Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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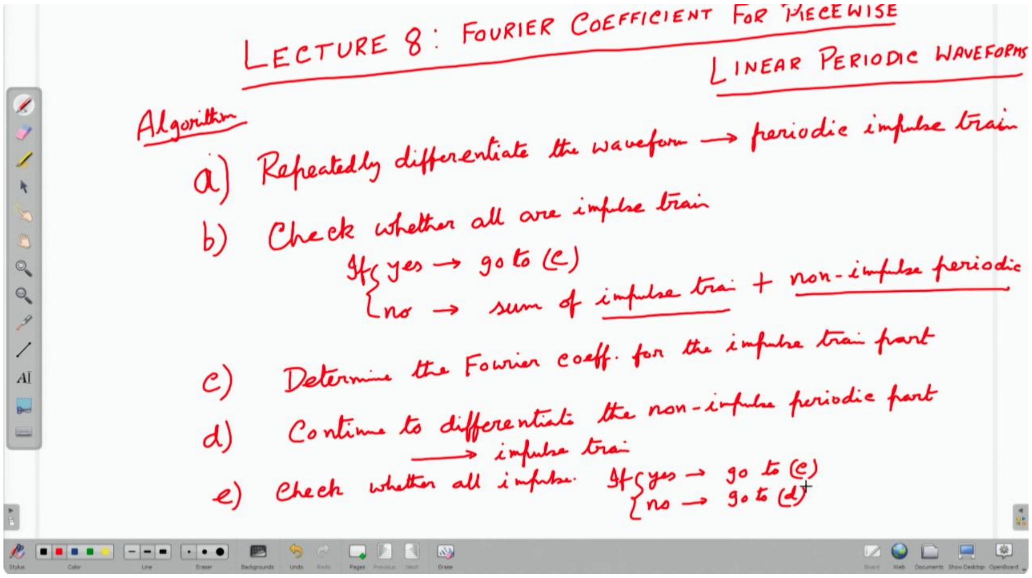
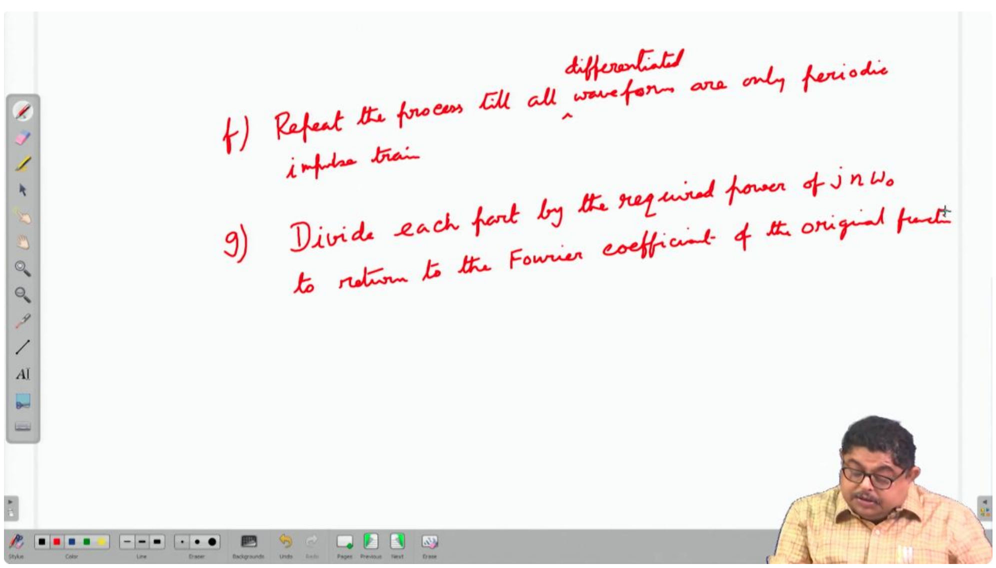
Week :02
Lecture 8: Fourier Coefficient for Piecewise Linear Periodic Waveforms

Welcome to the 8th lecture of the course on EMI EMC and Signal Integrity Principles, Techniques and Applications. In the last class, we have revisited some properties of Fourier series. Today, we will use those properties to find an algorithm for a class of waveforms, class of waveforms which are piecewise linear. So, piecewise linear means the waveform is composed of only line elements. It does not have any curve such periodic waveforms. Basically our clock etcetera can be always they are categorized in this class. For them this method is applicable. We will see an algorithm to find very elegantly with less computational labor the Fourier coefficients. We have already seen clock waveform, a semi-tappizoidal waveform and we have seen that considerable labor is required to find their Fourier coefficients. But today I will first tell the algorithm.

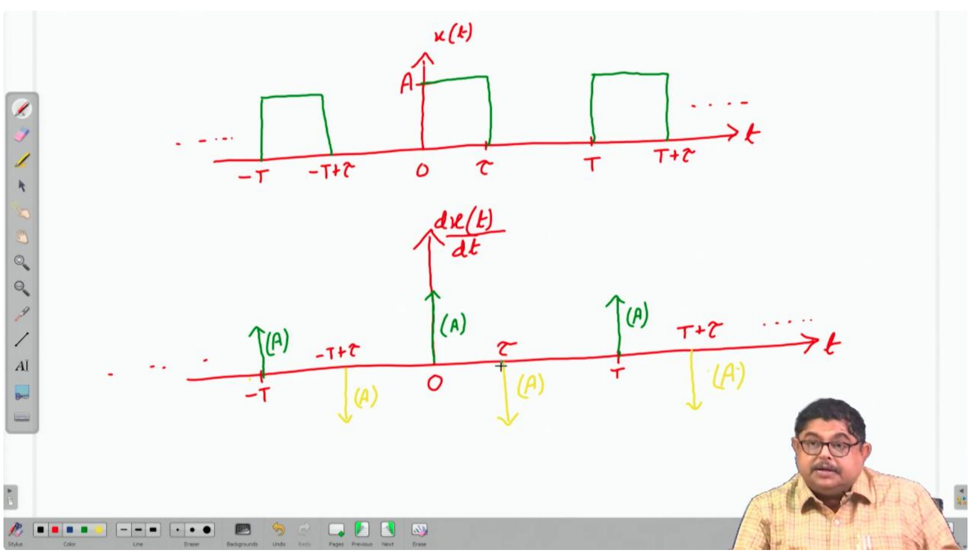
The first or the first part of the first thing is you repeatedly differentiate the waveform until the first occurrence of a periodic impulse strain comes. So, first thing is repeatedly differentiate obviously, differentiate means with time, differentiate with respect to time, differentiate the waveform. So, you go on differentiating till you get a periodic impulse train. Once you get a periodic impulse strain, then you see whether the differentiated waveform consists solely of periodic impulse strain or there are non impulse train periodic functions also. If such that means, if there are non periodic non impulse train functions also, then write the result as a sum of the part that contains the impulse train and another part that contains the non impulse train. That means, here you first check whether all are there may be many that is why I am writing all all are impulse train now. If this is yes, then your job is over we will go to the next step, but if not then you break the whole waveform as sum of one part is impulse train plus sum of impulse train plus non impulse periodic function. So, you break it into this part. If it is yes, then I will write later where to go. So, this is I can say the algorithm for finding this Fourier series. So, in (c) find the for this part which is impulse strain determine the Fourier coefficients for the impulse train part. So, I will now write that if yes, then go to (c) go to part C determine the Fourier coefficient and for this non impulse periodic continue to differentiate continue to differentiate the non impulse periodic part till how long till you get impulse train. Again you check that means, after every differentiation check whether all impulse if yes go to (c) if no go to (d).



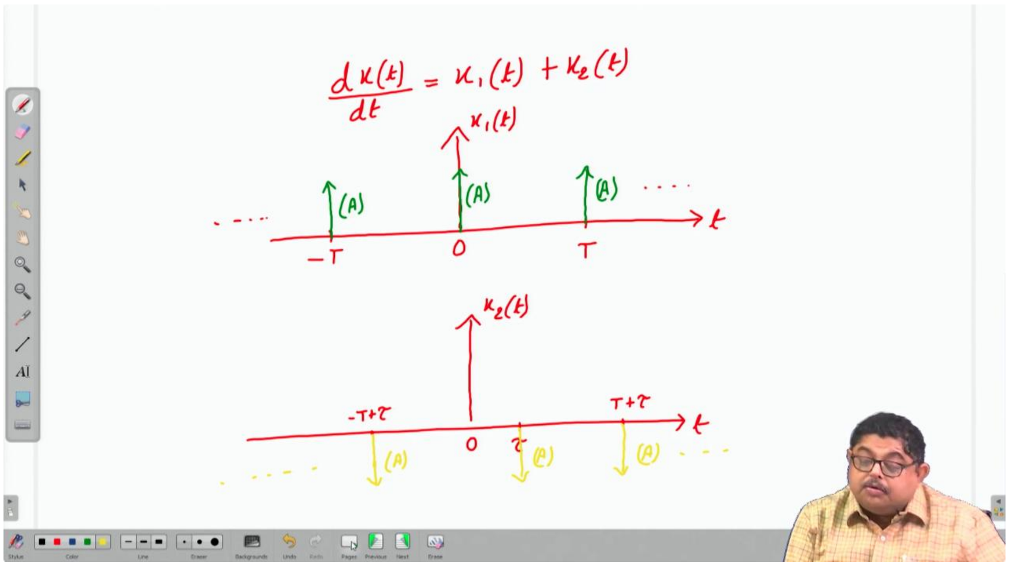
So, (f) is repeat the process till all waveforms all differentiated waveforms all differentiated waveforms are only periodic impulse train. This is guaranteed how because a line if you differentiate it then ultimately we will get an impulse. So, that is the that is why you only for piecewise linear this method is applicable and at the end you will end up with all periodic trains they are impulse trains. Now, the last part is that due to differentiation you know the Fourier coefficient change. So, you now appropriately divide each part by the required power of J n omega naught to return to the Fourier coefficient of the original function. So, this is the complete algorithm. Now without example this would not be clear.



So, I will take an example first let us take the example of the rectangular or square wave pulse that we have seen before that was our first problem. So, finding the C n's the magnitude of the Fourier coefficients. So, we will do that by this method and compare which one is more computationally intensive. So, let me start that. So, the problem is this that this is your x t, this is your t, this is your 0 and there are A is the amplitude and tau is the duty cycle and t is the. So, now I can draw the waveform that. So, this will be T plus tau, this will be minus T plus tau, this will be minus T. So, this was our thing that means, no rise time fall time rise time is 0 fall time is 0 etcetera. So, now this is dx dt the algorithm says you see the part A of the algorithm that differentiate these. So, if I differentiate these basically if I differentiate these what I will get d x t dt versus t. So, that will give me you see here there is a rising edge. So, I will get A here, this portion the flat portion that after differentiation it will be 0 and at tau I will get. So, let me use different colour. So, I will get a falling edge. Similarly I will get another falling edge. So, another impulse here at t plus sorry those things I will put later, but what is the strength of this one this is also A, but in the negative side or you can say minus A. Similarly here you will get another at minus t plus tau you will get any. So, these values let me write that this is minus t by tau you will get a falling edge at tau you will get a falling edge at T plus tau you will get a falling edge. Similarly at 0 you will get a rising edge at T you will get a rising edge at minus T minus T will be here you will get a rising edge. So, let me draw the rising edges now there will be an impulse of A there will be an impulse of A.So, I can say that this whole thing is continuing. So, repeatedly we will get this. So, you see that there are two pulse trains one is green coloured pulse train they are having strength A and there are yellow coloured pulse train which are having strength you can say minus A and there are different points all are periodic all are periodic with period T, but one is shifted from the origin.

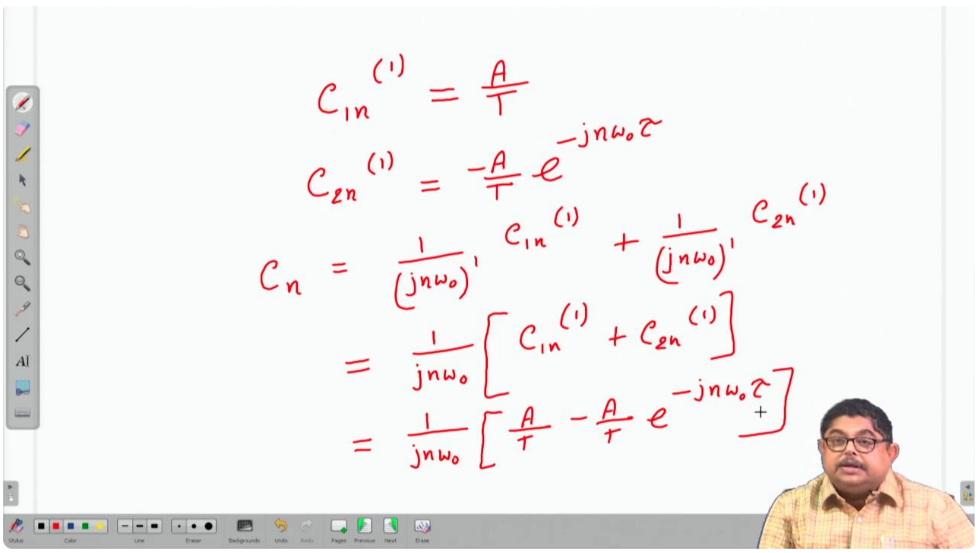


So, let us break this two pulse strain as d x t dt is equal to x 1 t plus x 2 t it was said that break all the things. So, fortunately in this case we do not have any non impulse type thing both x 1 t and x 2 t are impulse trains. So, our job will be easier. So, what is x 1 t? Let me draw x 1 t versus t. So, let me put 0 T minus Tetcetera etcetera in both sides this is a impulse train and what was the colour I think green. So, it will be A this will be A this will also be A. So, this thing will continue this is x 1 t and let me also plot what is x 2 t versus t. So, this is 0 and we will have the thing at tau there will be something then at T plus tau there will be something at minus T plus tau there will be something etcetera and so that colour I used last time yellow. So, it will be A this will be A this will be A. So, since we have finished we can now go on finding what is the expansion coefficient.

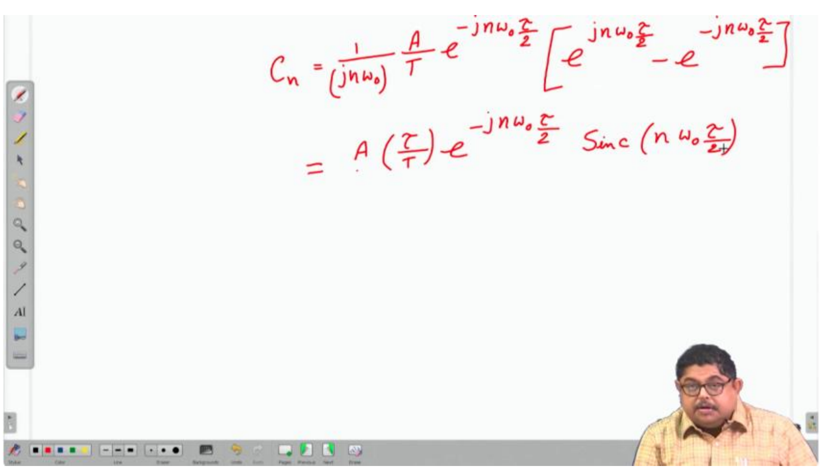


So, I can write like this sorry that I can write you see like this C 1 m. So, C n now the subscript 1 stands for the waveform x 1 will follow this nomenclature and it was obtained after one differentiation to remember that I am writing it the superscript is within bracket 1 that means after first differentiation it was obtained and the waveform we are calling x 1. So, its expansion coefficients or Fourier coefficients are C 1 n. So, what is C 1 n? It is easier to see if you look at what will be C 1 n it is an impulse train of strength A. So, can I write that this is A by t. Similarly, what will be C 2 n 1 because it is x 2, but it was also obtained after first differentiation. So, the superscript is still within bracket 1. So, it will be minus A by t because the impulses are negative side or I can say minus A by t, but also they are you see shifted from the origin by an amount time delay of tau. So, it will be to the power minus j n omega naught tau the time shifting property here we are using. So, now is our job to find total C n now with linearity property I can say that it will be something C 1 n. So, x 1's coefficient and x 2 coefficient they will be added, but they are differentiated. So, I will have to write 1 by j n omega naught to the power 1 into C 1 plus

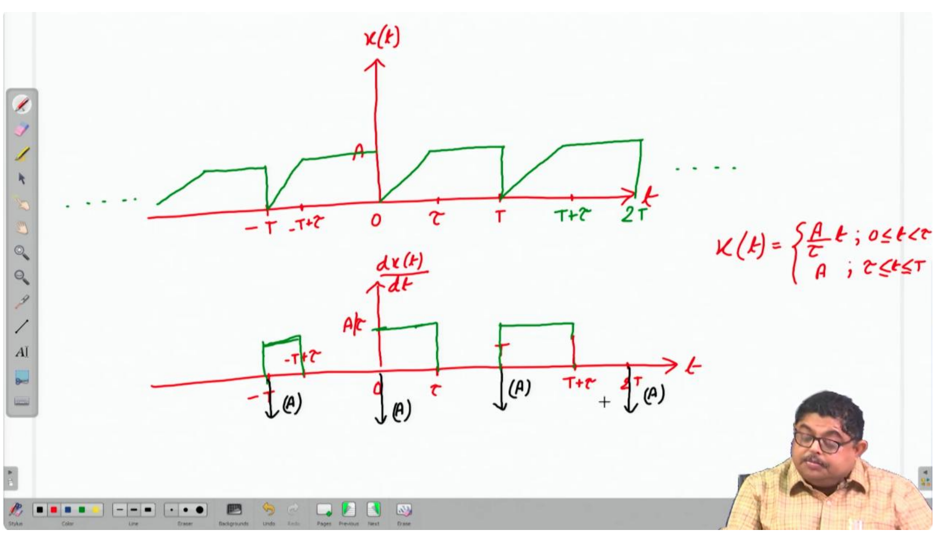
1 by j n omega naught whole to the power 1 C 2 n 1. This is the property that the derivative have with the original expansion coefficients. So, all these 4 properties we have seen. So, now it will be 1 by j n omega naught can be taken common and I can write C 1 n 1 plus C 2 n 1 is equal to 1 by j n omega naught A by T minus A by T e to the power minus j n omega naught tau. So, A by T you can take out as common and 1 minus e to the power minus j n omega naught t. So, I said all whenever there is a 1 minus something you take e to the power minus j n omega naught tau by 2 common. So, that there will be a sin or cos type of function coming.



So, I can write C n is 1 by j n omega naught into A by t into e to the power minus j n omega naught tau by 2 e to the power j n omega naught tau by 2 minus e to the power minus j n omega naught tau by 2. So, this can be easily written as this there will be you need to this j n omega naught that will go here. So, after some manipulation you will get A into tau by t e to the power minus j n omega naught tau by 2 sinc of n omega naught tau by 2. So, this is you can compare previously also we got this result, but that time we had to do much more mathematics here much simpler way you can do.



So, next let us take the another one we have already seen that one the trapezoidal clock or semi trapezoidal clock after that I will go to trapezoidal clock. So, semi trapezoidal clock means that I have this is x t t this is 0 there will be tau here there will be T here there will be 2 T here there will be minus t here. So, if I use green colour I can write that. So, this is A here. So, this was our trapezoidal clock. So, let us take its differentiation that is the algorithm says. So, I will take d x t d t versus t this is 0 as before and you will get tau you will get T you will get 2T you will get minus T here I can say this was minus T plus tau. So, we will get that point also that minus T plus tau etcetera. You see that this portion that when it is rising that portion will give you this is a linear portion. So, if I differentiated a line it will be a constant. So, what will be the value of the constant that you can easily calculate that this equation of the line is that here it is from 0 it is going to A in a time tau. So, the slope is A by tau. So, I know that this will be A by tau and there I will take that from here to tau there will be a constant function up to tau. Then this portion that means from tau to T that is a constant. So, after differentiation it will become 0. Now, but at T there is a sharp falling edge from a to t. So, there will be a there will be a of magnitude A. Then from here from T to 2 T we will get that same thing A by tau and that will be where it will come back here this value is T plus tau. So, this value will be T plus tau not 2 T and there will be a 2 T. So, at 2 T there will be again a falling edge. So, we will have similarly at 0 also you will have a falling edge. So, this then this portion from 0 to minus T plus tau after differentiation there is nothing, but at T minus T plus tau to t there will be same that this A by tau and at minus T there will be an A. So, again you see that here you have two waveforms one is a non impulse type waveform that is green and impulse time waveform that is shown in black both are periodic with period T. So, here we can write also that this equation original equation x t was A by tau t 0 to less than equal to t less than equal to tau and A for tau less than equal to t less than equal to t. So, now our job is to break this one this d x t d t into two parts.



So, d x t d t is equal to x 1 t plus x 2 t where we can draw x 1 t x 2 t and then we can find out the spectral coefficient today time is up next day we will start from here. Thank you.