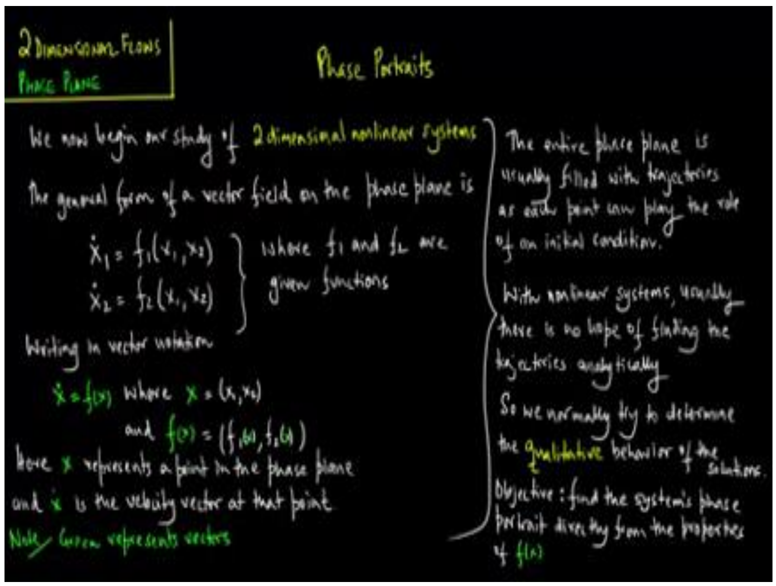
Introduction to Nonlinear Dynamics
Prof. Gaurav Raina
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module -06
Lecture-20
2-Dimensional Flows, Phase Plane, Lecture 1

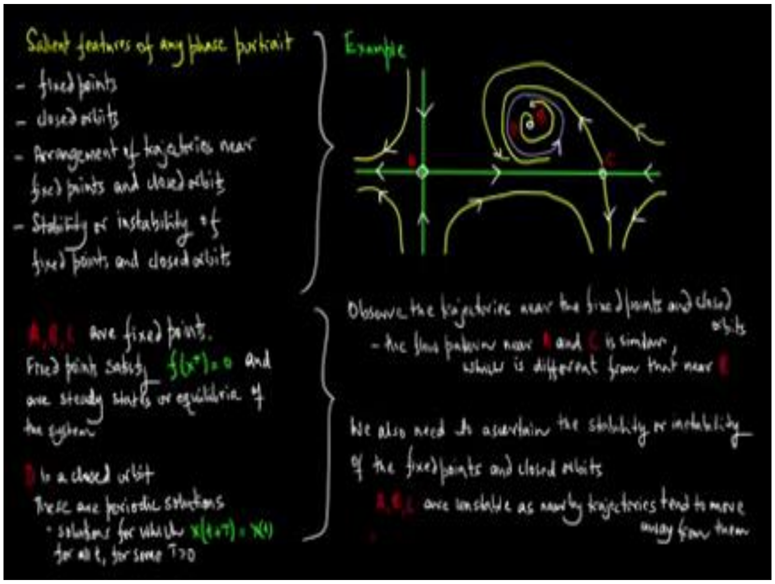
(Refer slide time: 00:01)



We are still dealing with two dimensional flows and here we deal with the phase plane. We start our study with phase portrait, so we now begin our study of two dimensional nonlinear systems. The general form of a vector field on the phase plane is x1 dot = f1 x1 x2 and x2 dot = f2 is the function of x1 x2, where f1 and f2 are given function. So, writing in vector notation, we have x dot = f of x, where x = x1 x2 and f of x = f1and f2 of x.

Here x represents a point in the phase plane and x dot is the velocity vector at that point. Note green represents vectors. So, the entire phase plane is usually filled with trajectories as each point can play the role of an initial condition. With nonlinear systems, there is no hope of finding the trajectories analytically. So, we normally try to determine the qualitative behaviour of the solutions. So, our objective is to find the systems phase portrait directly from the properties of f of x.

(Refer slide time: 02:27)

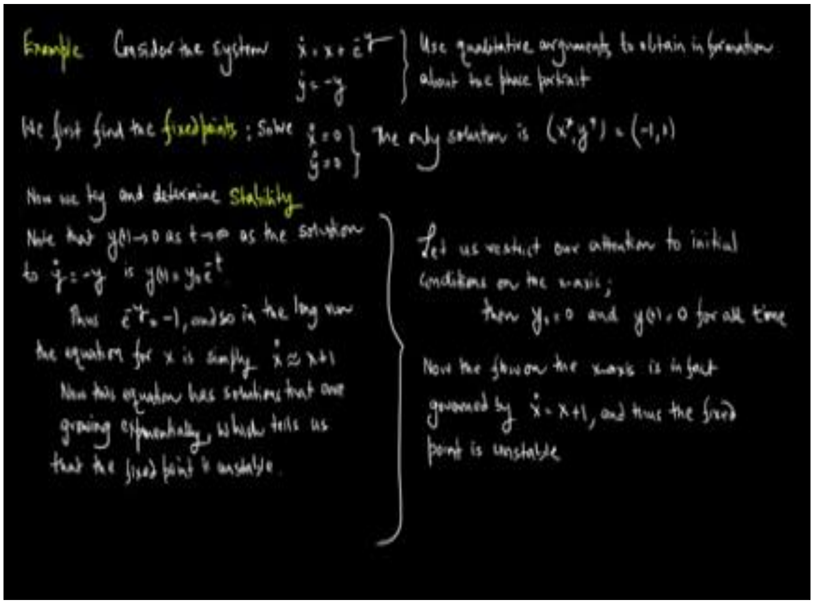


We outline some salient features of any phase portrait, these include fixed points, closed orbits, the arrangement of trajectories near the fixed points and closed orbits and finally stability or instability of fixed points and closed orbits. Now let us plot an example of a phase portrait, a is the fixed point, b is the fixed point, c is the fixed point and then we have the closed orbit d. So, a, b and c are fixed points. Fixed points satisfy f of x* = 0 and are study states or equilibrium of the system.

D is a close orbit, these are periodic solutions ie solutions for which x of t + capital T = x of t for all t and for some capital T greater than zero. So now what we do is we start filling out the phase portraits. See we get rather a pretty looking picture. Now observe the trajectories near the fixed points and the close orbits the flow pattern near a and c is similar, which is different from that near b.

We also need to ascertain the stability or instability of the fixed points and the closed orbits, a b and c are unstable as nearby trajectories tends to move away from them and d which is a close orbit is stable.

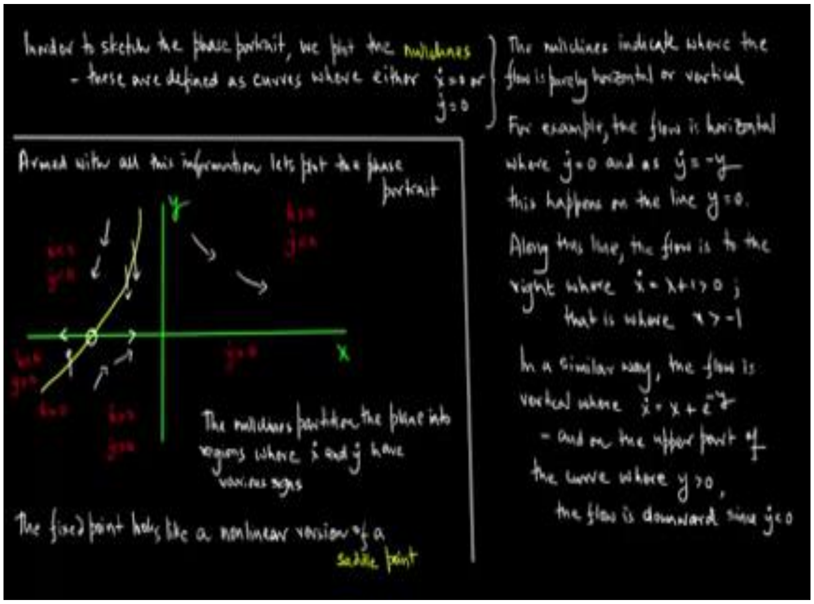
(Refer slide time: 05:43)



So, we look at an example, consider the system x dot = x + e to the -y and y dot = -y. So now we use qualitative arguments to obtain information about the phase portrait. So, we first find the fixed points. So, we solve x dot and y dot =0 and the only solutions is x* y* = -1 0. Now we try and determine stability, note that ys of t is tends to zero as t tends to infinity as the solutions to ys of t = -y is ys of t = y not e to the -t.

Thus, e to the -y = -1 and so in the long run the equation for x is simply x dot is = x +1. Now this equation has solutions that are growing exponentially which tells us that the fixed point is unstable. Let us restrict our attention to initial conditions on the x axis, when y not = 0 and ys of t =0 for all time. Now the flow on the x axis is in fact governed by x dot = x +1 and thus the fixed point is indeed unstable.

(Refer slide time: 07:57)



Now in order to sketch the phase portrait, we plot the null clients. These are defined as curves where either x dot = 0; y dot = 0. The null clients indicate where the flow is purely horizontal or vertical. For example, flow is horizontal where y dot = 0 and as y dot =- y, this happens on the line y = 0. Along this line, the flow is to the right where x dot = x +1 greater than zero that is where x is greater than -1.

In a similar way, the flow is vertical where x dot = x + e to the -y and on the upper part of the curve where y is greater than zero. The flow is actually downward since y dot is less than zero. Armed with all this information, let us plot the phase portrait, so that is familiar y versus x. We highlight the fixed point; the null client's partition the plane into regions where x dot and y dot have various signs. So, the fixed points actually look like a nonlinear version of a saddle point.

(Refer slide time: 10:43)

The topic of the discussion in this lecture was phase portraits in two dimensional systems. So, we had equation of the form x1 dot = to f1 is a function of x1 and x2 and x2 dot = f2 that is another function f2 as function of x1 and x2. And the objective is to plot x1 versus x2. There are four salient features that you need to keep in mind when plotting the phase portrait for the two-dimensional systems.

Number one identify all the fixed points, number two identify all the close orbits, number three pay close attention to the arrangement of the trajectories closed to the fixed points and close to the closed orbit and number four highlight the stability and or the instability of the fixed points and the closed orbits. And in this lecture, we have one particular example, but these are four salient features, that you should look for in any phase portrait of a two-dimensional system.