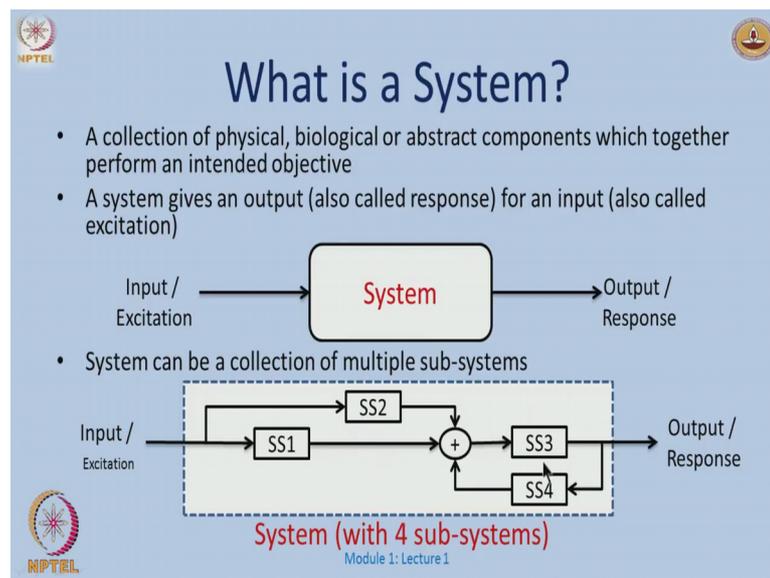


Control Engineering
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Module – 01
Lecture – 01
Introduction to Systems and Control

Hello everybody. Welcome to this first lecture on Control Engineering. So, what we will do is we will just keep it very short series of short lectures about 20-25 minutes each sometimes even less. So, we will begin with a little Introduction to Systems and Control, like what kind of tools you might eventually use in the course and some very basics or some very basic mathematical tools as well.

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So, let us start by defining what is the system. It could mean several things, but we will restrict what makes sense for us in the course. So, by definition it could be a collection of some physical components, biological components, sometimes even abstract components which talk to each other two kind of perform intended objective. From the control point of view we can say that a system when subject to a certain input also called excitation give us certain output. We can also call it as a response of a system and usually schematically we can draw it as a as a diagram like this.

Systems could also be composed of several subsystems, and we will eventually see physical examples of this. So, like one system could be connected this way S 1 you know it could be connected this way or this way you know S 3 to S 4 and so on. So, that overall input will give me some kind of an overall response of the system.

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Examples of System

- Motor**
 - Input – Electrical energy (voltage)
 - Output – Mechanical energy (Torque / Rotation)
- Air conditioner**
 - Input – Electrical energy (voltage)
 - Output – Heat energy (Changes the ambient temperature)
- Human body infected with a virus**
 - Input – Drug administration
 - Output – Drug distribution & effect on the body
- Vehicle (car or bus)**
 - Input – Acceleration or Deceleration
 - Output – Vehicle displacement

Module 1: Lecture 1

So, what could be examples some of them are like very obvious, it take a case of a motor where I just give a electrical input or and I just see the output as a mechanical energy in form of a torque. Air conditioner the input again is electrical energy, output you can sense in the terms of heat energy which changes the ambient temperature. Human body which is infected with the virus, the input could be the drug prescription which you take; an output could be the way the drug is distributed in the body its effect on the body in killing the virus.

We will see all this how this could actually be modeled mathematically. And another not very surprising example is that of a car where the input could be some kind of a signal which generates acceleration and the output which you could obviously see is the vehicle displacement.

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Nomenclature

- Some of the basic symbols to be used in this course:

S.No.	Variable Name	Symbol	Description
1	Time	t	Time instant
2	Input	$u(t)$	Input signal given to the system at time t
3	Output	$y(t)$	Output signal of the system at time t
4	Delay	δ	Time delay in a signal
5	Disturbance	$w(t)$	Disturbance affecting the system at time t
6	Function	f	A defined relation between a set of variables



Module 1: Lecture 1

So, before we start: I will just introduce you to some notations in the course, the small letter t would denote the time variable, input would usually be denoted by u of t , output would be y of t , if there is certain delay in the time signal we will denote that as a delta, disturbance would usually be denoted by w of t which is a time varying disturbance and f denotes any function right which may be maps one variable to the other.

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Classification of Systems

- Variety of classifications are possible based on system features and applications
- Some of the important classifications include:
 - Linear and non-linear systems
 - Static and dynamic systems
 - Time invariant and time variant systems
 - Causal and non-causal systems



Module 1: Lecture 1

So, how do we classify systems? Based on the nature of systems we can have several varieties or several classifications; some of them which you which you had already learnt

in our signal systems course or our circuit course and so on. So, the first one could be something like a linear and non-linear, static or dynamic, systems which vary with time, and system which do not vary with time as we call as we call them as time invariant systems. And lastly what we will deal with is causal or non-causal systems.

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Linear Vs Non-Linear Systems

Linear systems

- Output of the system varies linearly with input
- Satisfy homogeneity and superposition
- E.g. Resistor : $I = \frac{V}{R}$

Non-linear systems

- Output of the system does not vary linearly with input
- Do not satisfy homogeneity and superposition
- E.g. Diode: $I = I_0(e^{\frac{V}{\tau}} - 1)$

The slide includes two graphs: a linear graph of current I versus voltage V for a resistor, and a non-linear exponential graph for a diode. The text 'Module 1: Lecture 1' is visible at the bottom of the slide.

So, what is a linear system? The system for which the output varies linearly with the input; and we all know that where it just satisfies the principle of homogeneity and superposition. For example, a resistor; so let us see how this works.

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Handwritten notes in a Notepad window titled 'Note1 - Windows Journal'. The notes include the following mathematical expressions and diagrams:

$f(x) \quad f(x_1 + x_2) = f(x_1) + f(x_2)$ (Superposition)
 $f(ax) = a f(x)$
 $I = \frac{aV_1 + bV_2}{R}$
 $= \frac{aV_1}{R} + \frac{bV_2}{R}$

Two circuit diagrams are shown: a simple resistor circuit with voltage V and current $I = \frac{V}{R}$, and a more complex circuit with two voltage sources aV_1 and bV_2 in series with a resistor R .

$I = \frac{V_1 + V_2}{R} = \frac{V_1}{R} + \frac{V_2}{R} = I_1 + I_2$

So, if I just take a resistor right, so why do I call it a linear system. Let say first let see if we can define what is a linear function. If I say f of x then a function is linear if f of x_1 plus f of x_2 is f of x_1 plus f of x_2 ; is what we called as the superposition thing, a superposition property. As something call the homogeneity if I just take f a times x where a is just the constant I just have a a of x . What, why do I call a resistor a linear system is justly because; if I take a R I give it a input voltage V the output I see is I equal to V over R .

Now what happens if I have two voltage sources V_1 V_2 well of course with the appropriate sign conventions, and R what is the current. So, the current I see here is of course, V adds up like V_1 plus V_2 over R which essentially is V_1 over R V_2 over R and this is the response would I_1 which I would get only with voltage V_1 , and I_2 the response which I would get only with V_2 . So, this is like I could just take the individual voltages take the response and then combine them.

How does homogeneity work? So, instead of V if I say replace this by some constant a some constant b and I look at the current the current would just be a times V_1 b times V_2 over R , that simply would be again a V_1 by R b V_2 by R and so on. So, this is like again the response with voltage V_1 scaled by a factor a added directly to the response or the response with the voltage V_2 with scaling b . So, this is an example of a linear system.

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The image shows a screenshot of a note-taking application with the following handwritten content:

$$V = L \frac{di}{dt} \quad \frac{di}{dt} = \frac{V}{L} \quad \frac{V_1 + V_2}{L} = \frac{di_T}{dt}$$

$$\frac{di_T}{dt} = \frac{V_1}{L} + \frac{V_2}{L} = \frac{di_1}{dt} + \frac{di_2}{dt} \text{ (superposition)}$$

$$I = I_0 \left(e^{\frac{-Vt}{L}} - 1 \right)$$

$$I = I_0 \left(e^{\frac{-(V_1 + V_2)t}{L}} - 1 \right) \neq I_0 \left(e^{\frac{-V_1 t}{L}} - 1 \right) + I_0 \left(e^{\frac{-V_2 t}{L}} - 1 \right)$$

The NPTEL logo is visible in the bottom left corner of the note.

We could also see well what if I have instead an inductor say voltage is $L \frac{di}{dt}$ or in other words the current across an inductor follows this one V over L . So, what would happen if I have two voltage sources? So, my total voltage would be V_1 plus V_2 which goes across a resistor L and that would generate some amount of current is call this I total here $\frac{di}{dt}$.

So, what is this $\frac{di}{dt}$? This $\frac{di}{dt}$ is now can be written as $\frac{V_1}{L}$ plus again $\frac{V_2}{L}$; the same way as we did earlier for a resistance. This again should be what the $\frac{V_1}{L}$ is essentially if I just take the response with just V_1 as the source similarly $\frac{V_2}{L}$ would be the response which I would take with only V_2 as a source and you say the adopt right just define the super position thing. And similarly even the homogeneity would follow. So, what is the non-linear system do? Well, non-linear system well the response does not vary linearly with the input. And of course, it does not satisfy homogeneity and superposition. For example, a diode let us work out this as we did earlier for the same.

So, I was $I_0 e^{-\frac{V}{\tau}}$. And you see if I take again V as a combination of two sources V_1 plus V_2 I could never write I as some individual combinations. So now, the total I would be $e^{-\frac{V_1 + V_2}{\tau}}$, and this is not equal to $I_0 e^{-\frac{V_1}{\tau}} + I_0 e^{-\frac{V_2}{\tau}}$. So, superposition does not hold. And similarly you can check with the homogeneity as well say- if I just put a here it will not be equal to a times $I_0 e^{-\frac{V}{\tau}}$; just kind of a straight forward (Refer Time: 08:58).

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Static Vs Dynamic Systems

Static systems	Dynamic systems
<ul style="list-style-type: none">At any time, output of the system depends only on present inputMemory less systems$y(t) = f(u(t))$E.g. Resistor: $I(t) = \frac{V(t)}{R}$	<ul style="list-style-type: none">Output of the system depends on present as well as past inputsPresence of memory can be observed$y(t) = f(u(t), u(t-1), u(t-2), \dots)$E.g. Inductor: $I(t) = \frac{1}{L} \int_0^t V(t) dt$

Module 1: Lecture 1

The other kind of classification is between static and dynamic systems. So, for a static system at anytime the output depends only on the current value of the input. Other words they are also called memory less systems, whereas for dynamic systems the output depends on the past as well as the present inputs. And these are systems which are called systems with a memory. What does it mean? I can just work out a work out another example.

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Handwritten notes on a digital notepad:

Left side (Resistor):

- $V = IR$ (circled)
- $V = 0$
- $I = 0$
- $V = I \cdot R$ (circled)
- $I = 1A$ (underlined)

Right side (Inductor):

- Circuit diagram with a voltage source V , a resistor R , and an inductor L in series. Current i flows clockwise.
- Equation: $V = iR + L \frac{di}{dt}$
- At $t=0$, the circuit diagram shows the inductor with current $i(0) = i_0$.
- Equation: $iR + L \frac{di}{dt} = 0$
- Equation: $i(t) = i_0 e^{-\frac{R}{L}t}$ (underlined)

So, let us again take the example of V equal to IR a linear resistor. So, if I apply a voltage V it results in a certain current across my resistor; V is R and this is the output. And as soon as let say switch to V equal to 0, I just remove this guy. Then what happens? Well, I instantaneously goes to 0; which means the output is gone so the input is gone the output automatically is gone or is goes to 0.

Now, take an example where I have a V R and an inductance; this is my R and L if I write down the dynamics it would simply be V equal to IR plus $L \frac{di}{dt}$. And I say I just put this voltage source highlight the system go on for a while, a few second say. And later on what I do I just; sorry, I just take out the voltage which means I am now looking at the circuit which looks like this. I do circuits some time when I do the circuit when I switch it off for a sometime. You would know that the inductance accumulates some flux or its stores some energy.

Or in other words the initial current when I just say I do this at t equal to 0 is not 0 then will be some current. And I will write down the equations it will be IR plus $L \frac{di}{dt}$ is 0. And let me say I at 0 is some number I of 0. Then the current for all feature times would be I of 0 e power minus of R over L times t . So, what you see is that as soon as V goes to 0 I follow a certain pattern it does not go to 0 immediately, it will take some time to go 0 depending on what the value of R and L R and if these are usually referred to as a time constants of the system.

So, this is how we you classify between what are called as static and dynamic systems. And the way the reason we say memory is at well they store some energy and remember something and that they do continuously equal to 0. Mathematically it would mean that y if t is equal to u of t , just at that at that time instance. Or like for example, which we saw of the linear resistor. For the dynamics system y of t would depend on u of t its previous instance is what was like if we talk in terms of the stored energy, it would just mathematically look something like this. If I want to say I at 1 second it depends on all the previous instances starting from t equal to 0. For example, as we saw the inductor.

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Time Invariant Vs Time Variant Systems

Time invariant systems	Time variant systems
<ul style="list-style-type: none">• Output of the system is independent of the time at which the input is applied• $y(t) = f(u(t)) \Rightarrow y(t + \delta) = f(u(t + \delta))$• E.g. An ideal resistor $I(t) = \frac{V(t)}{R} \Rightarrow I(t + \delta) = \frac{V(t + \delta)}{R}$	<ul style="list-style-type: none">• Output of the system varies dependent on the time at which input is applied• $y(t) = f(u(t)) \not\Rightarrow y(t + \delta) = f(u(t + \delta))$• E.g. Aircraft: Mass (M) of aircraft changes as fuel is consumed• Acceleration: $a(t) = \frac{F(t)}{M(t)}$



Module 1: Lecture 1

Other examples could be the example of or the other classification would be in terms of systems which vary with time and systems do not vary with time; as we call time invariant and time variant systems. And this should not be confused by the dynamics of the system. Like for example, if in my earlier example here I was varying with time, but this is also a time invariant system. We are not really interested in how I varies. So, what we mean by time invariant is the following: that the output of the system is independent of the time at which the input is applied.

So, if I take the example of a resistance I do an experiment say today for a certain V, I get a certain value of I say- V is 1 volt R is 1 ohm and my I obviously would be 1 ampere. So, if I do this experiment say tomorrow the same time I should get the reading. And those are systems which are time invariant; time invariant, which means here we are concerned about the parameters of the system- the value of R.

Now, what does the time varying system do? Well, the output of the system depends on the time at which the input is applied. If I apply u I get a certain response y, and if I do the same experiment after a while the response might change. How does the response change? What does it mean? It means take an example of the mass of an aircraft: as the aircraft moves it keeps burning on fuel and its mass inherently reduces. So, if that the mass at take off would be different than the mass at landing, which means the parameters of the system are actually varying.

So, we are not yet classifying the time dependence in terms of the dynamics, but in terms of system parameters. So here the resistance would be the parameter and in the case of the aircraft the mass would be the parameter.

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Causal Vs Non-causal Systems

Causal systems	Non-causal systems
<ul style="list-style-type: none">• Output is only dependent on inputs already received (present or past)• Non-anticipatory system• $y(t) = f(x(t), x(t-1), \dots)$• E.g.<ul style="list-style-type: none">- Thermostat based AC- Motor or generator	<ul style="list-style-type: none">• Output depends on future inputs as well• System anticipates future inputs based on past• $y(t) = f(x(t), x(t+1), \dots)$• E.g.<ul style="list-style-type: none">- Weather forecasting system- Missile guidance system

Module 1: Lecture 1

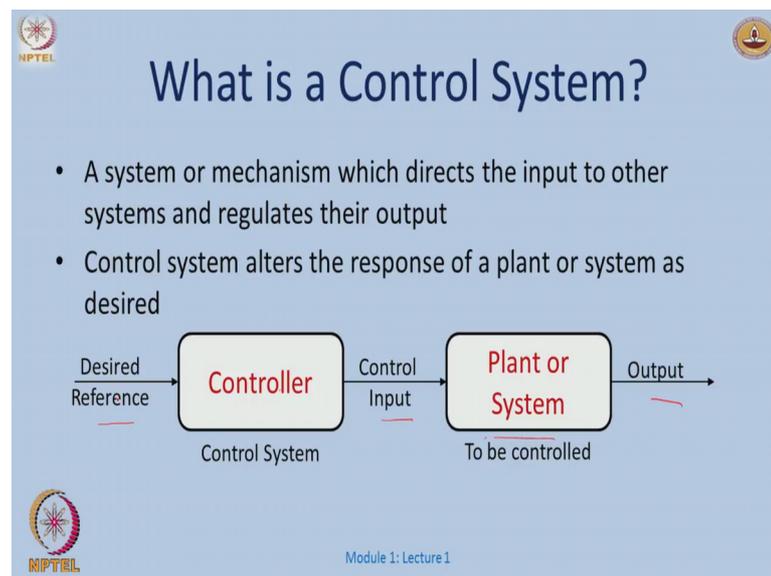
The last kind of classification which we would be interested in the course is that of causal versus non-causal, a very basic signal systems concept. A causal system is a one where the output is depending only on the inputs which are received either in the present or in the past. These are also called as non anticipatory systems. Non-causal systems they depend also on the future inputs. And the system anticipates the future inputs based on the past.

For example, well mathematically this would mean y of t will depend only on the inputs is it received at t it is received at t minus 1 and all if at all any inputs at 1 and so on. Well, a causal system could also do the reverse; it could also see what is what could happen at t plus 1 and get its outputs based on that. Well, a basic causal systems all systems we see around you know most physical systems are causal in nature a motor or a generator it does not; my AC does not react to see what might happened tomorrow to give me an output today. Whereas, if I look at the weather forecasting systems I would look at what could happen tomorrow to kind of predict today.

Or a more general example which we are which we are used to: when I prepare for an exam I prepare keeping in mind what could be the questions in the exam these are non-

causal systems we keep in mind or we anticipate what could be the future inputs or if I am playing a game or a cricket match. I decide what you know what kind of pitch I am playing, what kind of bowlers the opponents would have. And then I prepare myself based on that. These are essentially non-causal systems.

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Now, what are the implications of this on control? Well, what is the simple control mechanism? Well, a simple control mechanism could be mechanism which directs the input takes it through other systems and regulates their output. And a control system also alters the response of a plant or a system as decide. Typically, it could look something like this. You have a plant which is to be a controlled could be a motor, it could be car, a missile or whatever.

So, the controller gives it has certain control input and then we want to all this entire system the plant plus the controller to perform a desired objective; at I will say well I want to say set the speed of my car to a certain desired speed and I could measure that speed. So, this plant is controlled via a control input which the controller generates based on what is the desired reference.

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Disturbance

- Unwanted signals which affect the output of the system
- E.g. People entering and leaving an AC room disturbs room temperature
- Controller has to eliminate the effects of disturbance

Module 1: Lecture 1

So, what could this always work? Well, sometimes not you have disturbances, which are essentially unwanted signals, which affect the output of the system. For example, people entering and or a leaving a room while an air conditioner is on, sometimes external environments, so at morning 6 o clock my external conditions would be different then what they would be at 12 or 6 in the evening that could be modeled as a disturbance. And my controller should also be smart enough to eliminate the effects of these disturbances. And therefore, we need what is called as feedback in control.

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Feedback in Control

- Feedback senses the plant output and gives a signal which can be compared to the reference
- Controller action (control input) changes based on feedback
- Feedback enables the control system in extracting the desired performance from the plant even in presence of disturbance

Module 1: Lecture 1

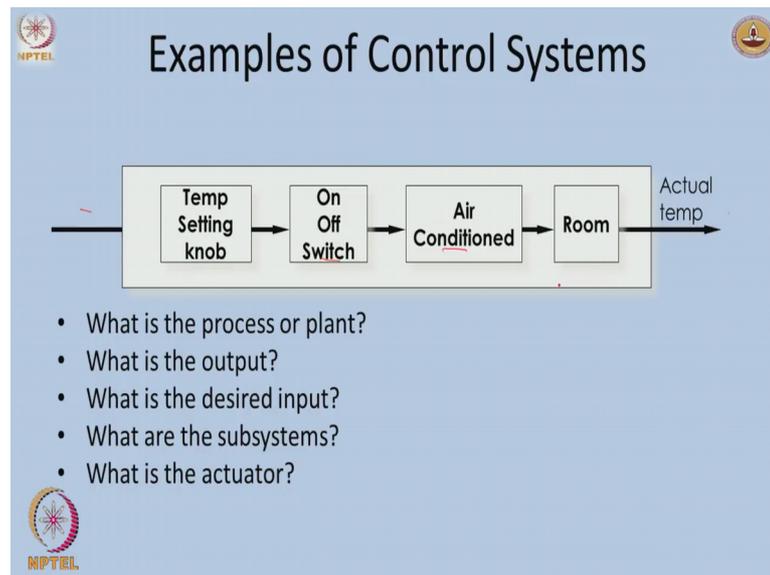
So, what is the feedback do? The feedback essentially it senses the output, it gives a signal which can be compared to the reference. Say this is 24 degrees- if I talk out of an air conditioner, this is 27 degrees. This guy will send the signal back to the desired sensible your output is not equal to the desired reference and this difference will tell the controller to give a certain input which might give which might set or eventually from 27 degrees I could go to 24 degrees.

So, the feedback senses the output of the plant and gives a signal which can be compared to the reference. And the controller action, the controller input is chosen or it changes based on the feedback for example decide is 24 and the actual is 21 degrees. The controller action would be different when the output is 21. Then it would be when the output is 27; so the controller action changes based on the feedback. And feedback enables the control system in exciting the desired performance from the plant even in the presence of a disturbance.

Say in (Refer Time: 19:16) is 24 and I say when my temperature is also 24 everything is perfect. Then I say there is some disturbance some 10 people enter the room or 10 people leaves the room. Then if 10 people enter the room then you would expect the temperature to go up say 26, then this is being continuously monitored here continuously being compared here. Then this guy will say ho hold on there is a difference in temperature so the controller should perform an action to give certain input to the plant to get back the 26 to 24.

So, this feedback also helps in eliminating the disturbance. And we will see the exact analysis of how to eliminate the disturbance in the future lectures.

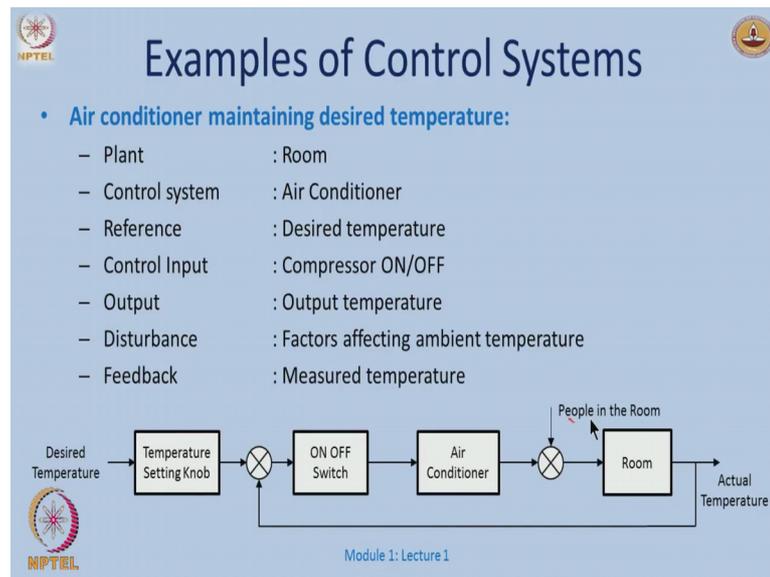
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What could be examples that we have been; of course, motivating ourselves to the examples. So, a typical open loop air conditioner would look something like this. So, this room is the plant to be controlled, air conditioner is my controller, the control action could be on and off switch off switch off the air conditioner, and I will have a temperature setting now or even as simple as simple as a regulator on my fan.

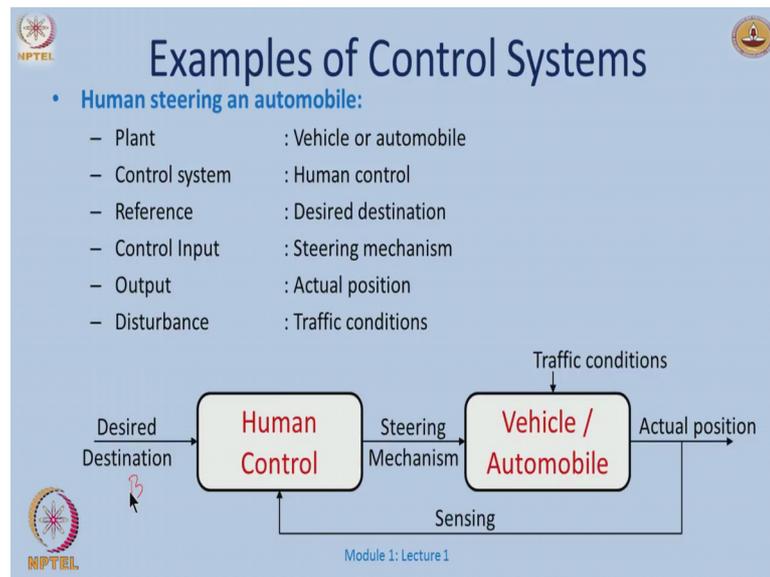
So, when I look at a control system I should identify what is the process or the plant in this case it is the room. What is the output? The output which I measure is the actual temperature of the room. What is the desired input? If I want the certain temperature what could be the desired input over here? Are there any subsystems involved in the middle and what is the actuator? So, we will slowly quantify or even model each of these and what see what these being in analysis and design of control systems.

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So, this was in the open loop case, now we could also or all the air conditions now we see are close loop in nature. We know that the air condition is it helps us in maintaining a desired temperature. The plant in this case is my room, the control system or the controller is the air conditioner, the reference signal is the desired temperature which I want the room go to, the control input could come as in terms of a on-off switch, the output which I measure is the output temperature, disturbance could be factors affecting the ambient temperature like people entering or leaving the room. And then the feedback signal is the actual temperature is compared with the desired temperature to perform the on off control action.

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Other examples could be well when I am actually driving a car I want to go from point a to point b. So, this is the desired so I can fix a destination to say a point b.

Now, at each point let see how far I am from point b, and then I have a control system which is essentially my control action when I drive the car. The reference is the desired destination, control input is the steering mechanism which what drives or steers my cars from point a to point b. And the output which I measure or I compare myself to is the actual position. Disturbance could be several in terms of traffic conditions, road conditions; you hit up slope or a slope and so on. And this constitutes a close loop control mechanism.

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Overview

Summary : Lecture 1	Contents : Lecture 2
✓ System with examples	➤ Model & its significance
✓ Classification of systems	➤ Types of math models
✓ Control system & examples	➤ Methods of modelling systems
✓ Feedback & its significance	➤ Steps of modelling systems

Module 1: Lecture 1

So, what we have done so far? We have looked at classification of systems which would be important from the control point of view. We have looked at a very basic notion of feedback and its significance. So, what will be doing in the coming lecture? We will look at how to build models and significance of models, types of mathematical models, and what are the other what are the steps involved in modelling of systems. So, we will do that in the second lecture.

Thank you.