Control Engineering
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Module - 03
Tutorial 2
Lecture - 14
Solving Problems on Time Response Analysis and Specifications

So, we just do some problems related to time response and its specifications. So, said with us very simple looking problem.

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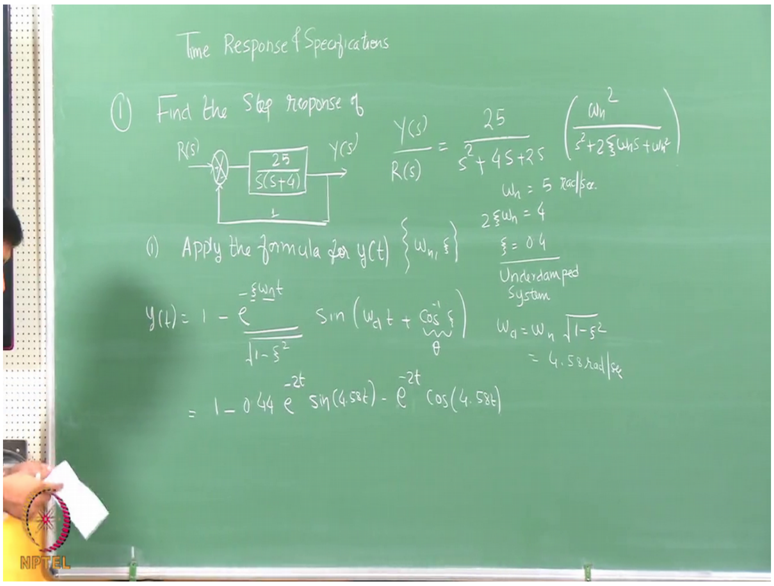

So, is to find the step response of the system. So, I have a reference which is a unit step 25 S S plus 4 there is a 1 here and this is my Y of S. So, there could be 2 ways to do this right one is right applying the formula for y of t if you remember this essentially was based on omega n and zeta. So, first let us we have a close loop looks like the close loop transfer function here Y of S over R of S is 25 S square plus 4 S plus 25 and this is of the form omega n square over S square plus 2 zeta omega n S plus omega n square right. Therefore, I can easily compute omega n is 5 and 2 zeta omega n is 4 and therefore, zeta can be easily computed to be S 0.4 and this will be radians per second.

And then we can clearly see that this is an under damped response or an under damped system and I know that the response just like this y of t is 1 minus e power minus zeta omega n t over square root one minus zeta square sine omega dt plus cos inverse zeta which I had this I had termed this as theta in earlier classes right. And then once I know so I know the zeta I know the omega n I could compute omega d as omega n square root 1 minus zeta square this turns out to be 4.58 radians per second and I do all the computations and I just end up with this expression 1 minus sorry that I will just skip the intermediate. So, there should be easy to easy to computed this way.

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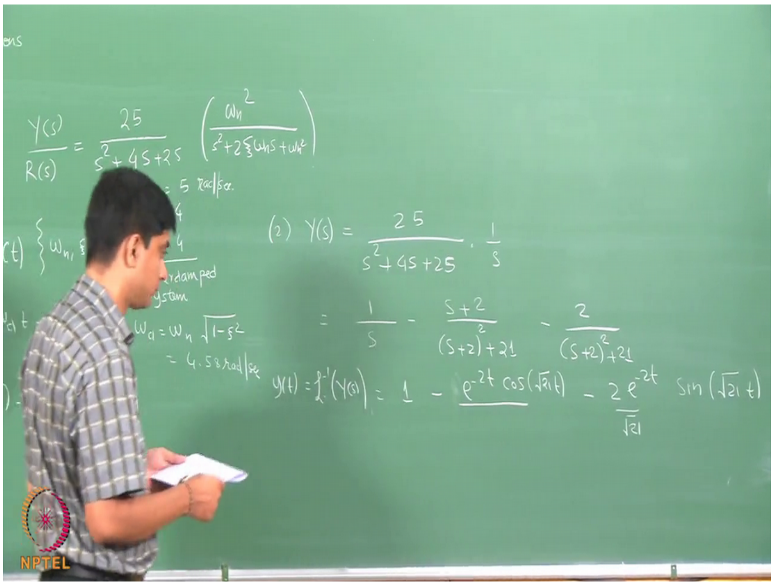

Another way to do it I know Y of S is 25 S square plus 4 S plus 25 and so on of S and let me just write this down in this way. And so this would be 1 the inverse Laplace transform y of t would be inverse Laplace of Y of S and this would be 1, this would be e power minus 2 t over cos and this is equal to this because I get compute directly by the formula or using the inverse Laplace transform right based on whichever is comfortable.

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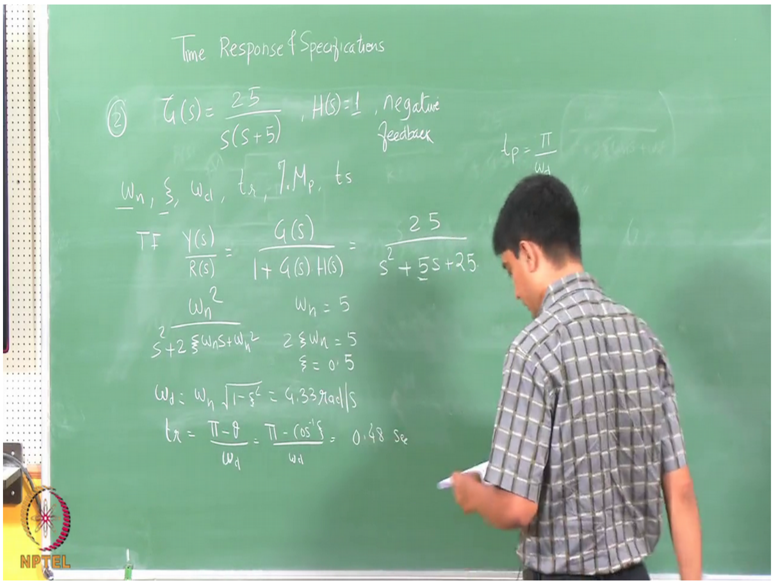

So, next problem I am given G of S is 25 over S, S plus 5, H of S is 1 and I have a negative feedback now here I have to find out what is omega n zeta omega d rise time percentage over shoot and settling time. So, the transfer function of Y of S to R of S is G S over 1 plus G H that would be 25 over S square plus 5 S plus 25. So, I just compare this with the general form of omega n square plus S square plus 2 zeta omega n S plus omega n square gives me omega n is 5 and 2 zeta omega n is 2 zeta omega n is 5 and therefore, zeta is 0.5.

So I know omega n I know zeta I know omega d is omega n square root 1 minus zeta square and that will be 4.33 radians per second and I have the rise time computed as pi minus theta over omega d and theta was defined as cos inverse zeta over omega d and this is 0.48 seconds. Peak time we computed this as pi over omega d.

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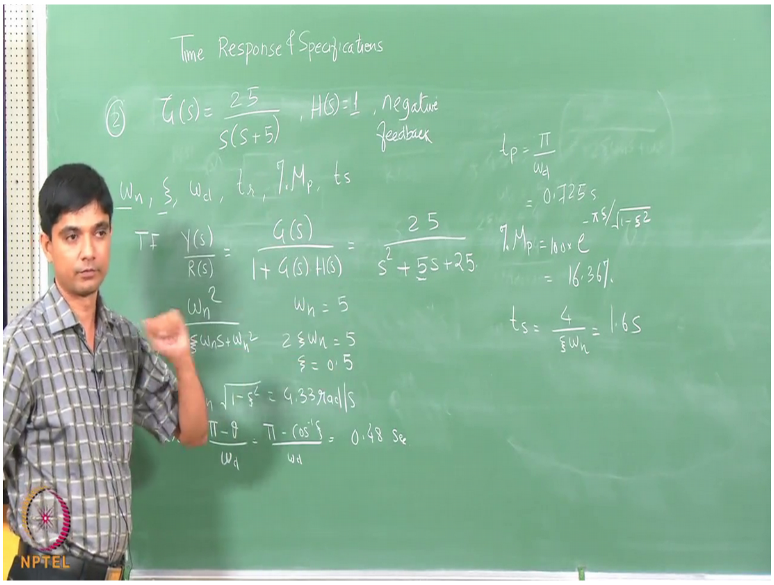

So, omega d is, so here and that will just be 0.725 seconds or and then the percentage over shoot was depently directly depending on the damping that was given by e power minus pi zeta 1 minus S square multiply by 100 together percentage. And I could compute this as 16.36 percent and finally, the settling time for the 2 percent tolerance criterion is 1.6 seconds and quite straight forward right. Once you right down the close loop and compute what is omega n and zeta then everything else is for fixed in to the formula right.

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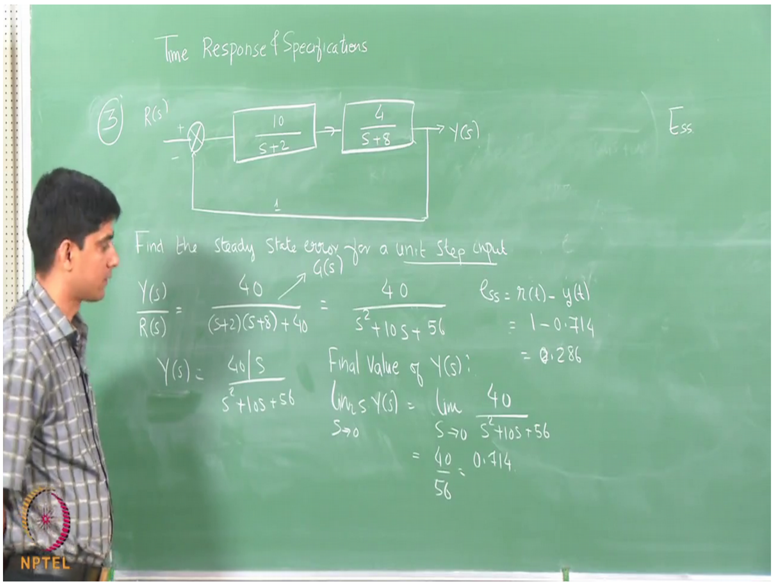

The third problem is well I am given a system like this 10 over S plus 2 4 over S plus 8 a plus a minus thus this is 1 Y of S. In the problem we use to find the steady state error for a unit step input, if I done this 3 or 4 lectures ago I would have done you know is calculate first the transfer function Y of S R of S is 40 over G over 1 plus G S that would be S plus 2 S plus 8 plus 40, 40 over S square plus 10 S plus 56. So, 2 weeks ago I would just do this right. So, 40 over S over S square plus 10 S plus 56 and then I would do, Y of S over R of S this is my transfer function and then I would compute what is a final value I am not really interested in finding the entire signal in with the time, but I am only interested in finding a several steady state error.

So, what is the final value of y? So, from the final value, you know a final value says limit S go into 0 S times Y of S for a step input this would be limit S tend in to 0 S times Y of S. So, Y of S sorry this (Refer Time: 13:07) just go right. So, Y of S would be 40 over S, S square plus 10 S plus 56. So, I have 40 over S square plus 10 S plus 56 and this would be 40 over 56 which is 0.714 and the steady state error is the actual signal r of t or the actual reference minus the actual output. So, is 1 minus 0.714 is 0.286, 0.286.

But since these are slightly longer process, but now I know directly how to calculate the steady state error given the type of the system right. So, I have if I look at this, this was called the G of S it is a type 0 system and I am trucking a unit step and I know that there will be some position error of course, this is tells me already that. So, how do I compute that?

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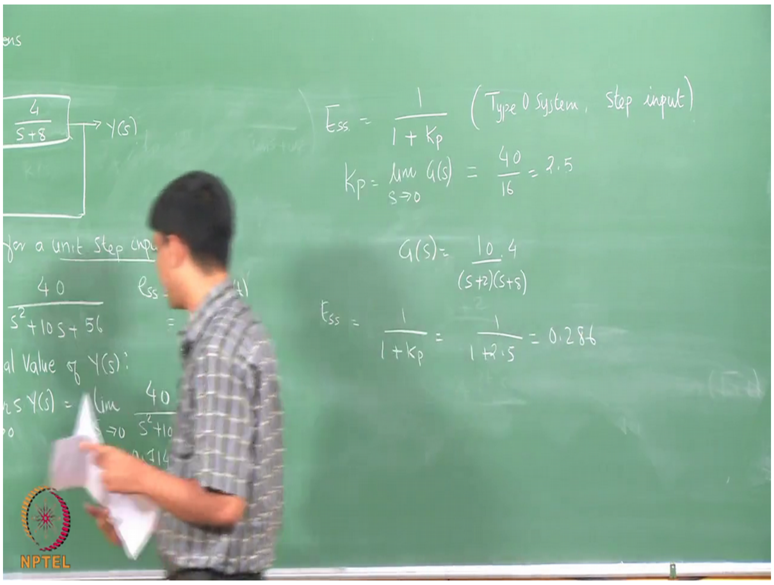

So, E SS is was 1 over 1 plus K p and this is for a type 0 system this is step input and then K p was limit S going to 0 G of S right that was what is G of S. So, is there 40, so G of S is this one right. So, G of S and this is looking at the open loop transform function this 10 over S plus 2 over S plus 8 and I have a 4 here.

So, limit S going to 0 G of S was 40 by 16 is 2.5. So, this is my K p. So, the steady state error is 1 over 1 plus K p is 1 over 1 plus 2.5 and that can be easily computed to be 0.286. So, the advantage here is I do not really need to compute the close loop transform function I can just look at the loop transfer function or the open loop transfer function and then compute this things directly. So, again this is the need to the same thing not surprise.

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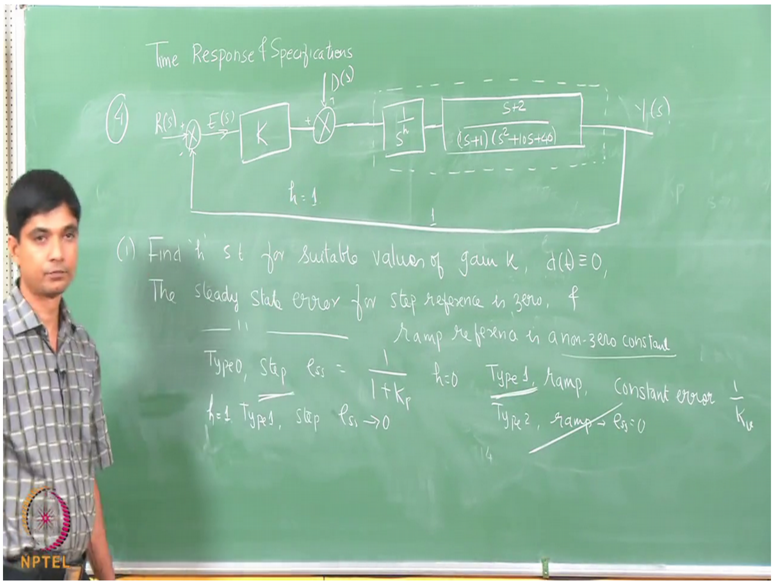

So, in the last problem. So, I am given a system is looks like this R of S, E of S I have given K I have a certain disturbance and then I have a part which looks like this 1 over S times H where H is any number or any integer value. I have S plus 2 over S plus 1 S square plus 10 S plus 40 right this. So, this is my entire part as looks and a negative feedback as usual a plus here a minus here which is so 1 here. So, this is my D of S this is my K and so on and this is my 1. So, in the first part I would want to find H such that for suitable values of a gain K additionally let me assume that the disturbance is identically equal to 0. So, the H should be such that this steady state error for step reference or step input is 0 and in steady state error again for a ramp reference is an on 0 constant which means we just interested in finding the type of the system.

So, let us recollect quickly. So, if I have a type 0 system and the input being a step my E Ss was 1 over 1 plus K p this was again for h equal to 0. So, this step input is what I am looking at that the steady state error for a step reference is 0. Now when h equal to 0 I would have some steady state error this is not 0. So, I should at least have h to be 1, h to be 1. So, that I have a type one system and then the step input then E S as goes to 0 that we know now is that the answer well if just this was the statement I could say that h could be one or even more h also be 2, h could be 2 and this statement would simply satisfied that is the steady state error was would be 0 for type 1 type 2 type 3 and so on. However, the second statements is that in addition to this steady state error for a step input to be 0 it should also have a nonzero constant error for a ramp input or for a velocity input.

So, what was the nature of for this thing? So, if I have a type 1 system and the input is a ramp. So, I know that a type 0 system will not will be able to track a ramp and the error is infinity and H is equal to 0 is ruled out anywhere. So, it has set out with h equal to 1 type 1 system a ramp input I know that there is a constant error given by I think 1 over K v or something and we know how to how to compute that K v. So, the answer would be that h well it of course, cannot be 0 it can be 1 or greater because of the second statement in a third statement restrict it only to be 1 because if I have a type 2 system then the error is not a one 0 constant it will go into 0 right at a type 2 if I have a type 2 system and the ramp input my E SS goes to 0 right. So, this is this is not allowed. So, I am just a type 1 system and therefore, the answer is h should be equal to 1.

So in the second part we will do something little more interesting.

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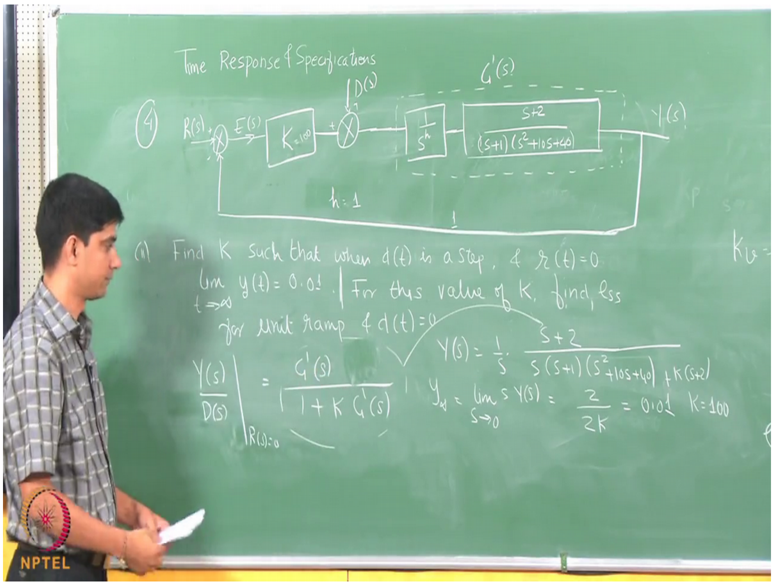

So, let me call this block as G prime of S and the second part what I am I want to find a K such that when there is disturbance signal d t is a step and the reference r t equal to 0 and I want to find the limit t going to infinity y of t should be equal to 0.01 right. So, I want to find a gain K such that when there is no reference and a disturbance which is a step that y of t should go to 0.01. Now after this for this value of K for this value of K find the steady state error for unit ramp and d t equal to 0 right. So, let us do the first part and here find a K which under a step disturbance and no input will give me an output at steady state at 0.01. So, this my Y of S over d of S when r is 0 text is formed is G prime of S, D of S plus 1 plus K G prime of S and we saw how to drive this transform function in the previous tutorial.

So, now, what I would found is d of S is a step right. So, Y of S is 1 over S which is my step disturbance and I have S plus 2 S, S plus 1 S square plus 10 S plus 40 plus K S plus 2 right this entire thing here expands to this one say for computation. Now why at infinity is again apply the final value theorem S 0 S times Y of S and I do all the math and I get this is 2 over 2 K and this 2 everything else disappear and this envier at infinity should be equal to 0.01 right. So, the question was find a gain K which for d being a unit step results in a steady state y of 0.01. So, we solve this and I get at the value of K is 100.

Now in the second part assume there is no disturbance I take this value of K to be 100 I will also this is a first part so I am not interested in what is the value of h at the moment, for this so this 100 was computed. So, we get h will actually be equal to one right we keep the value of h from the previous one and then compute the value of the gain K such that the output is 0.01 and that K was computed to be 100. So, I know what is the h now, I also know what is the K.

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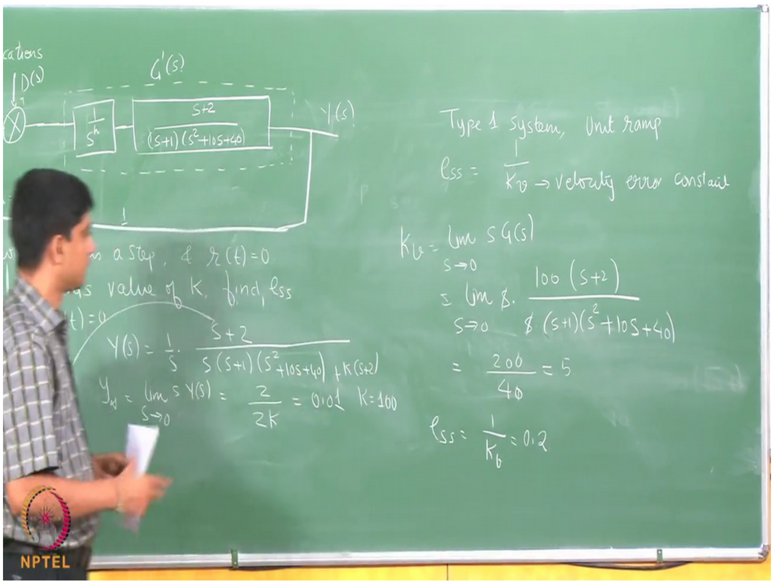

So, for these 2 values of H and K I want to find this steady state error for a unit ramp and assuming d t equal to 0. So, this is again a type one system and I have a input which is a unit ramp and I know that the error is definitely not equal to 0 and its some constant value. So, let us compute what is that constant value. So, our steady state error is 1 over K v and K v is the velocity error constant. And what was K v? K v was limit S going to 0 S times G of S K t into the math limit S going to 0 S times. So, if this entire G now, now this should be S times K is 100 in the numerator I have S plus 2 denominator H equal to 1. So, I have one pole at the origin S plus 1 S square plus 10 S is 40. So, I can do all the computation.

So, this guy goes away and in the numerator I am left with 200 and in the denominator I am left with the 40 and this is my K v right this is 5 and E SS therefore, is 1 over K v and that is 0.2. So, just say straight forward using the formulas you need trick is to how to get this transfer function form d to y putting r equal to 0 finding the appropriate value of H and then finding K, again by using the final value theorem and then using just the formulas for steady state error given that the system is type 1 and the input is a unit ramp. Yeah.

Student: (Refer Time: 28:40).

Sorry.

Student: (Refer Time: 28:43).

This is just Y of S.

Student: (Refer Time: 28:47).

Oh sorry d of S will go away. So, or I will just do this sum right this is Y of S over D of S is so there will be no D here and that D will be taken care of here and all this we assume here that that the R is 0 right.

Thank you.