Control Engineering

Dr. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 03
Lecture – 11
Tutorial - 01

Solving Problems on Block Diagram Reduction and Signal Flow Graphs

Hello everyone. In this tutorial session we will try to solve some problems in relation to block diagrams and signal flow graphs that we have studied in the previous classes. So, to begin with we will try to draw a simple control system with the plant and a feedback loop.

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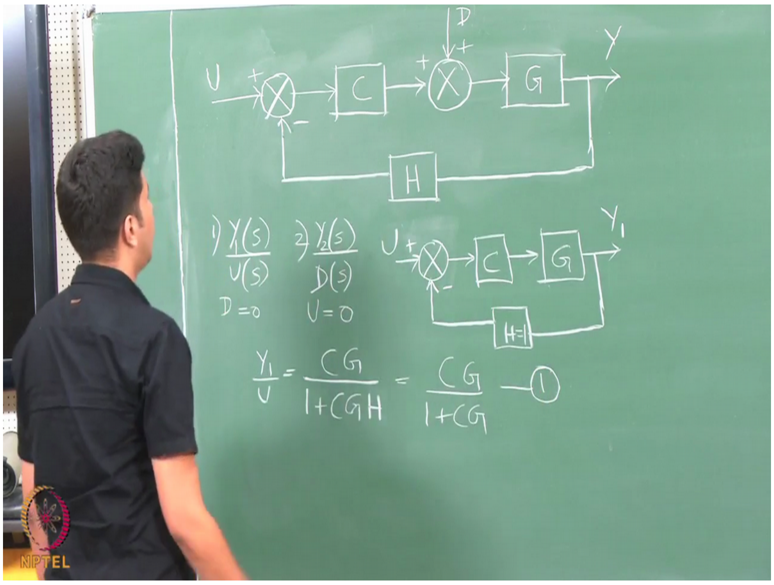

So, this is how the block diagram looks like. So, this is the input U which goes in the positive sign into and this is the control system with transfer function C and then we are adding a disturbance signal along with a control signal, both we take positive sign. And then we have the plant with transfer function G and we also have a feedback loop with transfer function H and this is the output and this node that for the sake of simplicity we are just ignoring the Laplace thing, but everything is in Laplace domain.

So, these are in fact, C of S G of S and H of S which we are representing as C H and G for the sake of simplicity. So, what we will try to do here is we will try to find the transfer function. So, here transfer function there are actually 2 input signals, that we have one is the normal input signal that you know, and there is another disturbance signal which you do not know. So, we can find the transfer function both with respect to both these inputs. So, that is we can find out what is Y of S by U of S and then we can also find out Y of S by D of S.

So, not only finding the transfer function what we will do is we will try to show that Y of S is actually a response due to the input and the disturbance, when they act individually while the other is completely 0. So, while doing this what we do is while finding out this transfer function, we will make D 0. And while finding out this transfer function we will make U 0. And then finally, we will try to find out the overall transfer function using (Refer Time: 03:12). So as these are parts, I will call them Y 1 and Y 2. So, this is Y 1 is the response when U L 1 is acting and Y 2 is the response when D L 1 is acting now. So, to find out Y 1 by U 1 we make this 0 and so our block diagram changes.

So, this part goes off completely and sorry Y 1 by U 1 we make D 0. So, this part goes off completely and what I will be remaining with is something like this. So, we are taking a negative feedback. So, this is Y 1. Now this is a very simple feedback loop and we know the transfer function of it directly. So, Y 1 by U is nothing but the forward loop transfer function C into G, by 1 plus C into G into H the feedback transfer function. Now what we will do is for the sake of simplicity we will even take H is equal to 1. Then our transfer function becomes C G by 1 plus C G. So, this is 1. Now we will find the other transfer function by making this 0, when we make that 0 we will get a diagram like this.

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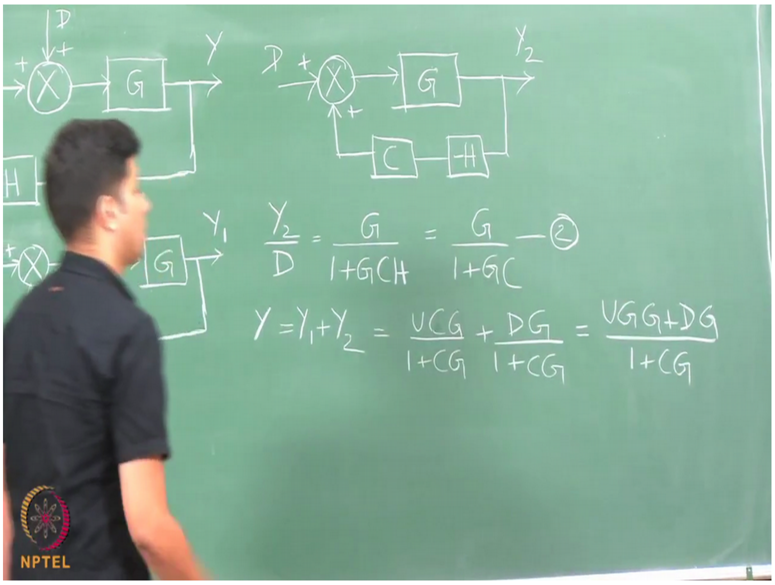

This as C because it is a negative feedback; I will take it as minus H. So, C and H go into the same loop because this completely vanishes and now this is Y 2. So, again those are feedback loop and you can take the transfer function to be Y 2 by D is equals to G by 1 plus G C H right. Now as we took H to be 1, we can just write it as G by 1 plus G C this is my second transfer function.

Now, my overall response Y can be actually written as Y 1 plus Y 2, by principle of superposition. So, if I take Y 1 plus Y 2 which will be. So, Y 1 is 2 times C G, by 1 plus C G plus D times G by 1 plus C G. So, this is nothing but U C G plus D G by 1 plus C. So, that is the overall response of the system. If we even do the same thing normally if you take the complete system and try to find the overall response that is what you will be getting. So, this is a kind of verification of the superposition principle using this block diagram.

Now what we will do is we will take up a slightly complex block diagram of a system, and then try to find it is transfer function. And then we will convert the block diagram into a signal flow graph, and then we will again find the transfer function. So, we will have a comparison of how difficult it is to find the transfer function using the block diagram directly and then using the signal flow graph using the using the mason gain formula. So, this is the block diagram that we will take.

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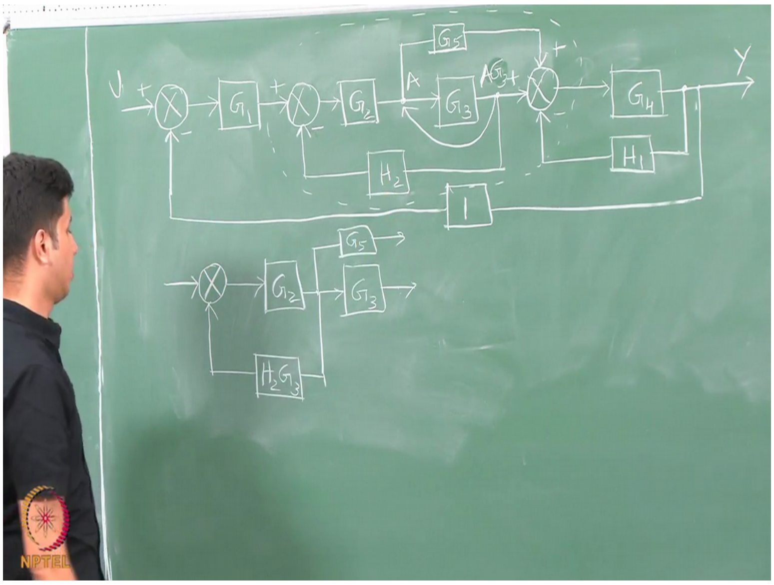

So, as you can see this is a pretty much complicated block diagram. And it has about 1 2 3 4 5 6 7 and 8 blocks. Each block has it is own transfer function, and you can see their 3 feedback loops and one parallel combination as well. Now what we will do is we will simplify this block diagram using the rules of block diagram algebra and so how do we begin with the first thing you can observe is this is a feedback loop which we can easily simplify. This is a parallel and this is also another feedback loop, but there is a small complication here.

So, this point and this point are slightly off positions, where as to easily apply the feedback formula that we know. So, what we can do is we can shift this here. And once we shift that we can easily find out or simplify these blocks and find out their overall transfer function. So, once we transfer that block this is what we get. So, I only draw this part of the transfer of the block diagram and see how it looks like once we shift that.

So, once I shift that point comes here, but for shifting that there is something which we need to compensate. So, look at here what is the signal that is coming out. Whatever the (Refer Time: 12:31) if I call this as some the signal at this point as A. So, the signal here will be a times G 3. Now if I shift this from here to here i, I will be getting only a I will be losing out on this G 3. So, what I do is I will put it here H 2 times G 3. So, whatever I lost this game, by shifting this point here I will be compensating it by including it in the feedback transfer function. So, I will this transfer function is now H 2 G 3. And everything else remains the same. So, now, we have a simple kind of a diagram this point here and this thing. So, what we will do is we will apply the rules that we already know we will find out the transfer function of this block this block and this block.

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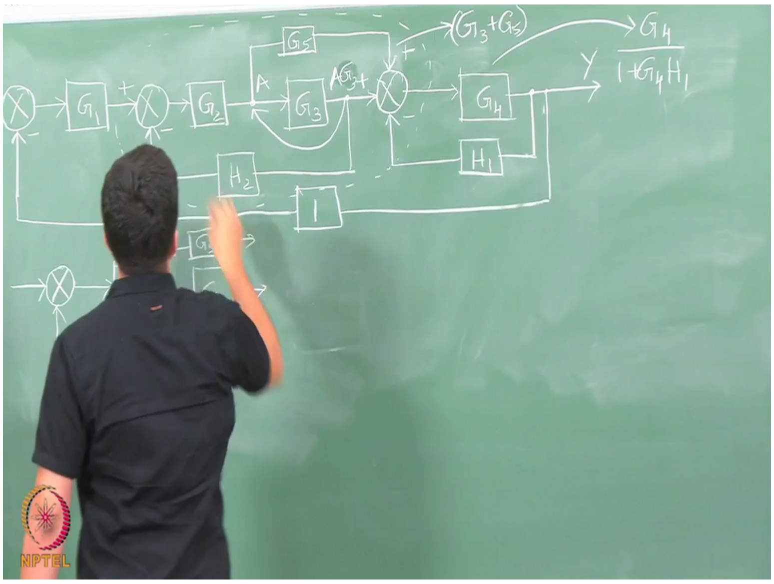

So, this will be G 4 by 1 plus G 4 H 1. And this will be rest the summation G 3 plus G 5.

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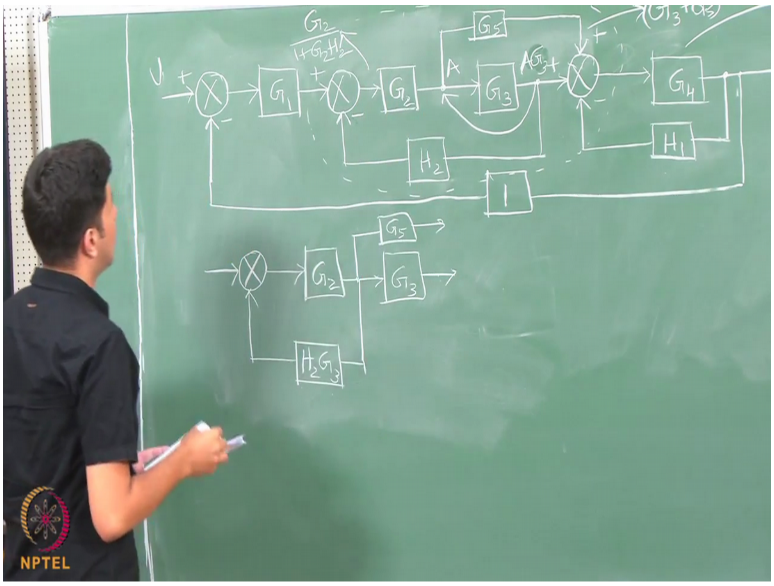

And this will be G 2 by 1 plus G 2 H 2. Once I do this, I can retry my whole block diagram in a much simpler fashion, which is looking like this.

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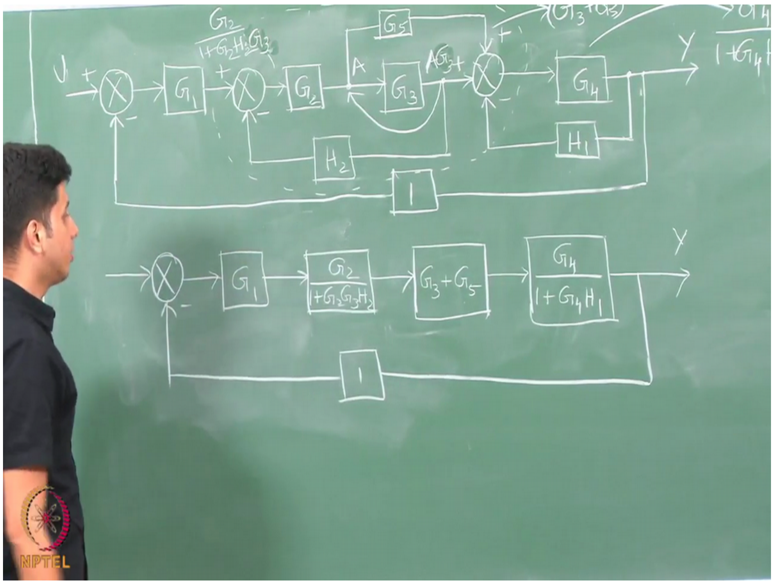

This will be G 2 and G 3 H 2 and G 3, because this G 3 is got multiplied here, when we shifted this point.

So, this is how the simplified block diagram looks like. So, we just have G 1 coming here and this reduction going into G 2 by 1 plus G 2 G 3 H 2 and this parallel block going as G 3 plus G 5, and this feedback loop going as G 4 by 1 plus G 4 H 1. So, now, we have 2 more simplifications to do. One is the cascade of all these 4 blocks and then applying the feedback law right.

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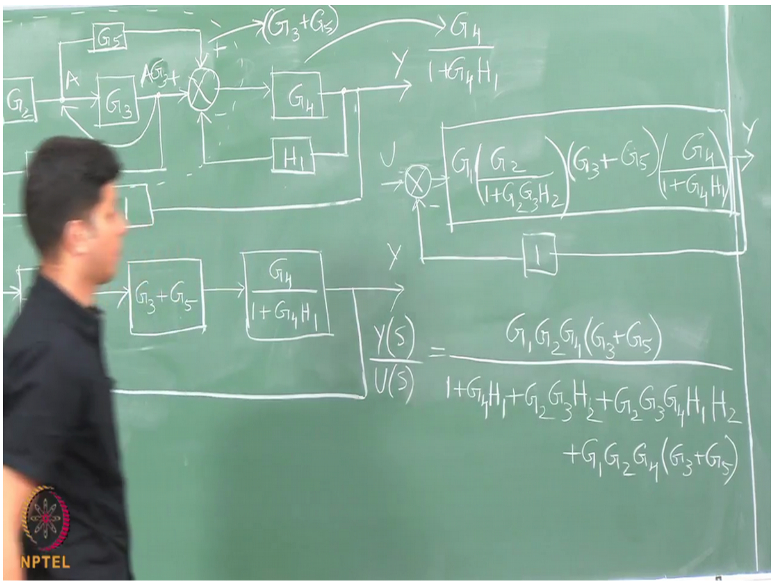

So, once I apply the cascade thing will be something like this. G 1 times G 2 by 1 plus G 2 G 3 H 2 times G 3 plus G 5 times G 4 by 1 plus G 4 H 4. So, this will be in a single block, now I can just add a unity feedback to this. So, now, I have very simple feedback loop. And I can apply the feedback formula and get the transfer function. So, the overall transfer function will be like this. So, I am doing all the simplification and then giving you the final answer. You can do the simplification yourself and check.

So, this will be the overall transfer function of the system. So, that is what we did was we started with a very complicated block diagram, and then we use the rules that we already studied in the previous class, and try to simplify the whole block diagram, and finally arrive it is transfer function.

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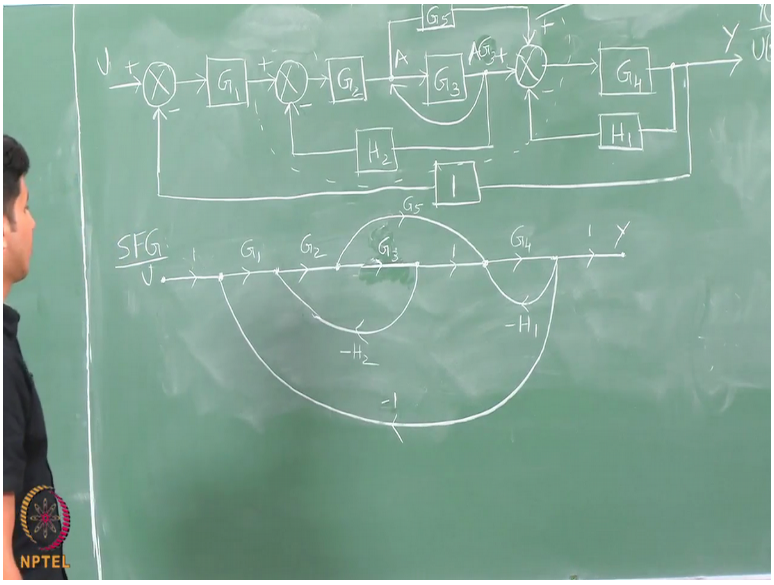

Now, what we will do is, we will draw this block diagram in terms of it is signal flow graph. And derive the same thing and see if both are equal or not. So, this is how the signal flow graph looks like. So, while converting a block diagram to signal flow graph. One should be very careful to ensure that some branches are not missed out like this. So, when you go from G 3 to G 5, this is one take off point followed by a summer, and so you cannot take it to be one node.

So, every take off point and every summer should have it is own node, and because there is no gain over this we will take a gain of 1. So, similarly when you write negative feedback, you should ensure that you put the minus sign here, because there is no summer kind of thing in the signal flow graph the negative sign goes into the gain, and this we was discussed in the class that you should add a branch to ensure that your input and output are as per the defined as per the definitions.

So, now this is how the signal flow graph looks like, and now what we will do is we will apply masons gain formula and try to derive the transfer function.

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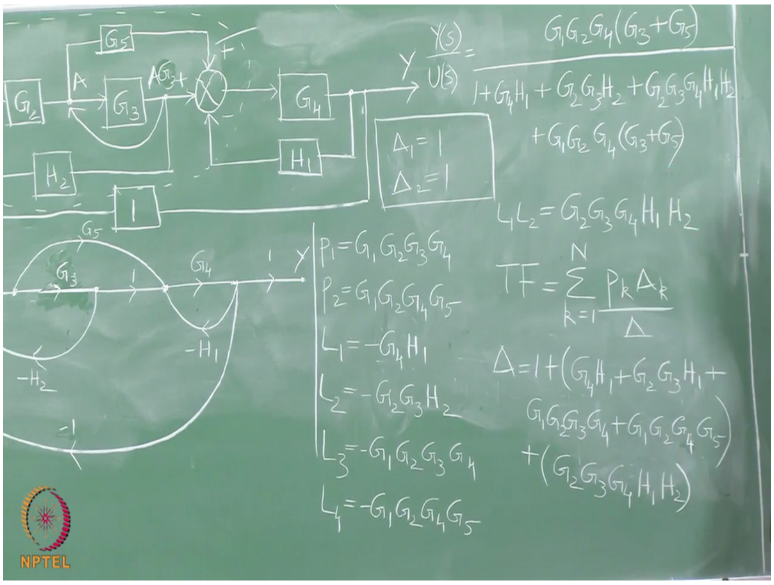

So, to begin with what are the forward paths? One clear forward path is U to Y, and we go we have G 1 G 2 G 3 G 4 and other forward paths G 1 G 2 G 5 G 4 and Y. And so there are no other forward paths this graph has only 2 forward paths what about the loops one loop is this. So, and another loop is G 2 G 3 minus H 2. And another loop is G 1 G 2 G 3 G 4 with unity feedback. And you should be careful that there is an also another loop with G 5, that is G 1 G 2 G 5 G 4 with negative feedback. So, generally people miss out on this. So, we should be careful here. So now, we have the forward paths and the loops now we will go to the non touching loops.

So, out of 4 loops if you look at these 2 loops these are not touching each other, they do not have any common node for the branches. So, among the non touching loops we have L 1 L 2 and G 2 G 3 G 4 H 1 H 2. And if we observe there are no other non touching loops because this loop and this loop are touching this. And this is touching and even with G 5 these 3 are touching each other. So, there are no other non touching loops, and if we even go 3 non-touching loops there is nothing. There are no 3 loops which are not touching. So, this is where we start now we all have all the required components to substitute in the mason S gain formula.

So, we looked at the mason S gain formula giving us the transfer function as given by sigma A is equals to 1 to n p k delta k over delta. So, now, we know both p k and before going to delta k we need what is delta. So, we will define what is delta is as per the definition it is 1 minus the sum of individual loops. So, I will take a plus sign because all of them have a negative and I cannot simply add them up. So, these are all the individual loops plus the products of the non touching loops. So, we have only one set of non touching loop I will just add it here.

So, this is delta. So, what about delta 1? So, if I take the first forward path there is no loop that is actually not touching that forward path. So, my delta 1 is simply 1. And even if I take the second forward path this case is the same. So, my delta 2 is also equals to 1. Now I can simply substitute everything into the masons gain formula, and what I will be getting is the same transfer function. So, you can simply observe thus this delta is nothing but the denominator which we got here. And if you just add up the forward path it is nothing but the numerator that we got because delta k is one input case. So, this is how you got the transfer function in both the cases.

And the most important point to note here is when you have a signal flow graph; it was actually very easy to derive the transfer function. In this case you have to do lots of competition lot of simplifications and before you could arrive this at this transfer function, but here it is very simple, and even if they are more complicated block diagrams it is always absorbed that signal flow graph does very well in finding out the transfer function. So, that is the advantage here. And now for the last problem of this tutorial, we will try to find out a signal flow graph from a network directly. So, we were told that apart from finding out from the block diagram we can also find out the signal flow graph from the network equations and that I s what we will try to do in the next problem.

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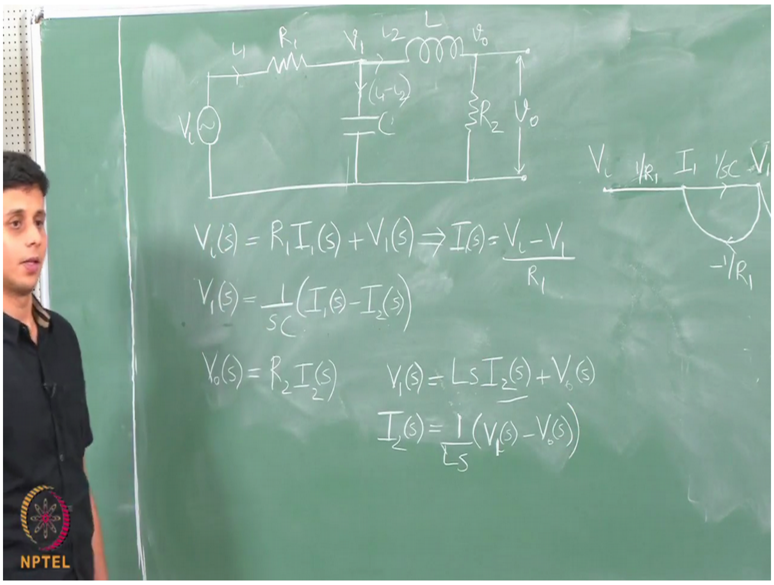

So, I will take a network with some RLC components. So, this is the network that I am taking, I have an input source V i 2 resistors R 1 and R 2 and one capacitor C and (Refer Time: 30:57) vector L and our output variable is we defined it as the voltage across the resistor R 2. So, here I will take the branch currents to be i 1 and this branch current to be i 2.

So, this will be simply i 1 minus i 2. So, we will be dealing with these variables. So, first what we will do is we will write down the equations based on Kirchhoff's law that we know and so the voltage here also I am defining it as V 1 this will be anyway V naught what is my V i of S I am directly writing everything in Laplace domain because now we are comfortable with writing down network equations in S domain directly. So, my V i of S is nothing but R 1 times i 1 of S plus V 1 of S. So, this is equals to this plus this. And my V 1 of S, I can write it as one by S C times i 1 of S minus i 2 of S and V naught of S can be written as R 2 times i 2 of S. So, we have these 3 equations.

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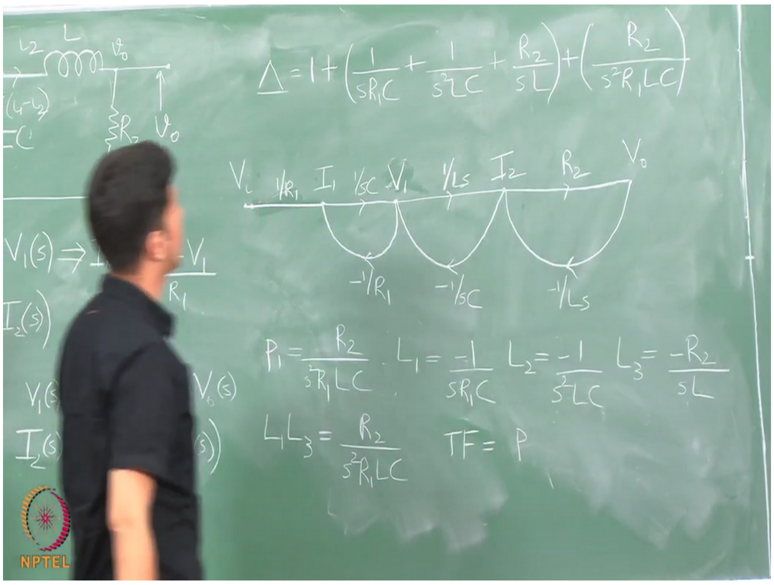

Now, while drawing the signal flow graph, we have to go from V i to V naught. So, in between as we said the nodes are actually the system variables. So, we include all these variable as nodes i 1 i 2. So, the easiest way to do it is always put it in the order in which they appear in the series i 1, V 1 i 2 and V naught.

So, (Refer Time: 33:29) i 1 V 1 i 2 and V naught. Again everything is in (Refer Time: 34:02) just using the variables directly without presenting as. So, how do we connect these nodes using a signal flow graph as we said we are using the equations to find out what will be the signal flow graph. So, I have certain equations which relate these variables now I need to find the right gains which will connect these things. So, fastly what is the connection between V i and i 1.

So, V i is nothing but R 1 times i 1. So, because I will be writing i 1 in terms of V i I need to reconvert this equation first, what is i 1 of S it is nothing but V i minus V 1 divided by R 1 right. So, my i 1 is depending on V i and V 1. So, what is the factor with which it is depending on V i is 1 by R1. So, my gain on this will be one by R 1 and with a negative 1 by R 1, it is depending on V 1. So, I will write it as feedback was this appearing after i 1, I will put it as a feedback. So now, what is the relation between i 1 and V 1. So, we know that V 1 is i 1 minus i 2 by S C. So, V 1 is depending on i 1 with a factor 1 over S C and it is depending on i 2 with a factor of minus 1 by S C. So, I will put minus 1 by S C.

Now, what is the relation of i 2 and V 1. So, we already know this relation. What we will do is we can take another relation as well which will actually give us a nicer this thing. We will take V 1 to be L S times i 2 of S plus V naught. This is fine right I can type I can because this is current i 2 and this is an inductor, I can write it as L times S into i 2 of S plus the voltage here which is V naught. So, why did I write it in this fashion because I have to observe that in the nodes I took i 2 is appearing between V 1 and V naught. So, it will be easy to draw the signal flow graph if I have i 2 in terms of V 1 and V naught right. So, what is i 2 in terms of V 1 and V naught. 1 by L S times V 1 of S minus V naught of S. So, i 2 is depending on V 1 with a facto R1 by L S. I can simply connect this and it is depending on V naught with a factor minus 1 by L S.

Now, finally, what is V naught V naught is simply R 2 times i 2. I can simply write R 2 here. So, my V naught will be simply R 2 times i 2. Now if you look at this signal flow graph. This will exactly represent all the equations that we have written here. And so there is nothing that is missing the complete. Network is actually coming into this signal flow graph.

So, now, what we will do is we will try to find out the transfer function. Earlier we used to apply all the this thing and do elimination and find out the transfer function now because we have a signal flow graph we can apply masons gain formula, and find out what is the transfer function. So, as you can see there is only one forward path, here. And it is gain is one over S square times R 1 and this is R 2 (Refer Time: 39:15) R 1 L and C. So, this is my forward path gain then what about the loops, are 3 loops here right. One is minus 1 by S R 1 C this loop another loop which is minus 1 by square L C and the third loop is minus R by R 2 by S L right.

So, we have 3 loops and one forward path. So, what are the non touching loops. So, these 2 are touching these 2 are touching, but this and this are not touching I can write L 1 L 3, which is R 2 by S square R 1 L C. So, now, simply we apply the masons gain formula to get the overall transfer function, which is before the masons gain we also need to find out what is delta, i will write down what is delta here. 1 minus because everything has a minus output I will put a plus 1 by S R 1 C plus 1 by S square L C, plus R 2 by S L plus this one, R 2 by S square R 1 L C. So, this is my delta. Now once we have and what about delta 1 delta 1 should be one because there is no loop which is actually not touching the forward path. So, when you go from here to here every loop is being touched. So, delta 1 is one.

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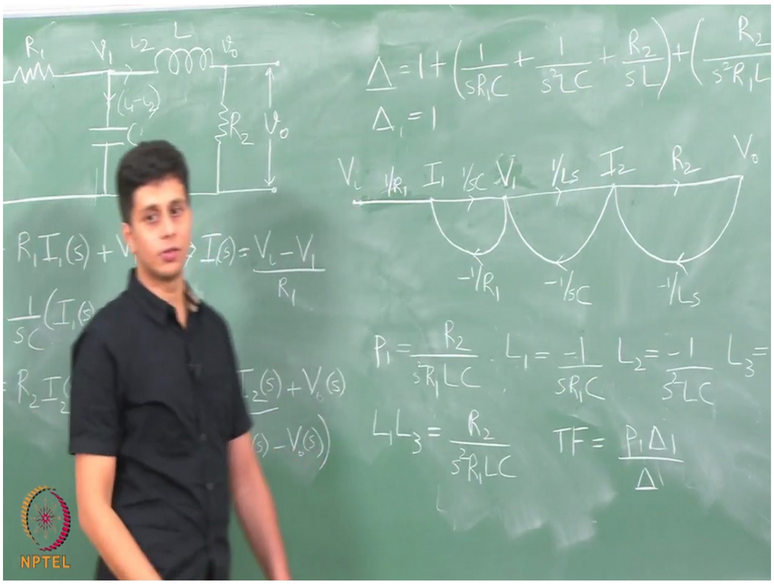

So, what will be the overall transfer function is simply P 1 delta 1 by delta, which is nothing but this is P 1 divided by this whole data. So, that will give you the; you can just simplify it and find out the overall transfer function.

So, instead of doing all the analysis and trying to eliminate everything and finding out what is V naught by V i, we can find simply draw the signal flow graph and directly get the transfer function. So, that is it for the tutorial today.

Thank you.