**Control Engineering**
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**Module – 04**
**Lecture – 03**
# Closed Loop System and Stability

Hello everybody, today we will talk a bit more details about closed loop system and stability. So, we are studied. So, far stability from the BIBO point of view or the bounded input bounded input point of view and also stability in terms of the equilibrium points and then we characterize those as the location of poles of my open or close loop system depending on wherever you are. So, what we will do today is to also look a little more details about feedback is feedback really helpful all the time or there are some drawbacks we will skip the drawbacks part or we will you know do not do not go to machine to the details.

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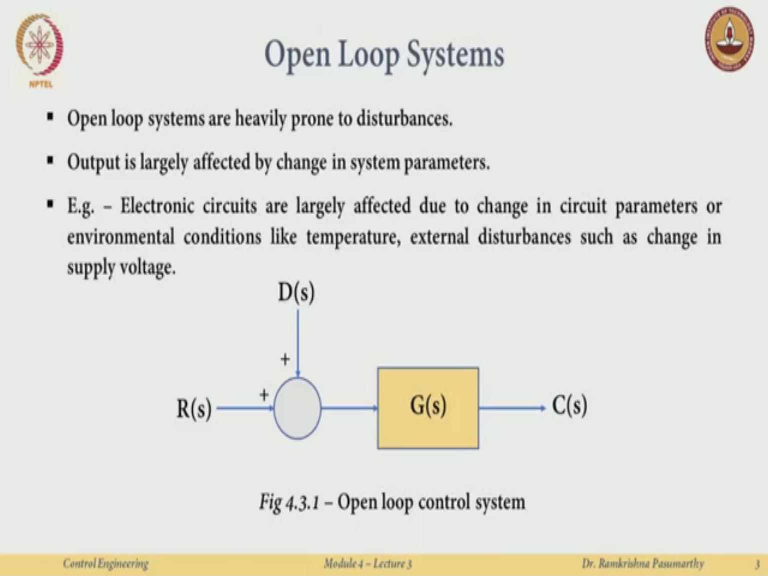


And we will just see how feedback helps us in lot of ways and also defined something called the relative stability.

So,. So, open loop systems are heavily prone to disturbances right. So, the output is affected by the change in system parameters system parameters could be like I am talking of a mechanical system a mass could change or any other thing could change in the system it could also change because of external environmental conditions like temperature sometimes disturbances act into the system and I would I really would not know how to how to bring my system back to the original state for example, if I am driving a car and the pedal is at constant position if I hit a ramp you know or an upslope that is a disturbance right. And if I keep the same pedal position and I keep going through the through the ramp I will not be able to maintain the desired speed and after a while I might even loose the speed and I must go to 0 speed right. So, there is no automatic correction if I said my pedal speed to its open loop value and just do not change it when the disturbance comes, right.

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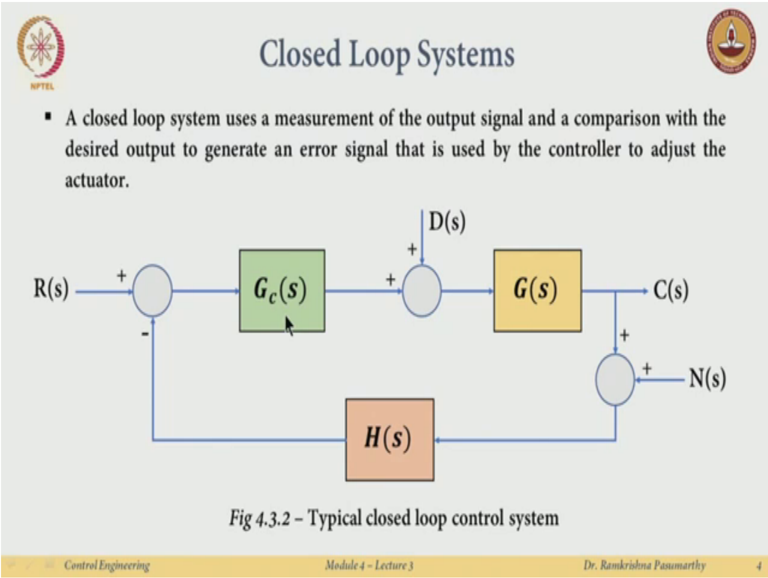


In closed loop systems the output is actually measured and compared with a reference signal and which generates an error signal by which I update my control law and through the control law I adjust my actuator which actually gives the actual control input to the system right. So, typical things that could happen are you have effect of disturbance and you also have measurement noise right. So, my measuring devices may not be very accurate and you know they might they might come with a bit of a noise and how do we deal with those these things the disturbance signal here and the measurement noise. So, we want to keep or design our controller or keep in mind such that I reject disturbances and I also should reject the measurement noise.

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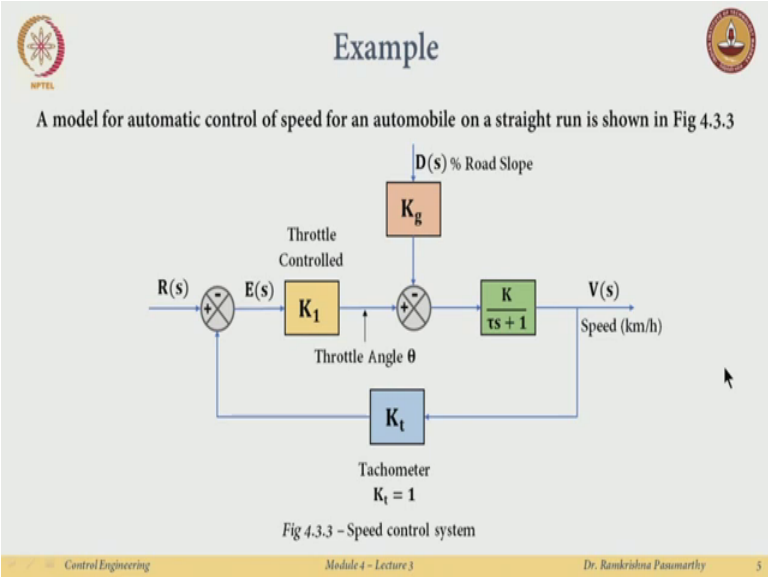


So, let us start with an example right. So, so this is a model for automatic control of speed for an automobile on a straight run. So, I have a reference speed I have the measured speed and this generates an error signal which will decide the position of my throttle the this is the throttle angle there could be disturbances in terms of the road slope bit could either be positive slope or a negative slope no disturbance would just mean no slope and this is a very simple model of my car right because car which now looks very big with lots of components actually you know looks very small here right just a first order differential equation or a system with just one pole and I just get the desired speed, right.

So, this is like the standard measuring devices like the tachometer and all and this will this will convert it to be appropriate speed.

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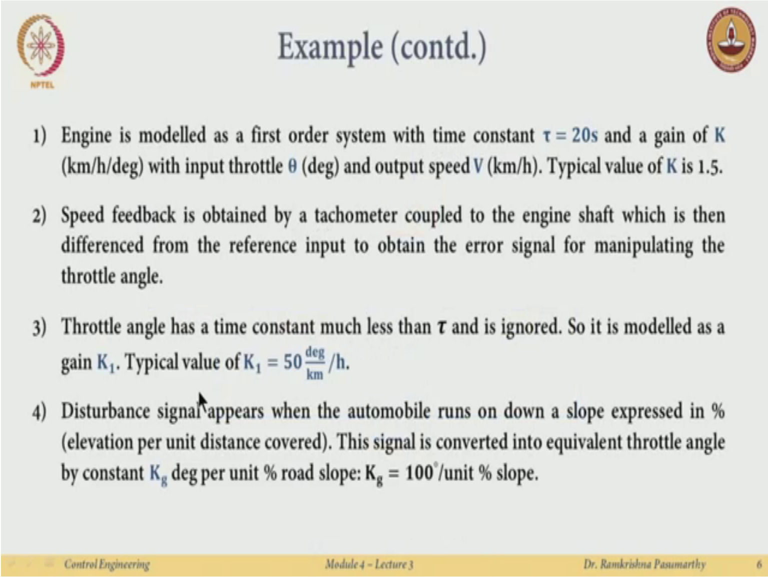


So, the engine for simplicity is modeled as a first order system with a certain time constant and a gain of K with a typical value of 1.5 the input is a throttle angle measured in degrees and the output is a desired speed right and the feed peed the speed feedback is obtained by a tachometer right somewhere here. So, this is comes a as in the feedback loop and this is used to obtain the error signal for changing the throttle angle right and for desired purpose says we says that throttle angle has a has a time constant much less than less than the system time constant and therefore, we could we could ignore it right and we just modeled as a small gain K 1.

So, the error signal via K 1 gives me the control input and some typical value we could assume it in this case to be 50 the disturbance signal appears on the automobile runs up or on a down slope right it is a it is expressed in terms of percentage of the of the of the road slope and then a 90 degrees would actually mean that you actually hitting a wall and all and just going vertically upwards. So, and this signal is converted into equivalent throttle angle and then you have this, this K G which takes care of the units and you have this like K gs 100 degrees per unit percentage of the slope.

So, why are we doing this? So, let us see if what happens if I just do not measure the guy right I have a reference and I just have the output right and there was no, no, feedback and also take the case of feedback.

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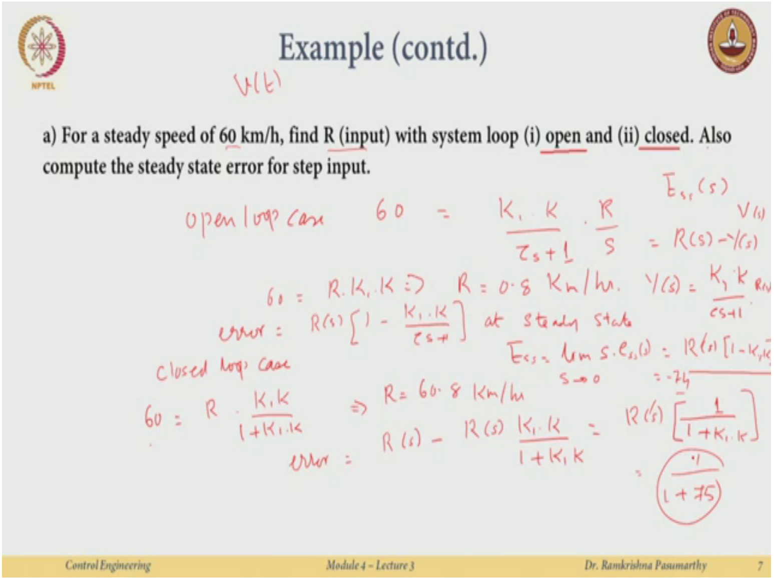


So, given the first condition or the first problem that my reference or not; reference my desired speed is 60 kilometers per hour that is in terms of the picture that would be v of s. So, V of s is 60 kilometers or say v of t more messy and then we have to find the R or the reference signal with the system being first open loop and second be closed loop and we also see how the steady state error changes with this, right.

So, let us say in the open loop case. So, at steady state steady state speed is 60 and how will my transfer function look like. So, it will be K 1 times K over tau s plus 1. So, this output is K 1 K tau s plus 1 and sum R of s this will be a step signal right surrounded (Refer Time: 07:07) in the size of the step. So, let me just call it some number say R over s and I do the final value theorem and I just get that 60 is this R times K 1 times K right and then K 1 was K 1 was this guy 50 K is 1.5 and I do this computations and I get that R is 0.8 this kilometers per hour.

Now, let us see what is the steady state error right. So, also if I if I look in the in the Laplacian domain right E s s E sorry E steady state the Laplacian is R of s the reference signal minus y of s. So, what we want to do is compare the errors; that is R of s minus y of s. So, y of s here or what whatever is so, y here is a velocity. So, v of s would be equal to y of s. So, y of s is again take this K 1 K tau s plus 1 s K 1 tau s plus 1 over this is by R of s. So, the error is R of s 1 minus K 1 times K by tau s plus 1 and at steady state sorry this error just becomes E s s I do the final value theorem as you know limit as going to 0 s times E s s of s that will just be. So, R is the step of some size 1 minus K 1 times K; this is this is my open loop error ok.

So, let us do the closed loop case. So, in closed loop the reference is still 60 is R can you just I will directly write the steady state expression at I just take you know get rid off all the transients 1 plus K 1 times k. So, I am just finding this 1 right I am finding R in the closed loop case when the desired speed is 60. So, I do all the computations and I get R as 60.8 right the reference is again the speed.

So, now what is the error here is again I look at the signal here right that is again R of s minus output here is R of s K 1 times K over 1 plus K 1 times k. So, this is R of s and this time 1 over 1 plus K 1 times k. So, just for simplicity assume that R is just 1. So, in the open loop case if I just say what is the steady state error this would be for the values of K 1 and K which were 50 and 1.5 this would be minus 74 and the and the steady state error here would be 1 over 1 plus 75 and this is a significantly smaller than here right. So, I have a big steady state error over here and I have a very small steady state error over here right. So, so you see how the feedback or closed loop reduces the steady state error.

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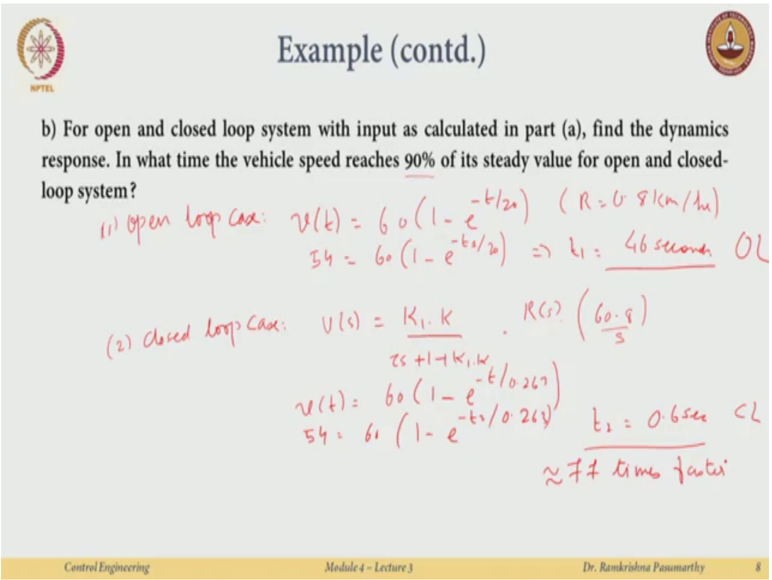


In the case, let us do some transient analysis right. So, earlier we had did we have done the steady state analysis. So, we want to find in what time does the vehicle reach 90 percent of its steady state value for open and the closed loop system. So, let us just do first for the open loop case. So, in the time domain I could write this (Refer Time: 12:40) v t is 61 minus E power minus T over 20, this is x straight forward to write from the expression. So, 90 percent of this value would be 54 is 60 and I want to find out the time what we call this T 1 at which this occurs. So, I have an equation here where there is 1 unknown. So, I can easily compute T 1 to be 46 seconds.

Now, let us see what happens in the closed loop case, so, in the closed loop case. So, my let me first write v s is K 1 times K over tau s plus 1 plus K 1 times K and this is multiplied by the R of s. So, I just do some manipulations and computations. So, in this case my R of s for the closed loop it comes from what I had computed over here. So, 60.8; 60.8 and then it is Laplacian; in the Laplacian (Refer Time: 14:10) going to be 60.8 where as in the in the previous case my R open loop when I use I will use this 0.8 kilometers per hour right, so, here when I do it here. So, in the open loop case my R is 0.8 kilometer per hour. So, just be careful of that.

So, now I just write down the final expression for v of t, I will skip a bit of computations. So, what I have 61 minus E power minus T over 0.263 and I just do nothing I just substitute this values and then compute the inverse Laplace transform and we in a by now know how to do these thing. So, again this ninety percent would be 68, I have 1 minus E power minus let me call this T 2 over 0.263 and I compute T 2 that is actually turns out to be just see this 0.6 seconds right. So, this is the open loop and this is the closed loop and you see that this is 77 approximately times faster right if I just have a closed loop my steady state error is much smaller and also my response of the system is pretty fast. So, this will motivate us towards you know analyzing what are the effects of feedback on the system.

First we saw well it helps in the steady state response it also helps in the transient response are there anything else associated with feedback.

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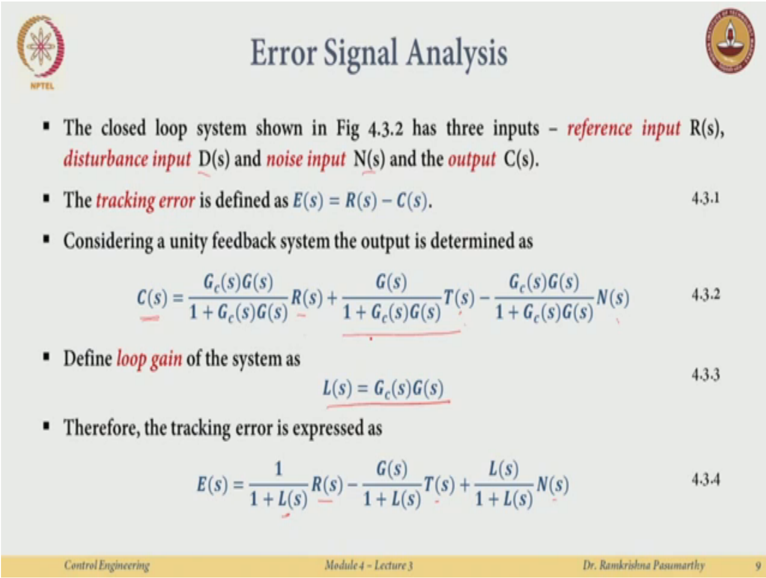


Now in the initial (Refer Time: 15:51) diagram we mentioned few other things in addition to the reference and the output. So, one was the noise sorry that the disturbance and second was a noise. So, in the presence of disturbance and the noise I can compute the overall output of system or overall responses of the system as this as the sum of response just to the reference signal and while computing this I set D and n equal to 0 then I compute the response by setting R and n to be equal to 0 this will give the response of the disturbance signal and lastly for the noise I said the remaining 2 inputs 0 and I just use the superposition. So, this is the total response to the reference signal and to the disturbance signal and the noise.

So, let me just define the loop gain as G c times G. So, if you have just forgotten where this G c and G comes from well we search these guys, right. So, G c is my controller this is plant this is H s is what happens in the feedback I have reference signal here the disturbance and the noise. So, the error now becomes of course, a function of what is the reference and something to do with the controller transfer function and the fund transfer function similarly with the disturbance and also with the noise. So, I can just compute this directly and these are I think in one of the tutorial classes we saw how to compute the effect of disturbance and then take the total response of the system as the sum of the response to the disturbance and is combined it to the to the response of the input right, just the superposition.

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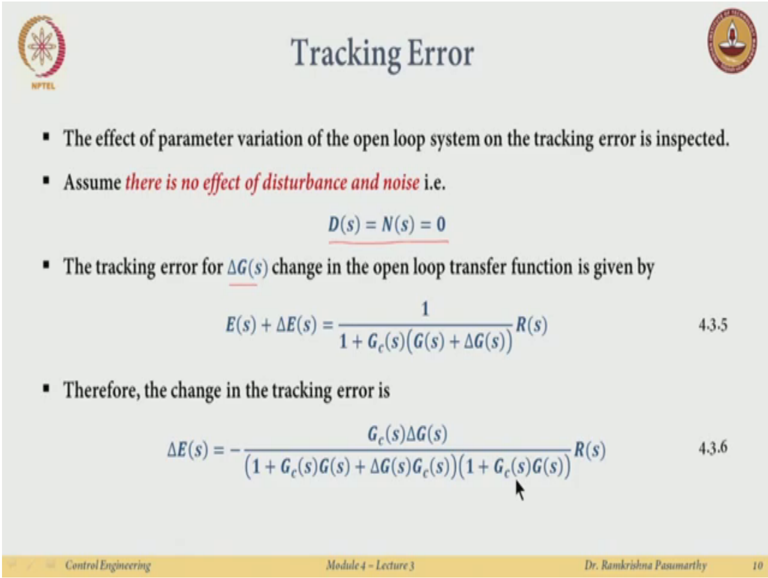


So, when we started this lecture we said well there could be some parameter variations right and then now we will see what is the effect of parameter variations on the system. So, assume there is no disturbance and noise we will come to this very shortly. So, say my transfer function or the plant changes from G to delta of G and then the error signal changes accordingly. So, I am not I am just doing something simple here. So, E which is 1 over 1 plus L times R L is G c times G I just substitute instead of G g plus a small number delta G. So, this is how my error looks like right.

Now the change in the tracking error can be computed you can just say what is delta E you take this guy here and then E I just substitute this guy E is 1 over L times R of s and I get this expression right. So, this is if again if delta G is 0 this goes to 0 and does the trivial case to check this the change in the tracking error is something like this G c and delta G and you have lots of terms of delta G influencing this particular number.

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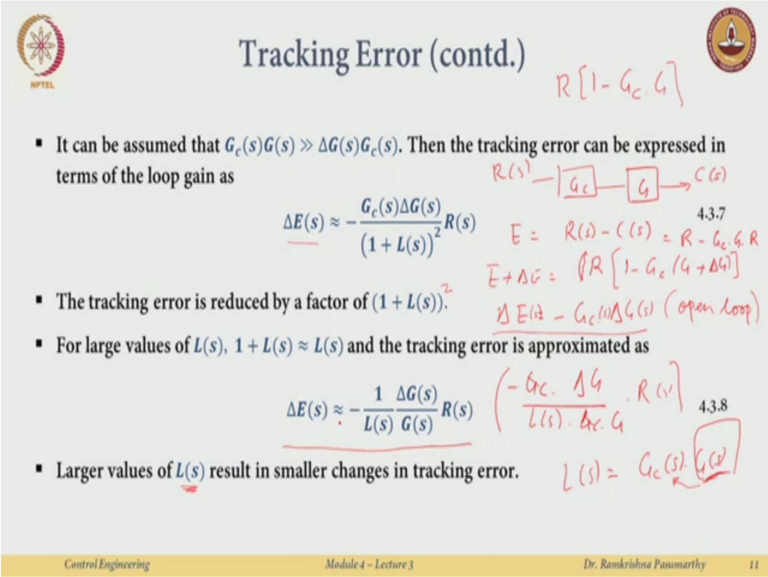


So, if I just take an open loop right. So, I have then this for convenience I will just draw it here and I hope I will use a write notation. So, this was R of s and this was c of s now the error is usually defined as R minus c of s. So, in this case this is R I just omit the s for the moment c G c times G times R now E plus delta E in our case turns out to be R 1 minus G c G plus delta G. So, this is how my error looks like. And therefore, delta of E would be this; guy minus the original error original error was what that was one the R 1 minus G c times c all with the s domain if I just compare these things. So, I just subtract this from this what I am left with is minus G c times delta G let me write this s here this is also in s. So, this is again in the open loop.

Now, just me compared to what happens in the closed loop. So, I have the error signal is of course. So, I do some approximation that G c times G is much larger than that that the very small change and I can I get an approximate expression like this that this that this is the change in error is G c times delta G 1 plus L whole square times R s. So, if you compare this with this you see that the tracking error is reduced by a factor of 1 plus L of s or the square and for large values of L of s. I can just again ignore this one and just write L s and I just get this expression, is it not this, this is not difficult to obtain right. So, the way you do this is have minus G c and you have a delta G just ignore this one I have just keep this L of s here and L of s is again G c times G right. So, this guys goes away and you have R of s and this is what remains right about delta G and a G and right and then. So, what does this mean that large values of L of s result in smaller changes in the tracking error is there something which I could change in L of s well. So, L l of s if we just recall was G c of s times G of s.

So, one could there could be well you know change the plant where I rid you do not want to do that right because the plant is already is already I have there is something which and given to control I just cannot change the subject or the plant. So, I can just manipulate my G c in such a way that I it results in small smaller changes in tracking error. So, this is about how feedback changes the error or helps in reduction of error.

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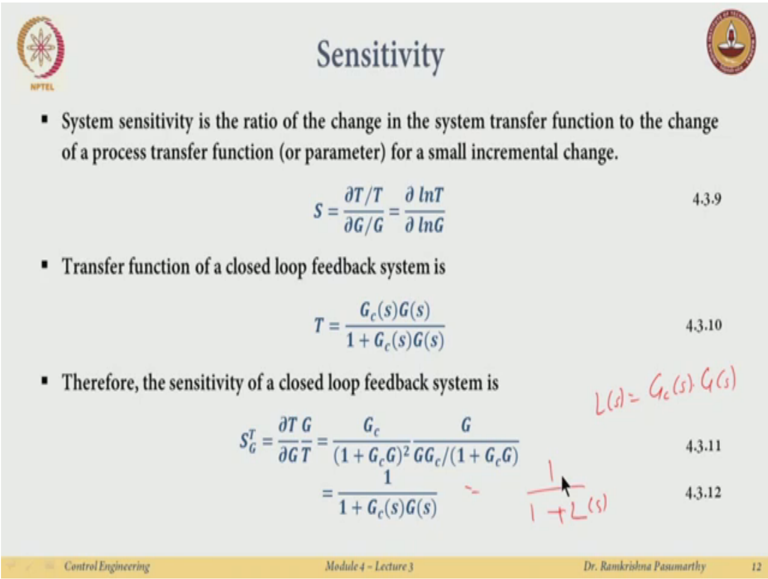


Now, so, what we saw was essentially this delta E related to small changes in the system parameter now let us try to quantify that a little more right. So, we call the system sensitivity. So, we will define this new term as the ratio of change in the transfer function to the change in the process transfer or some small parameter. So, I have the T as a transfer function. So, let us just understand from here. So, I want to check how my overall transfer function changes with small changes in G right. So, this is what I call as the sensitivity change this is s, how does it how does it how sensitive is a transfer function T to changes in g. So, what is T here this is my closed loop transfer function ok.

So, the sensitivity I define. So, the from here I just I assume D T by D G over T of G and this is how it looks like sensitivity of my transfer function with respect to G or changes in G is simply this one.

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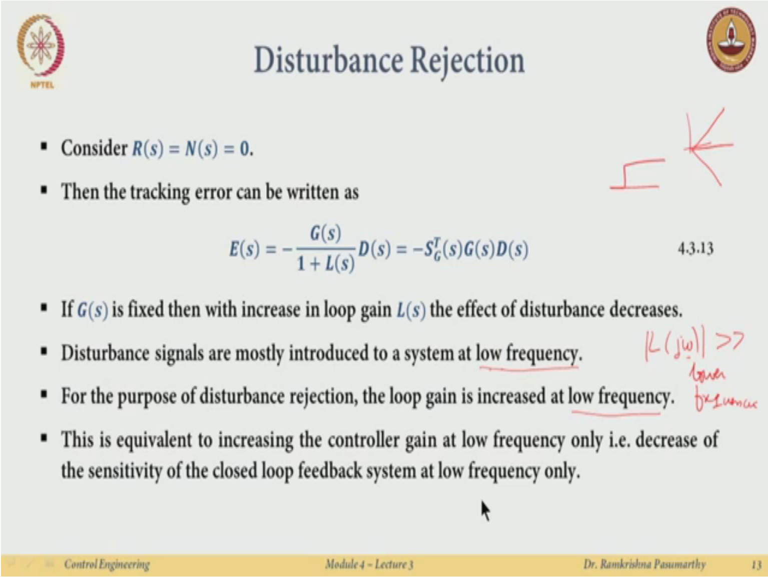


I will go here right again I just start with I am just looking at the sensitivity with respect to disturbance right when in which case I set the reference and the noise to be 0 and I can compute the tracking error right. So, this is E of s minus G 1 plus L of s. So, if I if I were just to write the sensitivity in terms of L this would just be 1 over 1 plus L where L is again what I call as the loop gain L of s and then just to recall again L of s was G c of s times G of s.

So, this is my error with respect to disturbance and it takes this system right. So, 1 over 1 plus L is the sensitivity; sensitivity of the transfer function which changes in G times G times D right if G s is fixed which is usually the case then with increase in the loop gain. So, I want to reduce the error a typically my error should be 0 if there is disturbance there should be no error right. So, how do I well, but then you know if I have a disturbance I would expect that there is some error. So, if there is some error I would want to minimize the error right. So, if G s is fixed then with increase in loop gain this guy the effect of disturbance can be decreased right.

So, something which I will briefly introduce now is this something related to the frequency will talk a little more of this when we talk of frequency domain. So, disturbance signals are typically signals of low frequency rights may be like step or even if there is oscillatory behavior in the disturbance they will just be at very low frequency in the car example my disturbance is are either I eta uphill or a downhill or something like will just be the constant thing which without any disturbance right. So, disturbance signals are mostly introduced or these usually occur at low frequency.

So, my L is if I just look how it varies with frequency I can just see the magnitude of L at frequencies right, so, at low frequencies. So, I want. So, what does it say that these expressions tell me that with increase in loop gain the effect of disturbance decreases now I do not have to increase it like forever right? So, L I know depends on frequencies and my disturbance signals are associated to certain frequencies or certain lower frequencies. So, I want G to be sorry this L to be increased at low frequencies. So, this should be bigger for lower frequencies. So, this essentially means that I increased the controller again at low frequency right or in another way I am just. So, if this increases my sensitivity decreases. So, I just I ideally I would want I do not want to be sensitive to disturbance right whatever happens I just I just should reject it.

But typically that does not happen right. So, we can only decrease the sensitivity. So, when I when I try to increase this at low frequency my sensitivity decreases right at low frequencies I do not really care what happens at high frequencies at the moment because there are no disturbance signals which come at high frequencies.

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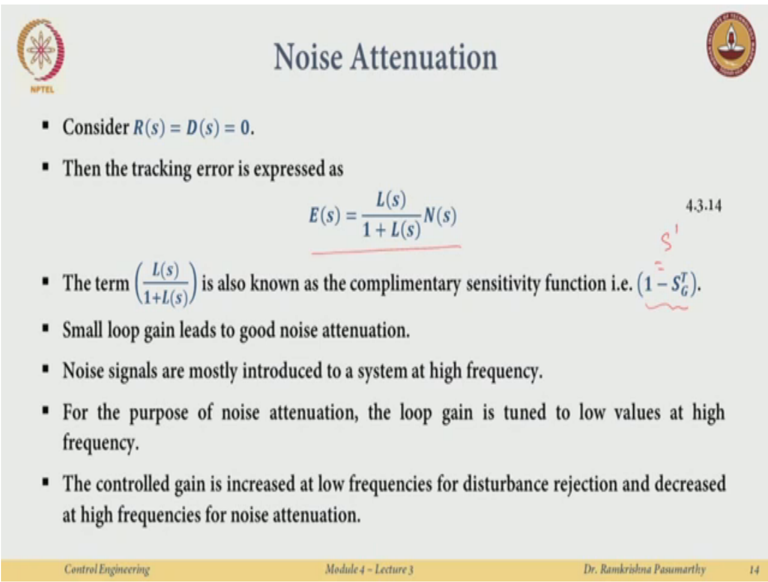


Now what about this noise right we. So, far have not talked about it, but then usually my measurement comes with some noise right.

The tracking error in terms of the measurement is something like this right again is this from the original expressions which we which we obtain. So, this guy L over 1 plus L is also known as the complimentary sensitive function. So, some people denote it as T some people denote it as c and since we have a already use a notation T and c will not we will just say this may be just for I can just call this s prime the complimentary sensitivity function and this noise. So, small loop gain if I L is very small it leads to a good noise attenuation right because this the effect of noise would be reduced and typically noise signals are signals which occur at very higher frequency or which get into the system these signals are typically have frequency signals.

So, for the purpose of noise attenuation the loop gain is turned to be low of or of low values at very high frequency right previously what we had that I want the loop gain to be higher at lower frequencies that will eliminate my disturbance for the purpose of noise attenuation the loop gain is tuned to have lower values at higher frequencies right. So, when I do this the controller gain is increased at low frequencies for disturbance rejection and decreased at high frequencies for noise attenuation, right. So, it is like some kind of a complimentary effect towards there right. So, these are the 2 ways which you know based on what is the L of j omega I can I can deal with disturbance and noise together or reduced the effect of disturbance and the effect of measurement noise.

We will do more of this when we do frequency response like you know what is actually mean what is low frequency what is higher frequency and so, on and how do I actually go about computing these things that will keep till a little later when we do frequency response analysis and something which we will use throughout from now on is what is called as the characteristic equation, right.

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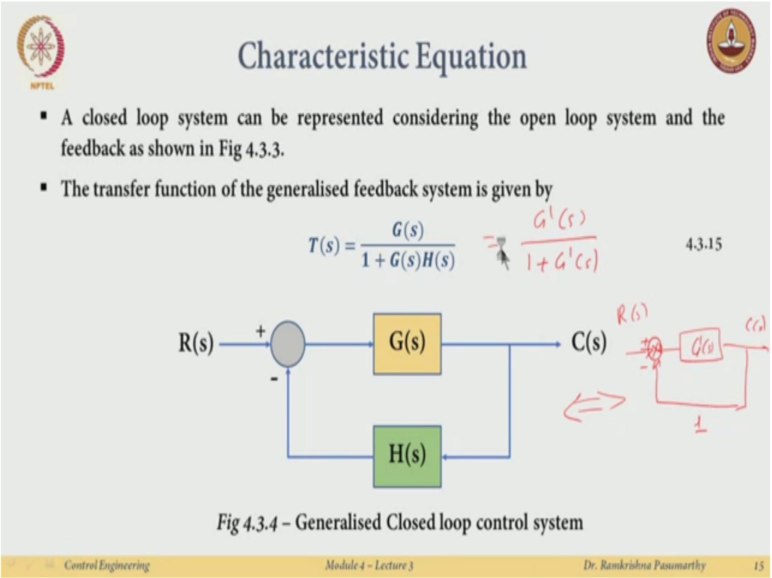


So, what is a characteristic equation well it just I take the transfer function and I look at what is in the denominator. So, I know how to compute this. So, at given G in the forward loop H in the in the negative loop the overall transfer function T of s from R to c is G over 1 plus G.

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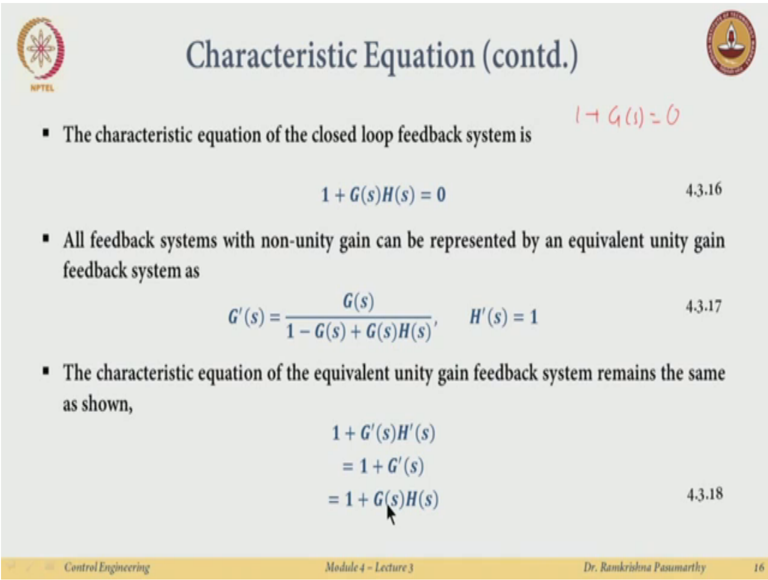


So, the transfer function sorry the characteristic equation is when I equate the denominator of this guy to 0 right 1 plus G times H equal to 0 and. So, if H equal to 1 then I am kind of something very simple in 1 plus G of s equal to 0.

So, now this is a system with a non unity feedback now is it possible that I do some magic and I convert this to a system which looks like this let me call this G prime of s sorry there is nothing here erase so, and then plus or minus with this to be one this guys still being R and this guy still being c right. So, I just want to convert this non unity feedback system to a unity feedback system can I do this well the answer is yes. So, what I do is I just equate. So, how will this transfer function look like that is simply G prime of s over 1 plus G prime of s and I find out what is G prime in terms of G and h. So, a non unity feedback system can be transformed into a unity feedback system by just using this transformation this should be very straightforward to compute because we are just dealing with very easy looking linear equations over here.

So, G prime would be this 1 H prime would be 1 right and while I do this the characteristic equation remains the same. So, if I just look at 1 plus G prime H prime of s this is 1 plus G prime of s and I will just get this; this is again simple computations will take you from 1 plus G prime H prime to 1 plus G times H right this is nothing special happening over here yeah.

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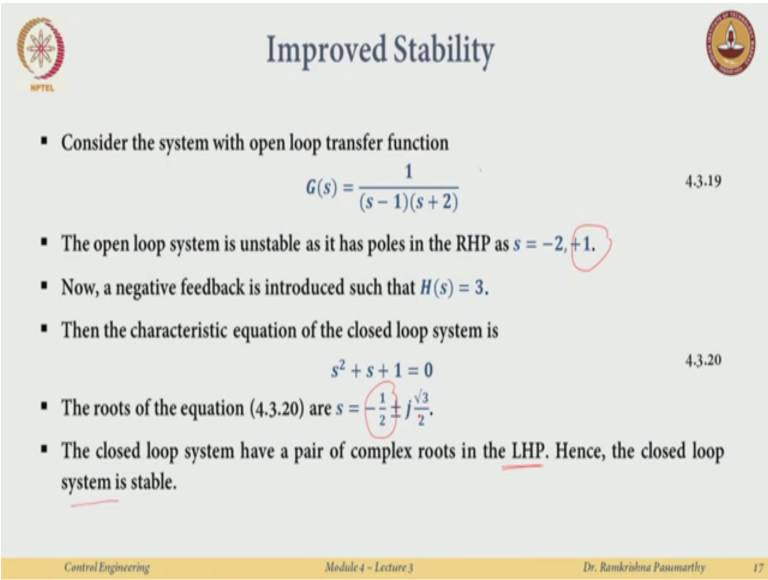


So, let us revisit stability for a while. So, I start with the; that the system that looks that looks like this write. So, this is I know that this table because I have sorry this is unstable because there is a pole on the right hand side. So, anything on the right hand side unstable right. So, can I do something with this right can I do something. So, as to make this system stable well I just say I just take the negative feedback H of s just to be a number 3 and I take the closed loop transfer function set the characteristic equation to 0 and the poles of the closed loop system are now minus 1 by 2 plus minus j root over 3 by 2 right.

So, these are complex conjugate poles, but which are again in the L H P right in the in the left half plane right so this guy; so, all poles are to the left half plane and therefore, the closed loop system is stable right. So, I start with open loop system which is unstable I can stabilize it with just some feedback right. So, this is one advantage which I get when I when I do feedback right I can make unstable systems stable is it possible always that give me any unstable system can I make it stable via feedback well that may not be always possible, but it in several cases it is possible.

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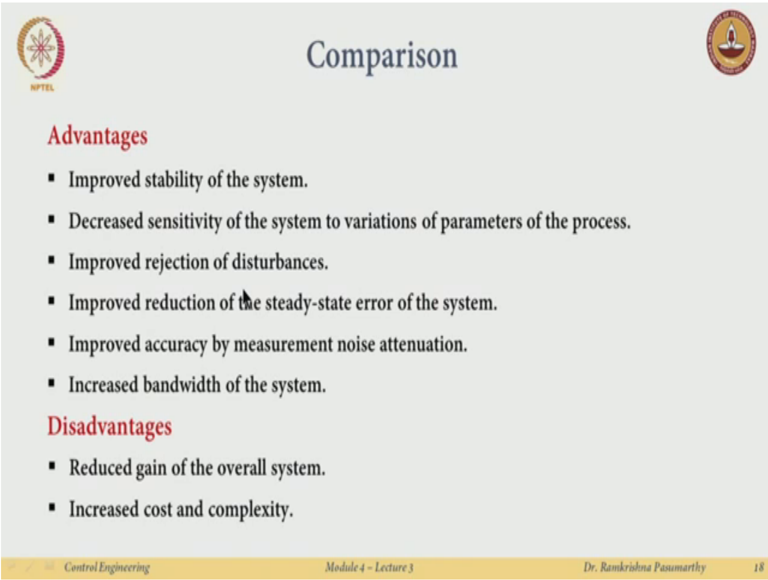


So, what does what are the advantages is I can see that I can improved stability I can have decreased sensitivity to parameters I can deal with disturbances I can deal with noise right and we could also increased bandwidth we will see this when we do the frequency response of the system. But then my overall gain of the system reduces right because I have some multiplicative factor over there so and go here and this kind of gives some attenuation which may not always be desirable. So, that is one of the drawbacks and of course, I would have increased cost to deal with and sometimes bit of complexity to, but of course, you know the advantages are lot more compared to the disadvantages.

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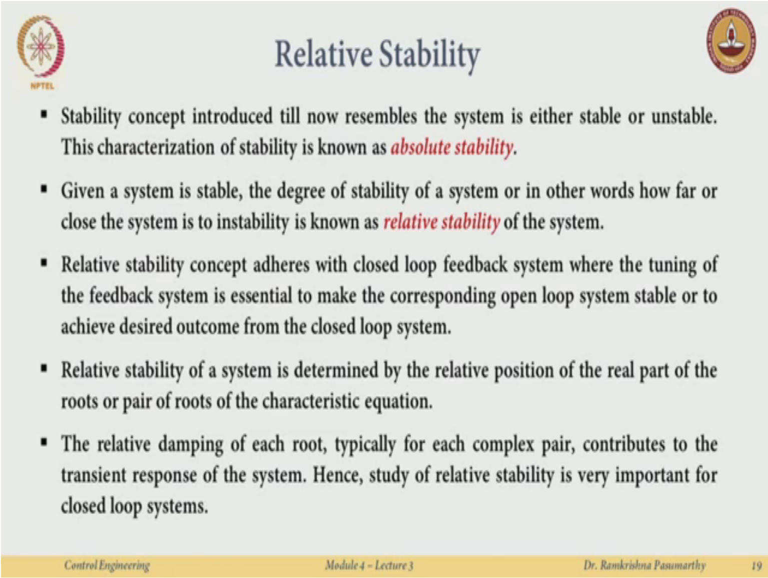


So, sometime now we have seen you know how the system could be sensitive to change in parameters right ah. So, we will see does that have any effect on stability and that is called relative stability. So, at the moment we just talked about is the systems stable or unstable right nothing more right. So, now, given a system can be defined some kind of a relative stability relative stability means can a small change in the parameter of the system make it unstable right and then that we will define it as the there also be some measures of stability that if my system parameters are between. And so I could still be stable right.

So, for example, if I am I am making cooking food and the you know I just download something from some [FL] dot com or you know all those all those websites and I say they say one table spoon of salt, but if I just put one table spoon plus 1 percent or minus 1 percent it does not really kill the taste right. But if I put 2 table spoons that might be that might be overfull if I put 0 table spoon that may still be very good so, but then there is some amount of relative stability right you can you can goes slightly higher or slightly lower and still maintained stability right stability in terms of the taste, but if I am making Tandoori chicken and it says put on the oven for twenty minutes, but if I say 22 minutes then I might you know not really get something very edible right. So, that little thing could be unstable over that 20 minutes 1 second could be ok.

But if I do 20 minutes plus 2 more minutes that could you know give me something very strange right. So, so these are these are things which you know changes in little parameters sometimes effect stability sometimes would not affect stability. So, if I am looking that I am know I am playing a cricket match and my team is chasing a score of something in some 20 overs and at the end of 15 overs I am you know I still need 40 runs with 5 wickets in hand I could still say well I can still win the game right because my required rate is not too much, but all of sudden if I lose 2 wickets right then that could be recipe for disaster I could go from a winning zone directly to a losing zone right.

So, these are things you know you are stable and you can know all of sudden become unstable and that is exactly what is the concept of relative stability. So, we do not really deal with food or sport here we deal with physical systems. So, what is what is it deal mean in our case right.

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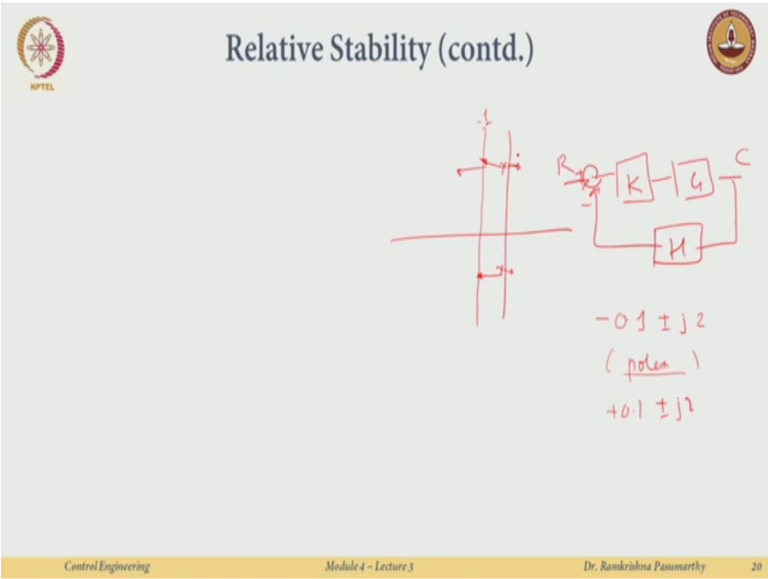


So, we look at our stability in terms of the location of the poles right well typically complex poles or real poles or whatever right. So, so what I said is in terms of systems if I say well I am I am here. So, these are these are my poles and I say I have system which typically would look like this K G H the plus a minus this is my R this is my C or the output and say I had design my system I work very hard and I say well my system is stable it has poles at plus minus 0.1 sorry not plus minus should only be minus.

So, I am given fantastic design problem and I just say this my poles are at 0.1 plus minus j whatever j 2 right I work very hard and I submit this as my thesis and I say my controller works because my poles are here, right. So, these are my poles and 2 days later you know my supervisor comes and checks it you know some something would have gone wrong there right you know my mass would have been a little different than earlier or you know some rats might have chewed on to something where my mass slightly decreases and I when I run this system again. So, my may be this you know these guys just jump here right or maybe I did any it any air condition environment the power to goes off and I am presenting this to the examiners or my supervisor in a different environment where the parameters might change and say oh what I see the response oh. So, it goes unstable right.

So, my poles could just be instead of minus 0.1 it would just be 0.1 plus minus j 2 this could happen right. So, and we will see the lots of cases where this will happen and therefore, when I when I design things I will say well I do not just I do not want stability I want thing like more stable right yes, yes, is like this right somewhere. So, so I want my poles to be somewhere here not just left of 0, but say at minus 1 or further left of left of minus 1. So, I want to be to have a little buffer for myself to do things right.

So, these are things which we can talk about you know in of relative stability now do we have any tools where I can say where we can we can actually quantify relative stability we have learnt the Routh table right. So, let us see if I could say something about relative stability may not be very useful in terms of exact analysis, but some preliminary analysis I could do about relative stability right. So, I have a closed loop characteristic equation where I just you know factorize at them in terms of n poles right. So, these are direct relation over here.

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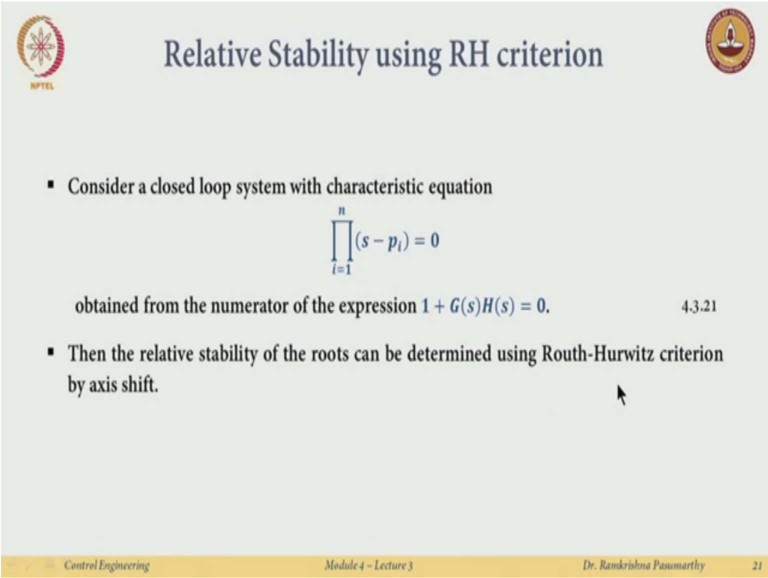


So, the relative stability can be determined by using Routh Hurwitz criterion by just shifting the axis right.

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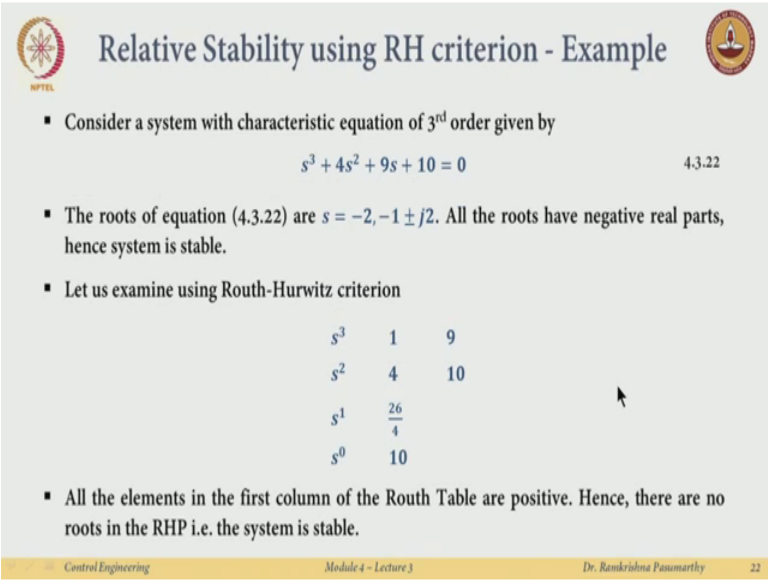


So, if I say start with this guys you know say consider a system a third order system and say is it stable I do the Routh thing and I say everything is positive no sign change and therefore, my system is stable.

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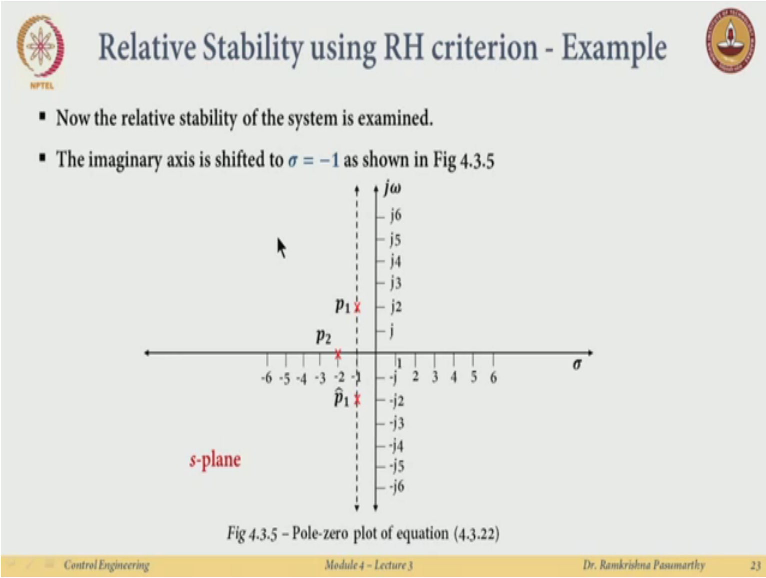


But these does not help because you know if my temperature changes slightly or some middle things goes wrong here and there then I go to unstable therefore, I will pose a slightly different problem to be a little more comfortable and say well R the plane. So, R all the poles over here to are they that left of minus 1 until now I was only interested in left of 0, but if I have at minus 1 I have a little margin here right. So, if the temperature increases which shifts the pole may be from minus 1 to 0.8 I am still stable right and therefore, I want to push them you know as slightly further to the left.

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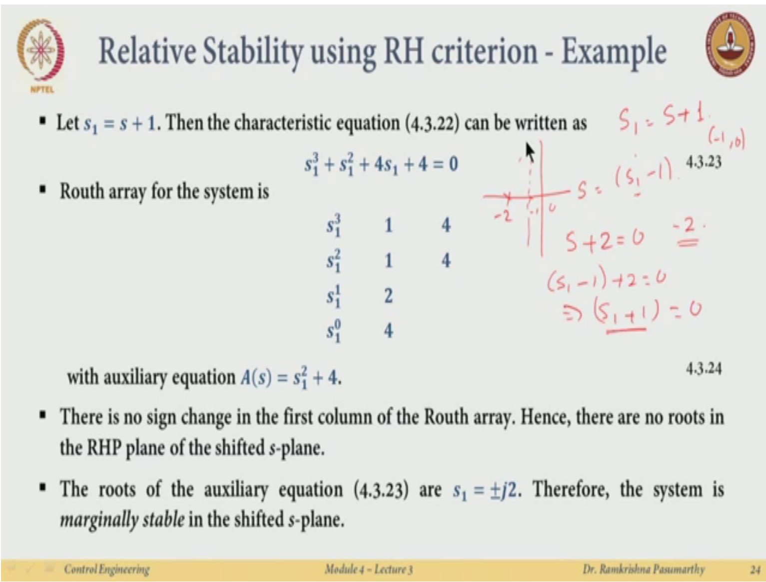

So, the way we checked is like this. So, let me define a new variable called s 1 which is s plus 1. So, my s now will become s 1 minus 1 and I rewrite. So, this is my new coordinate frame where the origin is minus 1 comma 0. So, if I take a characteristic equation which is s plus 2 equal to 0 which has a pole at minus 2 how will this pole look in like in the new coordinates. So, my s is s 1 minus 1 plus 2 equal to 0 this means I have s 1 plus 1 equal to 0 this is right. So, in my new coordinates sorry in my old coordinates I was at a distance of minus 2 from the origin if this is the new origin at minus 1 I should be at a distance of minus 1 from the new origin right. So, that is that is what this guy tells us.

So, simply put if I were to check for a general polynomial here. So, are the roots of this equation to the left of minus 1 just substitute this guy right. So, in the new coordinates s 1 I just show this substitution my characteristic equation looks like this now I do the entire Routh table thing for this right. So, there is a 1 4 1 4. So, this row will go to 0 and I will have auxiliary polynomial and I will just solve for the x 0 you know you know differentiate this and then I do all this things. So, first is there is no sign change and therefore, there are no roots on the shifted plane there are no roots which are on the right hand side of minus 1.

However there are 2 roots here. So, we can say this is in some sense called marginally stable with respect to this axis of course, in the original coordinates it will not have an have a have a sustained oscillatory behavior that we should be little careful of. So, I am just saying well it is not left of minus 1 whether a couple of guys were just sitting on minus 1 marginally stable with respect to this new shifted coordinate that is minus 1 and 0.

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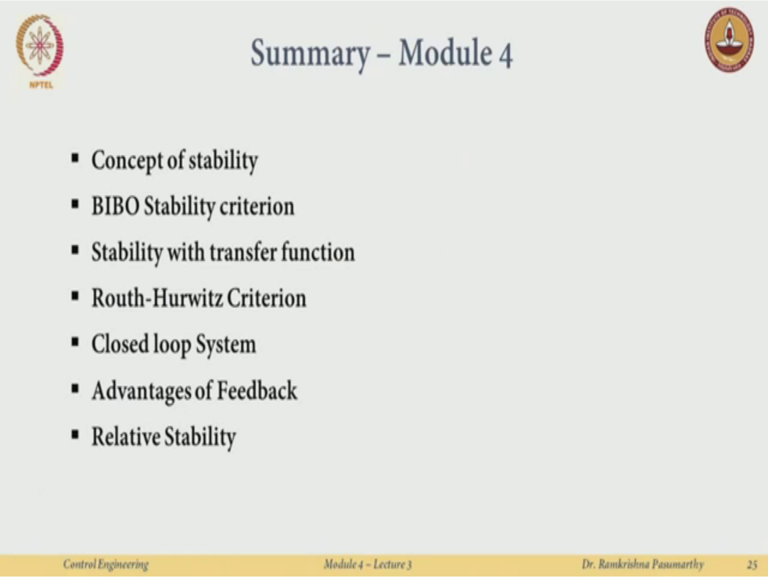


So, what we have done. So, far is we started with the concept of stability we defined some criterion for BIBO stability we also found out how we given a transfer function how I computed stability given then higher order polynomial we had an algebraic method to compute are there any unstable poles we looked at closed loop system advantages of feedback and some analysis on relative stability some very very basic analysis on how stable my system is. So, what we will do next is to focus on these kinds of things of relative stability and see if there are more sophisticated analytical tools which is called the root locus method we will see how my pole locations respond which changes to certain things in the systems.

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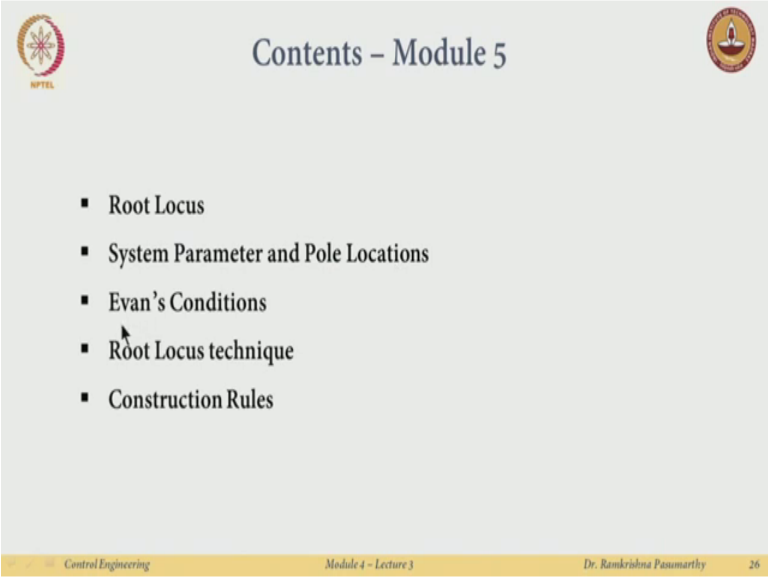


So, these are essentially you know conditions divide no derived by Evan and called the root locus techniques we will study some construction rules of this graph called the roots locus technique.

Thank you.