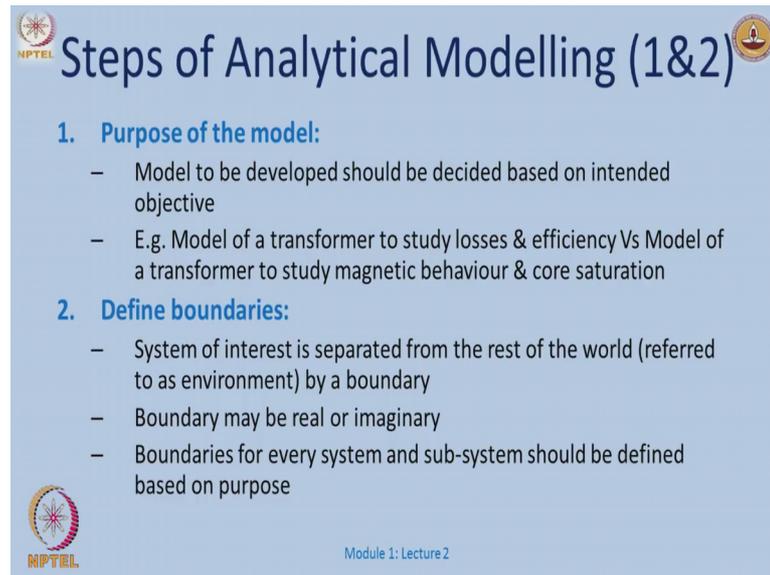
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If I would look at analytical model what could be the steps. First is, I should define for myself what is the purpose of the model. Like as I said the about the example when I am interviewing a candidate. Then I should define what are the boundaries: if I interviewing him for a control engineering post I should not go beyond what is the job specifications. Then I postulate a structure: structure is what we will talk about shortly.

Select the variables of interest; and look at the mathematical description of each of those elements applied relevant physical laws if any. Get some kind of a final form of mathematical model. Why did that to see if the behaviour is correct or not, and modify the model if necessary.

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Steps of Analytical Modelling (1&2)

- 1. Purpose of the model:**
 - Model to be developed should be decided based on intended objective
 - E.g. Model of a transformer to study losses & efficiency Vs Model of a transformer to study magnetic behaviour & core saturation
- 2. Define boundaries:**
 - System of interest is separated from the rest of the world (referred to as environment) by a boundary
 - Boundary may be real or imaginary
 - Boundaries for every system and sub-system should be defined based on purpose

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Let us go through these steps one by one. Purpose of the model: it should be first based on the intended objective. For example, if I were to study the model of a transformer to study its losses and efficiency I would use an equivalent circuit of the transformer, whereas if I were to study some kind of other properties of the system like its magnetic behaviour I would look at something else; that is, the core behaviour and the core saturation.

Next is to define the boundaries of the system. The system of interest is separated from the rest of the world. For example, if I am looking at say improving my own cricket team; I just set my boundaries as the set of players which I have, I do not really bother what is happening with the other team. In some sense I could do, but mostly not. This boundary could either be real or imaginary. And this boundary is defined again with some purpose in hand.

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Steps of Analytical Modelling (3&4)

- 3. Postulate a structure**
 - Systems store, dissipate, transfer or transform energy from one form to another
 - Identify simple elements which characterize these operations on energy
 - Model elements generally have two ports, sometimes more
 - Represent the actual system as an interconnection of these elements
 - Referred to as physical modelling
- 4. Select variables of interest**
 - First step of mathematical modelling
 - Assign variables to all system attributes of interest
 - E.g. current, voltage, velocity, temperature, etc.



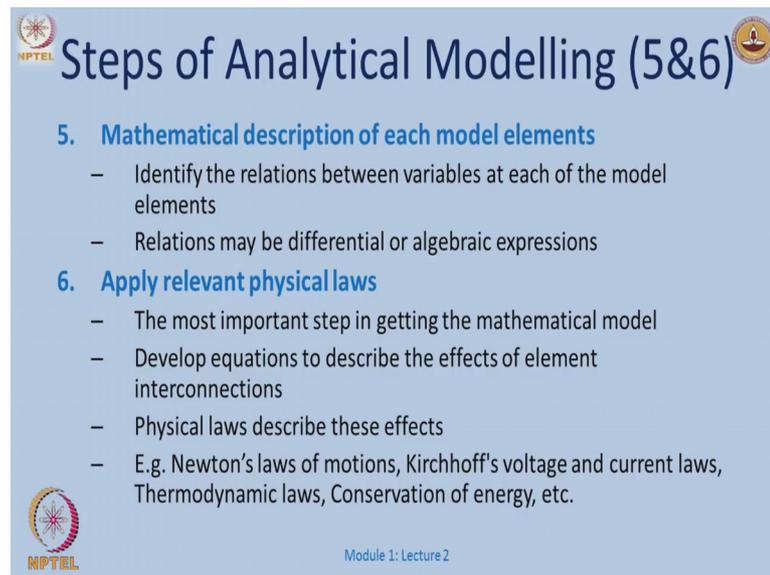
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Then the third step is to postulate a certain structure. Most physical systems if you look at what we have learnt, what we learn in circuits, what we learn in some applied mechanics or any of those courses is systems what do they do: the system components either store energy, they dissipate energy or transform energy from one form to the other. For example, storing elements could be an inductor a mass element which stores kinetic energy; dissipative elements are obvious resistors; and elements which transform energy from one form to other would be either generators or motors and so on.

And for all these things we identify simple elements which characterize these operations of the energy. And these basic elements would have some ports which with we could connect the systems to the outside world. And the overall system is now captured as a interconnection of all these small sub systems. And this is referred visually to as the physical modelling.

Next step would be to select the variables of interest. For example, how does my current build up for the inductor. And we assign variables to all these things which are of interest. For example, the current, the voltage, a certain velocity profile and so on.

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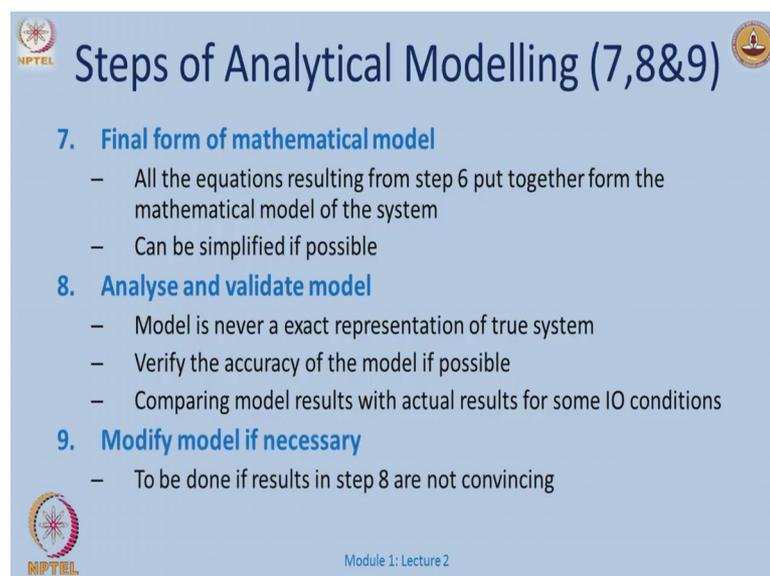
Steps of Analytical Modelling (5&6)

- 5. Mathematical description of each model elements**
 - Identify the relations between variables at each of the model elements
 - Relations may be differential or algebraic expressions
- 6. Apply relevant physical laws**
 - The most important step in getting the mathematical model
 - Develop equations to describe the effects of element interconnections
 - Physical laws describe these effects
 - E.g. Newton's laws of motions, Kirchhoff's voltage and current laws, Thermodynamic laws, Conservation of energy, etc.

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Now each of these elements would come with its own mathematical representation: could either be a linear relation, a static relation, or a dynamic relation. Once I have models of all these individual elements I interconnect them and apply relevant physical laws. For example, Newton's laws or even the Kirchhoff's voltage laws, current laws, Conservation of energy laws, and things coming from Thermodynamics and so on.

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Steps of Analytical Modelling (7,8&9)

- 7. Final form of mathematical model**
 - All the equations resulting from step 6 put together form the mathematical model of the system
 - Can be simplified if possible
- 8. Analyse and validate model**
 - Model is never a exact representation of true system
 - Verify the accuracy of the model if possible
 - Comparing model results with actual results for some IO conditions
- 9. Modify model if necessary**
 - To be done if results in step 8 are not convincing

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And, once I do all these things I just get my final mathematical form. If there are constraints which I could eliminate, I could eliminate and simplify the model as much as

possible I analyze and validate the model. If there are some inaccuracies in the model behaviour then I repeat certain steps which could be necessary.

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The slide is titled "Overview" and is divided into two columns. The left column, titled "Summary : Lecture 2", lists four items with checkmarks: "Model & its significance", "Types of math models", "Methods of modelling systems", and "Steps of modelling systems". The right column, titled "Contents : Lecture 3", lists four items with right-pointing chevrons: "Classification of physical systems", "Mechanical and Electrical systems", "Analogue between systems", and "Basic variables and elements of systems (Step 3 & 4 of modelling)". The slide includes NPTEL logos in the corners and the text "Module 1: Lecture 2" at the bottom center.

Summary : Lecture 2	Contents : Lecture 3
✓ Model & its significance	➤ Classification of physical systems
✓ Types of math models	➤ Mechanical and Electrical systems
✓ Methods of modelling systems	➤ Analogue between systems
✓ Steps of modelling systems	➤ Basic variables and elements of systems (Step 3 & 4 of modelling)

So, in this lecture what we have looked at is: why do we need a model, how to obtain a certain model, what are the type of mathematical models, and several steps or the various steps involved in modeling of systems.

In the next lecture we could classify our modelling process into a certain class of physical systems and we will restrict ourselves to mechanical and electrical systems. We will see analogue between this systems, is there any relation between some kind of mechanical models which we get and some kind of electrical modellings which we will get. We will see what are the basic elements in mechanical domain, in the electrical domain to arrive at models and how what could we do with those models how do we analyze them. So, those things will be a part of the next lecture.

Thank you.