Control Engineering
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Module - 05
Lecture - 01
Root Locus Technique

In the previous class we concluded with the concept of relative stability. And if I want my poles to be not in the left half plane, but left of minus 1 can I do that or does my Routh-Hurwitz criterion give me an answer to that: the answer was yes, you know I could find out if the roots are to the left of minus 1 exactly to the right or on the axis itself, but that may not necessarily be enough when I am handling design problems right. So, what we will do today is to learn a technique called the root locus, which will tell me little more about relative stability right. So, what is this root locus all about?

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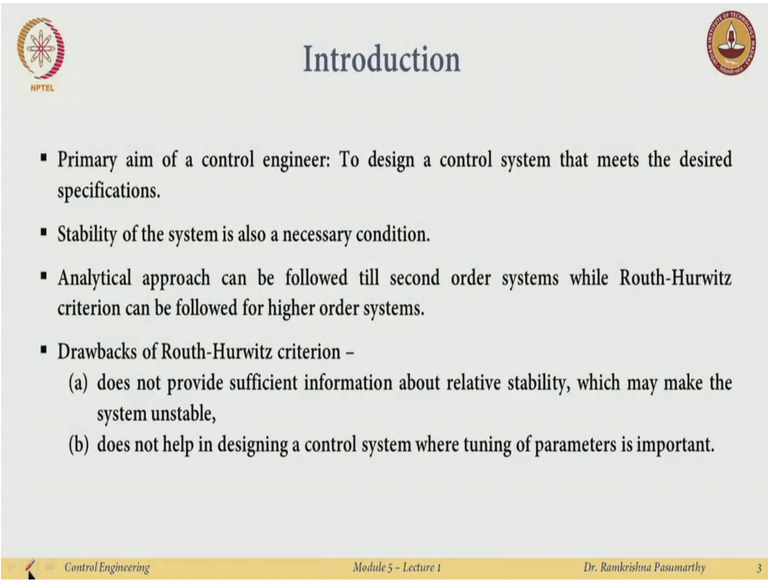

So, when I design a system. So, I want the system to be such that it meets the desired specifications, and if we go to our earlier lectures the specifications were usually either in the transient response or in the steady state response. So, in the transient response I wanted to speed up the response, but then I had a constraint of the overshoot. So, I wanted to speed up in such a way that I do not overshoot too much and I also wanted the system to have well very small or no steady state error at all, but before I do this I always wanted that the system should first be stable if it is stable then I can look at performance right. So, until now what we did we just followed some analytical approach while we do while we were doing the Routh-Hurwitz criterion.

So, it does not provide even though it gives me some information of relative stability it does not give me too much information I may need something more than just ask I just asking the question do the roots right to the left of minus 1 or left of minus 2 and so on right and it does not really help in choosing parameters in that way right, it does not help in choosing a control where you know I really want little more fine tuning of the system parameters.

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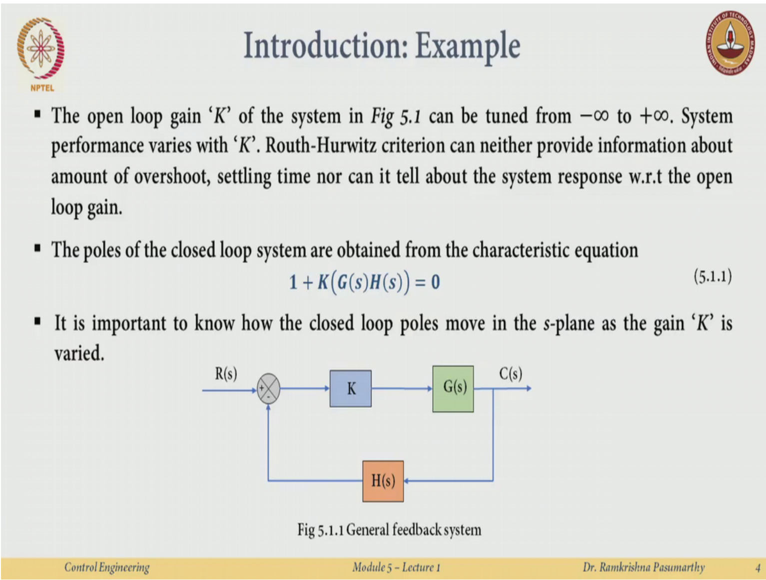

So, if I look at a typical closed loop system like this right and I just ask a question what happens if I change this gain? It could be from all values possible values from minus infinity to 0 and to plus infinity, what happens to the system well the Routh-Hurwitz does not really give me an exact answer to this question, neither does it tell me any information about the settling time or the rise time overshoot or even the steady state error. So, for those things I need something little more sophisticated or little more in detail. So, let us see are there any techniques like that right. So, as again what we will be interested is in the characteristic equation the solutions of which will give me the location of poles of the closed loop system. So, we would like to know how the poles of the closed loop system vary by changing the gain key.

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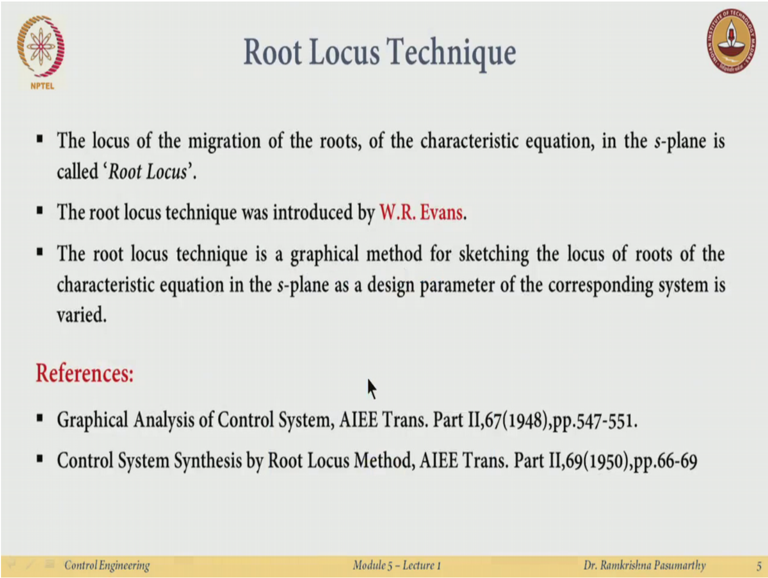

So, this thing or this locus of the migration of the roots of the characteristic equation as the gain K varies is called the root locus techniques introduced by W. R. Evans paper in 1948. So, what is this guy this is this root locus is a technique, it is a graphical technique for sketching the locus of the roots of the characteristic equation as a certain design parameter.

Here it is this gain K it could be something else also we will see how we deal with those, when this design parameter is varied and these are the first 2 two papers where this root locus was first introduced. So, let us not get in to that at the moment let us start with something very small.

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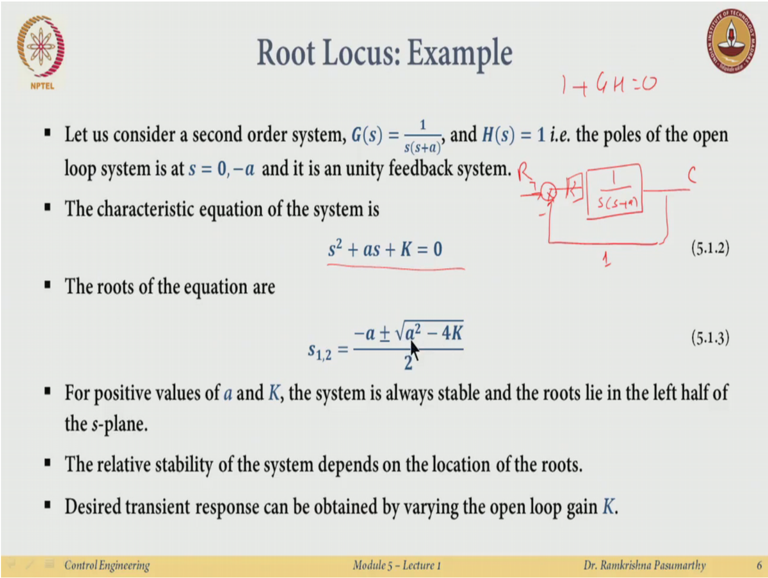

So, I start with a second order system if I were just to draw it I have in my forward as a function 1 over s s plus a, have h of s is 1 a plus a minus here again all these usual things of r and c as 1 right. So, the characteristic equation of the system 1 plus G H equal to 0 gives me something like this now the roots. So, I think I should also put a gain K here write another block. So, this will give me the characteristic equation in this way. So, the roots of the sixth equation second order equation I have 2 roots minus a plus minus square root a square minus 4 K over 2.

So, let us analyze what this how these roots change and what are the 2 parameters here a changes and K changes. So, for positive values of a right when a is greater than 0 minus a is always less than 0, additionally if K is also greater than 0 system is always stable right and because this guys because and the root always roots of the system always lie in the left half the relative stability now depends on the location of the roots.

So, then I could see that by varying K which is corresponds to my omega n in the second order analysis this guy corresponds to 2 zeta omega n dealing with the damping coefficient also right. So, the desired transient response can be obtained by just changing this may be if the a is fixed. So, given a transfer function a could be fixed. So, what happens? So, given a there could be few cases because the roots of the characteristic equation depend on what is sign of this guy.

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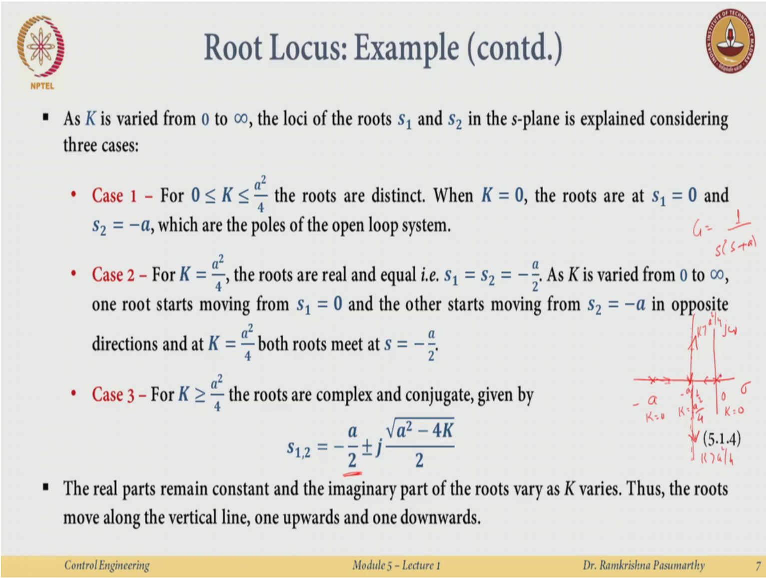

So, for case one when K is between 0 and a square by 4 the roots are distinct right I can just see very easily. When K equal to 0 the roots are at s 1 equal to 0 and s 2 equal to a. So, let us plot that for. So, this we plot it here is. So, I start with K equal to 0, I am here that is 0 and I am here this is minus a in the s plane sigma and j omega right; for K between 0 and a square by 4.

So, this guy would be real in that case. So, the roots would be distinct. So, there will be somewhere I know that that there will be distinct they will just be really some somewhere on this in this axis could be here and wherever, but I know that I a stability is always maintained because a and K are positive you do not even need to look at the Routh table right here may be you can just trivially see that the Routh condition is satisfied.

And therefore, when we were doing the second order system analysis for given zeta and omega n which is always positive we never worried about stability, because stability was very obvious in that case. You can now see that the Routh criterion satisfies this very trivially and therefore, we were really never worried about stability. For K equal to a square by 4 this guy disappears and I have equal roots minus a by 2 minus a that minus a by 2. So, let me just say that this r when K equal to 0 K equal to 0 and when K equal to a square by 4 well this is at minus a over 2 occurs at K equal to a square over 4. So, when K changes from 0 till at least it reaches a square by 4, what I see that this guy starts moving here and this guy starts moving here until they would meet at this point right at a square at K equal to a square over 2 and this point which is minus a minus a over 2.

Now, what happens after this? So, K was between 0 and a square by 4 when I just say that were these are somewhere here, I just well I knew that they were somewhere on the real line, but now I know that even K equal to a 4 you know they meet here. So, this guy travels here this guy travels here what happens for K being greater than a by 4? Well the roots are complex and conjugate and given by this number.

So, what you see that after K becomes greater than a by 4, a is given to us right. So, this guy remains fixed. So, all the real parts will be at minus a by 2 and the complex part will follow this guy which means well I am just. So, there will be all the real parts are a by 2 here and the complex parts will keep on changing. So, when K equal to a K equal to a square by 4 I am here, and then I see that I am just moving around this, this lines these lines and this line.

So, this is for K greater than a square by 4 similarly here, K greater than a square by 4 and we know that if there is a root here there always be a root here right because of the nature of the system right. So, the real part remains constant and the imaginary part varies adds the gain K varies, thus the roots move along a vertical line right here 1 1. So, they go this way and this way. So, what do we observe right?

So, as gain K for K equal to 0 and just at the open loop poles, as K increases both these guys meet up here and they again go away right to this keep going till infinity. So, something nice happens is happening here right. So, what information I used? I just used to this guy G is s sorry this is 1 over s s plus a right I am I am not doing anything with the transfer with the character equation and I just interested in how this K times z and then now this will be the closed loop.

I am not drawn this is. So, I am just taking this information right this is this guy. So, what I observe for K equal to 0, I am at the open loop poles and as K increases these guys tend to move and they direct they decide to move the decide to meet here and then again separate.

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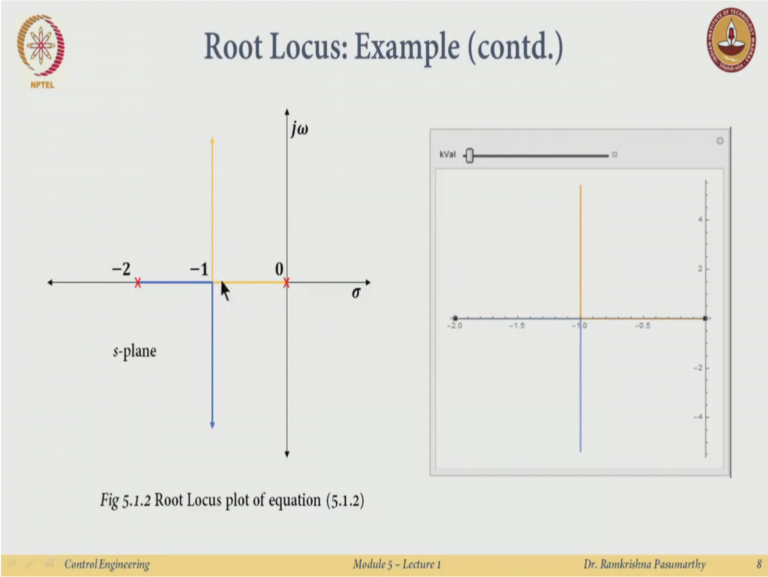

So, this is how I just play a very small video of this right. So, this is how the roots will go something's saw there right. So, if I give. So, again let us play this again K increases. So, this guy will go and then following well it is of K and then goes small and then I can come back to the position. So, this is how the roots will move. So, this guy will go here and move this guy direction this guy we go here and move for the direction right.

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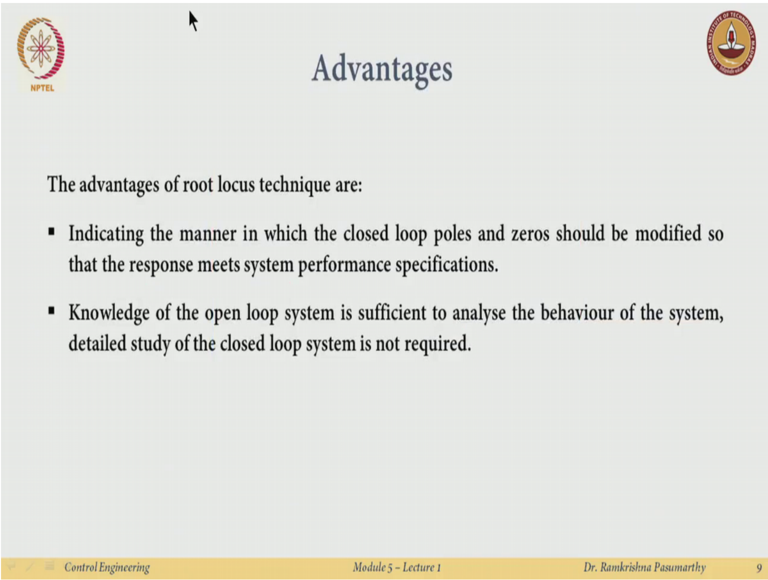

So, what did we know what did we see here that. So, the root locus it indicates the manner in which the closed loop zeros and poles move so that the response meets some specification just my specifications are usually in terms of zeta and omega n and these specifications usually translate 2 locations of poles. So, this could be some zeta and omega n which would be may be satisfied by K equal to a square by 4 plus 1 right then I would know.

So, exactly know where the roots are maybe somewhere here and somewhere here right and then I would actually exactly know what should be the value of K such that my closed loop poles satisfy some desired specifications or some desired performance criteria. So, I would know how to modify the gain K such that my performance is met and performance is usually mapped to some pole locations.

And I am just doing this only with the knowledge of this guy and I started with s the open loop poles s at 0 s at minus a and I am just now increasing the gain K we see how will formalize this a little later. So, just by looking at these 2 I start with the open loop poles and these are all the closed loop things now at as K increases I am already in the closed loop right; and then how the closed loop poles are modified starting from the open loop poles. So, I just need to know how the open loop system looks like and I can just do a good study of the closed loop system. So, I do not really need to compute the closed loop poles and then analyze things and so on.

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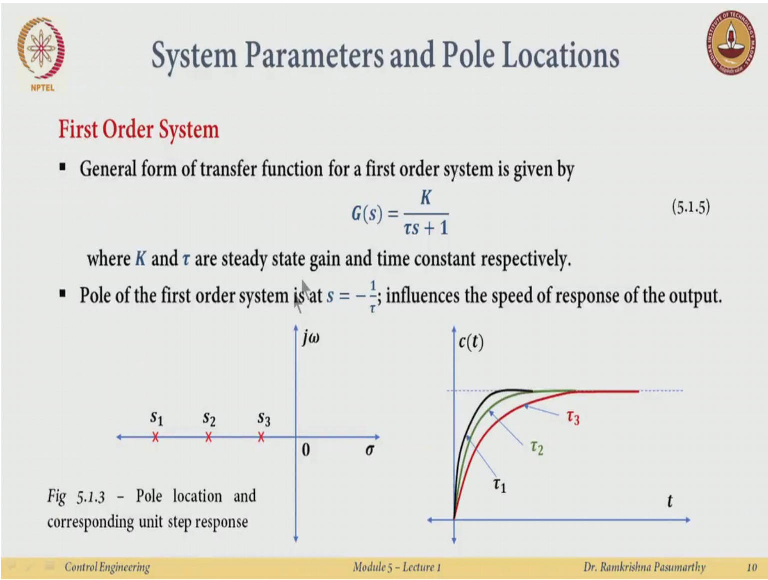

So, how are poles related to pole locations related to the performance well we analyze this while we were doing first order systems. So, just say if for different values of poles at s 1 I will have a faster response s 2 slightly slower, and the slowest will be for s 3. So, this is like how the how the location of poles influence the performance this we did while we were doing analysis of first order systems.

Now, in general the first order system was very easy right. So, all the poles are on the real line then based on the location I would determine how are they fast or they slow the slowest would be here and the fastest would be very you know further away from the origin.

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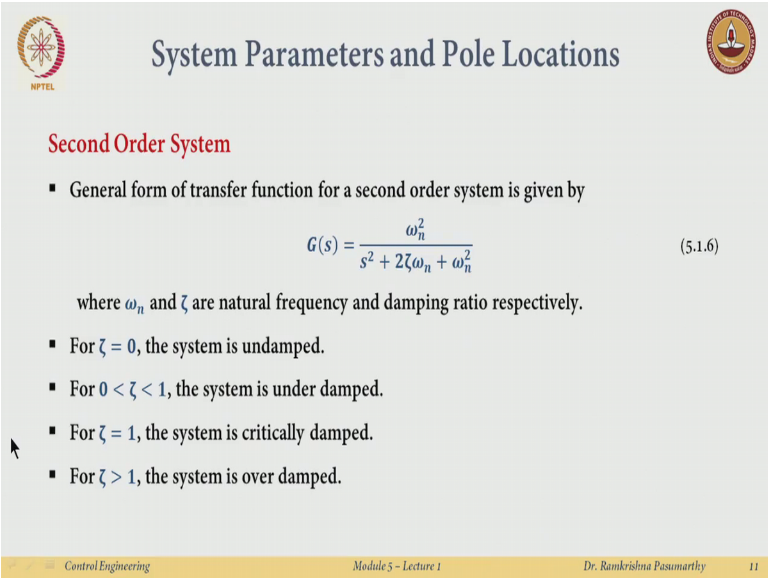

However, for second order systems there were little little more detail said we started well we had this un damped systems where we had the maximum overshoot the settling time was infinity, then we had the case of under damped systems where we which quantified the rods or we define under damped, nature in terms of zeta right and then we had also when the system was un damped the frequency of oscillation was omega n, for sigma equal to 1 the system was critically damped and then we also had the notion of over damped system where this zeta. So, it was it was greater than 1.

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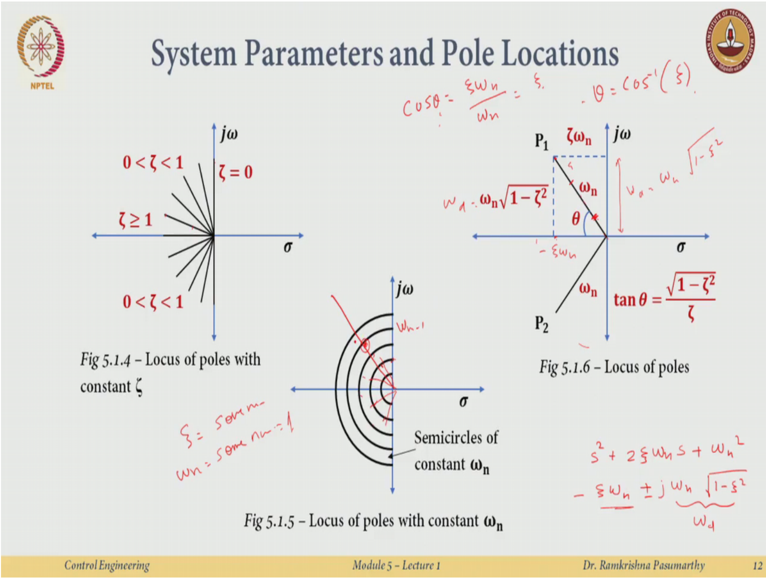

So, if I just look at you know the poles I typically well I am just looking at solution to this equation at s square plus 2 zeta omega n s plus omega n square. So, if I just look at the under damped case my roots were minus zeta omega n plus minus j omega n square root 1 minus zeta square for this is the for zeta less than 1 if we remember properly and this guy I said this is omega d. So, this is the real part here. So, if I just take a complex conjugate pair right p 1 and p 2 the real part this distance would be this will be at minus zeta omega n, and then this guy would be omega d this distance omega d is omega n square root 1 minus zeta square right this is just like. So, this is a right angle triangle and I could say that this omega n is just you know square root of this guy plus this guy right from the from the Pythagoras theorem; now something else right.

So, this is what we also call this the omega d right now look at this guy theta right and if I say how does cos of theta look like. Cos of theta is this guy zeta omega n over omega n this is zeta right. So, from this cos of theta is zeta and therefore, theta is cos inverse of zeta. So, let us look at this right. So, if I am at this point theta remains the same whereas, omega n changes right if I go here if theta is the same then zeta is the same.

Now if I go from here till here this distance which is omega n changes what cos of theta still remains the same right slightly here also if I go from here till here what happens my zeta remains the same, because 30 times the 70 from this moving along this line right and then omega n changes right for some given zeta. So, all these lines are lines which I call as constant zeta lines.

So, I can go here right if I just take any arbitrary line here right for which a theta would be fixed, if I move along this line right what happens is my theta is fixed, whereas omega n changes. So, this behaviour could be seen from zeta equal to 0, I am here I keep on increasing zeta and there are all these all these lines are the lines for which the zeta remains constant. So, at this point you will have different omega and this point is different omega n, but the zeta here is same similarly on the other side. So, these are all constant zeta lines or we just say how the locus of the poles looks like for constant zeta.

Now, let me keep omega n fixed and keep moving right and I just want to vary theta. So, what will I do when omega n is fixed? So, omega n is fixed and I just move around some semi circles, I do not want to do a circle because I do not want to go in to the unstable region. So, if you look at this guy over here omega n this is same omega n is same omega n the same same, but each of these will have different zeta right this will have a different angle, this would have a different angle and this is have a different angle.

So, if I keep on moving omega n then I just see I just move am just moving along the circle and this guy also you know since all poles exist in complex conjugate pairs. So, this guy will also move just according to how the omega n varies. So, you just have some semi circles here. So, this is a pole here there is also put here and this will correspond to certain zeta if I go here this pole will move here and this pole will keep on we move here right the complex conjugate, but the zeta will be.

So, these are semi circles where omega n is constant and zeta is changing, these are lines where zeta is constant and omega n is changing. What is the importance of this? When I say well design something such that zeta is some number and omega n is also some number. So, zeta is say 45 degrees and omega n is 1. So, I have this constant omega n circles and then I just draw a line of say zeta or also the theta being 45 degree. So, I just draw a line from here till here and if say this is omega n equal to 1 then my pole should be here.

So if I have again I will repeat if I want a design specification where zeta should be say some number which corresponds to angle of 45 degrees given I given theta I can always compute zeta right and then add a certain omega let us say this is 1. So, I just find these 2 lines. So, this is my zeta right and then this is my omega n equal to 1 and the point where they intersect is where I want my closed loop poles to lie and this I know how you know from the previous example that I could very this closed loop poles just by no varying the game. Now I just investigate is there any gain which will do me this now how do I test. So, that is exactly what we will study in the in the root locus case.

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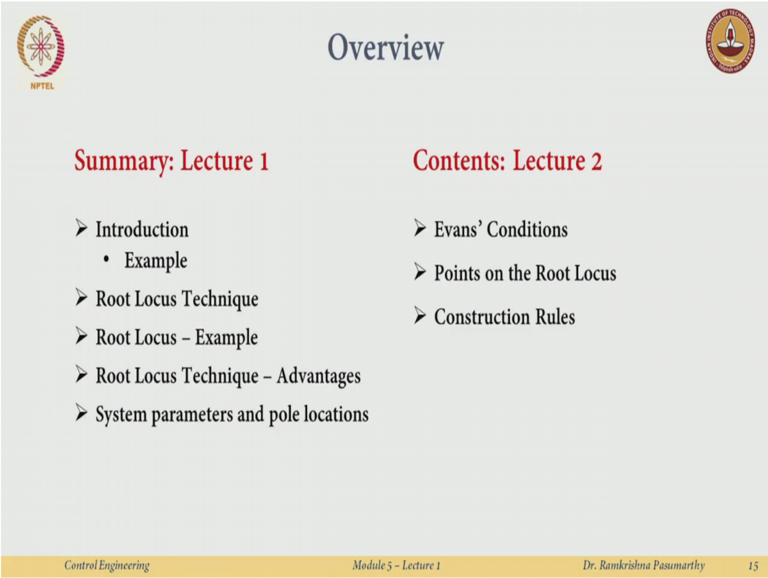

So, this is just a little little example right, and now will see how we can generalize this right do the roots. So, what we saw that the roots were travelling towards each other they meet and then they go away do they always meet and if they meet do they again have to go do they have to go to the imaginary axis or they could possibly just stay over. There was no 0 in our example right it was just s 1 over s plus a, what happens if there are zeros in the system.

So, all these things we will try to analyze a little more, this will lead us to some conditions on the root locus. So, all these roots is what we will do in our next lecture.

Thank you.