Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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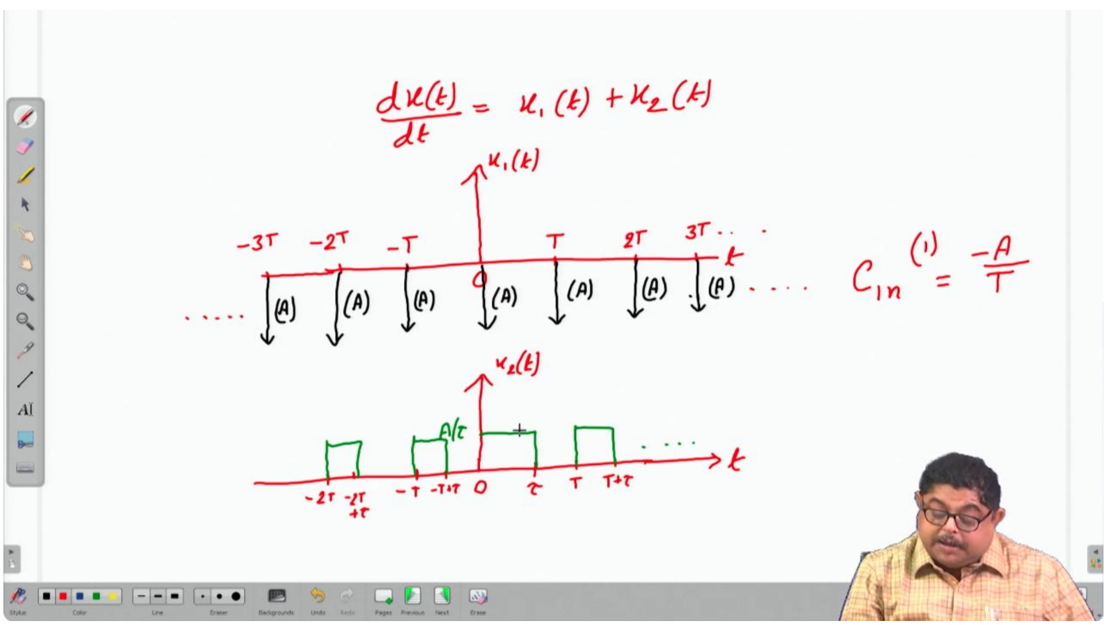
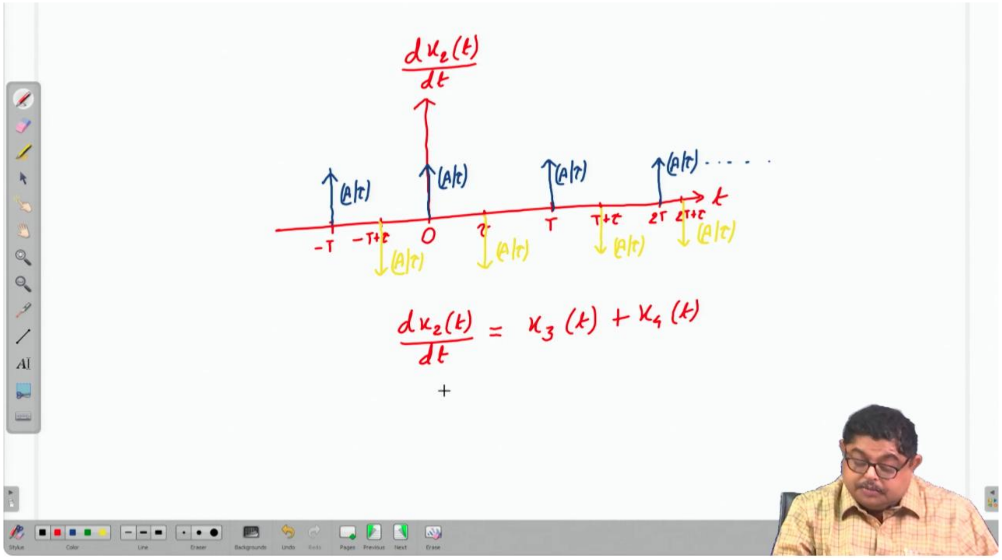
Week :02
Lecture 9: Fourier Coefficient for Piecewise Linear Periodic Waveforms (Continued)

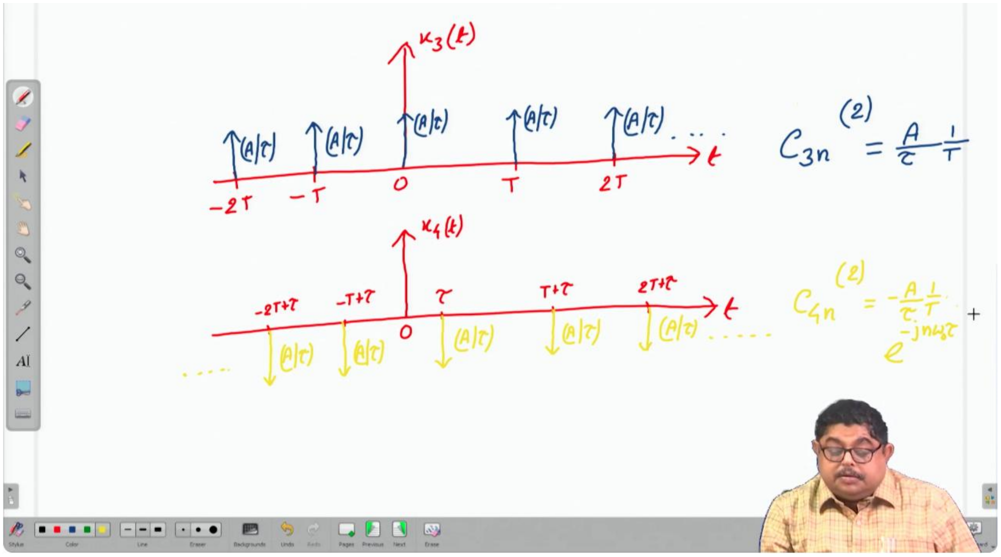
Welcome to the ninth lecture of the course on EMI/EMC and Signal Integrity Principles, Techniques and Applications. We will be continuing with our previous lecture, we were seeing semi trapezoidal clock by our new algorithm. So, we have seen that after first differentiation, we have landed with one impulse train and one non impulse periodic function. So, the impulse train we are calling as x 1 t and the other part we are calling x 2 t. Here we are having only one non impulse type function, if we had another non impulse type periodic function, we could have written x 3 t etcetera. So, what is our x 1 t? Let us draw it, x 1 t, this is t, there will be 0, then there will be, if you see that the impulse strain is coming at every 0, then T, then 2T, here minus T minus 2 T etcetera. So, we are writing this 2 T, 3 T etcetera, here minus T, minus 2 T, minus 3 T etcetera and we are getting that here there is an impulse of strength and what will be x 2 t. So, it will be occurring up to tau, then the next one will come at T, T plus tau, then here minus T, here minus T plus tau, here minus 2 T, minus 2 T plus tau etcetera and I think previous one I have used green colour. So, I am using green colour, so 0 to tau and the magnitude is A by tau, similarly here like this, here, my drawing may be a bit wrong but the proportional things have gone, but all are basically square wave. So, you see that already I can write the c 1 n from here, so let me write it that for the impulse train, this is x 1, so I am writing c 1 n, this is after first differentiation, so I am writing c 1 n 1 and that is nothing but minus A by T, simple, but here I cannot write because here still I have not got the impulse, so I will have to differentiate it again.



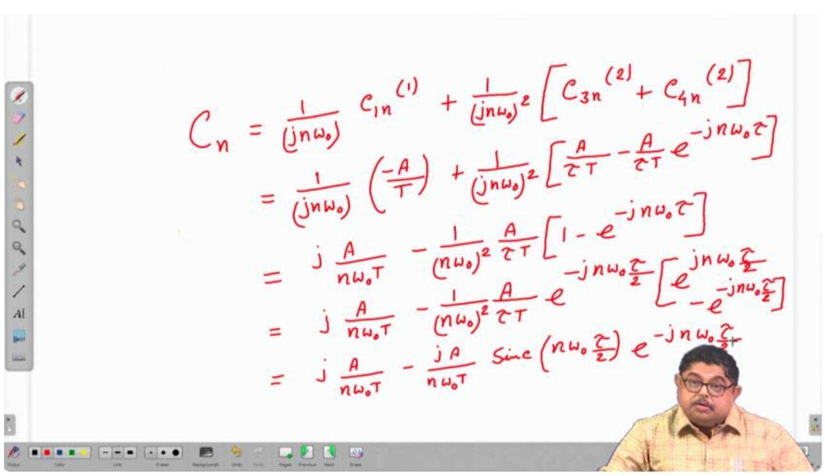
So, I will have to make dx 2 dt, this is 0, so dx 2 dt, there you see that here again let me write tau, let me write T, let me write Tplus tau, let me write 2 T 2T plus tau etcetera and here let me write minus T plus tau minus T then etcetera. So now, you see that there is a, sorry, there is a rising edge at 0, so I will get a positive impulse here. So, there will be, let me take this colour, that there will be a positive impulse of magnitude A by tau, here A by tau, here A by tau, similarly here A by tau etcetera and at all tau T plus tau to 2Tplus tau, there will be a negative impulse. So, let me take this colour that at tau I will get A A by tau, here I will get A A by tau, here I will get A A by tau, here I will get A A by tau etcetera. So now, I have broken these into two pulse trains. So, after this second differentiation it has come, so we can write, sorry, d x 2 dt is equal to x 3 t plus x 4 t. What is our x 3 t?



I am elaborately doing so that you understand, later you will be able to, in short you can do it, x 3 t at every 0 t, 2 t etcetera minus t, minus 2 t etcetera. You will be getting this A by tau, here you will be getting A by tau, here you will be getting A by tau. This is your x 3 t, similarly your x 4 t will be this is 0, then there will be tau, there will be T plus tau, there will be 2 T plus tau, there will be minus T plus tau, there will be minus 2 T plus tau and there you will be getting negative impulse train A by tau, A by tau. This will continue A by tau, A by tau, this will continue. So, directly we can write since we have got the impulse train, so now we can write, so what we will call this, sorry. So, I can call that C 3 n, this we got after second differentiation. So, I am writing C 3 1 2 is equal to, this is a pulse train of strength tau and impulse means it is Fourier coefficient will be 1 by T. Similarly, here we will be writing it at C 4 n 2 is equal to minus A by tau 1 by T and it is displaced by delay tau. So, it is e to the power minus, sorry it is not accommodating, e to the power minus J n omega naught tau. This is the time shift. So, now we have, now in total 3 expansion coefficients,



so we can the total, sorry. So, C n will be now, you see for the first differentiation, I will divide by 1 by J n omega naught C 1 n 1 plus there are double differentiation. So, I will write this into C 3 n 2 plus C 4 n 2. So, I know the values of C 1 n etcetera. So, now I will 1 by 1 put them 1 by J n omega naught minus A by Tplus 1 by J n omega naught whole square. This is A by tau T minus A by tau T e to the power minus J n omega naught tau. So, now it is only I will have to just simplify. So, the first one I will get as J A n omega naught T minus 1 by n omega naught square A by tau T 1 minus e to the power minus J n omega naught tau. So, this J A by n omega naught T minus 1 by n omega naught square A tau by T e to the power minus J n omega naught tau by 2 T to the power J n omega naught tau by 2 minus e to the power minus J n omega naught tau by 2.So, you know that this will lead to a sine function and by properly adjusting you can make it a sinc function. So, I can write that it will be J A n omega naught T minus J A by n omega naught T sinc of n omega naught tau by 2 e to the power minus J n omega naught tau by 2. So, now we can note that omega naught t is nothing but 2 pi and omega naught is 2 pi by t.



So, I can now write that C n is equal to J A by 2 pi n 1 minus sinc n omega naught tau by 2 e to the power minus J n omega naught tau by 2. So, finally, if I put the values of omega naught then it will come in terms of T not in terms of omega naught will be absent. So, J A by 2 pi n 1 minus sinc n pi tau by T e to the power minus J n pi tau by T. So, this is the same result that we obtained before, but here I think you will appreciate that my labor is less I do not know whether you appreciate that or not, but to me it appears that it is a much elegant way of dealing with this just you will have to have differentiation properly account for the how many differentiations took place you for impulse strain the Fourier coefficient is very well known 1 by t. So, that you put you get the result. Thank you.