Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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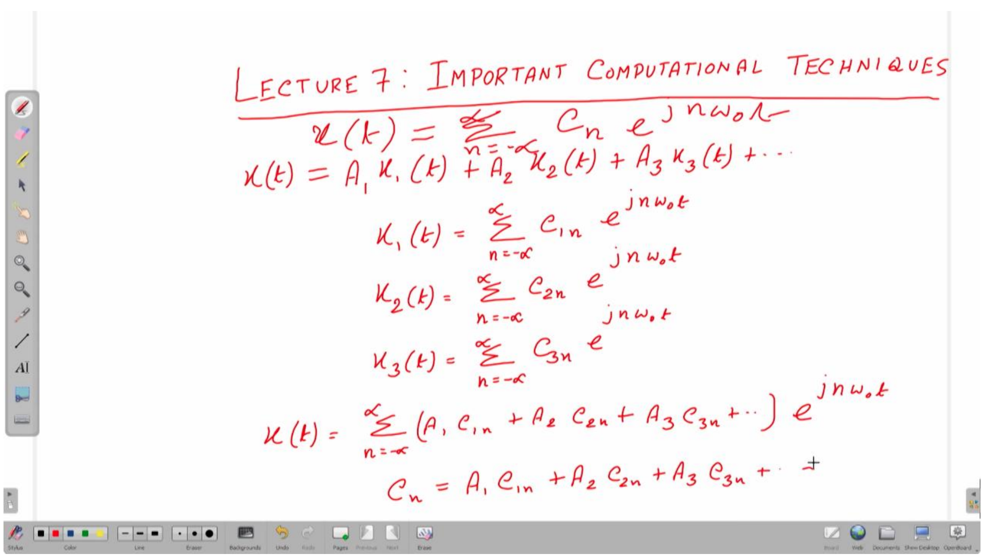
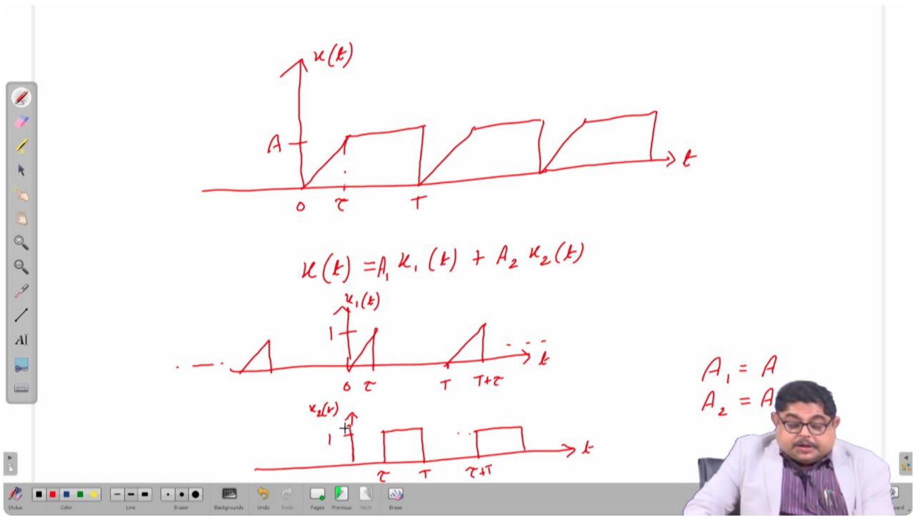
Week :02
Lecture 7: Important Computational Techniques

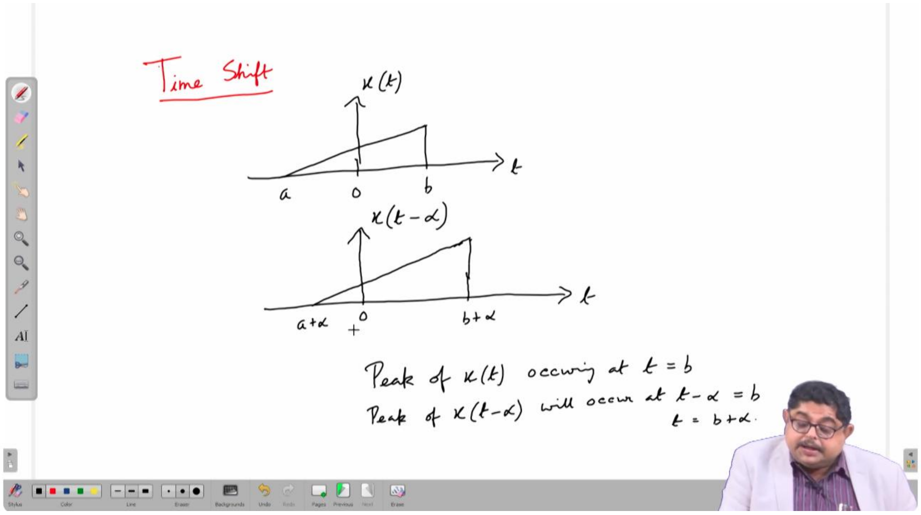
Welcome to the 7th lecture of the course on EMI EMC and Signal Integrity Principles Techniques and Applications. today we will discuss some important computational techniques. As I said in the last class that calculating the spectrum of the output signal is quite mathematically intensive. So, can we have some shortcuts to our class of signals or systems. So, I will again go back to your basic knowledge of signals and systems Fourier analysis. So, there we will I will recall the property of linearity of any waveform. So, and it is linearity of your it is Fourier coefficients. So, if a waveform is decomposed into 2 waveforms x 1 t and x 2 t with some coefficients that means, if I can write x t is equal to a 1 x 1 t plus a 2 x 2 t plus a 3 x 3 t etcetera. Then if x 1 t has it is spectral components as c 1 n and x 2 t has it is spectral components as c 2 n and x 3 t has it is spectral components as c 3 n etcetera. Then x t can be written as. So, you see that the spectral component c n can be now decomposed as. So, we can say that if x t is spectral components are c n e to the power j n omega naught t, then we can write c n is a 1 c 1 n plus a 2 c 2 n plus a 3 c 3 n etcetera. This can be easily proved I think you know that also.



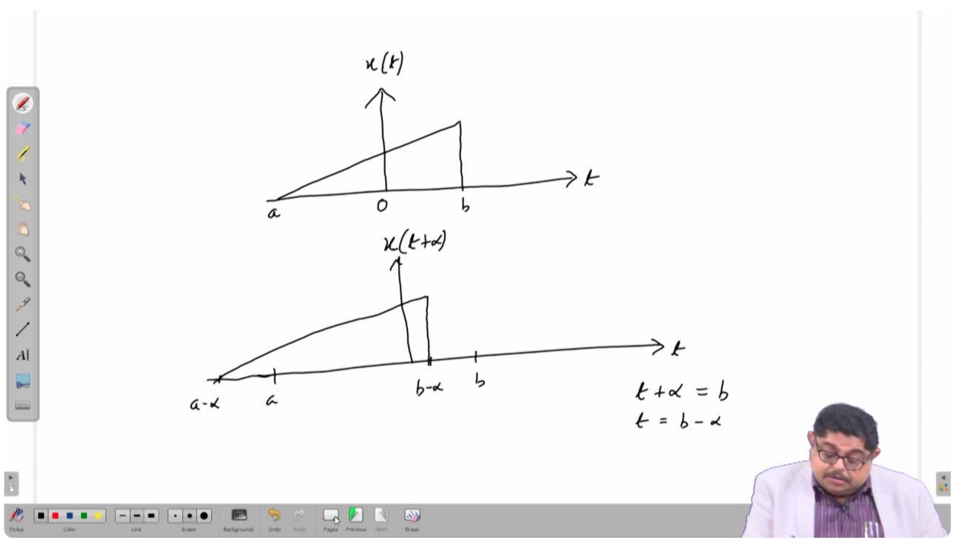
So, you see that when we have this type of signal we have already seen this clock signal. So, here you have noted that this x t over one period it was broken into or it can be written as x 1 t a 1 plus a 2 x 2 t. So, I can write now that all are t x 1 t is. This is x 1 t obviously, there were this side also they were there. So, I can write this easily as x t is equal to a 1 t a 1 x 1 t plus a 2 x 2 t where a 1 is a a 2 is also a, but then I will have to take these ones as 1. This one 1 if I take that then a 1 is a a 2 is also a. So, like this. So, this is the first property linearity property that we have seen.



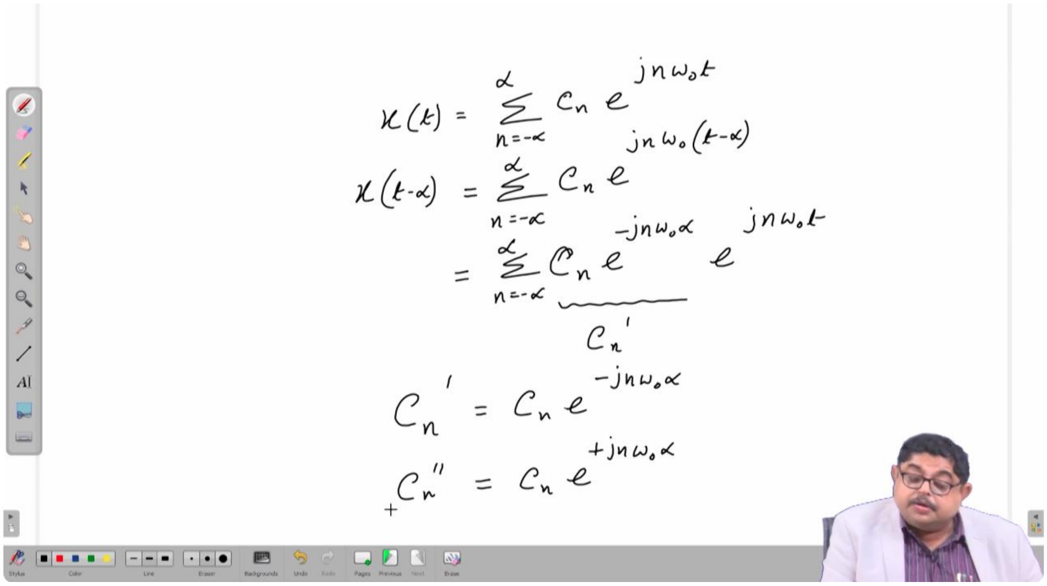
The next one is the time shift property time shift. So, suppose I have a signal x t which is when I give it a time shift. So, let me call it I am giving a time shift of alpha actually I generally we call it tau, but since in pulse strain we have tau reserved for on time of the pulse. So, I am calling it alpha. So, it will be b plus alpha, it will be a plus alpha this is 0. Now, how you find this? You see that the corresponding points on the waveforms of x t and x t minus alpha occur when the arguments of x t and x t minus alpha identical. The meaning is what is the corresponding point? Suppose this is the peak, this is also the peak. Now, where it will occur? When the arguments of x t and arguments of x t minus alpha are identical. So, peak of x t is occurring at t is equal to b. So, peak of x t minus alpha will be occur at. So, I can write peak of x t occurring at t is equal to b. So, peak of x t minus alpha will occur at t minus alpha is equal to b or t is equal to b plus alpha. You can think corresponding point as the nulls. So, the null or 0 of x t is occurring at a. So, that means, at t is equal to a then 0 is occurring. So, for this function the time delayed function the delay null will occur at t minus alpha is equal to a that is a plus alpha.



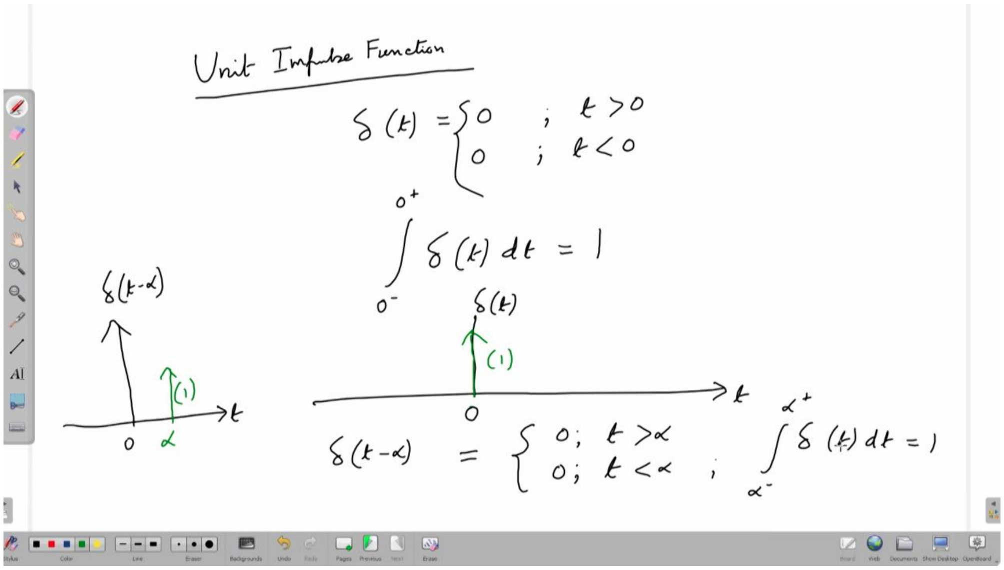
Similarly, this if I have again I am drawing x t x t versus t a b this is 0 x of t plus alpha. So, now, you will be able to say that where will be the peak. So, for x t the peak is occurring at t is equal to b. So, for t plus alpha the peak will occur at t plus alpha is equal to b that means, t will be b minus alpha that means, now if this is a this is b it will be b minus alpha. So, something may be here b minus alpha. So, and so, you can say that peak will be here. So, and this is a minus alpha provided alpha is positive if alpha is negative then it will be other side ok. Now, so, this is a example of time shifted signal.



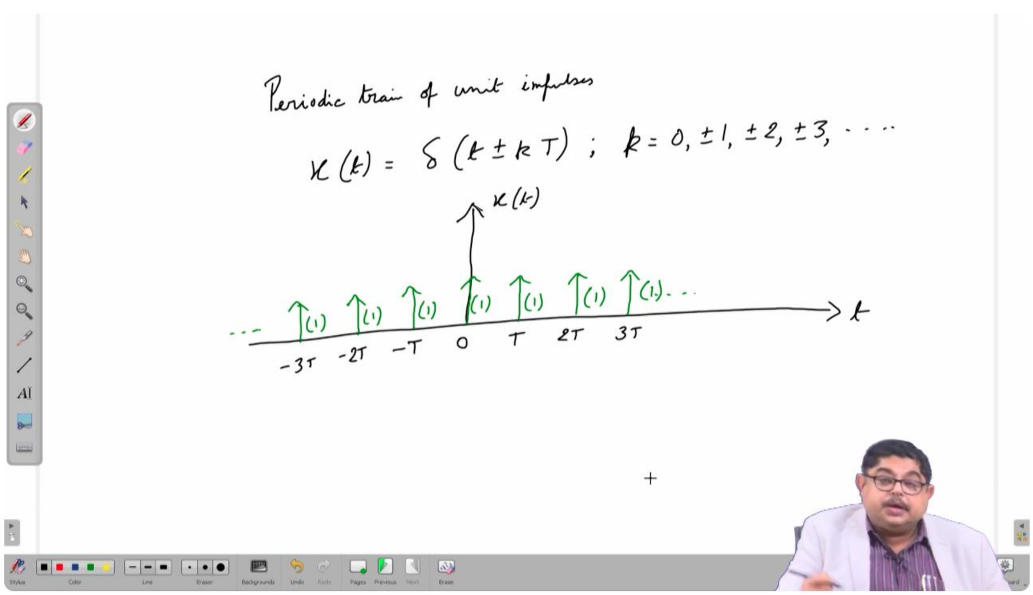
So, from Fourier series we know that if x t is its spectral coefficients are C n then what will be spectral coefficients of x t minus alpha that will be C n e to the power j n omega naught t minus alpha. So, n is equal to minus alpha to alpha. So, what is happening C n e to the power minus j n omega naught alpha into e to the power j n omega naught t this thing is now our C n dashed if I call it for this thing. So, we can say that C n dashed is nothing, but C n e to the power minus j n omega naught alpha. Similarly, for x t plus alpha if I call that as C n double dashed then it will be C n e to the power plus j n omega naught alpha. So, this is the time shift property this also you know easily from Fourier series analysis you can prove this.



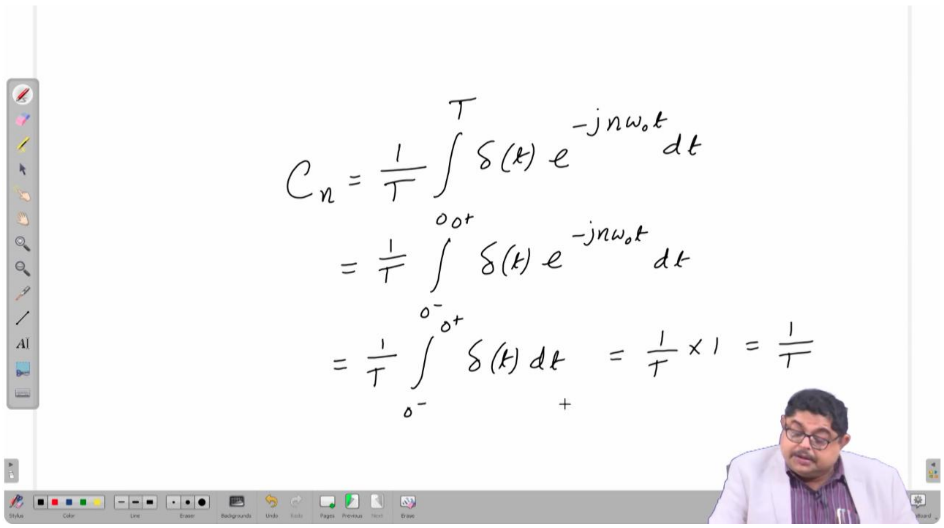
The next one I am I think you know or every electronics engineer should know what is unit impulse function. Its definition is del t is equal to 0 t greater than 0 0 t less than 0 and and you see that it exist only at t is equal to 0. So, it is not a function actually in the proper sense what we learnt in our basic courses on calculus that a function means it is continuous. So, it should have its left hand limit is equal to right hand limit is equal to the function at a point. So, at t is equal to 0 you see its left hand limit is 0 its right hand limit is 0. So, limit exist, but none of the limit is equal to the function at t is equal to 0. So, it is not continuous, but this is in physics physical reality it comes that is why it was defined as a special function and the this definition is not complete you will have to define that its 0 minus to 0 plus that means, integration of del t dt is equal to finite is equal to finite since we are having unit impulse it is this its representation is like this that this exist here and we denote its strength here. Now, if it was not unit impulse it could have been some constant that we should have written. So, this is your 0 this is your t this is and this is your delta t ok. If I have delta t minus alpha then that definition will be again let us say unit impulse. So, 0 for t greater than alpha 0 for t less than alpha and also alpha minus to alpha plus delta t dt is equal to 1. So, all that that means, this 3 makes the definition perfect and its representation will be that time. So, this is the delta function. It is represented by a vertical arrow pointing upward if it is positive if it is vertical arrow going downward it is its value is negative the height of the arrow is immaterial the strength of the impulse is denoted in parenthesis adjacent to this arrow. So, instead of unit impulse if you have a impulse of strength a you will write it as a here.



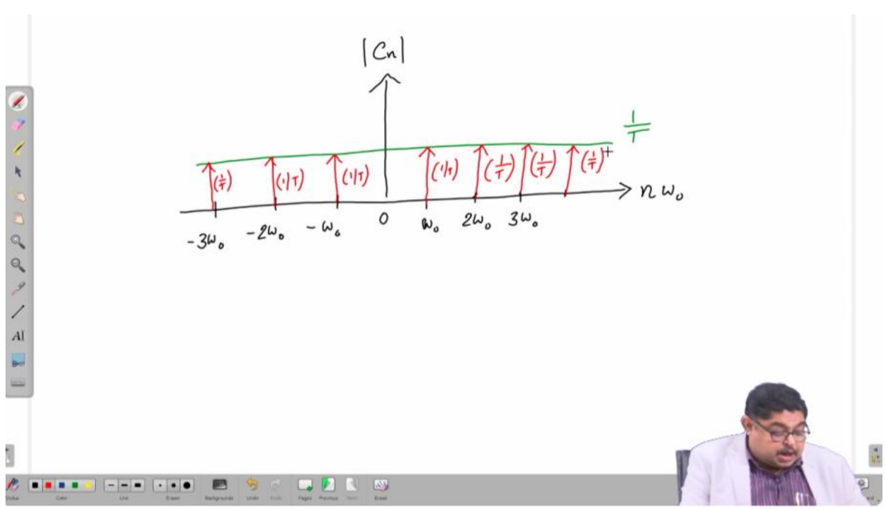
Now, consider a periodic train of periodic train of unit impulses. So, that means, I can write x t is equal to delta t plus minus k t k is equal to 0 plus minus 1 plus minus 2 plus minus 3 etcetera. So, then what is it delta sorry x t. So, the waveform will be look like. So, delta t is equal to delta t plus minus 3 delta t plus minus 3 delta t plus minus 4. So, this is the periodic pulse train. So, again the complex exponential Fourier it periodic pulse train means it has Fourier series. So, we can determine its complex exponential Fourier coefficients.



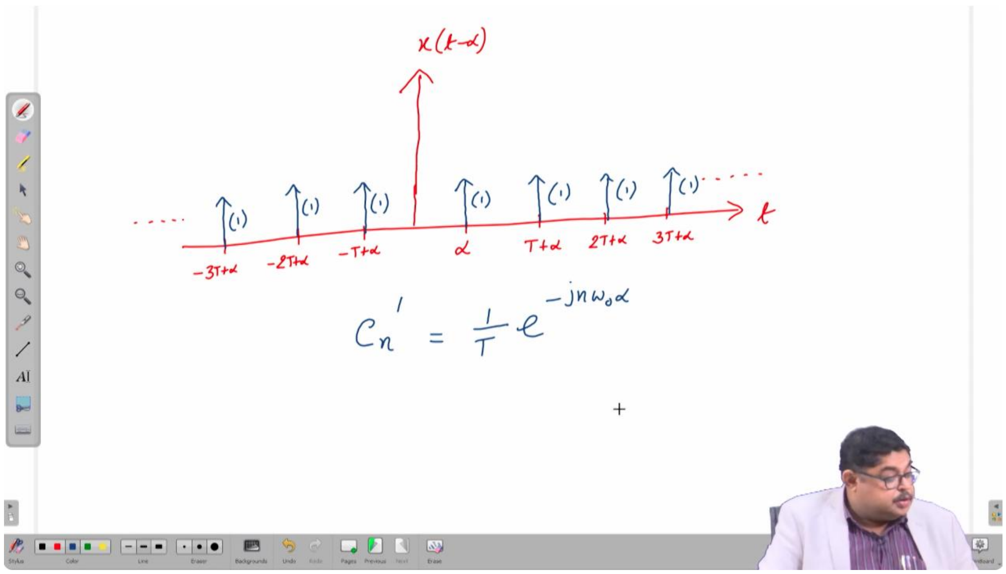
So, that will be C n is equal to 1 by t 0 to t that is its period delta t e to the power minus J n omega naught t dt is equal to.So, everywhere delta t is 0 except at t is equal to 0 that is why I see what happens at 0 minus to 0 plus delta t e to the power minus J n omega naught t dt. So, it exist only between 0 minus and 0 plus, but there what is the value of e to the power minus J n omega naught t t put t is equal to 0. So, this is 1. So, we are left with 0 minus to 0 plus delta t dt and that value we know is 1 by T into 1 is equal to that is the definition. So, it is 1 by T.



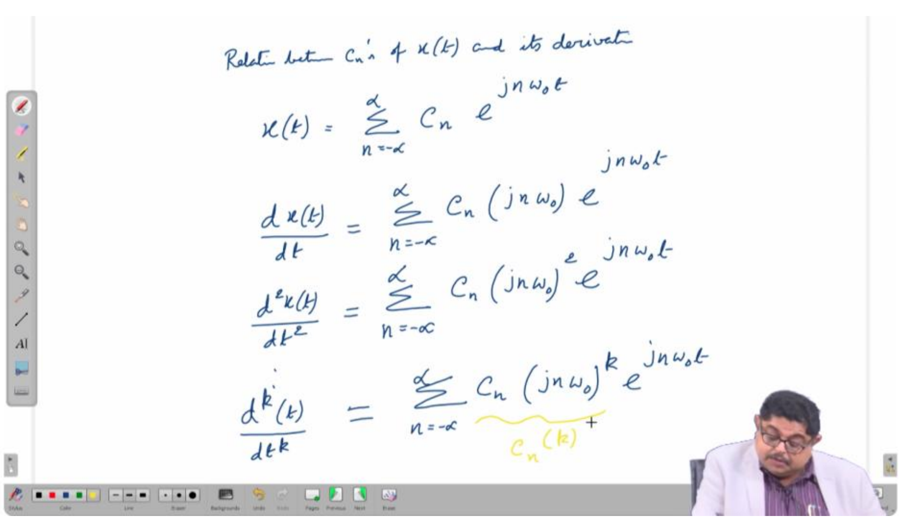
So, it is Fourier coefficient that means, if I plot the C n magnitudes versus n. So, n is 1 or you can call n omega naught. So, omega naught 2 omega naught 3 omega naught etcetera minus omega naught minus 2 omega naught minus 3 omega naught etcetera the value will be same for all and that value is 1 by T. So, you see spectral coefficients are all same actually it should be discrete spectrum actually I have this is the envelope of that. So, the discrete spectrum will be something like this 1 by T 1 by T 1 by T 1 by T 1 by T 1 by T 1 by T. Now, let us say the this pulse train is shifted ahead in time by alpha.



So, this pulse train so x t minus alpha by t and so the pulse train sorry. So, so this is the shifted ahead in time by alpha a periodic unit impulse pulse train. So, using time shift property we can say that the C n shifted version of these is 1 by t e to the power 1 by t was original pulse trains C n. So, it will be minus J n omega naught alpha.



And finally, I have one more property to recall from Fourier analysis that relation between C n's of x t and it is derivatives. Let x t is equal to n is equal to minus infinity to infinity C n e to the power J n omega naught t and so what will be d x t d t? It will be n is equal to minus infinity to infinity C n J n omega naught e to the power J n omega naught t. What is d 2 x t d t? It will be n is equal to minus C n J n omega naught square e to the power J n omega naught t up to that what is d k k th derivative sorry. So, let us call that this one as C n k then can I say that sorry C n k is nothing, but C n J n omega naught k.



So, we can say that so this is the property that the expansion coefficient of x t and expansion coefficient of k th derivative of x t they are related by these.So, C n is 1 by J n omega naught whole to the power k C n k. So, with this we have seen the preliminary properties needed to go to our shortcut. As I said that all this we are doing that we will find an algorithm to derive for our clock pulses periodic pulse trends for them how to find the Fourier coefficients because we will have to estimate the harmonics as an EMC engineer we should know that. So, how to calculate it very fast that will be the topic of our next lecture. Thank you.