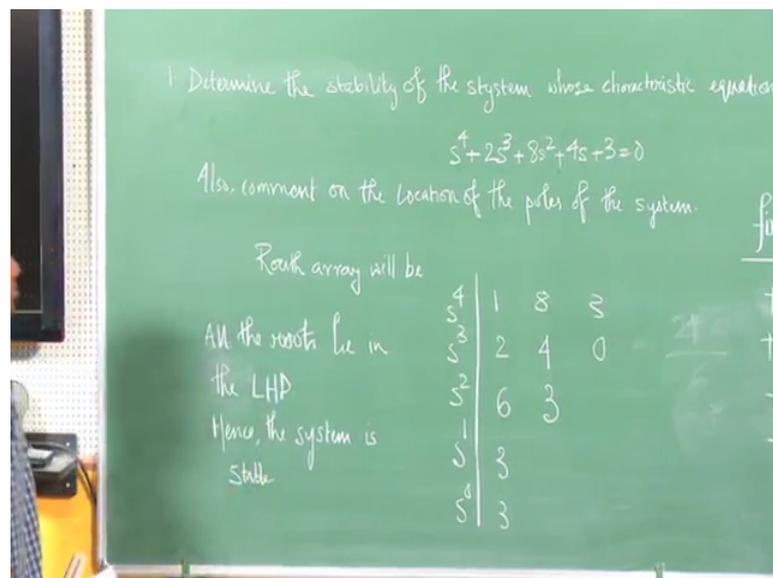


**Control Engineering**  
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**Module – 04**  
**Lecture - 17**  
**Tutorial**  
**Routh Hurwitz Criterion**

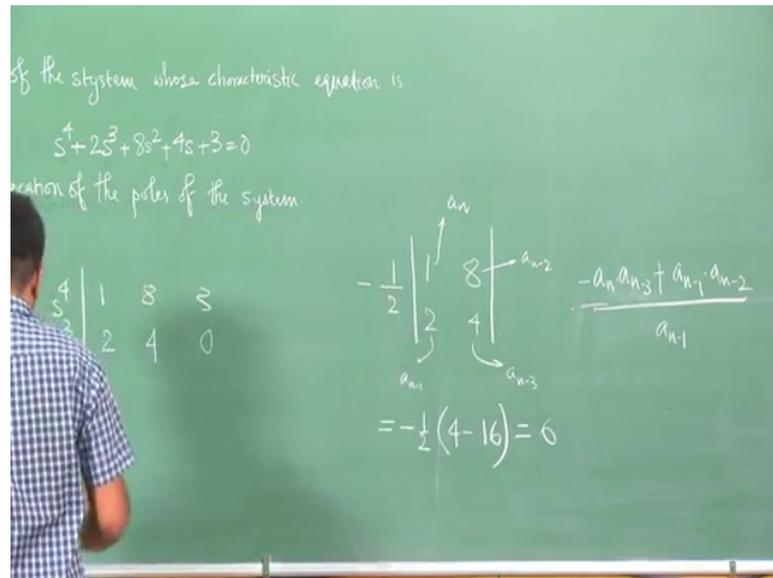
Hello everyone. Today we are going to do a tutorial session on Routh-Hurwitz stability criteria. So, we are going to do some problems and based on the theories, we studied till now like how to determine stability, how to find out unstable systems in using Routh-Hurwitz criteria. We will also see special cases one as zero as the first element and also zero in the entire row now. So, we start with the basic problem how to determine stability of a system.

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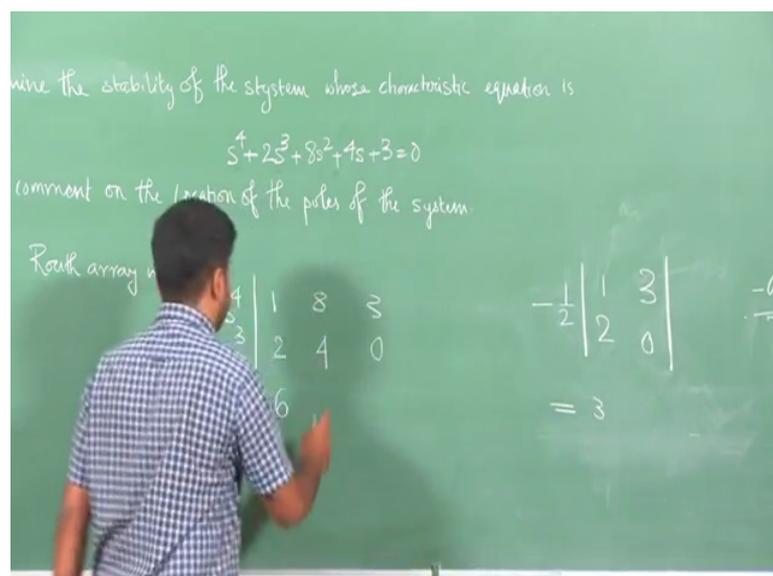
So, determine the stability of the system whose characteristic equation is  $s$  to the power 4 plus 2  $s$  cube plus 8  $s$  square plus 4  $s$  plus 3 is equal to 0. Also comment on the location of the roots of the system the poles of the system. So, this is our characteristic equation. Now, for the characteristic equation, the Routh array will be so we take first  $s$  to the power 4 then  $s$  to the power 3. Now,  $s$  to the power 4 - first one, alternatively 8 and then 3; and for  $s$  to the power cube,  $s$  cube, so 2 and then 4 and then 0; so as we saw that this term comes from multiplying these 2 minus multiplying these 2 divided by this factor.

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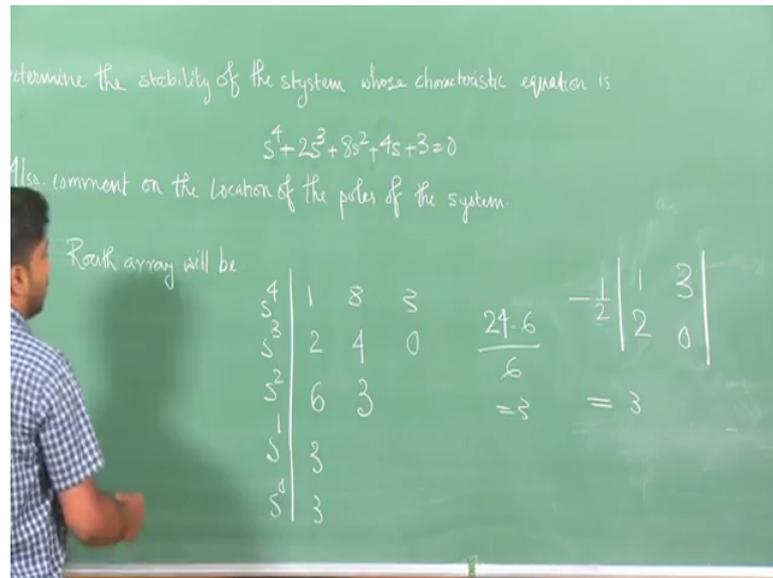
So, we can do this way that 1 by 2 divided by I just take this is the determinant; so 1 by 1, 2, and 8, 4. So, what we are doing basically this is a n. this is a n minus 2, this is a n minus 1, this is a n minus 3. So, what we are doing is a n into a n minus 3 minus a n minus 1 into a n minus 2 divided by a n minus 1 minus. So, basically the minus will come here and this will be plus, so that is what we will and. So, here we get minus half 4 minus 16. So, here it will be 3.

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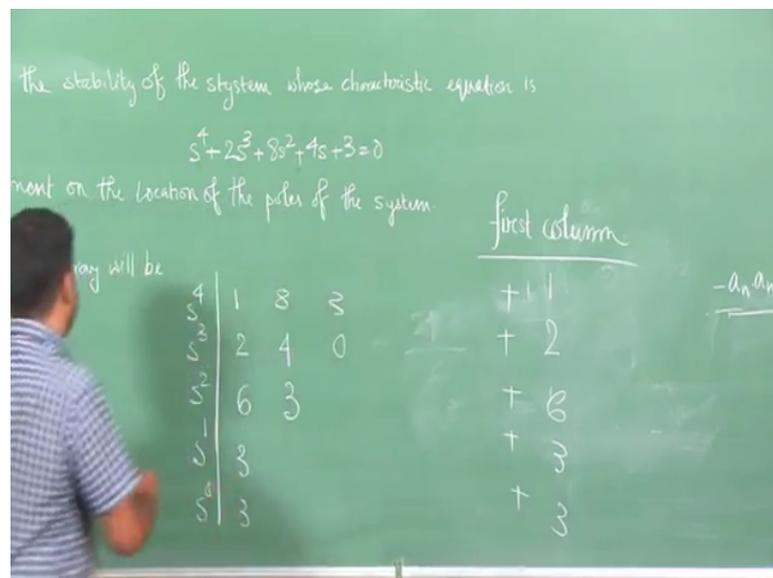
So, similarly if we do 1, 2, and 3, 0, we will get 3. So, here you get 3.

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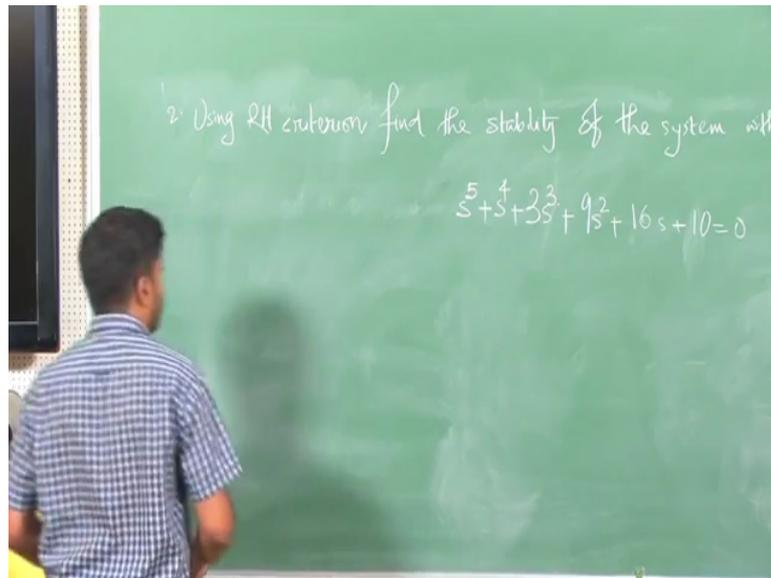
Now, s to the power 1; so similar way 6 into 4 minus 3 into 2 that means, and s to the power 0, we will be just 3. So, we see that there is no sign change. So, just let us write the first column only.

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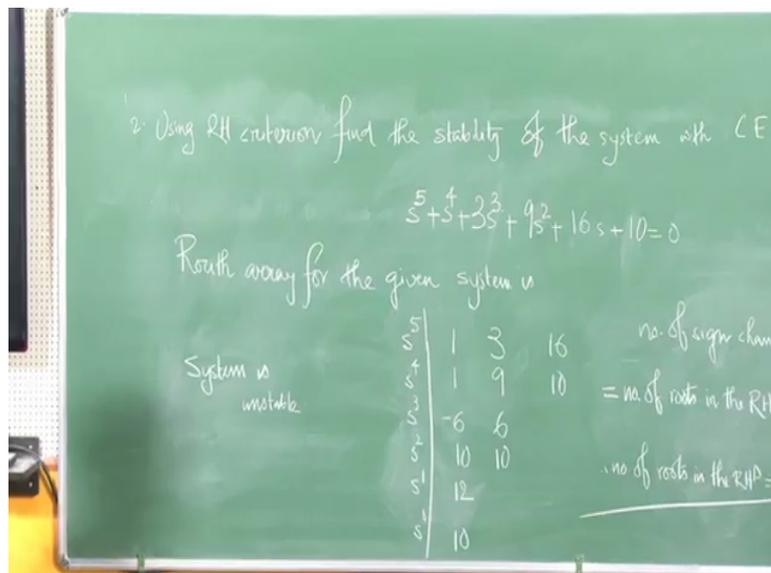
So, the first column is 1, 2, 6, 3, and 3. So, the signs are positive. So, all are positive. So, there is no sign change. So, we can say the system is stable, there all the roots lie in the left hand plane hence the system is stable.

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So, second problem we are going to do one the how to find out if the system is unstable or not. So, second problem will be, so using Routh-Hurwitz. So, this is RH means Routh-Hurwitz, find the stability of the system with characteristic equation  $s$  to the power 5. So, it will be 3 s cube.

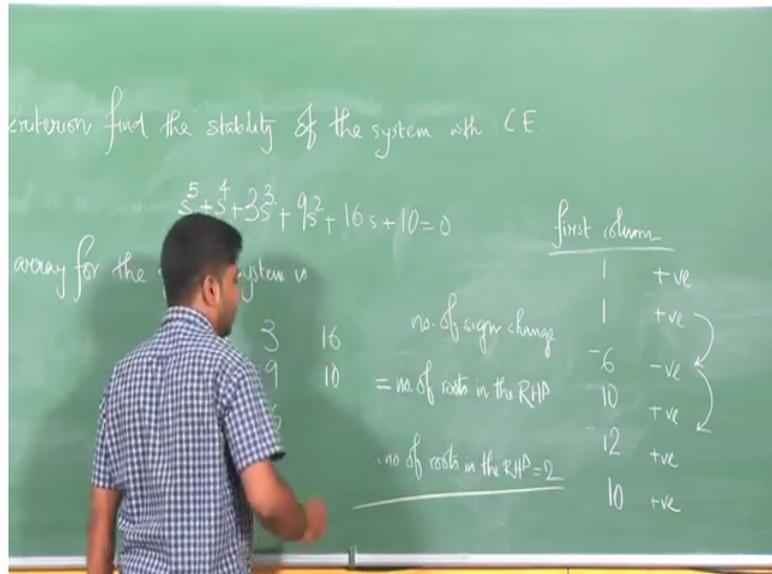
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So, Routh array for the given system is first  $s$  to the power 5, then  $s$  to the power 4,  $s$  to the power 3,  $s$  to the power 1. So, 1, alternatively 3, then 16 and here comes 1, 9, 10. So,

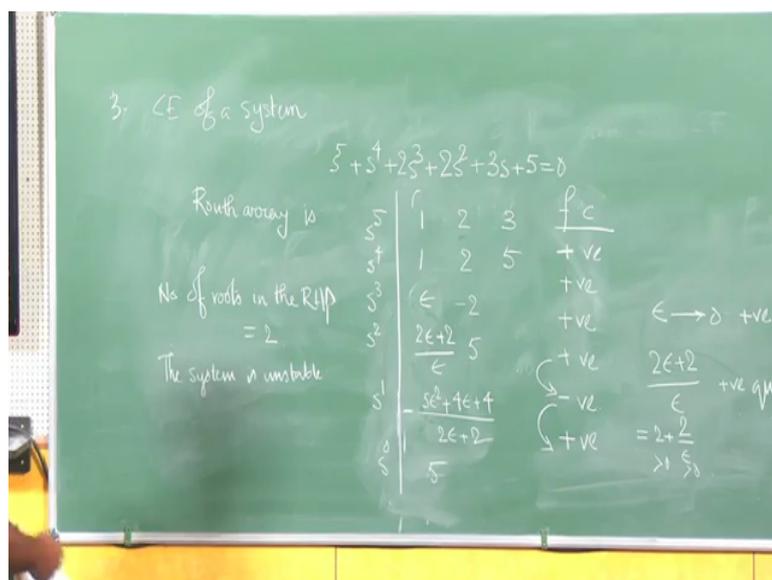
this point will be, so this is easy 3 minus 9, so it will be minus 6, 1 into 16 minus 10. So, this will be 6. Now, this is 10 into 6, 60 minus, so 12 and this is 10.

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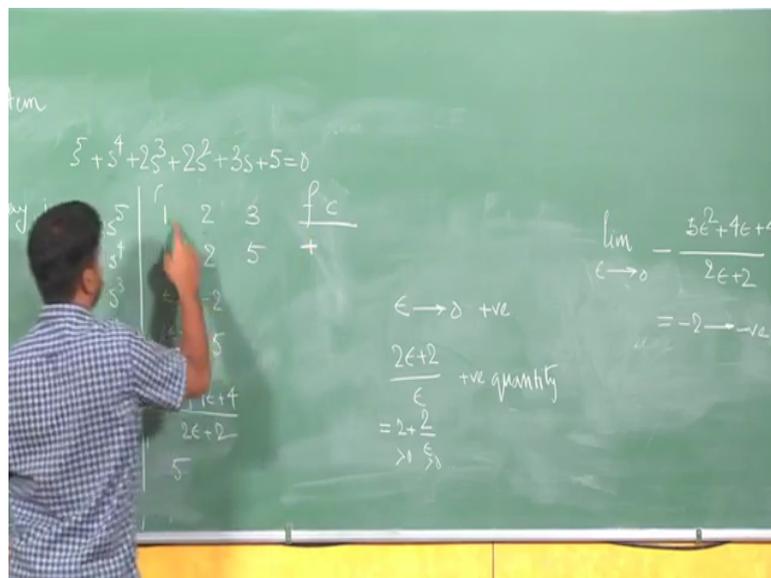
So, now the first column is 1 1 minus 6 10 12 and 10. So, positive, positive, negative, so there are two sign changes from here and from here. So, we learn that number of sign change is equal to number of roots in the right hand plane. So, number of therefore is 2. And the system is unstable.

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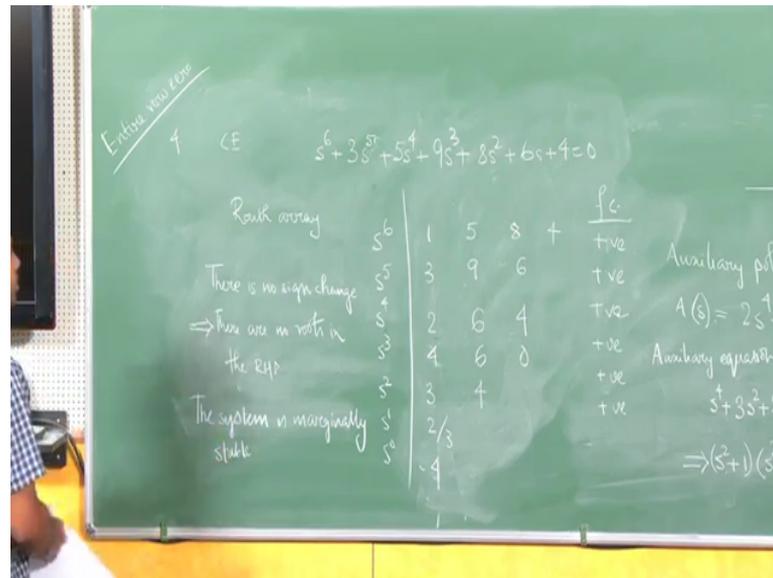
Now, we solve a problem if the first element of the row is 0. Let us consider the characteristic equation of a system  $s$  to the power 5 plus  $s$  to the power 4. Now, the Routh array is, so first will be 1, then 2 then 3, and will be 1 2 and 5. So, 2 minus 2 this will be 0; and 3 minus 5, this will be minus 2. So, we see that this term is becoming 0. So, what we learnt is we will replace it by a positive number, small positive number epsilon here. So, the 0 is replaced by small as positive number epsilon now. So, this will be epsilon. Now, so the next term become 2 epsilon plus 2 by epsilon, and here we get 5. So, calculating we find this will be 5 epsilon to the power 2 plus 4 epsilon plus 4 by 2 epsilon by 2, and  $s$  to the power 0 it will be 5.

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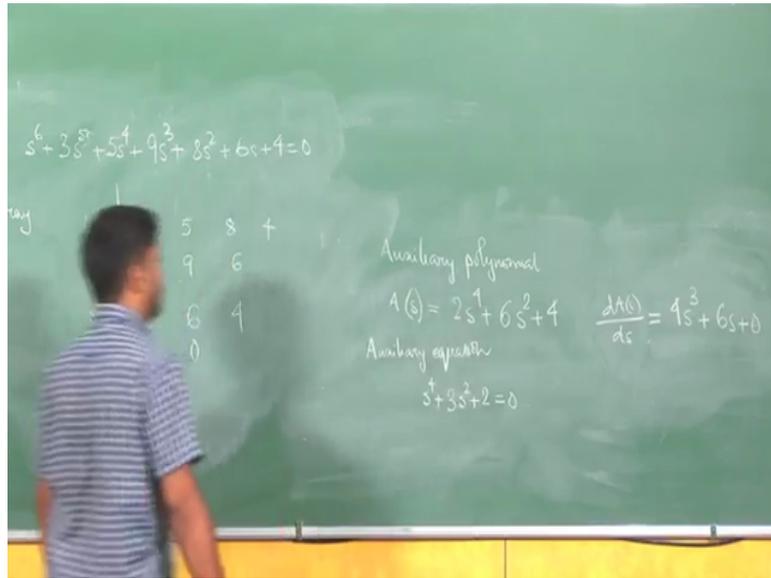
So, what we do is now we consider these terms, we check the signs considering that epsilon is a small positive number tending to 0, but it is positive then 2 epsilon by plus 2 epsilon, this is a positive quantity. Now, you can write it as 2 plus 2 pi epsilon. So, this is greater than 0 and epsilon is positive. So, this is also greater than 0. Now, the other term this one limit epsilon tends to 0 minus of 5 is equal to minus 2 that is negative. Now, we consider the first column first one is 1, so positive; second is positive, third one is epsilon we assume that epsilon is a small positive quantity, so this is positive; this is also we found positive. Now, this quantity we found that this is negative, so this is negative; and finally, 5 so positive. So, there are two sign changes here and here. So, number of roots in the right hand plane is equal to 2; and the system is unstable, yes.

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So, let us consider one example where the entire row is 0. So, how we use the auxiliary polynomial and find out the roots and determine the stability of the system. So, let the characteristic polynomial equation be given by  $s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$ . So, construct the Routh array  $s^6, s^5, s^4, s^3, s^2, s^1, s^0$ . Now, first comes 1, then 5, then 8, and then 4. And here is first element is 3, then 9, and then 6. So, here will be first element will be  $2 - 15$  minus  $9$  by  $3$  are  $2$ , and then  $6$  and last term is  $4$ . So, now, for  $s$  to the power  $3$  all the terms are  $0, 18$  minus  $18 - 0, 12$  minus  $12 - 0$ . So, I get here, so  $0$ .

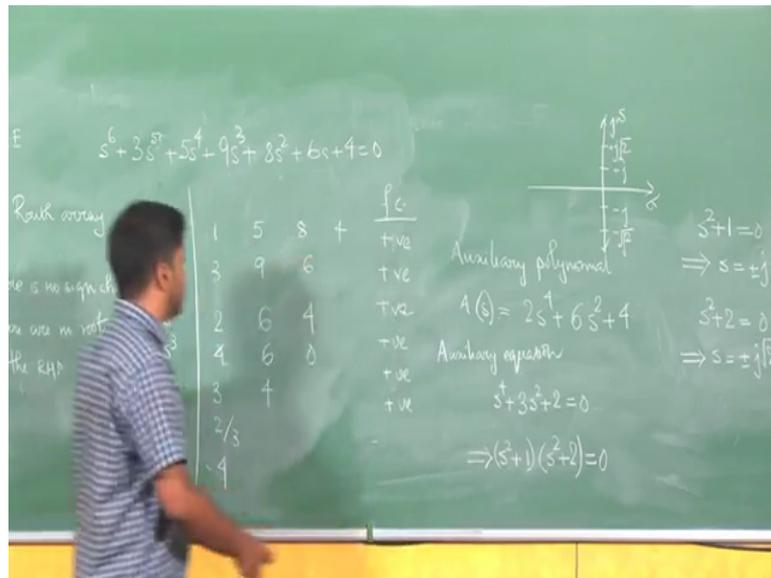
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So, what will learnt is we consider auxiliary polynomial  $A(s)$ , and we go to the immediately upper row, so which is for  $s$  to the power 4, so here it will be  $2s$  to the power 4 plus  $6s$  square plus 4. And the auxiliary equation  $s$  to the power 4 plus  $3s$  square plus 2 is equal to 0. So, we take common 2, so divide all other terms by 2 and we get this polynomial equation. So, what we now do is we find the derivative of the auxiliary equation that auxiliary polynomial that is  $dA/ds$  with respect to  $d/s$  that is  $4s$  to the power 3 plus  $6s$  plus 0. So, the coefficients where will be replaced by 4 and 6. So, 4, 6 and 0; for  $s$  to the power 2, the terms will be 3 and here will be 4. Now, so 18 minus 18, this only 2 by 3 and here it will be 4.

So, just consider the first column. So, there are no sign changes. So, all the terms are positive, positive, positive, positive. So, there is no sign change that means, there are no roots in the right hand plane, but for determine the stability of the system we need to find out the roots of the auxiliary equation, because we learnt that the roots of the auxiliary equation is also the roots of the characteristic equation. So, the roots will determine the stability.

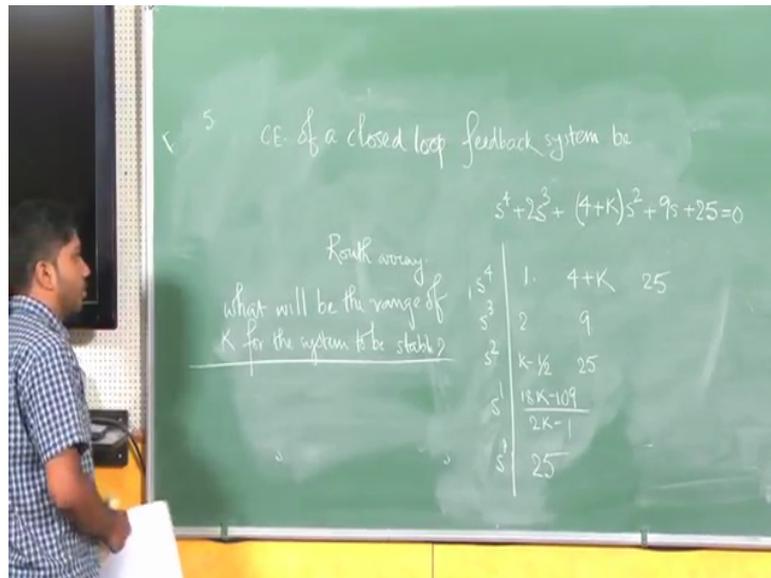
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So, here we can see that  $s^2 + 1$  into  $s^2 + 2$  equal to 0. So, for  $s^2 + 1$  equal to 0, we get  $s$  is equal to plus minus  $j$  into 1; and for  $s^2 + 2$  equal to 0, we get  $s$  is equal to plus minus  $j$  into  $\sqrt{2}$ . So there are 4 roots on the imaginary axis. So, if we construct we just draw the  $s$  plane and this is  $j\omega$  this is  $\sigma$ . So, and there is one at plus  $j$ , one at minus  $j$  and one has  $j\sqrt{2}$ , one has plus  $j\sqrt{2}$  square root.

What we learnt in the lecture of module 4 that if there are non-repeated roots on the imaginary axis, then the system will be marginally stable. So, here we find that there are 4 roots on the imaginary axis, but none are repeated. So, the system is marginally stable, so done; now, some problem on feedback. Now, we do some problems on the determining the gain of the feedback systems what we learnt in lecture 3 of module 4. So, how we will how we can use Routh-Hurwitz criteria and apply it in feedback systems to determine the gain. So, quickly we will do.

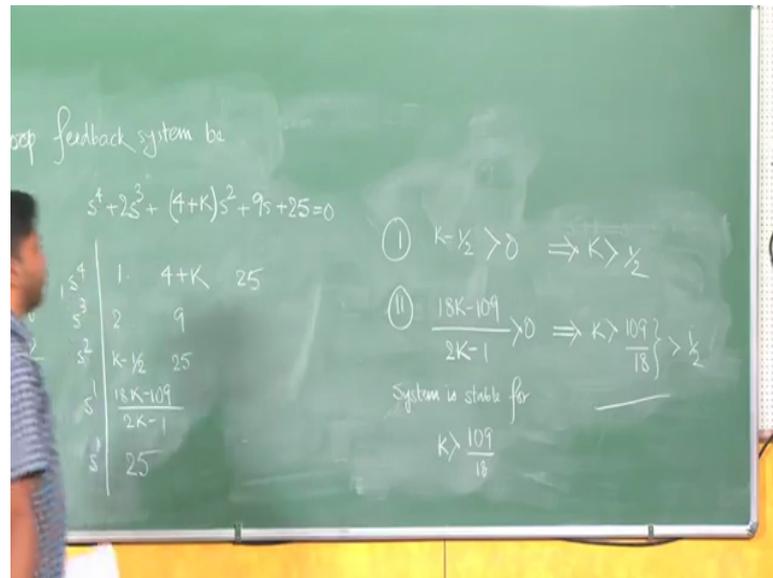
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So, this is problem number 5. So, let the characteristic equation of a close loop feedback system be  $s$  to the power 4 plus 2  $s$  cube plus 4 plus  $k$   $s$  square. So, first we construct the Routh array  $s$  to the power 4. So, first term will be 1, second term is 4 plus  $k$  and third term is 25. Now, here first term will be 2 and second term will be 9. So, here we get is  $k$  minus half and here 25.

Now, just after calculating we will find that this term is 18  $k$  minus 109 by 2  $k$  minus 1, and this term will be 25. Now if we want the system to be stable, so let us find out to what will be the range. So, what will be the range of  $k$  for the system to be stable? So, for stability, what we know is that all the terms should be positive.

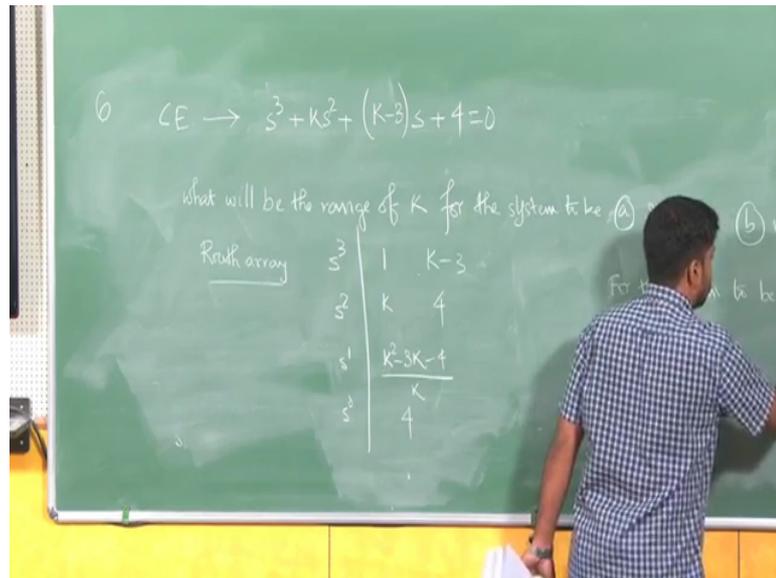
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So,  $k$  minus half first condition is  $k$  minus half should be positive that means,  $k$  is greater than half and second condition is. So, this term will also be positive. So,  $18k$  minus  $109$  by  $2k$  minus  $1$  is greater than  $0$  that implies that  $k$  is greater than  $109$  by  $18$ . Now, definitely this value  $109$  by  $18$  is greater than half. So, the system will be stable when the value of  $k$  is greater than this. So, system is stable for  $k$  greater than  $109$  by  $18$ .

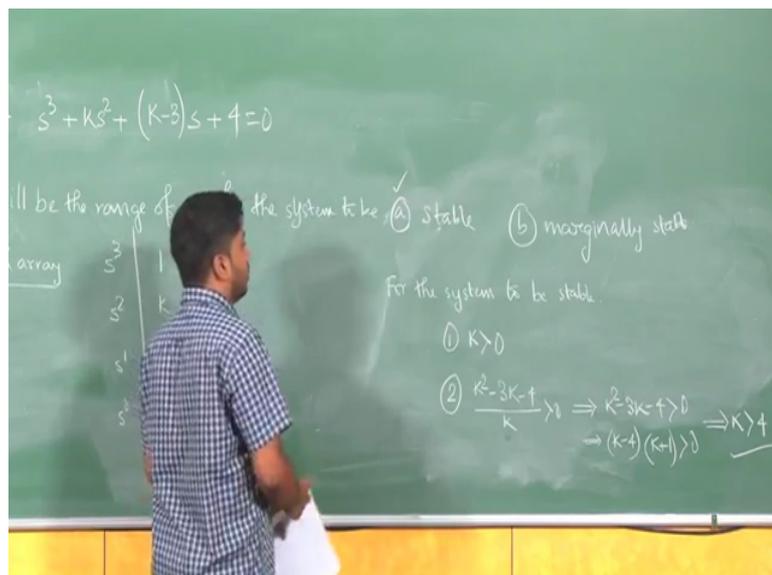
So, so similar problem if we just give a parameter to the system, and we tune it and find out how the parameter what will be the range of the parameter such that such that the system will be stable or marginally stable or unstable. So, we consider another example like that.

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So, characteristic equation  $s^3 + ks^2 + (k-3)s + 4 = 0$ . So, we have to find out what will be the range of  $k$  for the system to be stable or marginally stable. So, Routh array  $s^3$ , so first term is 1, then  $k-3$  and this is  $k$ , this is 4. So, this term will be  $k^2 - 3k - 4$  by  $k$ , and this term will be 4. So, for stability the conditions will be for the system to be stable.

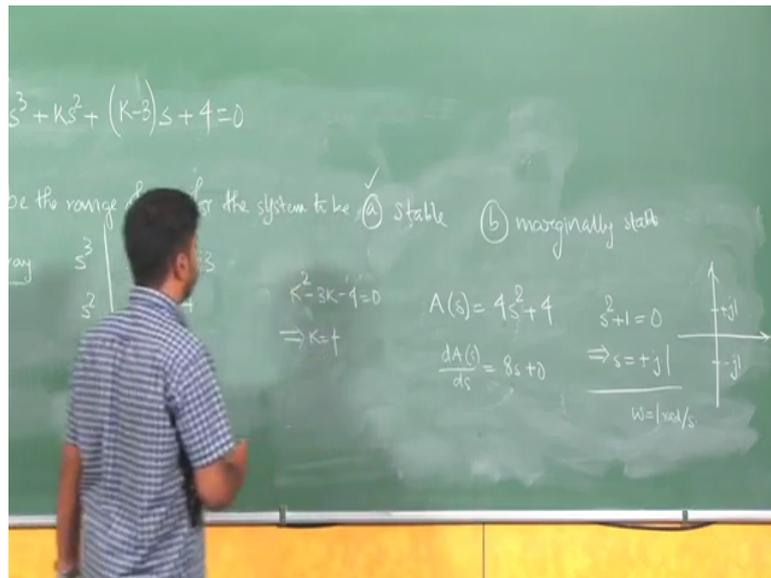
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So, all the elements of the first column should be of the same sign. So, it should be positive here. So,  $k$  first condition is  $k$  is positive and second condition is  $k^2 - 3k - 4 > 0$ .

$3k - 4$  by  $k$  is greater than 0. So, since  $k$  is positive, it implies that  $k^2 - 3k - 4$  is greater than 0 that is  $k^2 - 4$  is greater than 0. So, here we find that  $k$  is greater than 4 because  $k$  cannot be greater than minus 1 because that condition it lies between minus 1 and 0, so it does not hold first condition. So,  $k$  will be greater than 4 if the system is to be stable. So, part a is done.

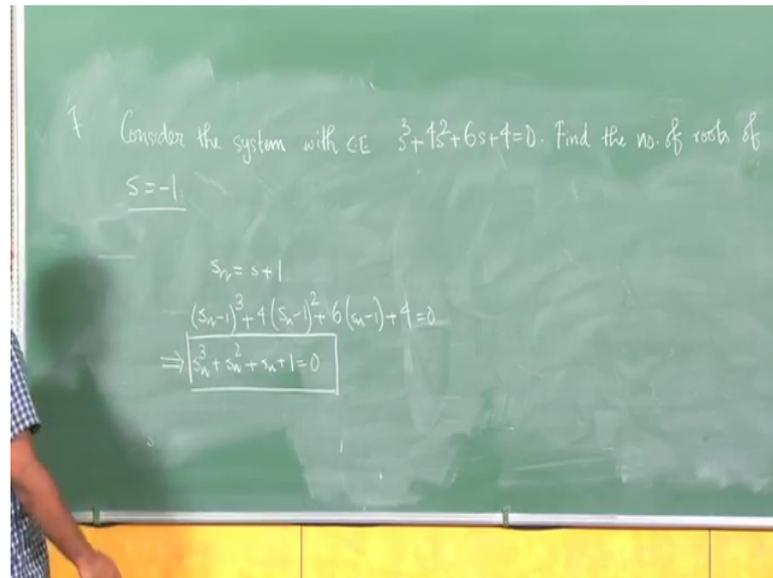
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Now, part b; so we now start we do part b. So, for marginally stable, this term will be 0 that  $k^2 - 3k - 4$  is equal to 0 that is when  $k$  is equal to 4. Now, the auxiliary polynomial  $s^2 + 1 = 0$  because  $k$  is equal to 4 and  $dA(s)/ds = 8s + 0$ . So, replace  $k$  by 4, replace this by 8, and this will be 4. Now, we find that the all the sign elements of the first column are of the same sign.

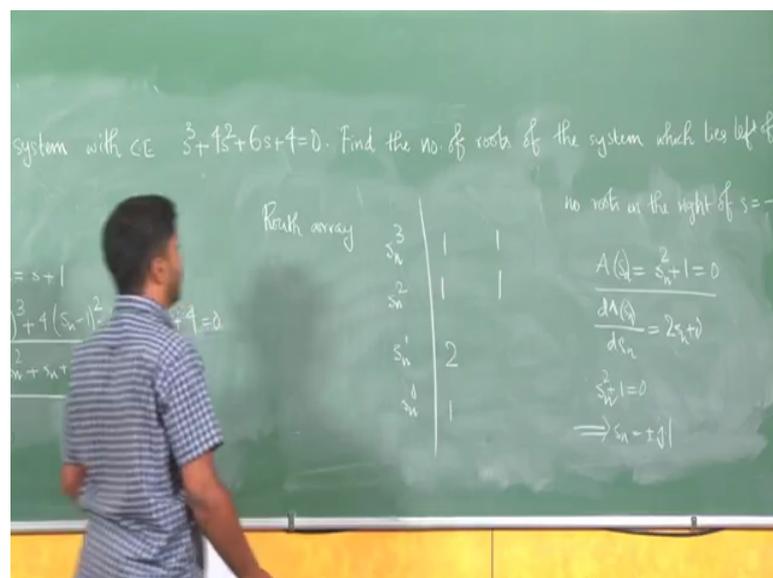
So, there are no roots in the right hand plane and the roots of the auxiliary equation  $s^2 + 1 = 0$  taking 4 common equal to 0,  $s$  is equal to plus minus  $j$ . So, there are two roots on the imaginary axis at plus  $j$  and that minus  $j$ , and hence the system is marginally stable with oscillating at the frequency of  $\omega$  is equal to 1 divided by second. So,  $k$  will be 4.

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Problem number 7: so you find the number of roots which are in the left of the axis,  $s$  is equal to minus 1. So, how to do that, so we have to find out how many roots are there in the let  $s$  is equal to minus 1. So, we consider a dummy variable  $s_n$  such that  $s_n$  is equal to  $s$  plus 1. Now, if we replace, if we right the characteristic equation in terms of  $s_n$ , so it will be  $s_n$  minus 1 whole cube plus 4  $s_n$  minus 1 whole square plus 6  $s_n$  minus 1 plus 4 is equal to 0. So, if you calculate you will find that the characteristic equation in terms of  $s_n$  will be  $s_n$  to the power 3 plus  $s_n$  to the power 2 plus  $s_n$  plus 1 is equal to 0.

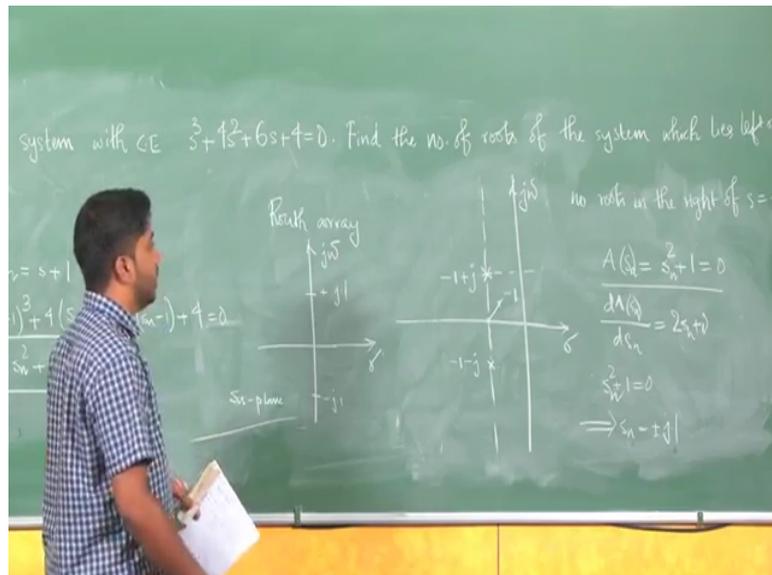
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Now, Routh array for this system, Routh array for the new equation is  $s^n$  to the power 3  $s^n$  to the power 2  $s^n$  to the power 1 was 0, so 1, 1, 1, 1. So, the entire row is going to be 0. So, we consider the auxiliary equation  $s$  is equal to  $s$  to the power 2 plus 1 is equal to 0. So,  $dA/ds$  is equal to  $2s$  plus 0. So, this is 2 and this will be 1. So, all the elements of the first column are positive. So, there are no roots on the right hand side of  $s$  is  $n$ , it is no roots,  $s$  is equal to minus 1.

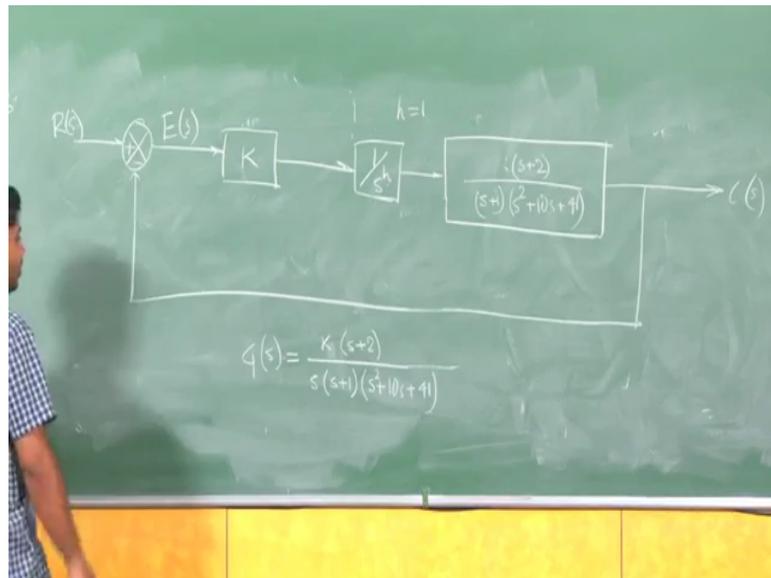
Now, for the auxiliary equation if we find the roots, so we will get sorry, so  $s^n$  square plus 1 is equal to 0 that is  $s^n$  is equal to plus minus  $j$ . So, what we get is that there are two imaginary two roots on  $s^n$  is equal to  $s^n$ ; so imaginary axis of the shifted plane  $s^n$ . So,  $s^n$  plane  $s$  plus this is  $j\omega$  sigma here. So, there will be two roots.

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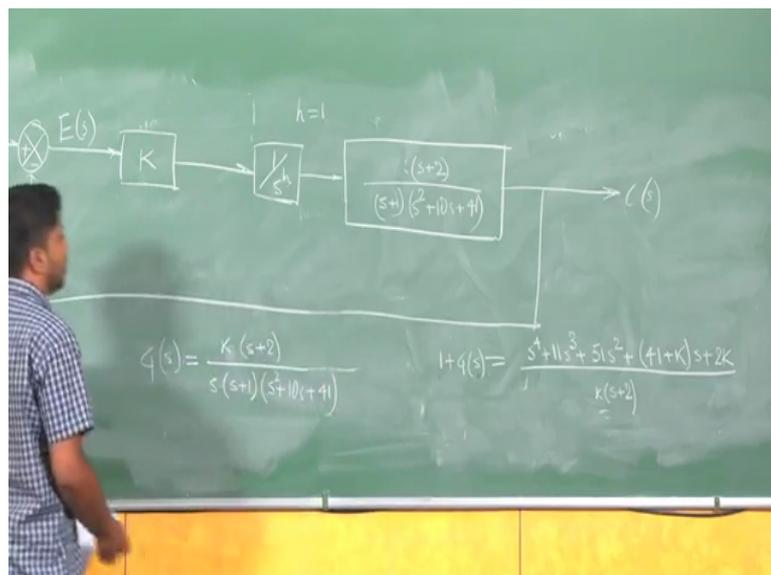
Now, if we see the entire picture meaning the  $s$  plane that is not in  $s^n$  plane, but in  $s$  plane what we are going to see find that. So, this is our original sigma, this is our  $j\omega$ . So, we have shifted the axis to minus 1, and there are two roots at minus  $j$  plus minus 1 or better to write minus 1 minus  $j$  minus 1 plus  $j$ . So, there are two roots here. So, this kind of axis shift is very useful in phase systems where there are number of imaginary sorry complex roots. So, more than one complex pair of roots are there. So, if we are drawing to determine the relative stability of the system based on how the roots affect on the damping of the system, so this axis shift method using Routh-Hurwitz criteria will be helpful.

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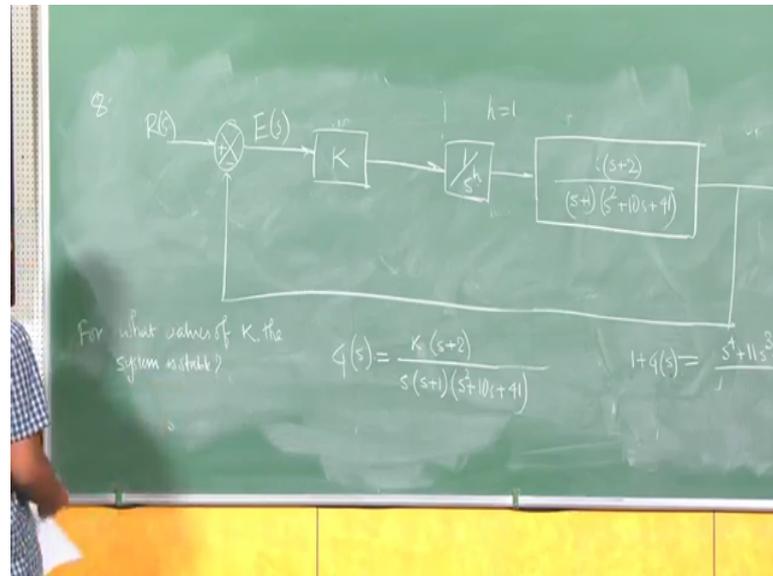
And the final problem for today's tutorial. So, final problem for today's tutorial. Now, let us consider the same problem we did in the module 3 for time response. So, this is the error  $E(s)$ . So, in that problem for time response what we found is if the steady state error is going to be 0, but not for ramp input. So, each will be one. So, we can modify the loop gain of the system as  $G(s)$  is equal to  $k$  into  $s$  plus 2 by  $s$  say  $s$  is equal to  $1$   $s$  plus  $1$   $s$  square plus  $10$   $s$  plus  $41$ .

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And the characteristic equation will come from 1 plus G s. So, the numerator of this 1 plus G s it is so the numerator of this term, so this part. So, this is the characteristic equation of the system.

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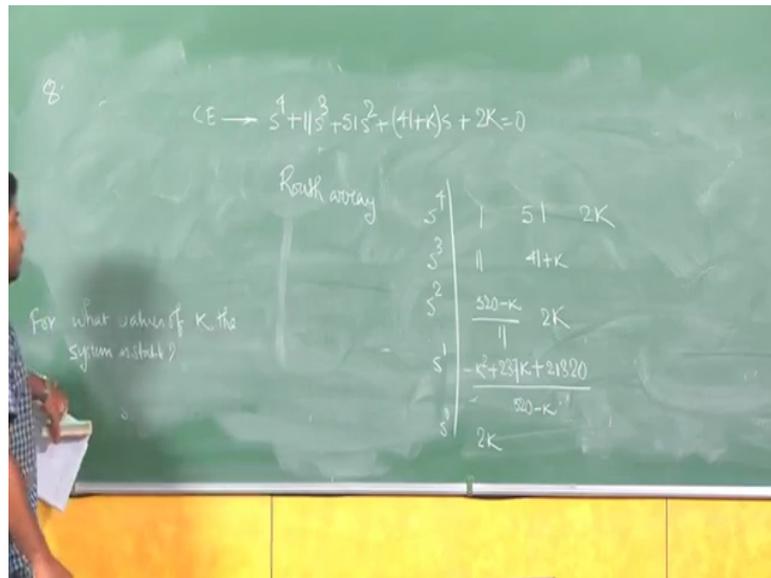


So, you have to find out what will be the range of k, for what values of k the system is stable. So, numerator will be the characteristic equation.

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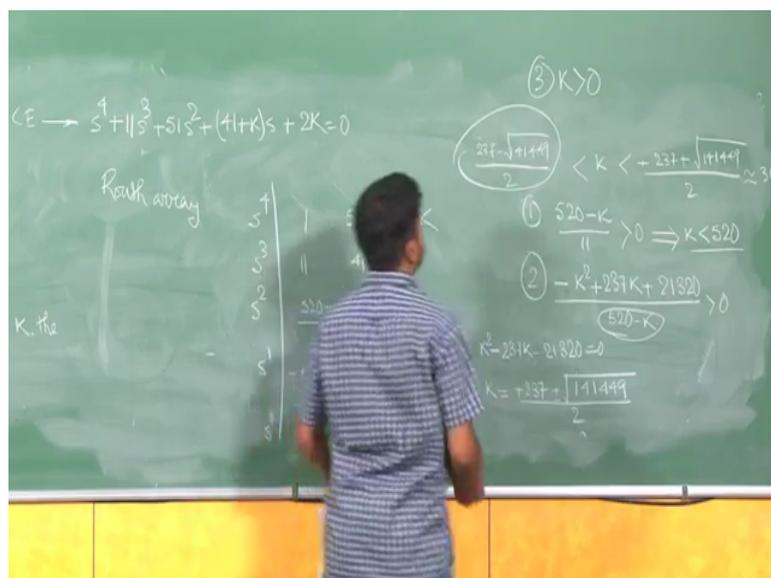
Yeah.

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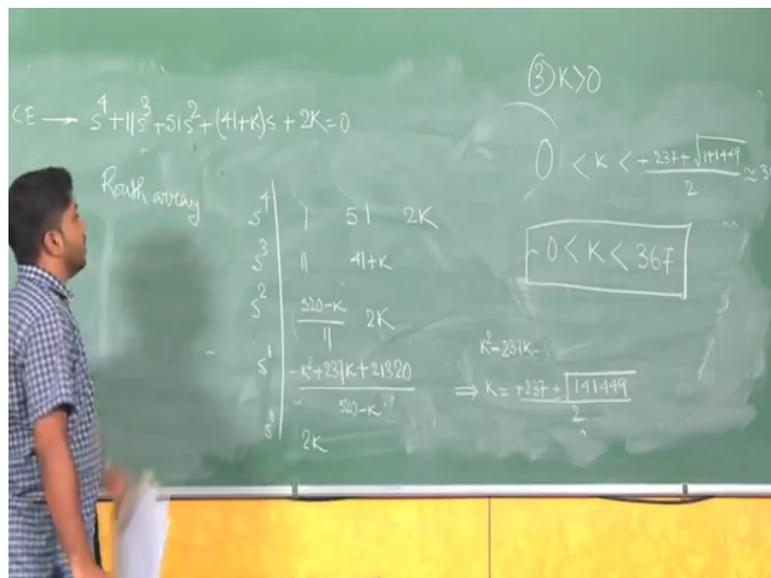
So, the characteristic equation is the numerator of the part of 1 plus G s that is CE is  $s$  to the power 4 plus 11  $s$  cube plus 51  $s$  square plus 41 plus  $k$   $s$  plus 2  $k$  is equal to 0. So, we form the Routh array. So, the first term will be 1, second 51 and finally, 2  $k$  and here it will come eleven and 41 plus  $k$ . So, this term will be 520 minus  $k$  by 11 and this is 2  $k$ ,  $s$  to the power 1 will be minus  $k$  square plus 237  $k$  plus 21. So, we have to find the value of  $k$  for with the system is stable. So, there are three terms related to  $k$  in the first column.

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So, the first condition will be 520 minus k by 11 greater than 0 that implies k is less than 520. Second condition is minus k square plus 237 k plus 21320 by 520 minus k will be greater than 0. So, from condition one this term is greater than 0. So, we have to make this term greater than 0. So, let us consider the quadratic equation k square minus 237 k plus minus 21320 is equal to 0. So, the roots of this equation plus 237 plus minus just calculate this you will get 141449 divided by 2. So, if this term is going to be greater than 0, so what we need is k should be k should lie between plus 237 plus root over 141449 by 2 and 2 237 minus root over 141449 by 2 what we find that. So, this is almost equal to 367. So, the upper limit of k now reduces to from 520 to 367.

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Now, the third part third condition comes from write it here third condition is k is greater than 0. So, this part is negative. So, this part will be replaced by 0. So, the range of k is now 0 greater than less than k less than 367. So, within this range, the system will be the entire close loop feedback system will be stable that is it.

Thank you.