Control Engineering
Dr. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 03
Lecture - 02
Time Response Analysis of Systems

Hello everybody. So, in today's thing, we will start a new topic called time response analysis of systems. So far what we have learnt is given a system how would you model that, model that using basic conservation laws from voltage laws to current laws of force and so on. And from that we could derive models based on transfer functions if we have systems which are interconnected transfer functions then we had techniques of how to get the overall transfer function via block diagram reduction or the signal flow graph techniques. So, what we will learn today is once I have a model. So, what could I do with the model or how does a model behave.

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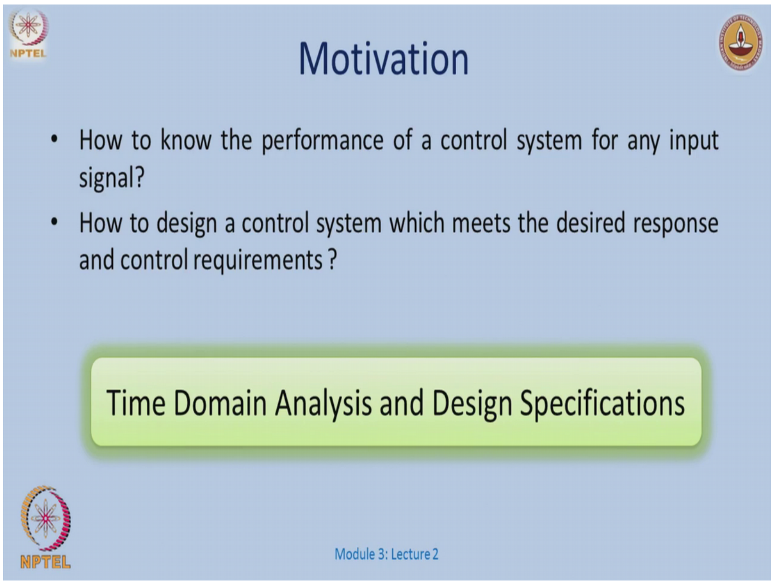

So, the first thing is after I obtain the model how to know the performance of a control system for any input signal. And next is how to design a system which meets some desired response it could just be some natural response or even to fit a control requirement.

So, how do we visualize this? So, given a system or given a model. So, let me say when we are subject to driver's license test right. So, the driving inspector does not really test us for all possible conditions on the road, which could include high speed on free way it could include heavy traffic on the city lanes, it could involve chaotic traffic with buffalos, running around kids playing cricket on streets we do not get asset for all those things right. So, instead what the driving test does is under standard Indian conditions. So, we will just have a little pre defined track which you have to drive to pass your learners test. In that track you have several things right I have to go straight you have to take a sharp turn you have to make a figure of 8, you have to take a reverse you have to park and several other things these are the standard things which we will encounter while we drive on roads.

In a similar way if I am being interviewed for software engineer right. So, I go as a model my model could be my b tech degree with a CGPA of 8.65 with a several list of courses. Now I want to hire you as a c programmer right. So, I will not give you a real time project to work on immediately right or I would also want to test how would you react or how would your performance be on a certain project or a certain deliverable. Then I would ask you some basic questions on sorting algorithms, it could be things related to pointers link lists some functions interconnection with a database and so on and based on. All these things I will get a estimate of how your performance could be while handling a real time project.

Now, this could differ from person to person right. Either you could be a fast programmer right you could write a program code very fast, you could some other person could take a longer time, but run write a code which is more efficient. So, all these I would get to know with some basic standard test signals. Similarly, what will happen is when we design a control system or we analyze a control system or a model we will subject it to some standard test signals, because we would not be able to emulate a real time control input kind of signal. So, these tests signals will tell me how my system will react under changing conditions. The conditions for a system could be well I have kind of a shock which could be an impulse, I could have a time varying signal which could be modeled as a ram signal I could have just a sudden disturbance which could be tested with the step signal and so on.

So, throughout this we will see how my system would behave, system could be of different nature right. So, how my system would behave based on certain standard test signals. And this one broadly constitutes the entire time domain analysis and design specifications.

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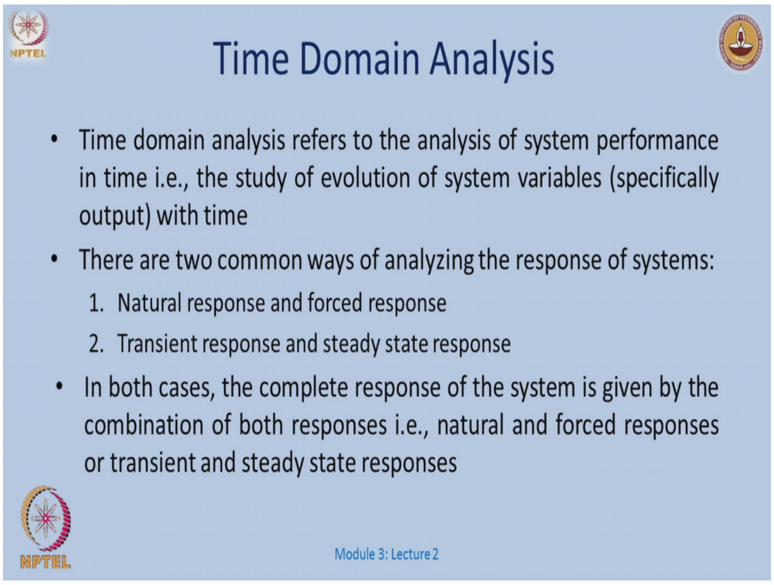

So, the time domain analysis refers to analysis of the system performance in time that is how my system variables particularly the output will vary with time right. So, how long does it take for a signal to reach a certain value which I want like a speed of a car? You say whenever you see this car related programs in this news channels usually they come on Sunday morning, you say oh this you know Audi a 4 or a 6 they go from 0 to 100 in 5 seconds, somebody goes in 6 seconds my own little car would take over 30 for 40 seconds that sometimes it possibly would never reach.

So, how; what describes the evolution of the system variables? So, in standard terms there could be 2 kinds of response, right. The natural response and forced response also classified as the transient response and the steady state response. So, I will shortly define what these things mean, but in both cases the complete response is given by combination of both the natural response and the forced response as well as a transient and the steady state response.

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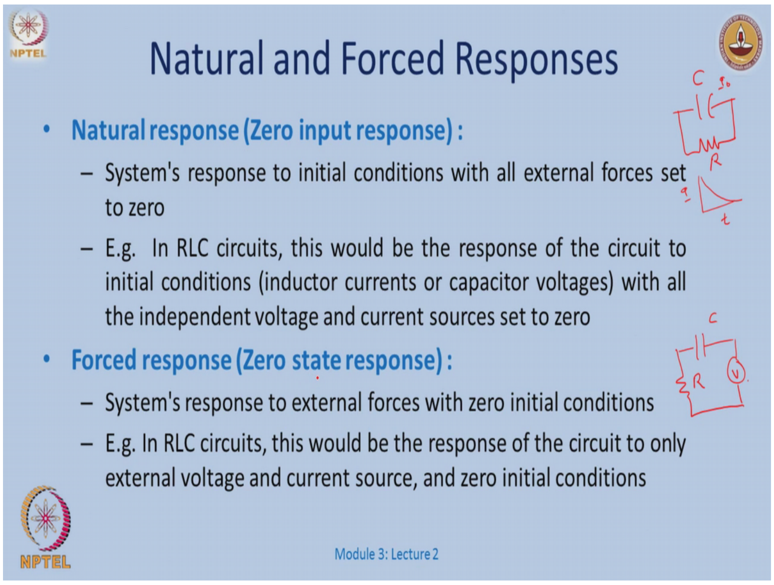

So, what is the natural response? It is it also is referred to as a 0 input response which means the systems response to a certain initial conditions with all external forces set to 0. So, if I take a simple RLC circuit, this would be the response of the circuit with initial conditions that inductor could be could have some initial current or a capacitor could have certain voltage, with all independent voltage and current sources set to 0 which means there will be no SML sources.

Say for example, I have a capacitor and I have R C with some initial charge I would call Q 0, the natural response would be that the charge if I plot with time, no possible when the current that entire charge gets dissipated through heat energy in this systems. That is the natural response I get (Refer Time: 07:12) for the elements an inductor with initial charge and so on. The force response is the systems response to external forces again assuming that all initial conditions are 0. So, again if I look at this circuit and I add a voltage source here, together with the resistance it will have a different kind of response, we will shortlist about these kind of systems mean and how these kind of responses would look like.

So, here there is this external voltage source here there is no external voltage source right. So that is a little distinction between the natural response and the forced response it is also called the 0 state response.

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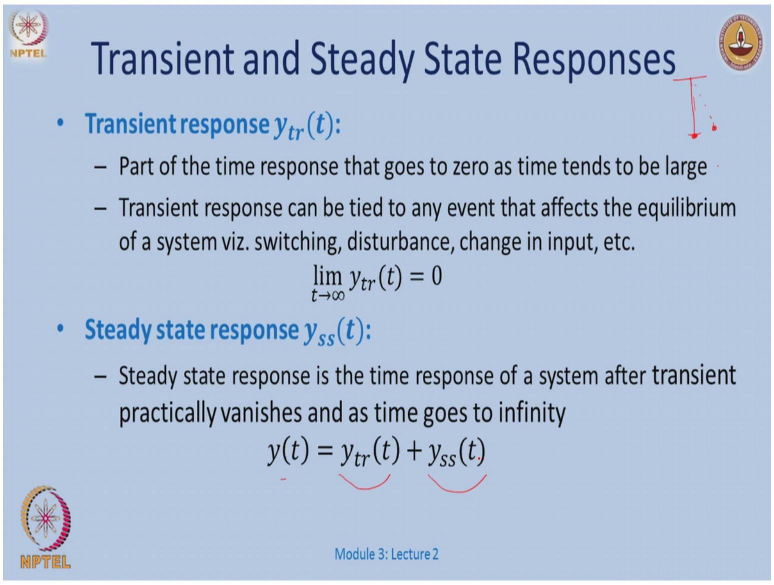

Similarly, I have 2 classifications in terms of the transient response. So, all this analysis when I make this for the statements I would be referring to systems which are stable right. And I will define formally what is stability a little later, but for the understanding of this particular module it is sufficient to know the following definition that stability means. So, if I am at an equilibrium position take my pendulum is just at this equilibrium position, if I pertubate slightly or I start from this initial condition it will eventually come back to this it is natural state or the equilibrium position.

So, that so all systems which are subset to some external perturbation which then retain their original configuration would be called as stable systems. That definition is you know for now right. You can think of it again the simple as the example of a pendulum right if just somebody comes and just pushes this it just goes back to the equilibrium position same like an alarm clock which wakes you up in the morning right your natural equilibrium position is the sleeping position, and if you have your alarm clock waking you (Refer Time: 09:07) a little disturbance or a little shock and then you switch off the alarm put it to snooze, and then go back to sleep which means your stable equilibrium position which is sleeping position is always stable.

So, we will talk of know those kind of systems. So, in this context the transient response is the response that goes to 0 or this, response this vanishes as the time tends to be large. So, we will shortly see an example with this it could also be written or it could be tied to any events that affects the equilibrium. So, as I said in the case of this pendulum correct that the response it just vanishes as time goes to a very large value. Steady state response well this is the response of the system after all the transients have died down right or as time goes to infinity again we will see this with the help of example. So, total response of any system would have 2 components, the transient response and the steady state response a steady state response could also be just the 0 thing itself.

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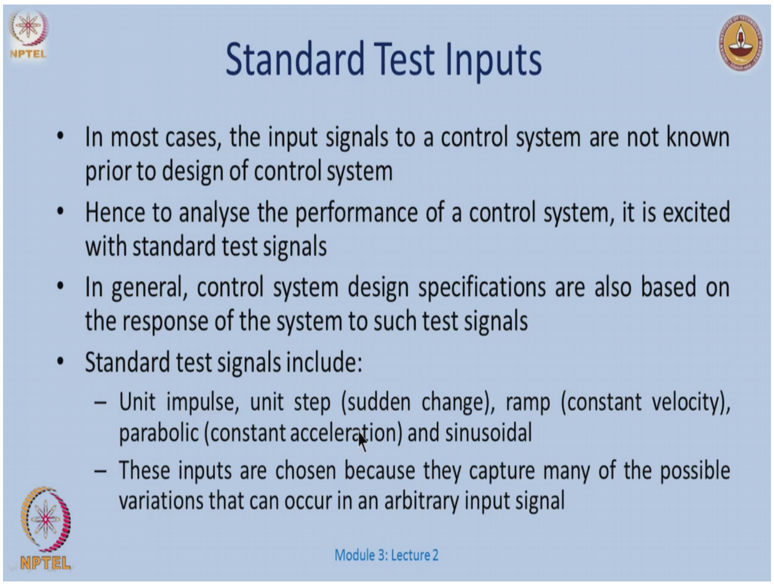

Now, as I said earlier that we will or based on this measure of responses, we will subject our system or the models which we have or even the control system to some standard test signals right. So, this standard test signals would be a unit impulse, which could be just a shock a unit step which could be just a sudden change in the system. For example, if I have 10 people in my room with my air conditioner set to 24 degrees 10 more people add up to the room and just stay there that is a sudden change right it is like a shed disturbance my occupancy is going from 10 people all of sudden to 20 people. Or a ramp or a constant velocity where you know one person is being added every minute or parabolic right which has like kind of units of or you can visualize it as something moving with a constant acceleration. And of course, you can have periodic signals like for example, sinusoidal signals. So, for the movement we will do analyze impulse step ramp and parabolic signals and postpone the sinusoidal analysis when we will do the frequency domain analysis. Again these inputs are chosen because they capture many of the possible variations that can happen in a real time scenario as the driving example or even the c programming example and then your; the design specifications could also be based on the response of my signal to a certain inputs.

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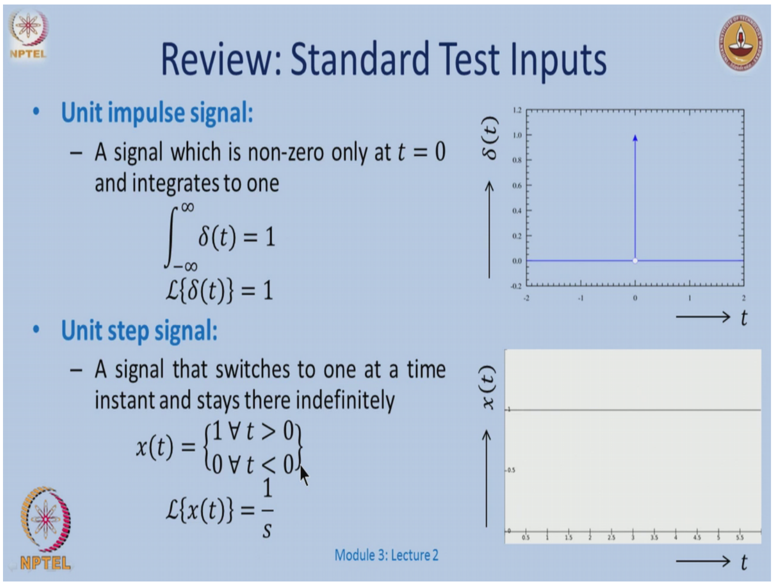

So, just to recall how these signals would try when the unit impulse as we saw in our earlier lectures, is just signal for which I integrate from infinity to infinity the area goes to one and it just looks something like this correct. Similarly as step is a signal that switches to one sorry, just at t equal to 0 plus I am at one and I then stay at one and for t all t less than 0 I am just at 0.

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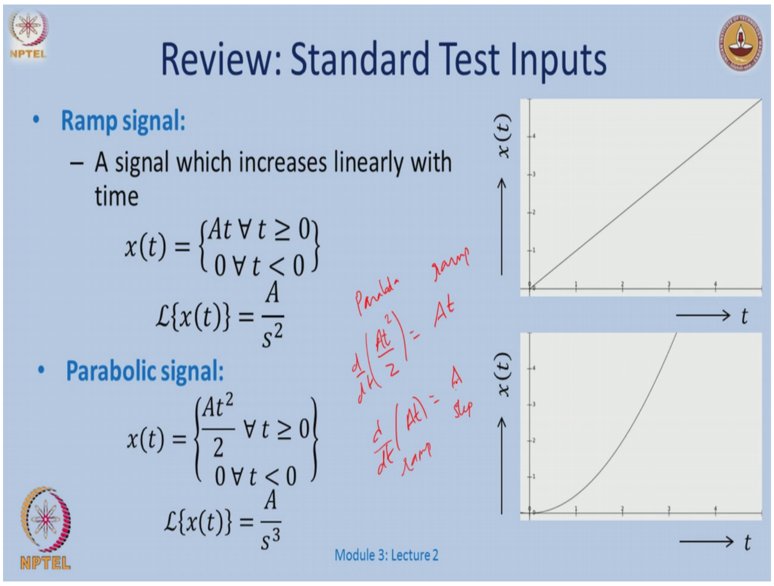

So, the Laplace transform we know of an impulse is one and of a step I know it is 1 over s ramp is something which is just you know with constantly increasing with time at may be a slope of one or it could also be a, I would define slope depending on what is the value of this a, and then the Laplace transform of this would be a over s square similarly with a parabola. So, we have a t square by 2 and then the response increases actually faster than what it was here.

So, one thing which you could notice is I take this parabola a t square by 2 and I differentiate it what I get is a t similarly I differentiate. So, this is a parabola I differentiate to get a ram, this is we can see it here now I differentiate a ramp and I get a step right now just keep this in mind for a while that I differentiate a parabolic signal to get a ramp, I differentiate a ramp and I just end up with a step this is type of size a. We will see does the response also have certain behavior like this right. We will just see that is true or not.

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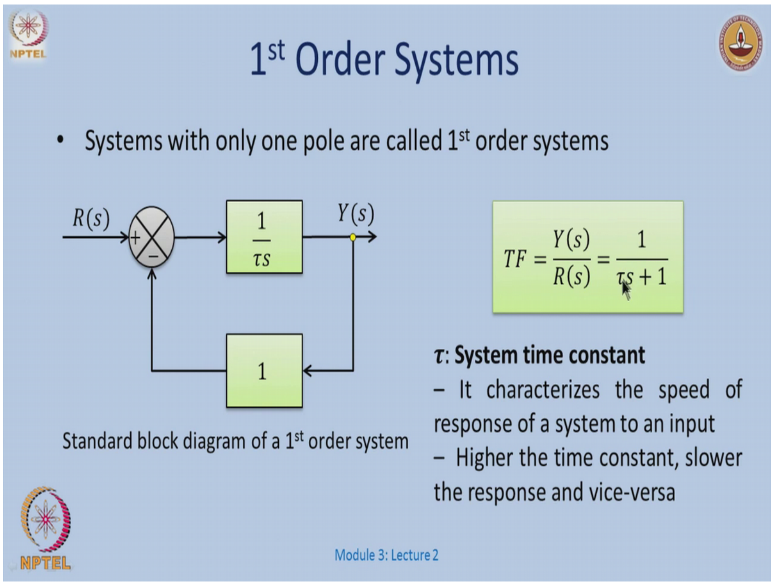

So, the simplest of all systems are systems with only one pole and are called first system there could be some simpler systems, where I just go from here to here this is a number one and I am not worried about anything. So, that those are not of interest there is some dynamics here right in terms of tau times s.

So, if I just do the standard feedback interconnection I compute the transfer function I just get that the transfer function from R to Y is Y over R is 1 over tau s plus 1 and usually R is it could be the reference signal which my system is subject to and I will see how my system behaves for different reference signals right. The tau is a system constant and it characterizes the speed of the response of a system to an input and we will see this with the help of an example. So, let us just start with dealing with first order transfer functions which would like this 1 over tau s plus 1.

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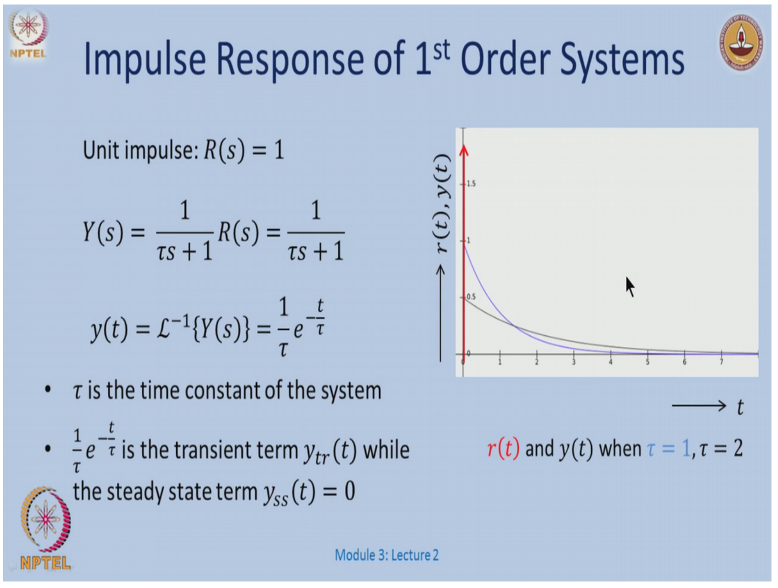

So, if I just take the impulse response now we will use all the tricks which we learnt you know Laplace transforms solving of differential equations and so on.

So, my input the reference is just 1, Y of s which is the transfer function g times R looks like this Y of t I just take the inverse Laplace of this guy and I get this guy so, 1 over tau E power minus t over tau where tau is the time constant of the system. So, if I just look at the plots quickly right. So, if I just choose a particular value tau equal to 1, then you know my response this dies this way for tau equal to 2, I have something like this. So, you can see that with changing in time constants your response also changes for example, the blue line which has a smaller time constants approaches the horizontal line faster than the black line which has a higher time constant.

How does it look in terms of the steady state or in the transient response? So, this guy 1 over tau E power minus t over tau is called the transient term. If you see that you know as how did we define the transient term that these guys tend to vanish as time becomes very large right. So, as time t goes to infinity this guy goes to 0. And of course, the steady state term here is this 0 right as I said earlier the steady state could term that that the Y eventually so, what the term or the value where which the transient term settles down could actually be the steady state value which is true in this case.

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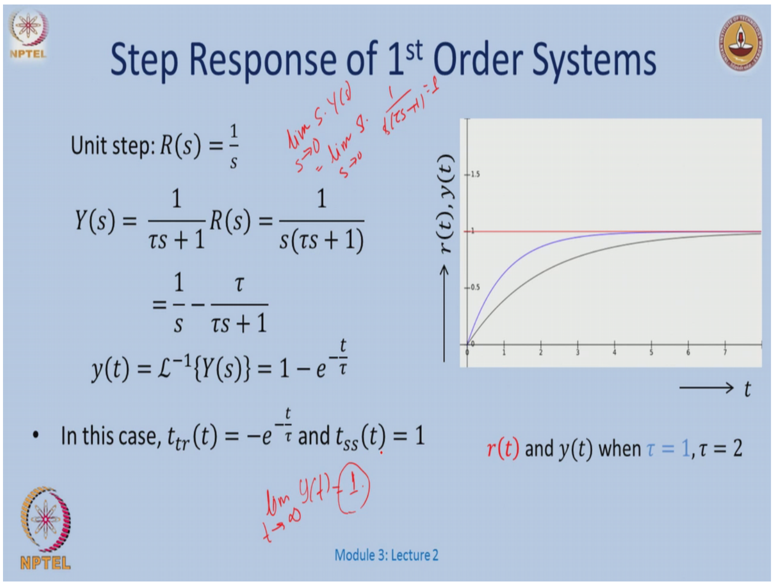

Now I go straightly further right, I look at the step response or more simply a unit step I can change the magnitude, but nothing much changes in the analysis.

So, Y of s is again the transfer function is just a first order transfer function the reference is a step. So, my overall output would look like this. Then I do some tricks here and get the inverse Laplace transform as 1 minus E power minus t over tau that is something different here than what we had learned earlier. If I just plot this for 2 values say at tau equal to 1 I start at 0, and I go and I end up at one intuitively or eventually. At tau equal to do I 2 I still end up at Y equal to 1, but a little slower than before right. So, in this case there are like 2 terms. So, this is the term which vanishes as at infinity right, the transient term and this is the steady state term a steady state what remains is only this guy 1 and at the steady state if I just write that down or Y of t limit t going to infinity is 1.

So, I can also compute it directly from here right just to see how the steady state looks like I can just use the final value theorem, and this goes limit s tending to 0 s times Y of s is limit s going to 0 s times 1 over s tau s, plus 1 this guys vanish and I am just left with one and also is an agreement with what I get here and system is stable. So, I can always use the final value theorem. So, I have here I have a a transient term and I can see very explicitly and the steady state term which is actually non 0 in the previous case the entire response was just going to 0.

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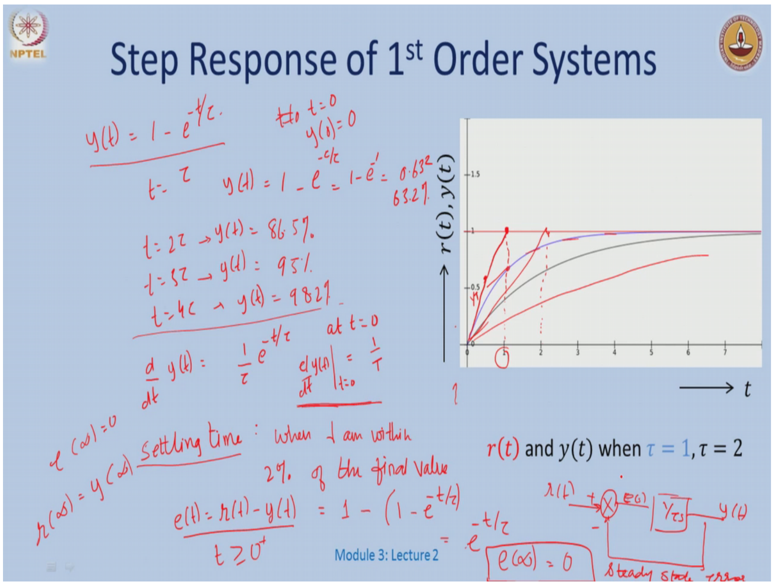

Now, something good happens here right. So, look at this signal right. So, what I have is at Y of t is 1 minus E power minus t over tau and say t equal to 0, Y of 0 is 0 and that is what even the plot says. And at t equal to the value of the time constant I have Y of t is 1 minus E power minus tau over tau. So, I am just looking at 1 over E power minus 1 is like point 632 or 63.2 percent of the final value right. So, what I could observe here is let us say what happens even at t equal to 2 times tau I reach. So, my Y of t would be 86.5 percent of the final value, I do it here a 3 tau. So, Y of t at this for this t would be above 95 percent and at 4 times the time constant, I have Y of t is like 98.2 percent of the steady state value which is one right. So, I can just plot this here at t equal to 1 I can just for the blue line this will work nicely here. So, I will just be somewhere this is this is like 0.632.

So, few things to observe here, so, one is let us see what decides this who goes fast and who goes slow. So, let us try to differentiate this signal right. So, I have d over d t of Y of t that is. So, I have 1 over tau E power minus t by tau now at t equal to 0. So, d over d t of Y of t is 1 over t. So, the initial velocity or the initial speed here is of a slope 1 over t. And then this goes here and just meets this point here. So, if I keep on going at the speed of 1 over t I will hit this point exactly at t equal to tau right. At this point this is my time constant similarly if I go. So, here the speed it will be at t equal to 2 that I am putting this value right like this, this and this one you can compute try it for different time constants this is a general formula for that and of course, the slope keeps on decreasing and so on.

So, where is this important and why cannot I you know maintain the speed all the time and just go to one in 0.6 seconds than just you know going this way. This phenomena essentially is because of the dynamics of the system which gives you a response like this or this curves could be explained by what is in here. So, this we could this something what we also experienced in our daily lives. So, if I take an elevator from say floor number one to floor number 6 you see that the initial velocity is a little higher and then as it goes up and up you do not have the same velocity all the time until you reach the sixth floor and then abruptly you come to a stop you go keep on going up and the velocity is keeps on going down, and then you just stop at the sixth floor very gradually same even when I am driving a car from point a to point b right I build up a velocity and then I just do not go and hit that point in the stop abruptly right I just stop gradually.

So, this is a phenomena that could be explained you know that kind of fashion right and then this if you look at this the slope also decreases monotonically as t goes to infinity and until it reaches slope of 0. Now what about these guys here how do how would I say that I have actually reached some kind of a steady state value right. I do not have to wait till infinity. So, how do we define is if I say of what I will slowly define a term now call this settling time, or when I would say that my system has actually reached the steady state is when I am within 2 percent of the final value. So, if I reach 0.98 for example, after 4 time constants then I say that my system has actually reached a steady state. This is this is very useful for analysis purpose right this definition of settling time.

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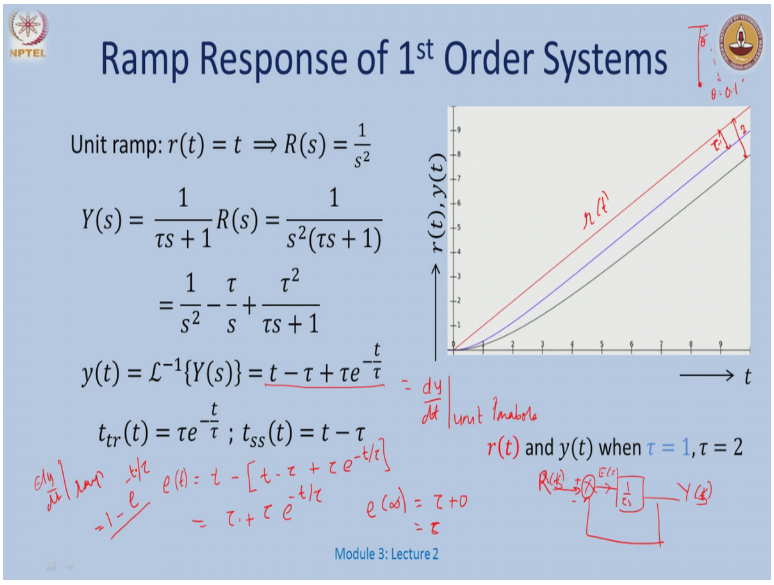

So, after this well you can look at the unit ramp response. So, even before I do this right. So, let us let me do something else also here. So, here my reference was a unit step, if I just look at the standard configuration of the plant and controller. So, I had nothing (Refer Time: 24:12) remember correctly, it will be 1 over tau s I had a unit step here which was my R of t this. So, Y of t plus minus; so, if I call this a (Refer Time: 24:31) E of s, the difference between the reference and the output. So, let me define something right. So, the error E of t has R of t minus Y of t. So, what is R of t is just a unit step 1, and what is Y of t Y of t is 1 minus E power minus t over tau. So, this is simply E power minus t by tau, this is all times greater than or equal to say this is 0 plus.

Now, what happens to error at infinity right, as t goes to infinity this is just this is my term right that it goes to 0 means at infinity. So, if E of infinity is 0 from this expression I can say that R at infinity is Y at infinity. And see this is exactly what is happening here right at infinity both signals become the same that my system I can say is asymptotically tracks a unit step. It is no matter what the value of the time constant is this guy does a little slower, but it is here you could have even slower guys which could do here, but they would eventually again go to go to the value one. So, this system I would say again I am slowly defining these terms, but we will elaborate this a little later, has a steady state error of 0. Right I can just call this I am running out of space here, but let me write it here.

So, how do we define this as a difference between the response and the output and how does it go at infinity. So, now, what happens when the input is a ramp or an actually passing my system it is like a ramp? 1 over tau s plus 1 R of s and I do all these things and I see do the inverse Laplace. So, a response to a ramp would look like this t minus tau plus tau E power minus t over tau. Again how do we classify or how do we differentiate between the transient response and the steady state response where the transient response the one which goes to 0 for very large time. So, this guy and a steady state this guy still remains. So, it need not be a. So, earlier we saw in the first example that the steady state was 0, the second example when I was tracking steady state was 1, and now you see it is actually depending on time also.

We will see what these things actually mean. So, let us see first what is the error right does at t equal to infinity are my responses just meeting the original signal or not again I just draw it for the sake of completeness here, that here I have R of t, I have 1 over tau s plus or minus and I have Y of s. Sorry Y of t better Y of s because this is s this also be s capital R of s. So, at infinity does my output track the ramp? So, that depends on what is the error signal here E of s. So, what is E of s, or E of E of t E of t is R of t which is my t minus Y of t right (Refer Time: 28:15) t minus tau plus tau E power minus tau over t, this about I am left with this tau plus another tau E power minus t by tau. Now if I do these things E at infinity would be well this nothing will change here tau plus this guy at infinity will go to 0. So, nothing will be the same tau the 0 is tau.

Now, what does that mean? So, this is my R of t is my reference signal right. So, as t goes to infinity, I am not touching this guy, but I am actually just at a distance of tau, which is one in this case right. Here at a distance of 2 right. So, there is some kind of a steady state error here, which is just governed by this number tau. So, smaller this numbers smaller the steady state error. So, what I conclude is that well first order system, if it has a ramp it will have some steady state error. How do I encounter the steady state error let us say I can take this example of a pendulum right? And I start it with some non initial condition and I expect it to go to theta equal to 0 this is theta, you know there could be something in the system (Refer Time: 29:45) may be they stops at theta I equal to say point one degrees right. So, that is the idea of this steady state error.

So, it does not really reach the final the desired value, but some number very close to that there is an error, but there is a way to determine that error, how small it is how large it is I can directly quantify by just looking at the dynamics of the system.

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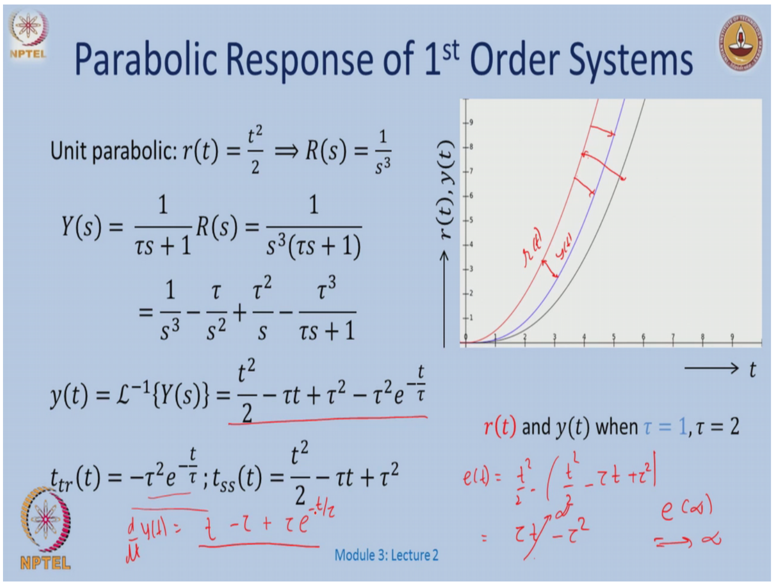

Now finally, we will see another test signal it is the parabolic input R of t is t square over 2 the Laplace transform is 1 over s cube, and I do all these things and I get signal which is t square over 2 minus this terms plus tau square and so on. Again this has a steady state this transient term which we will just write down as time goes to infinity and a steady state term well this actually looks possibly equal to (Refer Time: 30:36) right. Then the additional one it has a t square term now right this is no longer constant.

What does the error do here right? So, can my system track a unit parabola? So, the error E of t is my reference signal that is t square over 2, minus the output that is t square over 2 minus tau t plus tau square. I would I would exclude this exponential term because I know it will not contribute to the steady state error right. We will go to 0 anyways. So, this will be tau of t plus or with a minus tau square. So, E of infinity is actually a little strange here right E when t goes to infinity this guy also goes to infinity. So, the error will be very large. So, if you see that the distance between these 2 this is my actual reference signal and this is my Y of t as I go to infinity, this distance will keep on increasing right it will keep on increasing and then if you keep plotting this forever, you will see that the distance actually becomes unbounded similarly with here the different time constant of tau equal to 2.

Before this so, when one of my you know earlier slides, what I had shown was that this. In fact, I take a parabola I differentiate it I get a ramp. I take a ramp I differentiate and I get the step. Now just see if the responses also follow some kind of a pattern like that. So, this is my response here. So, this is this, this entire thing this, this guy. So, just differentiate this with time. So, d over d t of Y t in this case would be t minus tau, and then here you have plus E power minus t over tau 1 over tau here something like this now just remember this t minus tau plus tau E power minus t over tau, and this looks something like this right. Exactly this how right this is just the differentiation of d Y by d t for a unit parabola. So, I differentiate the response I get, now the response to a unit ramp. So, the differentiation holds until now.

Now, let me differentiate this again, d Y by d t, d Y by d t when the input is ramp is this will be 1, this will go to 0 this should be a minus E power minus t over tau. Now this E this is a 1 minus E power minus t over tau you go back slides, and how this is this is the same right this one and I can keep doing this right. So, you see that if the signals are preserved under differentiation, in this way that I differentiate a parabola to get a ramp the responses are also follow the same pattern. If I know the response to a parabolic input how do, I find the response to a ramp I just differentiate this again from here to here one more differentiation here to here one more differentiation good.

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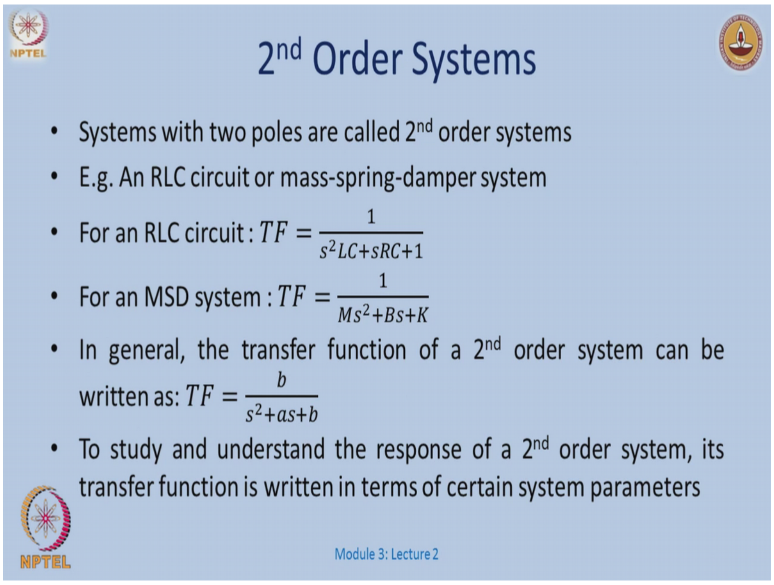

Now, we go to something slightly complicated, but still not very complicated, but very useful. So, second order systems, these are the systems which have 2 poles we have done. So, examples on this said a typical RLC circuit would have a transfer function like this if you remember vaguely this was a transfer function between the voltage of the capacitor and the input voltage. Similarly, for the mass spring damper system where I was taking the input was a force output was a position or a velocity one of those. And then in general I can write transfer function as well a second order polynomial in the denominator and some number in the numerator and else slowly we will learn why these are equal what are these numbers and so on.

So, the response here will be a little more tricky than what we saw earlier. So, how do I derive the system let us we start with a open loop thing which looks like this. I have a unity feedback and I go to a closed loop transfer function which looks like this. Some terminology which will shortly be clear is that omega n is what I call as the natural frequency of the system. Zeta is a system damping ratio.

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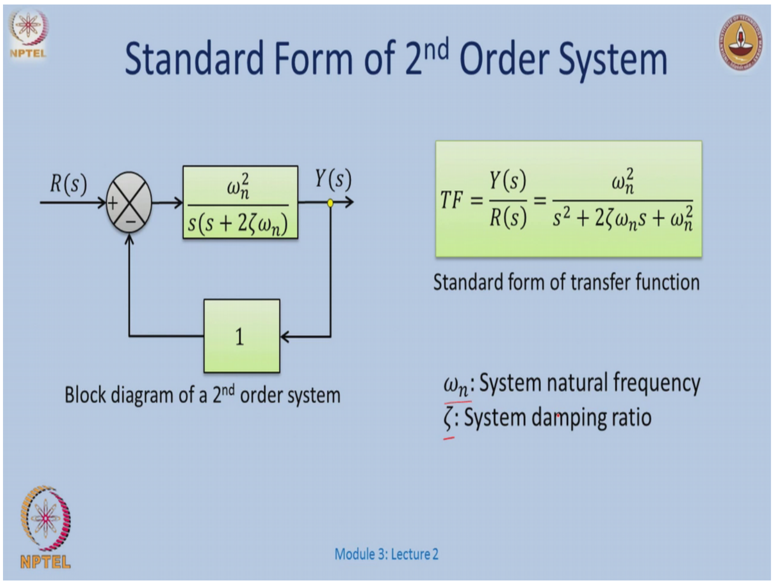

Let us just remember it with these names at the moment and we will see slowly what they could be.

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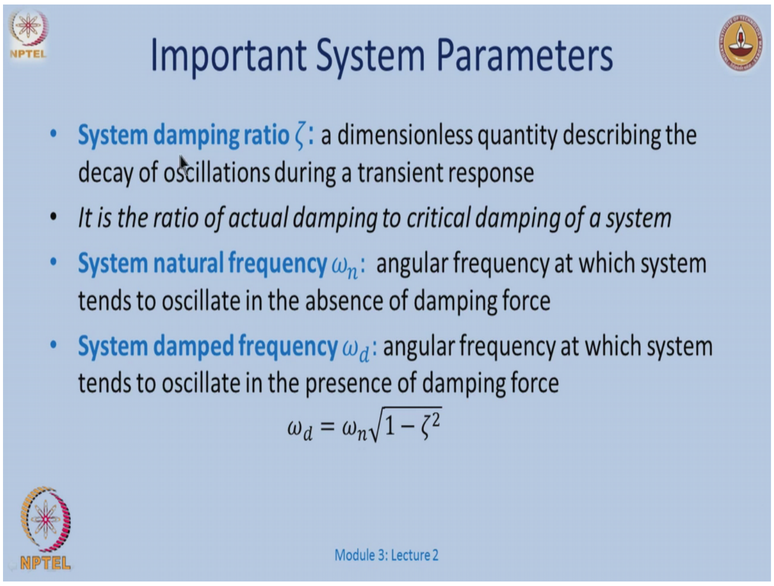

So, just some properties of it the damping ratio is a dimensionless quantity and it describe some kind of damped oscillation during the transient response of a system. So, this we will leave to the next lecture or the lecture after that. That is, it is the ratio of the actual damping to critical damping no need to understand this statement at the moment.

But we will recollect this when we go and then you know try to justify what this means. Natural frequency is the frequency, at which the system tends to oscillate in absence of a damping force, this will be clear shortly. So, again together with omega d which is the damped frequency of the system. So, we will again come back to this as we learn it through some examples.

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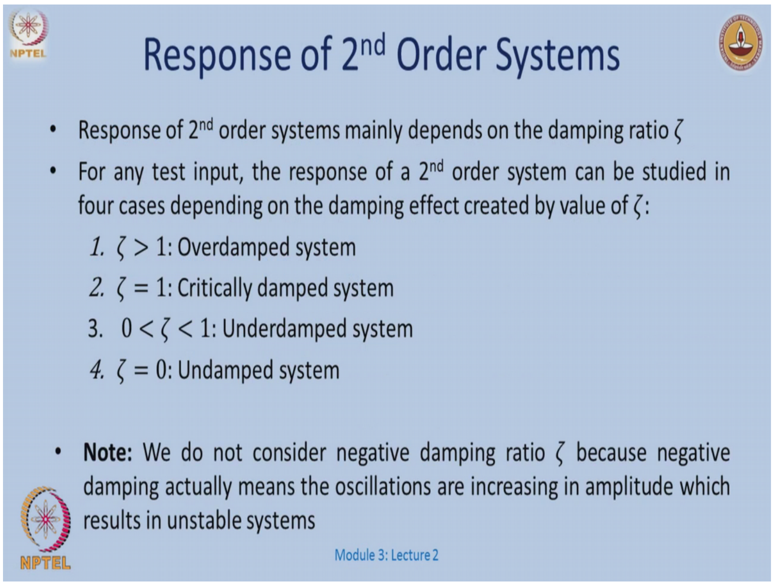

So, the response is essentially now depending on the damping ratio zeta. Some classifications before we understand them physically is when zeta is greater than 1, I called my system to be over damped. Zeta is one it is a critically damped zeta between 0 to one underdamped zeta equal to 0, undamped system you might have encountered these words somewhere earlier in some of the courses. If not do not worry we will try to define all those things and of course, we are not really interested in negative damping at the moment, because again all the things we are doing are restricted to stable systems.

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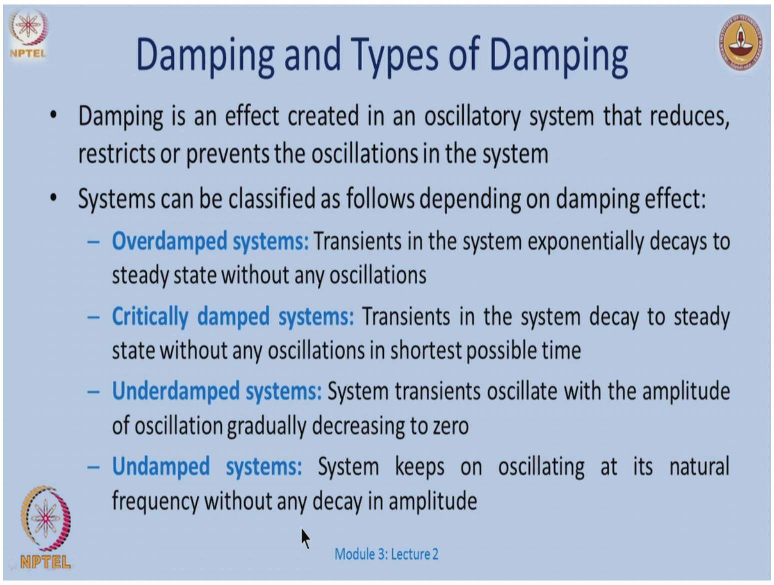

But you know a system with a negative damping ratio could resemble a behavior of a unstable system, which we are not interested in at the moment what is a damping right. So, damping is something which kind of reduces or prevents oscillations in the systems. And so I could have systems which are over damped in a way that transients exponentially decay without any oscillations I would have critically damped where you know the systems decay to the steady state value with or in the shortest possible time I could have oscillations which actually go to 0, and I could have undamped systems which keep oscillating at it is natural frequency without any decay in amplitude.

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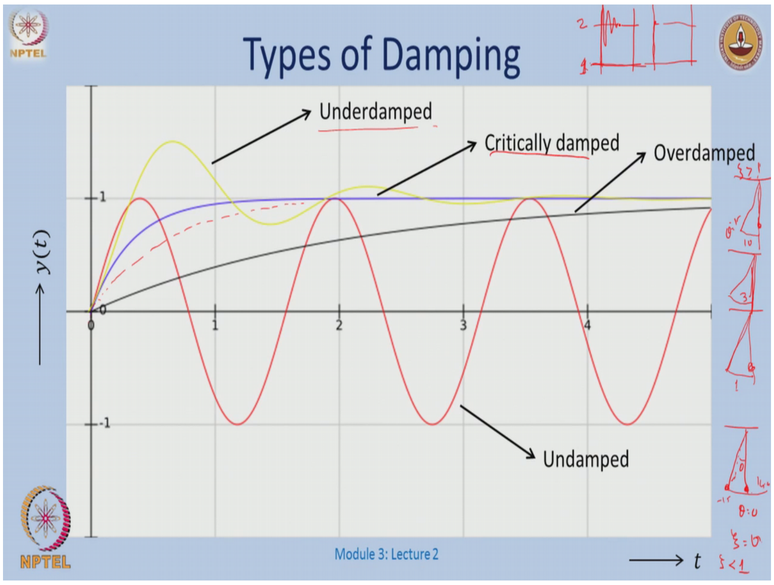

So, a typical response the red color here would be an undamped system. So, let me see if I could explain you with a simple pendulum example right. So, I am here this is my theta is equal to 0 position, and say I have an initial condition starting at say theta equal to some 15 degrees right. Some other theta and I am starting here if the system does not have a it is not subject to any sort of friction either here or here. So, what we will be we expect is we just keep oscillating right. So, we just go from theta equal to 15 degrees. So, this is a plus if this is a minus 15, it will go to plus 15 and just keep on doing this right. So, this will correspond to the behavior when zeta is 0. And I will explain you this mathematically very shortly. There could be cases when I could have a situation where this guy is goes from 15, may be it will go to a 14.9 here come back and it will keep on you know oscillating go back and forth back and forth until it reaches this point. So, these are called underdamped systems right. When zeta was less than one there could be a situation where I start from when theta equal to 15, I go and I just slowly settle here I do not go to the other side I just go and I just stop here.

There could be say like this is sometimes in one second there could be situations, where I start from here again and I go very slowly say it I am going in 3 seconds there could be even worth situations. So, I am not going you know I am not crossing this line I am not crossing this this line the lakshmanrekha there, could also be situations where I start at theta is again 15 or minus 15 depending on the convention, that I am going very slow very slow very slow and I just stop. This could take me 10 seconds. So, few kinds of responses theta equal to 0 I keep on oscillating right for some value of zeta less than one I am oscillating, but my oscillations are dying down dying out before I reach this steady state position, there could be situation where I just go from here till here in one second and just stop I do not go beyond theta equal to 0 it could take 3 seconds it could take 5 seconds.

So, the fastest value of and these things will vary with zeta right zeta equal to 0 zeta less than 1, and then all this behavior would be zeta greater than or equal to 1. So, the fastest even I can reach here and stay is the critically damped response, it is like the blue line here and for damping anything larger than that I have just keep taking a longer time I could have a response also here right some something like here which could be zeta still greater than one right. So, these are typically the 4 kinds of responses which we will encounter in designing a system. If I take for example, I want to design a lift or an elevator and I want to go from say this is level one to say level 2 I am lazy, and I do not want to walk and I take this elevator and I go up I do not want my system to go to 2, 2 point you know half it is the third floor come back here and then do this and I do not want you to do this.

Now, that that will be very unpleasant right even if you know, like you see in the in t v even if Deepika Padukone is stuck with music she will be very unpleasant. And therefore, what we design this systems are to be critically damped or like slightly over damped right then I just go here and I just slowly settle down here right. So, again depending on my requirement I would just want to you know have my system to be either be under damped critically damped or even sometimes over damped.

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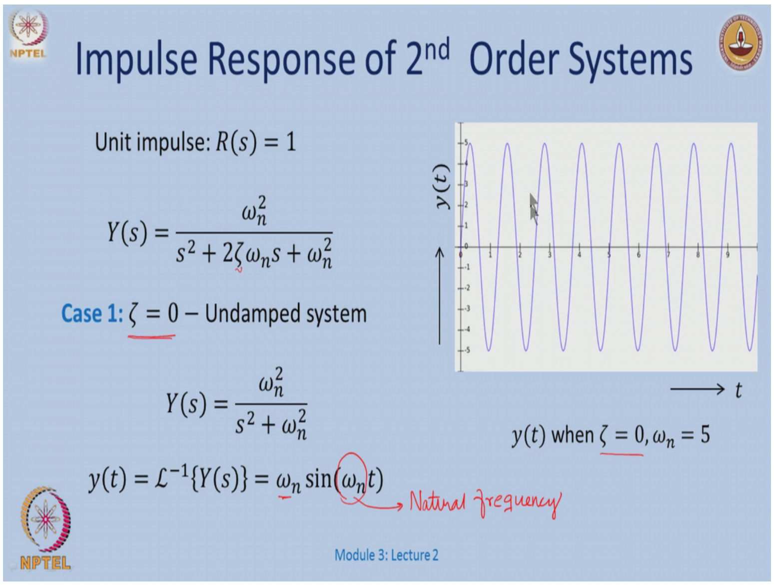

We will characterize all this very shortly. So, how does my system behave again with the standard test signals? So, here I have to be careful that there is another thing just not just the test signal, but something depends also on the value of zeta right. So, earlier my formulas were very easy to remember, but here you know this zeta adds more complexity to right or more it presents me with more test cases.

So, Y of s is omega n square s square plus 2 zeta omega n s plus omega n square. So, zeta equal to 0 would be this guy would go away. Y of s is this one and I can do the inverse Laplace and I can just say I just have a sinusoid of some frequency, omega n let us just highlight this very nicely omega n how does the response look like well the response is just a sinusoid I say what is it is amplitude well amplitude is this one could, now what I am interested is the frequency zeta is 0 and if there is no damping it will oscillate at omega n and therefore, I call this as the natural frequency as defined earlier.

So, here so, did we define that yeah yes the system natural frequency the angular frequency at which the system tends to oscillate in the absence of damping force. That is exactly what we did now there is no damping force zeta disappears and then my naturally my system just oscillates

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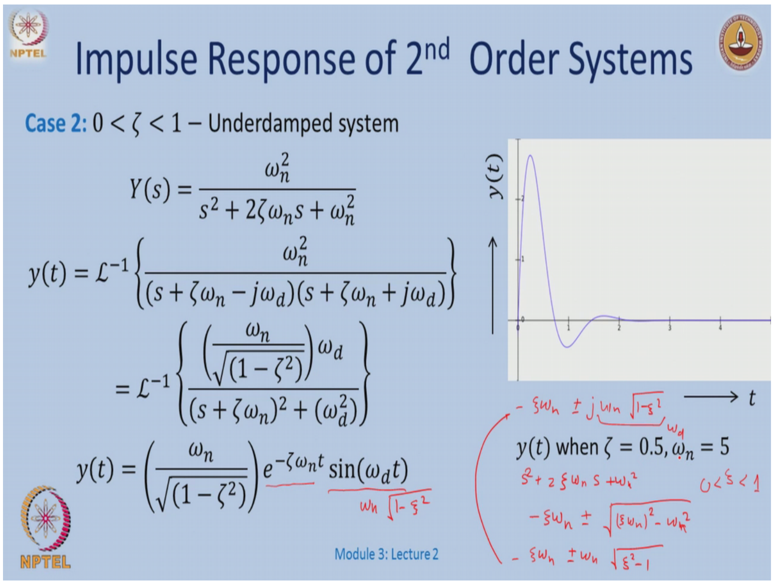

So, this is the natural frequency when there is no damping. So, let us make things a little complicated. So, sigma between 0 and 1 so, before I do this I would just do some very short calculation here. So, look at the denominator I have s square plus 2 zeta omega n s plus omega n square. So, just you say and if I look at you know how do I find the roots of this. This roots would be; so, minus zeta omega n plus minus square root of zeta omega n square minus omega n square right. So, this is how roots will look like you know divide by 2 and you know saw the standard formula, or this is minus zeta omega n plus minus omega n square root of zeta square minus 1. Now when zeta is less than 1, and of course, greater than 0 this will become a complex number, and therefore, the roots will look something like this. So, here and I call this guy. So, if this is complex I will just would here minus zeta omega n plus minus j omega n square root of 1 minus zeta square.

And this guy I will call as omega d thus, how this roots now expand right s plus zeta omega n minus j omega d and plus j omega d exact what is here.

Now, I just want to see the natural response or the impulse response and I see I just do all the Laplace transforms inverse Laplace transforms which I learnt earlier. I see that Y of t is some number here omega n bla bla bla, then I have an exponentially decaying term and this guy I say exponentially decaying because all these guys are positive and with time you know just falls in value and goes to 0. It still now earlier here I had sin omega n t whereas, here now I still have a sin curve or a sin wave, but with a different frequency that is omega d right and therefore, I called this as the damped frequency. The system damped frequency angular frequency at which the system tends to oscillate in the presence of a damping force and this zeta was greater than 0, but less than one and this is exactly the formula we use there this omega is not because of any magic that I defined it just because it just sit is in beautifully here. In the term of sin omega d t right this is why I call this omega d t, as the damped natural frequency this is omega n square root of 1 minus zeta square.

So, this is the damped frequency. And of course, have this exponentially decaying term and if you see the response where it will it will keep oscillating and eventually go to 0. And this is the response of the pendulum which we are talked about when zeta was greater than 0, but less than one right and thus the sin here says that will there will actually be oscillations and I will just go to whatever this number here is come back and of course, this numbers are depend on what are the specific values of zeta and omega n know.

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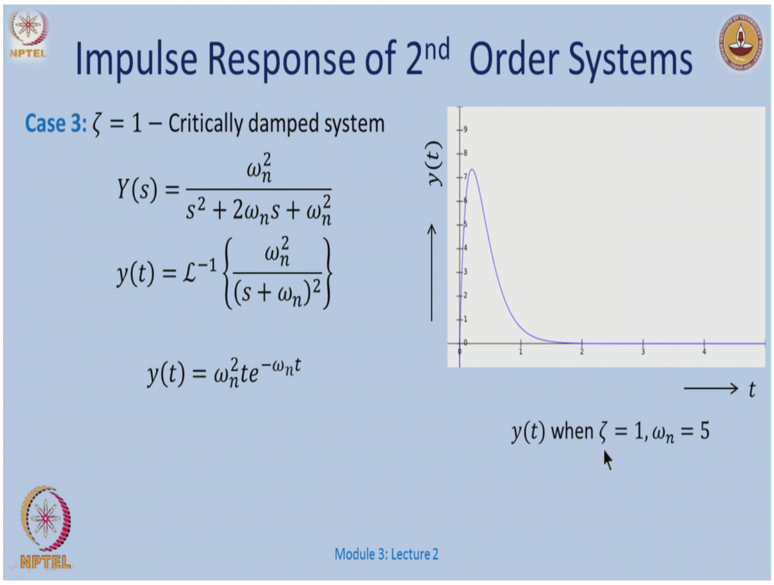

Now, other case could be when zeta is 1. So, when zeta is one my expression for Y just looks like this s square plus 2 omega n s plus omega n square, and the inverse Laplace would just be like this right. And if you see there is a sinusoidal term has disappeared which means there will not be any oscillations; however, you will see that as time goes to infinity I just come back to 0, this is again for a particular value of zeta equal to n and omega n, we will we will later investigate what happens for larger values of zeta and so on.

So, this is; so, here for a system which we actually called now as a critically damped system. The transients in the system decay to a steady state without any oscillations and in the shortest time possible as we are defined in the case of the pendulum this is the shortest time and this is the critically damped case.

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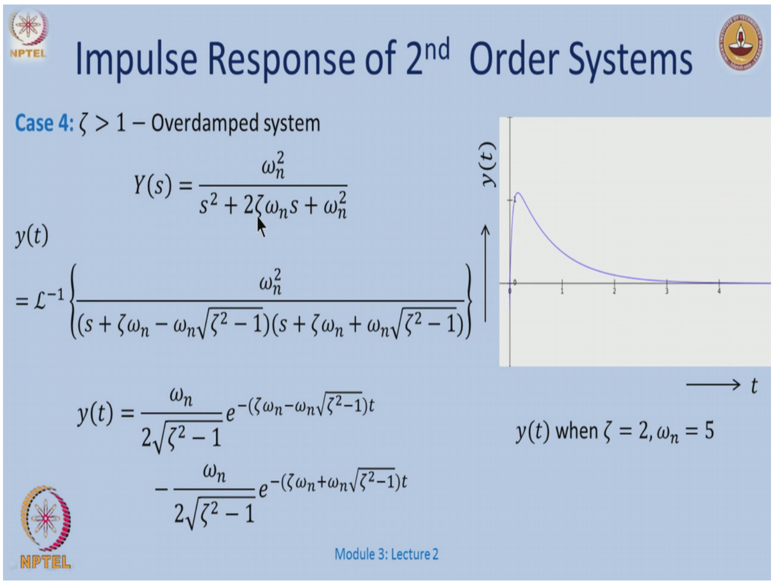

Now I could go over damped system right this zeta is greater than 1, in which case this roots will just be real. So, zeta less than one I had complex roots, but I will just have real roots. So what happens here? So, omega n square plus s square 2 zeta omega n s omega n square, I can just write it as a composition of 2 real roots like this for which the response looks like 2 exponentially decaying signals right zeta omega n bla bla bla and zeta omega n. And again this response you could see there is no oscillating terms there is no sinusoidal term. So, you just go here and then just come back possibly when zeta equal to you know 3, you might have even much slower response it could take even much larger time like this and so on.

So, we saw 3 cases now right first is the or I have actually 4 cases. The undamped case where I have the system just oscillates at the natural frequency. Then I had well I had oscillations, but they just died down. They died down because of this exponentially decaying term here right. And this will oscillate at now a new frequency which is my zeta defines, again you can just put zeta equal to 0 and say it will again go back to the damped natural frequency, and then a critically damped system and what we call as an over damped system.

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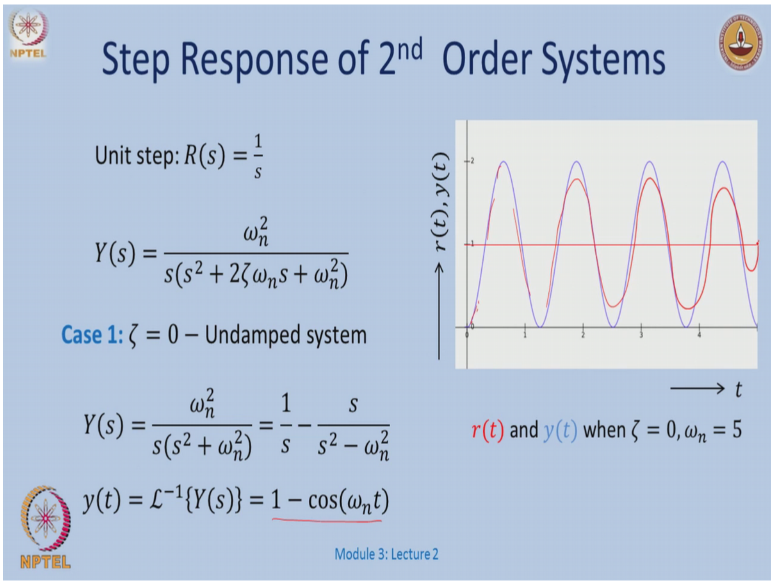

Now, this was all about the impulse response. Now how will these guys behave when there is a step response? So, step is in my R of s is 1 over s, then Y of s would be something like this s over here and bla bla case of zeta equal to 0, undamped system well nothing should change in principle as what was observed from the case when I had just an impulse response.

So, Y of s I just do all the math and I see well Y of t is 1 minus cos omega n t. So, this system is just 1 minus cos omega n t, it will just be like this right. And you can ask me sir what is the transient and steady state for this guy will never reach the steady state right. Where as this guys here this guys will have you know this kind of transient terms which go to 0, even the previous guy would have a transient term which goes to 0, initially the steady state is again coming back to 0 correct, here also you will have the transient term which actually is going to 0 this entire term could be viewed as a transient term and you know the steady state term is again 0, here well there is no nothing like that right there is nothing which is actually vanishing with time.

So, there is no possibly we cannot know nicely classify what is a transient and what is a steady state it is transient behavior is actually it is steady state behavior and it will never converts to any fixed value like a 0 or a one as in the previous case. So, same thing happens when I have a unit step response, I am still doing this one right none of the term actually disappears as time goes to infinity or terms are just intact this is again zeta equal to 0 omega n equal to 5 and the reference just being a step.

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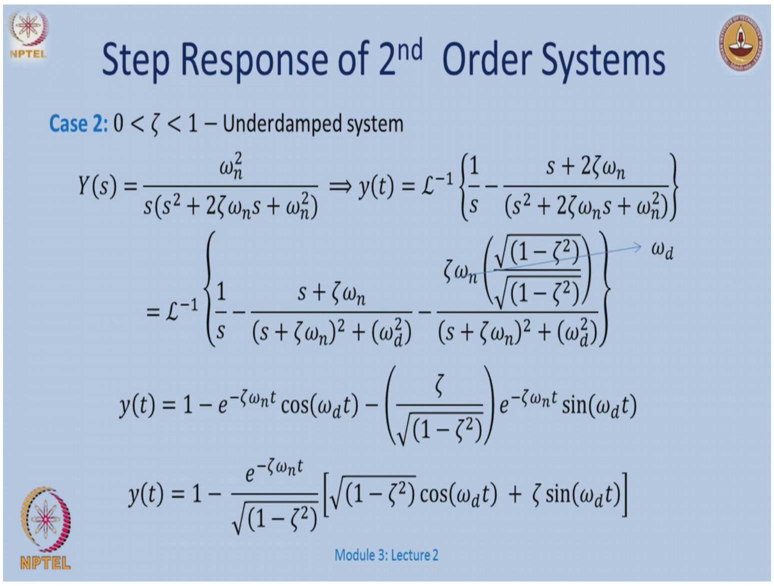

Now, we will do all the same cases which we did earlier, I will just keep all the math here, but we will see how my final expression looks like.

So, for zeta equal to this guy I will start with this and then I will have this one I will of course, have complex roots and all this will be taken care of here. So, my response has 1 minus some exponentially decaying term again. You have you have some oscillating terms here which could be further simplified to look something like this.

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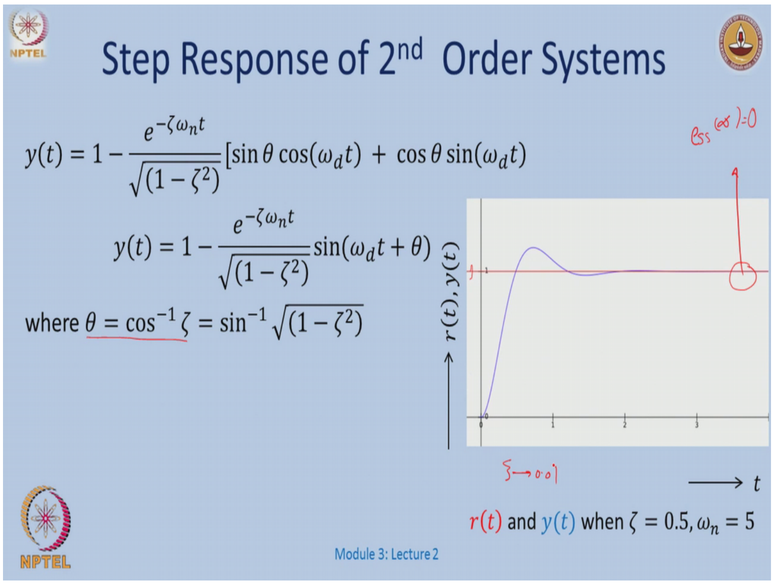

That is 1 minus E power all these exponentially decaying terms and this one or I could call this as my transient response. And this as my steady state response right and, but theta is we will we will you know at the moment we will just say that well the theta comes as cos inverse this theta. Of course, we are not really worry about why what this means, but all the math would tell me it is just this one.

So, how does the response look like Y of t is goes. So, I want to track this number one and I go up and overshoot and then I will go below one and then I eventually go to this one. As I can directly say here now I do not really need to compute what the steady state just by looking at this thing I can say that the steady state error at infinity is just 0. We will do more analysis on this, but just as a passing observation I am just introducing you to this this terms at the moment right and of course, we can have different behaviors for different zeta right as zeta is. So, intuitively you can see that as zeta goes to 0 you know is very zeta is very close to 0 say 0.1 then you could actually expect a response which is like this, even zeta is very close to 0 you could expect and response like this, but which is very slowly decaying right. So, it might actually take a very long time to decay.

So, (Refer Time: 54:23) draw something here and I will see this oscillations for a very long time and about they will slowly die down right. And of course, as zeta goes to very close to one then you will see that in oscillations are also (Refer Time: 54:33) again this omega d guy just it is here where the damped frequency of the system. Now this is again because of this guy zeta right a justification to Y, we introduce these terms specifically and called it the damped frequency of the system, because this is the frequency of the signal when there is in the presence of damping.

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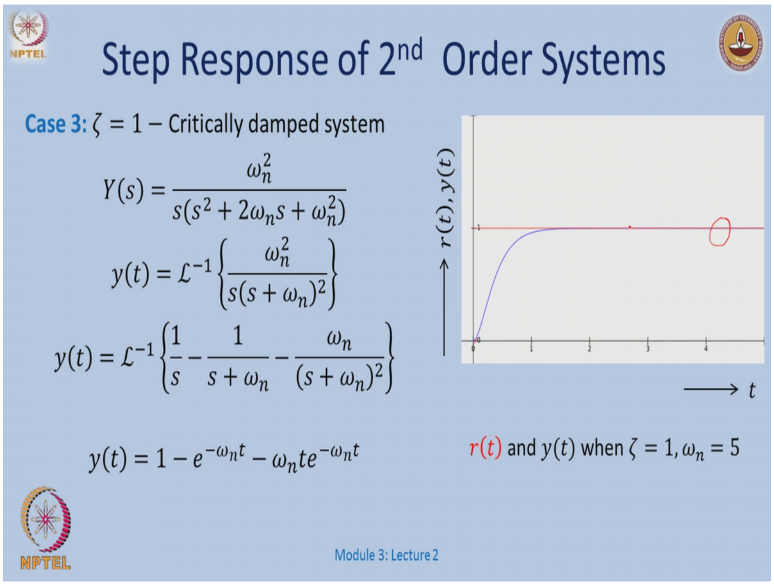

So, next what happens when I am critically damped, I will just keep all this all this math and just see that well the sinusoidal terms disappear, and I have just this one. Y t is 1 minus E power bla bla bla bla bla right. And I can just compute this response either with the final value theorem here or just put t equal to infinity and I can just compute the error is again 0. So, this is like it goes from 0 to Y in the shortest possible time I just takes there.

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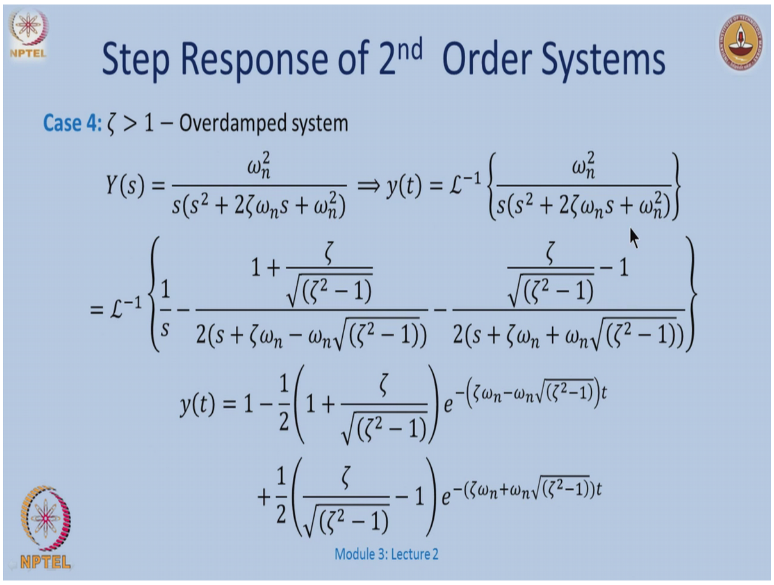

The next one is the case of an over damped system. Again we will have 2 exponentially decaying terms which is just be like this it will go a little slower than the previous one, and you could just plot you know things zeta goes to 3 and possibly it will have a much slower response zeta equal to 4 and much slower response and the limiting case you will say well there is a very large zeta then you just refused to move.

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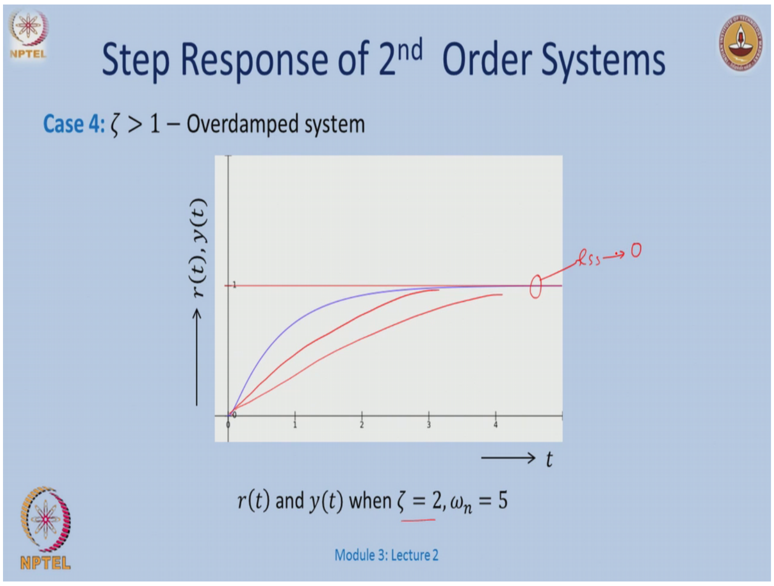

So, these are also a literature referred to as very sluggish systems, which are like very slow to respond right. So, also in this case you can just you know naturally see that the steady state error goes to 0. So, what we have we have learnt. So, far today is what we do with the models which we obtained from our previous classes.

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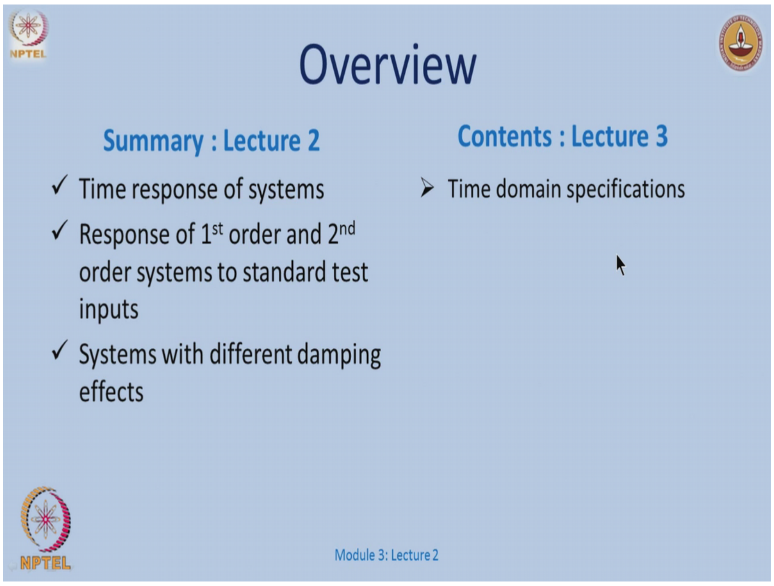

So, once I have the models I need to subject them to some real time conditions which I possibly who will not be able to emulate, when I am just having a system at hand. So I just give it a certain test signals which will be closer to what I can see in reality. And therefore, I have a series of signals in my case I have the impulse the step the ramp and the parabolic kind of signal, and we just saw very saw what how we could define what we call as the settling time.

What are the transients what are the steady state, and in the; we did it for the first order second order case whether it will more interesting because the damping actually paid a role in the dynamics of the system? If I have oscillations if I have sustained oscillations if I have decaying oscillations or if I have no oscillations at all right. So, that was a summary of what we did today, and we will do a little more analysis or we will elaborate more on this in in the next lecture, when we do more analysis on this time domain specifications.

Thank you.