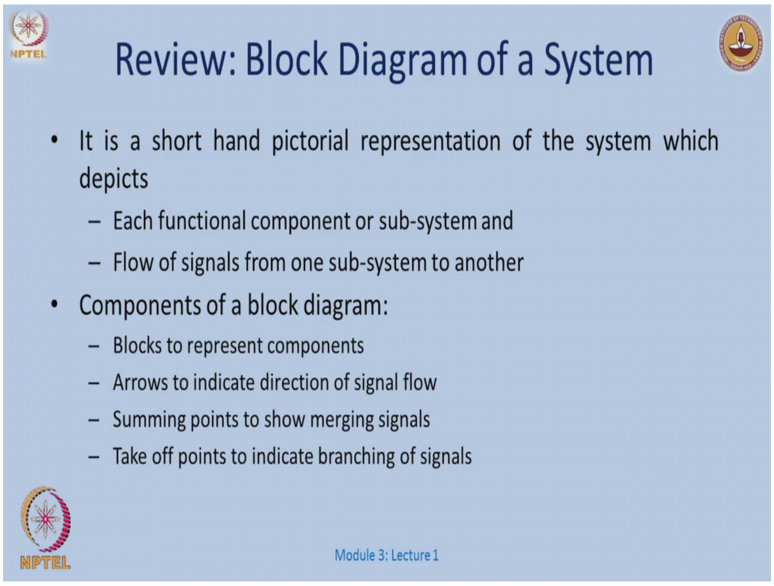
Control Engineering

Dr. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 03
Lecture - 01
Block Diagram Reduction Signal Flow Graphs

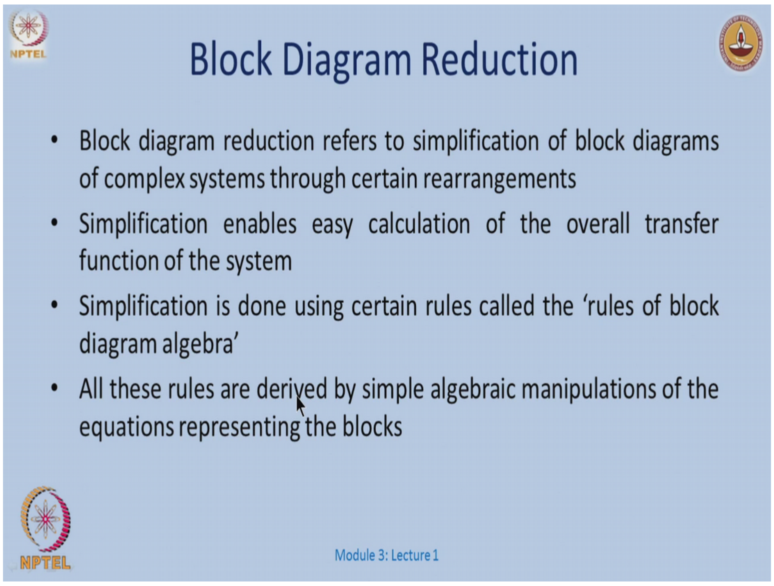
Hi guys. Today we will learn simplification techniques for block diagram. So, if you have a system with a lot of interconnected components connected in series or parallel or feedback interconnection and so on, are there any methods where we could simplify and get the overall transfer function. We will also see; what is also known as signal flow graph method for, these are very small tricks which you need to learn and I will quickly run you through those tricks before we do some examples.

(Refer Slide Time: 00:55)



So, what is a block diagram? So, (Refer Time: 00:57) it is a short hand representation of the system where you have each sub system or each component represented by that it is transfer function. And the interconnection is captured as flow of signals from one system to the other. And well what was the components (Refer Time: 01:20) did we draw the block diagrams. So, the blocks represents the components, equivalently represented by a transfer function, arrows indicates the direction of which the signal flow summing points to show, do signals add up or do they know it is a 1 plus and a minus and so on, and take off points indicate how we branch of signals.

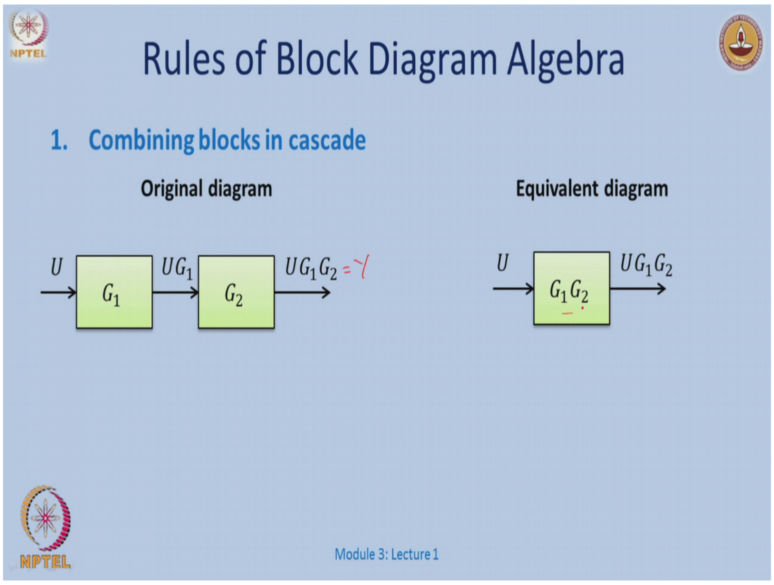
(Refer Slide Time: 01:41)



So, what is this reduction business then? So, we are aiming to reduce or simplify the block diagram in order to get the overall transfer functions. We do not need to really write down equations all the time and then eliminate each variable one by one, that was what we did in yesterday's examples right then we have the block diagram representation, but then if it is a complex one we just have to simplify it you know we need to get the overall transfer function.

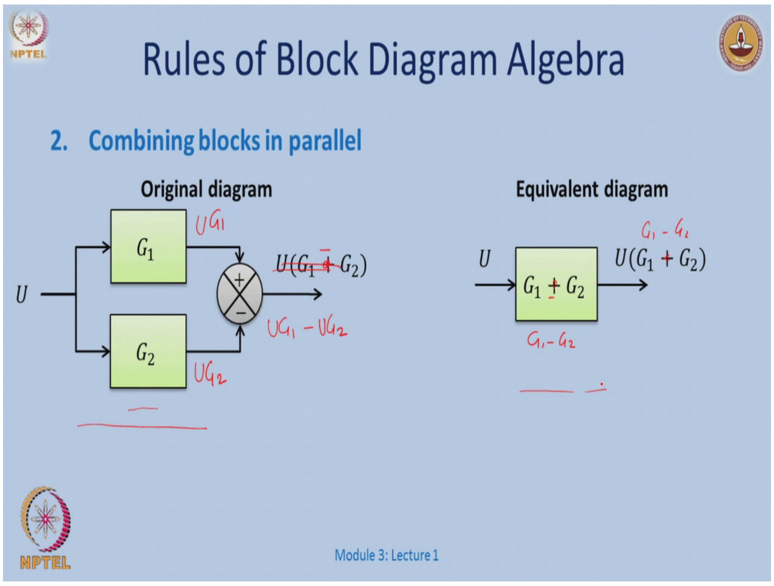
So this simplification is done by some very basic rules of block diagram algebra. And these are very simple algebraic methods when where we just reduce our variables or reduce the blocks and so on. Just very basic things like addition and subtraction and so on.

(Refer Slide Time: 02:30)



So, we learned this thing yesterday. So, if I have 2 systems in cascade. So, if the 2 systems are in cascade then I think the overall transfer function is just G 1 times G 2, and the output Y is simply U times G 1 times G 2 other transfer function is just this guy G 1 times G 2. This we learnt yesterday and it is just kind of a very straight forward straight forward proof. So, this is one of the basic building blocks which we will use.

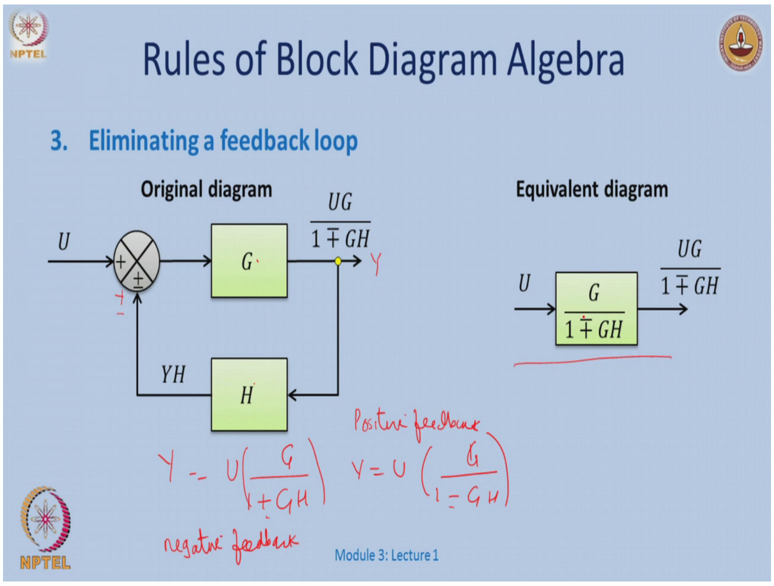
(Refer Slide Time: 02:58)



So, similarly if I have things in parallel then I can just start with this know this, this diagram which looks like this is a block diagram and I know that you know these 2 signals this guy here is U times G 1, this guy here is U times G 2. And then they just add up according to this thing. So, this is A plus here or minus here. So, it will be U times G 1 and so this should be a minus here and this a minus here. So, U G 1 is what comes from the top and then U G 2 comes from the bottom (Refer Time: 03:38) minus sign it will be U times G 2.

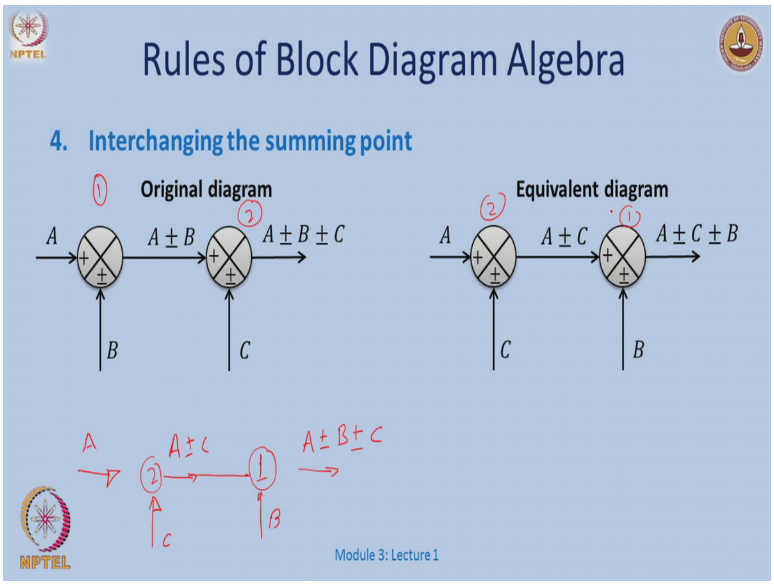
So, it is very straight forward to do and then the equivalent representation like this U times G 1. So, if this is a minus here. So, it will just be G 1 minus G 2, and similarly for the G 1 minus G 2; so apologies for this little thing here. So, if it is A plus everything will be plus and then things will be fine. So, this guy on the left is looks a little you know things with a bigger component, and the guy on the right is something which is simplification of the block diagram. And we will we will again these very basic building blocks.

(Refer Slide Time: 04:11)



So, we also studied yesterday about the feedback interconnection right. So yesterday we started with feedback interconnection this is my U my output signal Y if I have a negative feedback. So, this is a minus here then my overall transfer function would be G over 1 plus G H right. And then Y would be U times this entire thing this is with the negative feedback. So, I can still have plus sign here also right, A plus here this would be called as a positive feedback. In which case my overall transfer function of Y would be U times G over 1 minus G H right. So, the lazy way to remember this is a negative feedback here you will end up with A plus and if it is a positive feedback you will end up with a minus here. And equivalently I can write these entire things as just one component here as just one single transfer function.

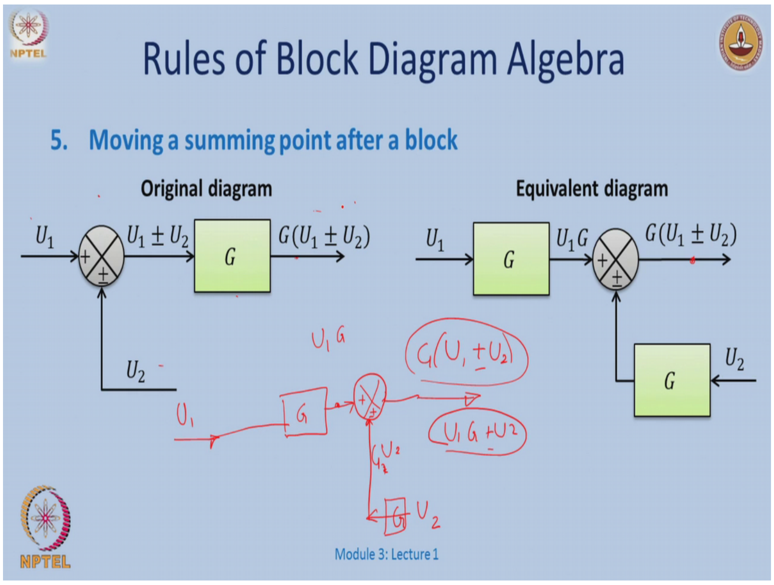
(Refer Slide Time: 05:28)



So, if it is negative feedback I have plus if it is a positive I have a I have minus right. So, now, some more things which we could do with this block diagrams. So, say I have a summing point here you can call this say one, and then I have a summing point 2 and if I want to interchange them. So, in a way that I want to put a one here, and I want to put a 2 here what should not change is the output signal right if I just change them arbitrarily then what is coming out here may change and I do not want to do that I want to (Refer Time: 05:58) of the entire input output structure of the system. So, what should come out here would still be A plus or minus B plus or minus C depending on what is the sign of summing block here.

So, I start with a still goes in, but now it will go to the summing block 2 and then what is the input to the summing block 2 this C over here and then well the resulting signal here would be A plus or minus C. That will go all the way here, and then I have input here. This is C this is A and I can just interchange them this is my summing block 2. So, this is my summing block one right this is very kind of straight forward.

(Refer Slide Time: 06:45)



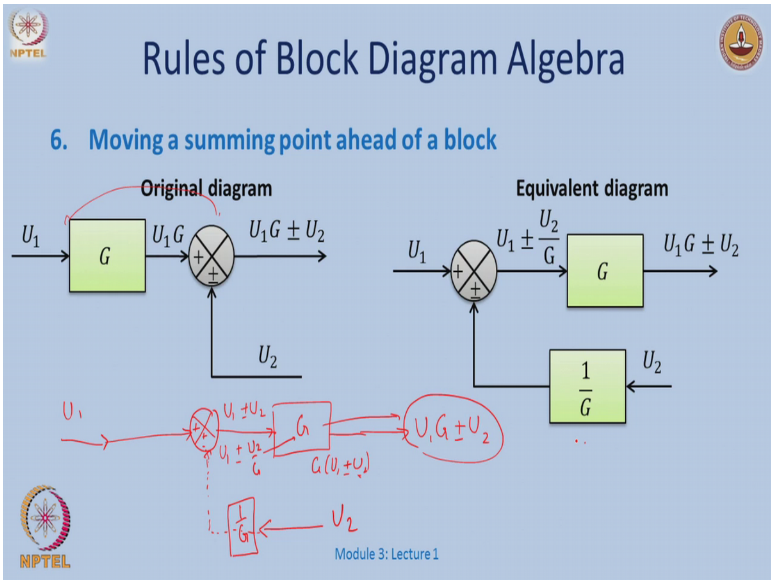
This is a little, but tricky, but again you know it is kind of easy to understand and we will know why we are doing this kind of tricks is, we will want to get things either in the feedback form like here or things in which are interconnected in parallel, or which are interconnected in series and this we formulas I know very well and these are very you know easy to derive and even easy to handle.

So, all this tricks I will use to just simplify into one of these 3 basic configurations right. So, this is my summing point and then I have a block. So, what I aim to do here is I aim to move the summing point somewhere here, such that this guy moves backwards somewhere here. So, what should happen here is that the signal should still be the same. So, what does not change is the way U 1 is given to me it is given to me from outside. So, I just cannot change anything here and then U 2 comes to me from here. And what is measured at the output at this take of point if I call this a output that would still be the same this will be G U 1, plus or minus U 2. And I want to move the summing point ahead which means the summing point should come somewhere here and then there should be a G here.

So, let us try to do this in a just blindly replace it and see what happens. So, this guy goes here U 2 goes into the summing point and I have this I have plus and plus minus. So, the signal here is U 1 times G. The signal coming from here is U 2 what is the overall signal now, that is U 1 G plus U 2. This is not what I want right what I want is a signal something like plus or minus what I want is G U 1 plus G U 2. So, what do I do? So, what I see here is at U 2 should be multiplied by a block with G. And if this is G this guy is the G times U 2 G times U 1 and then I preserve my original signal that is what is exactly happening here.

So, U 1 goes to G I have a U 1 G here U 2 goes via G and I have your signal G U 1 plus minus U 2, it is the same signal here the inputs of the same signals and the outputs are also the same.

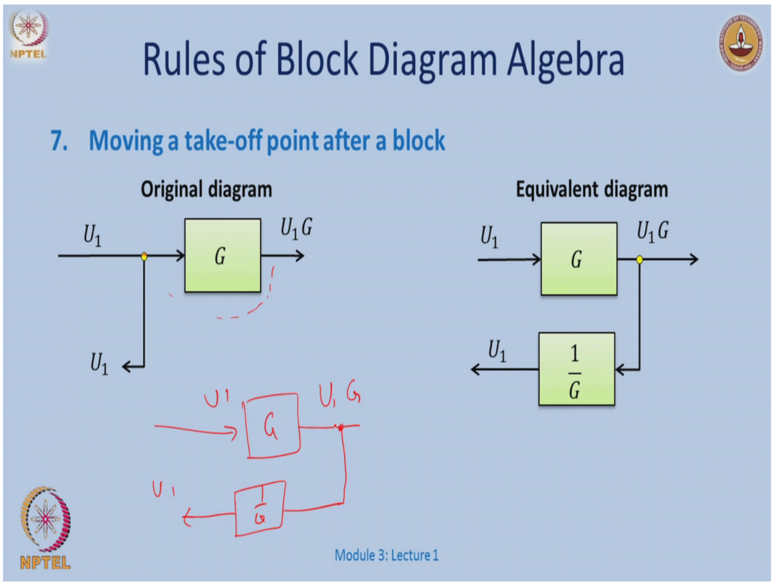
(Refer Slide Time: 09:23)



Now, if I want to do the reverse, I want to move this summing point somewhere here again we just follow the same structure right. U 1 is given to me as it is U 2 comes from outside. So, this is also the same and I want to even preserve what is the output U 1 G plus minus U 2, and then I want to move G to the right. So, let us say I have a G here and then I move into summing point here, and then I connect this with a plus sign and say just I just directly connect this U 2 here without doing any change and this way.

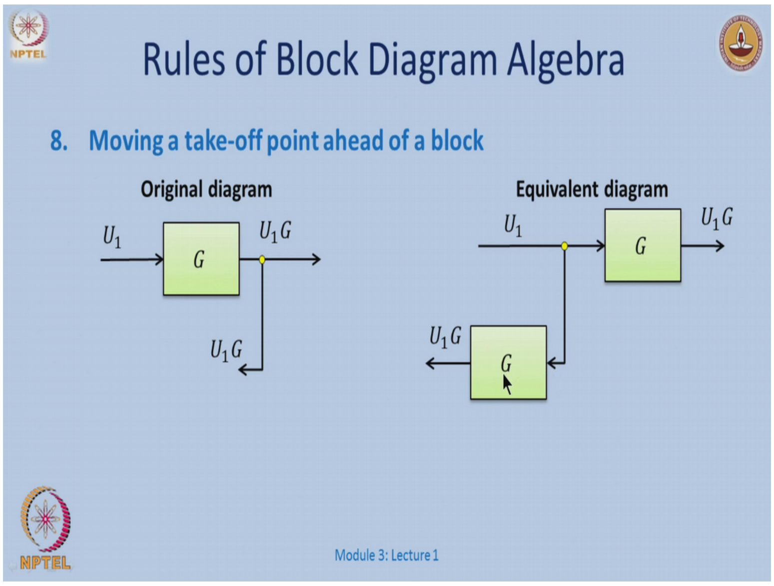
So, what I am a left with. So, from here I have. So, my signal with this connection would like the following this is U 1. So, here I will have U 1 plus minus U 2 and what comes out here is G times U 1 plus minus U 2, but you said what I want no I want something like this I want U 1 times G plus minus U 2. So, I have here U 2 is multiplied by G I do not do not want that I want U 2 just to be by itself, therefore I add a block here which is just 1 over G now the signal looks little bit alright. So, here this U 1 plus minus U 2 over G this will go via G to give me this guy what is happening here.

(Refer Slide Time: 11:06)



Now, this is kind of straight forward type. So, if I have this take off point I just want to be clear right. So, again I have U 1 going in and what should come out is also U 1, but now I want this guy to go after the G right. So, this is U 1 times G, where I say this is U 1 times g, but what I want to measure is U 1. So, I just divide it by G, so this kind of straight forward to do this one.

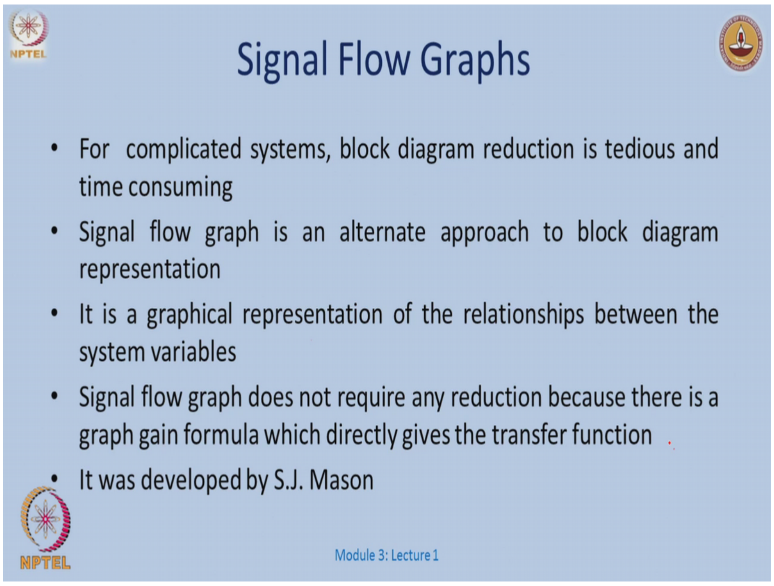
(Refer Slide Time: 11:40)



The reverse I do not think I need to do the proof. So, if I have one to move this guy over here again. So, the input is U 1 the output here us U 1, G and I still want to you know just take off something the signal coming from this take off point still to be U 1 G if I get this guy here I just have to multiply it by G.

So, these are some if the very basic tricks which we will use and we will do some examples later in the lecture.

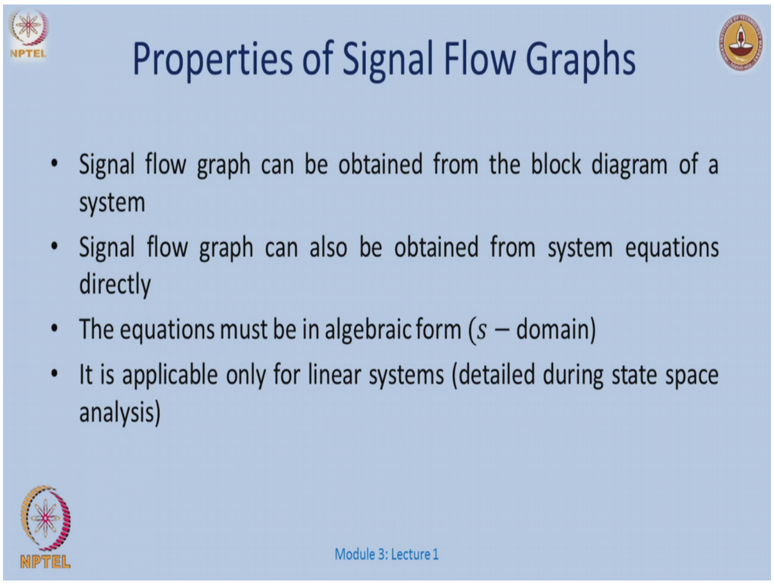
(Refer Slide Time: 12:06)



So other: sometimes it may be very cumbersome to do this kind of exercise right that you know then it might get a little messy and you might forget which id to be moved (Refer Time: 12:16) you know and so on. So, is there any simpler representation and the answer is provided by the signal flow graph. And this is an alternate way or one to represent the block diagram or the physical system which I write the set of physical equations, I interconnect them and then have an equivalent block diagram representation. It is a simpler representation of the relationship between the signals between the system variables.

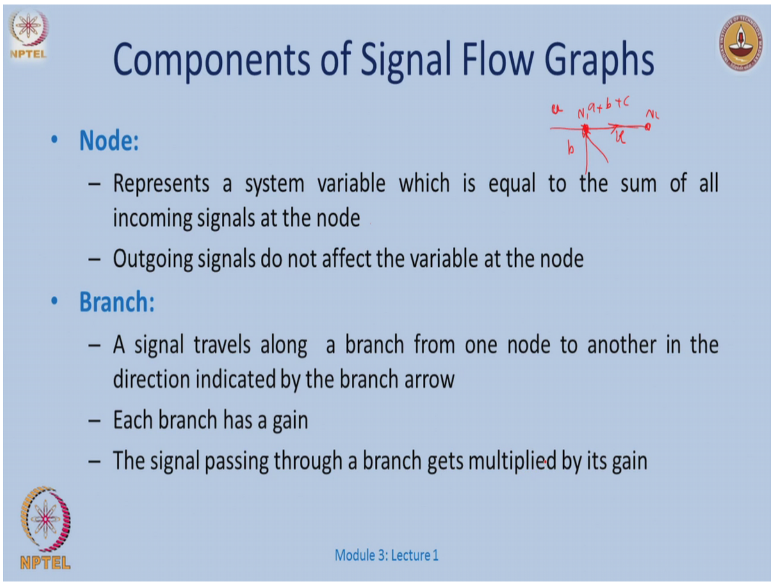
And I do not really need to do any reduction I just kind of memorize this small formula. We not derive that formula, but we just try to use that formula, formula developed by mason and therefore, it is called the mason s gain formula. So, how does the signal flow graph look like?

(Refer Slide Time: 13:02)



So, I start with a block diagram right. I could also start directly from the system s equations I can even skip the block diagram part of it I can just start with system equations which are essentially linear equations, if I do it in a laplacian domain right the equations in the laplacian domain and of course, as said earlier all our analysis for the Laplace base things was based on linear system this, is also for linear systems and we will also do a bit of this during the state space analysis which should be towards the end of the course, but for the move time we will just concentrate on the first 3 points here.

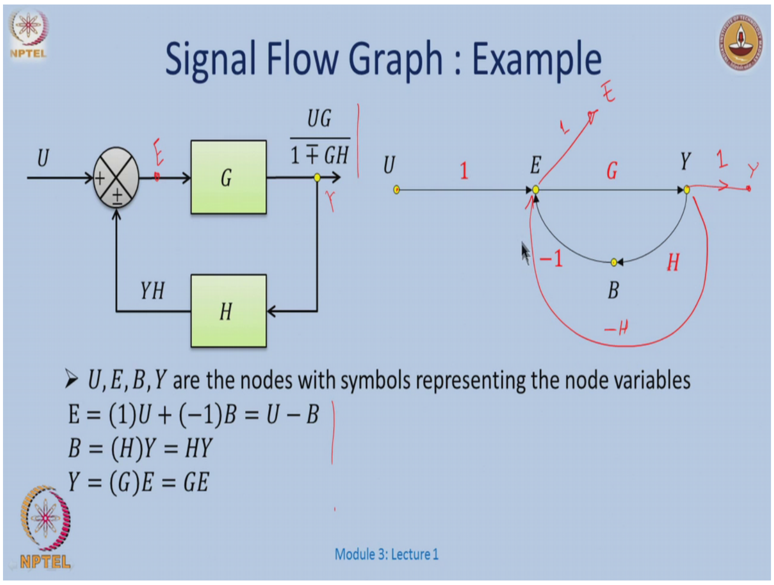
(Refer Slide Time: 13:36)



So, what are the components? So, I have node which represents the variable which is equal to sum of all the incoming things at the node as. So, this you can relate to circuit side if I have a node and then if I want to compute the currents, and you just sum of take the sum of all the elements which you know enter into that and then outgoing signals. So, in addition to that the outgoing signals would not affect the variable at the node for example, if signal here if I have something coming here right. So, this would be say-signal A, signal B what is going out will not be affected. So, this will be kind of A plus B this C will be A plus B plus C and so on. So, what is the computation here we just depend on what is the incoming of the outgoing signals do not affect the variable at the (Refer Time: 14:20).

This is we will also see this from some kind of this case C L kind of stuff right. Branch well a signal travels along a branch from one node to other. So, if this is a node and well this is signal travelling here this is node this n 1. So, this is node 2 and this this guy will be called a branch. And each branch could possibly have a gain right if I talk in terms of circuit elements will be some elements sitting over here right some gain let us call this gain k right. And then if a signal is passing from here to here the signal passing through a branch gets multiplied by it is gain this is not very surprising see.

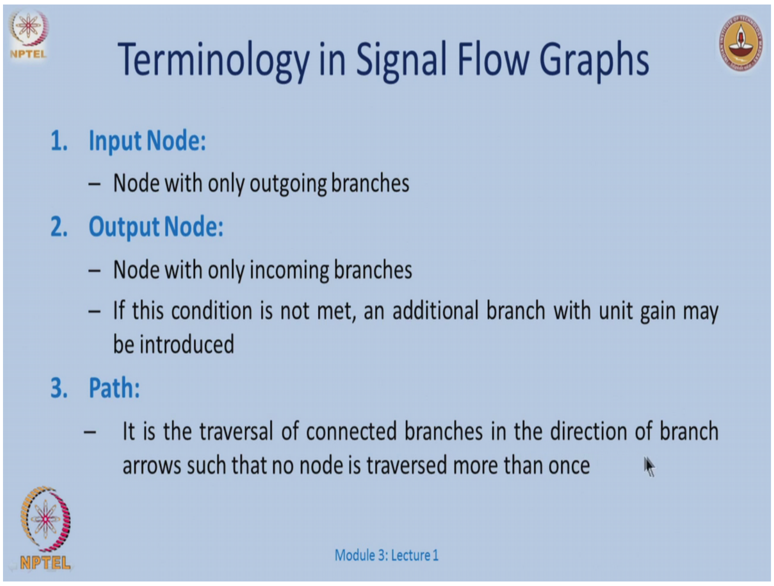
(Refer Slide Time: 15:04)



So, if I come back to my original feedback configuration. So, I have a input signal, I have a output signal here Y this transfer function G H and I know how to derive this transfer function by now. So, what could be the nodes here, when I can say something starts from here this is a node which just has one branch going out of it here. So, this we denote it as the error signal. So, E has U coming in from here and something coming from here right here in this case it is Y times says depending if the sign is A plus or a minus. So, I add another node. So, I can go from here till here I have a H, and this plus or minus I just represent it with a minus sign if it is a minus 1 I could also keep this process and this call this entire branch directly as minus H nothing changes I just multiply these 2. They also like if you look at a transfer function H multiplied by this again minus 1, it will just be minus H and from E to this node Y I have a gain G.

And then of course, these are very straight forward computations right. So, what is happening at B is Y times H which is what we said here at the signal passing through a branch gets multiplied by it is gain, that is exactly what is happening here. And then So, these are the same things at B is H times Y this Y is G times E and so on and E is so E has a combination of signals one coming from U with a gain of one this guy B coming with a gain of minus or you can also talk about this directly depending on what kind of analysis you do.

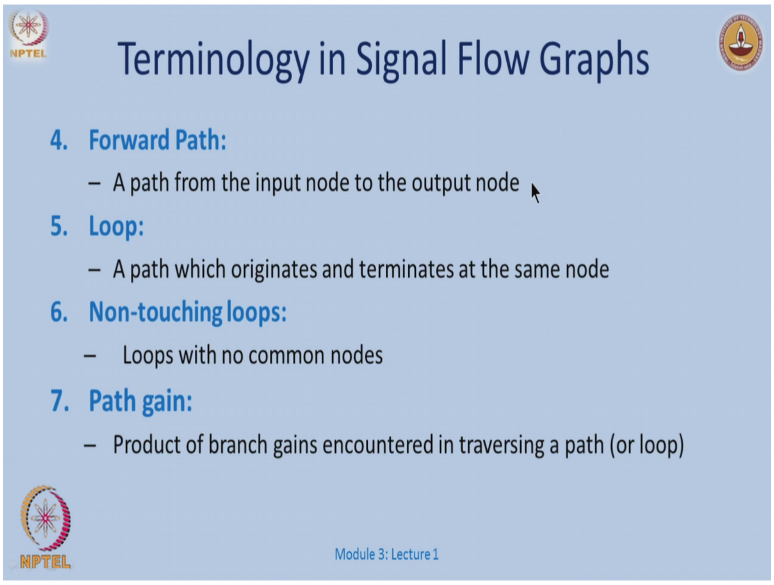
(Refer Slide Time: 16:54)



Some basic terminology what is an input node, input node is the one which has only outgoing branches. For example, here the U is the input node it just has outgoing branches now output node. So, output node is the one which has only incoming branches and if this condition is not met then we can just draw an additional branch with unit gain. So, what does this mean in the context of this standard feedback system let take. So, here I really do not satisfy this definition directly, the node with only incoming branches and this condition is not met what do I do is I just take this guy and I put a gain of one put a arrow here this will be Y. Now this will satisfy the definition now. Because it just has branches which are just incoming into it and I think no other branch here satisfies that criteria.

You could also may be pull out something from here right this is nobody stops you from doing this also this also you could call as the output node. So, the path the definition is not surprising it is the traversal of the connected branches in the direction of branch such that, no node is traversed more than once. Here a path would simply be if I want to go from U till Y just go from U to E to Y and so on. This is not allowed I cannot go from U to E E to Y again come back wear this path and go. So, each node should be should be traversed only once that is what we will be careful about.

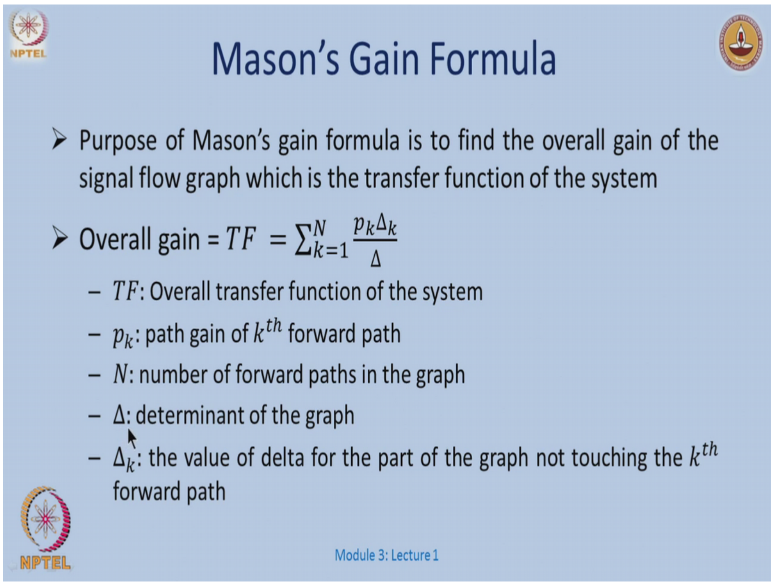
(Refer Slide Time: 18:38)



A forward path well is a path from the input node to the output node and then a loop is a path which originates and terminates at the same node. So, this is a forward path this is a loop starts from E and ends at E this, this outside guy is also a loop, and then a non touching loops could be the loops which have no common nodes. So, in the previous example there are no non touching loops and then the path gain would be just a product of branch gain encountered in traversing a path or even a loop.

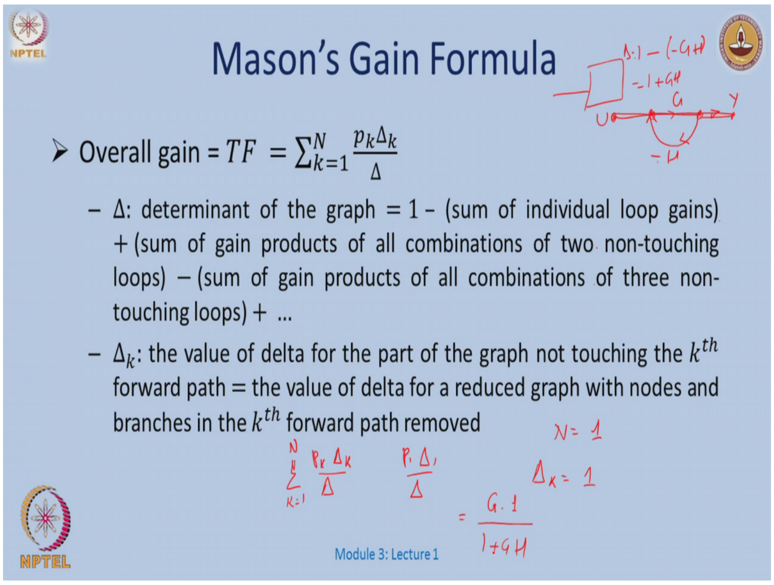
So, for example, if I traverse from U till Y, the gain is one times G times one, it will just be g, but if I traverse around this path or along this path the gain would be G times H times minus 1, it will be minus G H or this one G times minus H also minus H.

(Refer Slide Time: 19:40)



So, now we are ready to learn the masons gain formula, what is the purpose is again to find the overall gain of the signal flow graph. And why do we find the gain and this gain we will see is just a transfer function of the system right. So, what are the things which we need to know. So, the formula looks kind of simple it is the transfer function is summation from k to 1 p k delta k over delta. So, what do these mean t f is of course, the transfer function p k is the gain of the k the forward path. There could be more than one forward path we will shortly see with the help of an example.

(Refer Slide Time: 20:29)



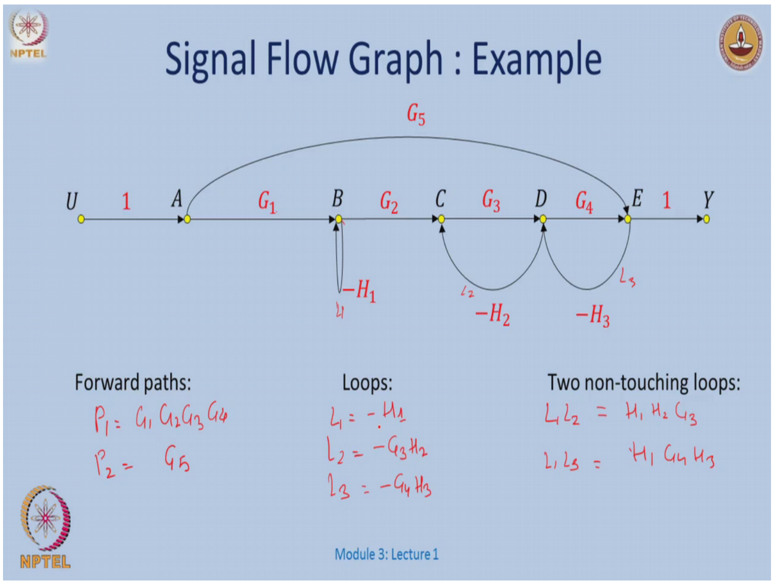
N is the number of forward paths delta is the determinant of the graph, and we will again define what this delta means we will just go to the definition of the delta. So, delta is by definition which is also called the determinant of the graph is just 1 minus sum of all the individual loop gains. So, let us keep that example in mind in that example. So, I have I just draw the basis I have a G, this way I have a minus H I am talking of a negative feedback, I have U and I have Y and this goes in this direction right.

So, the determinant is 1 minus sum of all individual loop gains how many gains here I have 1 minus, there is just one loop right. Minus G H this is 1 plus G H does not stop here, it goes further and says sum of gain products of all combinations of 2 non touching loops, well there are no 2 loops here this guy will go away minus the sum of products of all combination of 3 non touching loops of course, there are no second guy is absent the third guy will also be absent and then this formula just goes by right.

So, this is how we determine the delta now delta k, delta k is the value of delta for the part of the graph not touching the k-th forward path. What does that mean in this case there are number of forward paths is 1, just this is the forward path. Now if I remove this right. So, this value of delta will again be computed this way. So, just give me a graph I can just look at 1 minus loops and the non touching loops 3 non touching loops and so on. And this ones of I remove this node this node this node nothing remains right therefore, the delta k here is just 1.

Now, let us see the transfer function by the formula summation k equal to 1 to n p k delta k over delta and equal to 1. So, I will just have p 1 delta 1 over delta what is this p, p is the gain of the k the forward path. There is only one forward path what is it is gain from here till here that is G times delta 1 is 1 over the overall delta this is the overall delta 1, plus G H and this is exactly what we derived earlier for a negative transfer function if it is A plus, here this will just become a minus. Now let us see a little one more example which will help us understand these concepts of loops and forward paths and so on.

(Refer Slide Time: 23:20)

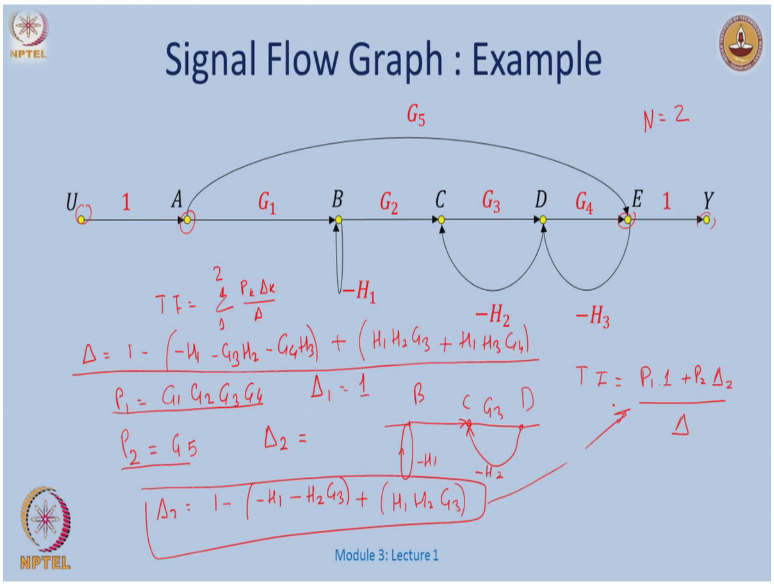


So, from U till Y how many forward paths I have when I can goes this here, the forward path let me call this as p 1, p 1 would be G 1 or the gain of the forward path would be 1 times G 1 times G 2 times G 3 G 4 and 1, G 1 G 2 G 3 and G 4. Now is there any other forward path where I can reach from U till Y where of course, I can go here I can jump all the way where G 5 come to E and go to Y. So, that is the second forward path p 2 with the gain of G 5. How many individual loops are there, well this one. So, loop one which has a gain L 1 is minus H 1 this is also a loop I start from C, I go to D and I come back again to C here I just start from D and again come back to b. So, like a self loop here L 2 this is G 3 times minus H 2 that is G 3 times H 2.

And look at the third loop there is also a loop here. So, this is D to E and again back to D again with G 4 times minus H 3 that is G 4 H 3 with a minus sign are there any other loops no right. So, I cannot do this way just 3 loops and then here are there any non touching loops. So, this loop is touching this loop this is not a non touching loop. So, this is one loop which does not touch this loop for example, again the same loop does not even touch this loop. So, what are the non touching loops that is L 1 if I call this loop as L 1, I call this loop as L 2, I call this loop as L 3. So, L 1 and L 2 are non touching and their gains would be minus H 1 times this guy that will be H 1 H 2 G 3,

Similarly, L 1 and L 3 are not touching each other, is the total gain is H 1 G 4 H 3. Then the minus gets cancelled out. So, we have 2 forward paths 3 loops and 2 non touching loops.

(Refer Slide Time: 26:41)



Now I want to write down the entire transfer function for this. So the formula read let us recollect the formula, this is summation k equal to one to n p k delta k over delta. So, here my n is 2. So, my transfer function is summation 1 to 2 p k delta k over delta. Now first thing is to find delta. So, let us see what the definition says the definition of delta is 1 minus sum of all individual loop gains, let us do this first this is 1 minus sum of all individual loop gains what were the loop gains H 1 this and this, 1 minus the individual loop gains would be minus H 1 minus G 3 H 2 minus G 4 H 3.

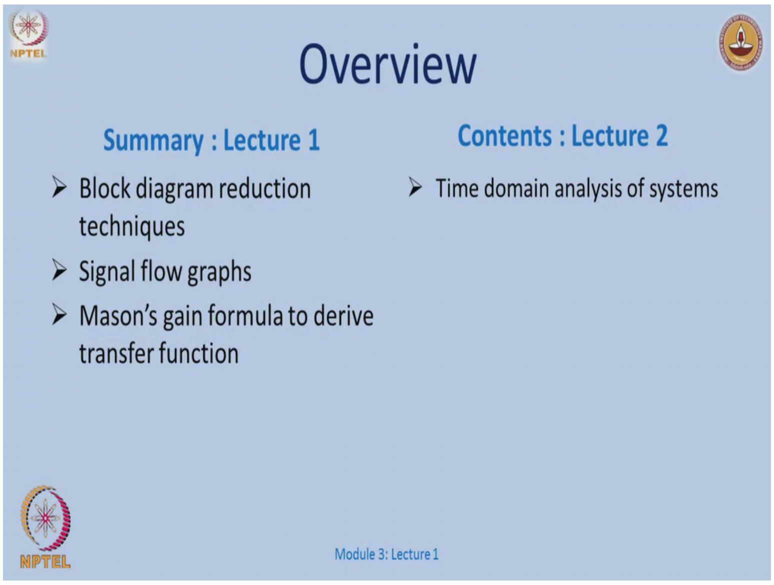
So, what does the next step say plus sum of gain products of all combinations of 2 non touching loops gain products if I have 2 loops L 1 and L 2 which are non touching. So, I will just take the product gain of each of this L 1 and L 2. So, this is this one H 1 H 2 G 3 1 H 3 G 4. So, I have plus here (Refer Time: 28:24) this guy H 1 H 2 G 3 plus this times this that is H 1 H 3 and G 4 and so what does it say plus the sum of gain products of combinations of 3 non touching loops there are no 3 non touching loops here right the no 3 loops here. So, there are 3 loops here, but this loop is touching this loop. So, there are no 3 non touching loops. So, this is my delta.

Now, next I want to find out what is p 1 and it is corresponding delta 1 p 1 as we followed earlier just G 1 G 2 G 3 and G 4. What is the corresponding delta 1. Delta 1 again by definition is the value of delta for the part of the graph not touching the k th forward path. So, how does the graph look like? So, this is the first forward path and all the nodes go away right this is nothing left right. So, delta 1 would simply be 1. Now look at p 2 p 2 again they have this path this here and this one that is G 5 now delta 2. So, delta 2 looks how does it look like. So, this node is gone this node is gone, and this node is gone this node is gone. So, I am left with B to C D, and I have this one with minus H 2 G 3 and self loop here minus H 1.

So, this how my graph looks like without the nodes U A E and Y because they touch the second path now again I apply the definition of delta for this guy. So, that will be 1 minus again just recollect the definition. Sum of individual loop gains how many loops are there this is one loop, and this is one loop 1 minus H 1 minus H 2 G 3. These are all delta 2 and then plus I have to look at non touching loops. So, this loop is not touching this loop. So, I have around one pair of non touching loops plus overall gain that is H 1 H 2 G 3.

So now, I am pretty much ready with my formula the transfer function is p 1 times 1 plus p 2 times delta 2 with the overall delta this this guy is the overall delta right this is p 1 this is my p 2 and this is the overall delta 2 I just plus all these guys here, to get my transfer function as simple as that just that you have to be alert in finding are there any non touching loops and so on.

(Refer Slide Time: 31:53)



So, that like ends the lecture for today, where we have just learnt block diagram reduction techniques, construction of the signal flow graph and the masons gain formula to derive the transfer function. What do we next focus on before we do some problems, or after we just finish some problems in block diagram (Refer Time: 32:14) graph is to look at time domain analysis of the system. I will not go to details of what will be there, but that will be explained in the next class.

Thank you.