Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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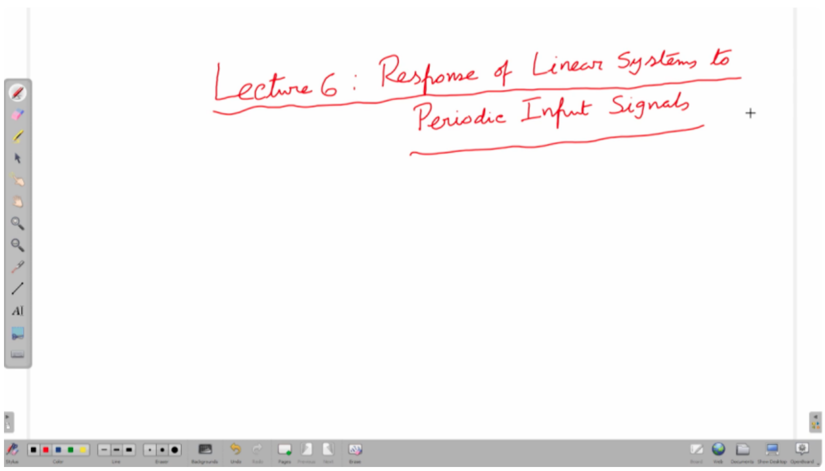
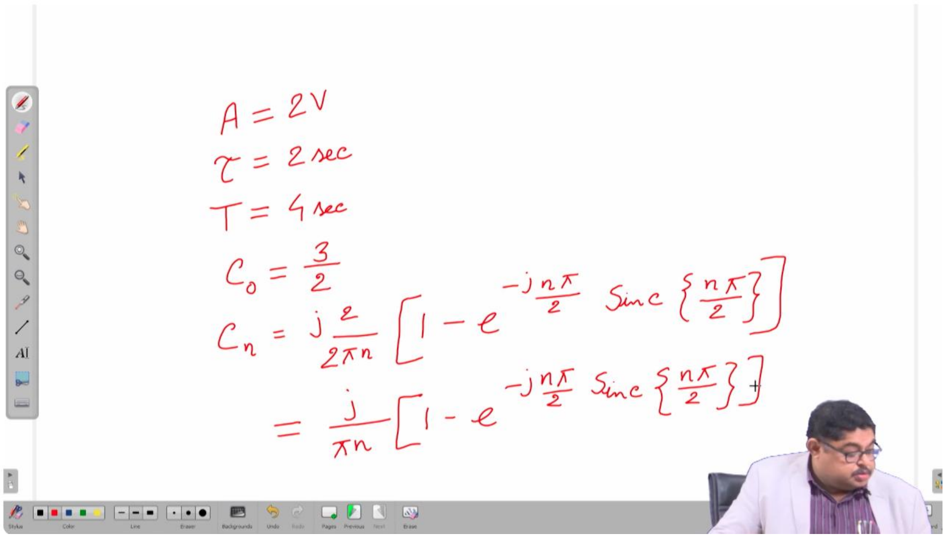
Week :02
Lecture 6: Response of Linear Systems to Periodic Input Signals

Welcome to the 6th lecture on EMI/EMC and Signal Integrity Principles, Techniques and Applications. We will we were discussing the one sided spectrum in the earlier lecture. Now, today we will continue that and then go to this today's lecture this lecture's topic that response of linear systems to periodic input signals ok. So, first let me go back to previous day.



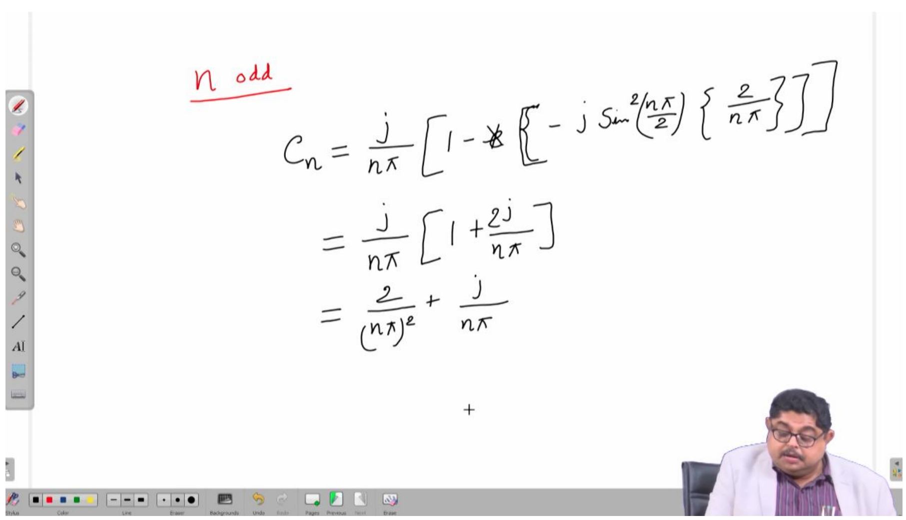
So, we have derived the double sided spectrum C n. So, actually for EMC the phase spectrum is not so important.So, we will be concentrating on your magnitude spectrum. Now, we have got an expression for C n. Now, let us take some typical values that A let us say 2 volt amplitude. That means, signals periodic rectangular pulse strain that is amplitude after it reaches the after is crosses the rise time it is staying at 2 volt.

Now, let us say that tau the rise time is roughly rise time actually rise time is 10 percent to 90 percent to reach the steady state, but let us say from 0 to 2 volt it takes 2 second and the time period is 4 second these are typical values. So, we are just taking. So, what will happen to our C naught? C naught that means, the DC value of the spectrum that will become 3 by 2 and C n will become J 2 by 2 pi n 1 minus e to the power minus J n pi by 2 then sinc of n pi by 2. So, this is actually J by pi n into this one 1 minus e to the power minus J n pi by 2 sinc of n pi by 2 .

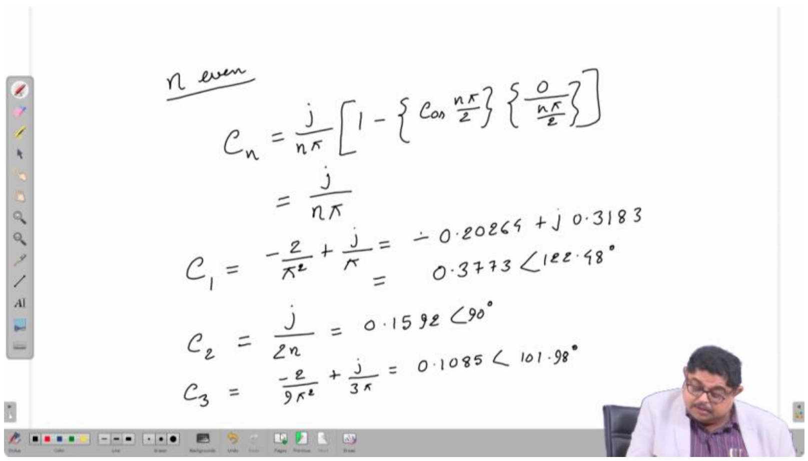


So, again like before let us first take n_{odd}. So, for n_{odd} we will have C n is equal to J by n pi 1 minus e to the e to the power minus J n pi by 2 if it is odd then it will be 1. So, I can write that it will give me minus. So, that thing if I absorb here. So, it will be minus J sin square n pi by 2 then this sinc value for sin n pi by 2 for odd values it is 1. So, I will get basically 1 by n pi by 2. So, that is 2 by n pi .

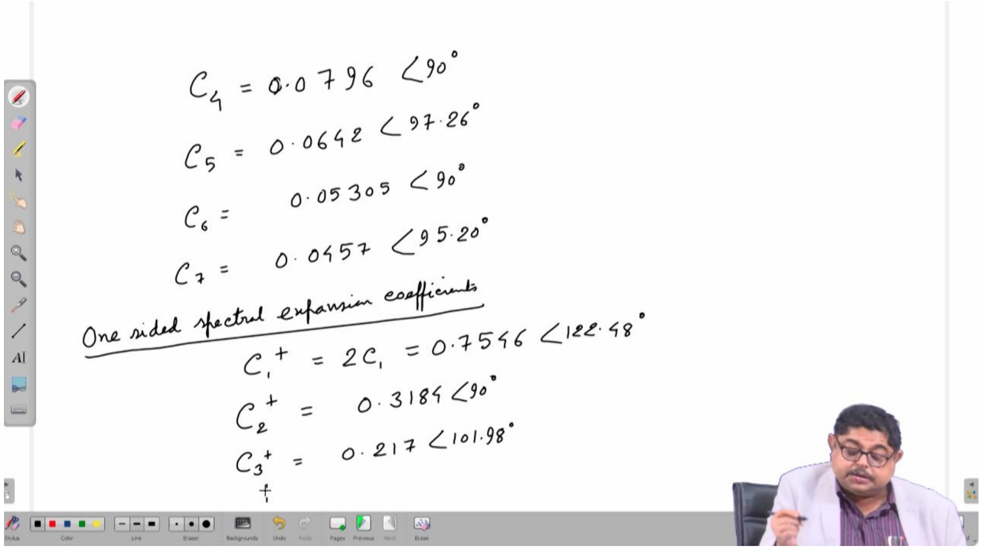
So, let me so, if I simplify I will get J by n pi then 1 plus 2 J by n pi . So, if we write it in rectangular form it will be 2 by n pi whole square plus J by n pi .



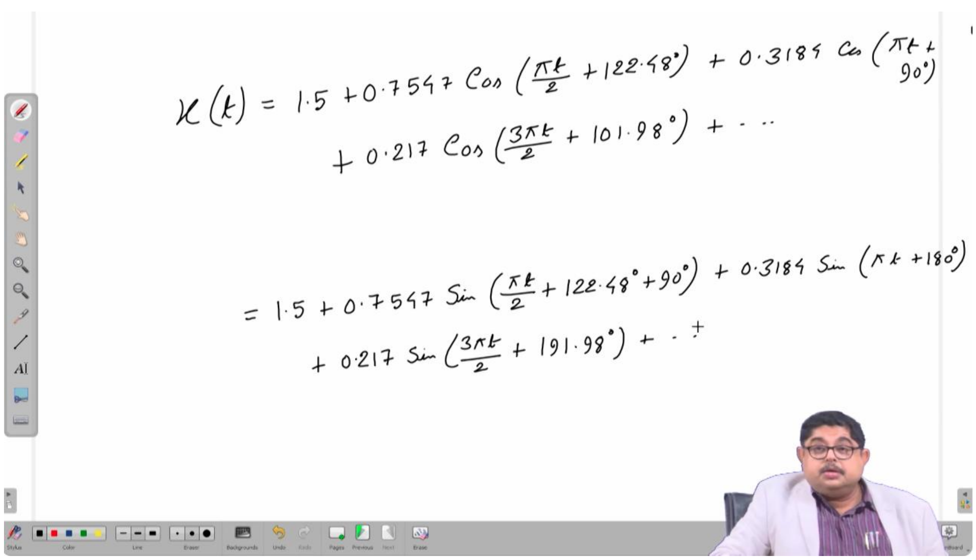
For n even I have C n is equal to J by n pi 1 minus cos n pi by 2 if you expand e to the power minus J n pi by 2 then you get this and for sinc term you see for n odd it is 0 by that n pi by 2 . So, you will get simply a J by n pi . So, now, we are in a position to write the first 7 harmonics. So, what will be C 1? C 1 is a odd. So, it will come from the odd formula. So, it will be put n is equal to 1 there. So, it is minus 2 by pi square plus J by pi .So, this is a constant. So, that one if you write it will be minus 0.20264 plus J 0.3183 . So, you can also write it in magnitude and phase form that means, polar form 0. 3773 122.48 degree in your calculators all these functions are available. So, you can easily convert it polar coordinate similarly C 2. So, C 2 will be J by 2 pi and that in polar coordinate is now I am writing directly to polar coordinate 90 degree C 3 that will come to minus 2 by 9 pi square plus J by 3 pi. So, that is 0.1085 101.98 degree.



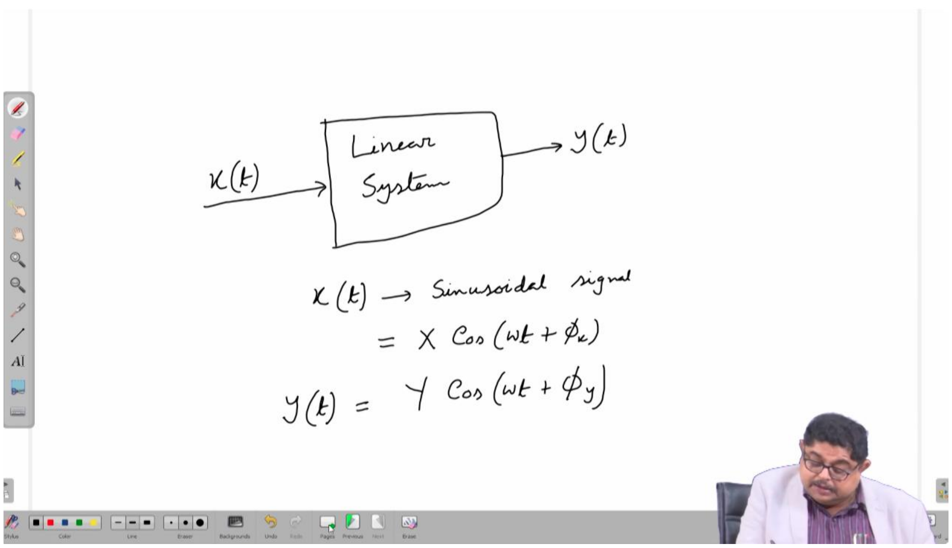
you see there are nothing new, but you probably never done these actually EMC engineers want to do this because they want to see what is the third component what is the fourth component as you have seen in the EMC standards there were written that up to 10 thermonic up to fifth thermonic. So, you need to take that value and see what is the value coming suppose you have given a 2 volt pulse with that tau and t. So, what will be the third harmonic value what will be the fifth harmonic value that is why they want to write like this similarly C 4 will be you see all the harmonics are gradually decreasing 0.0796 let me clearly write it 0.0796 90 degree C 5 is 0.0 this I am writing. So, that you can check yourself that whatever whether you have learned the practice C 6 is 0.05305 90 degree C 7 is 0.0457 95.20 degree. So, now we can write one sided spectrum which was our requirement spectral actually these are called spectral expansion coefficient that will be C 1 plus is equal to 2 C 1 plus is equal to 0.7546 122.48 degree C 2 plus is equal to 0.3184 90 degree C 3 plus is equal to 0.217 101.98 degree etcetera etcetera I am not writing the other values.



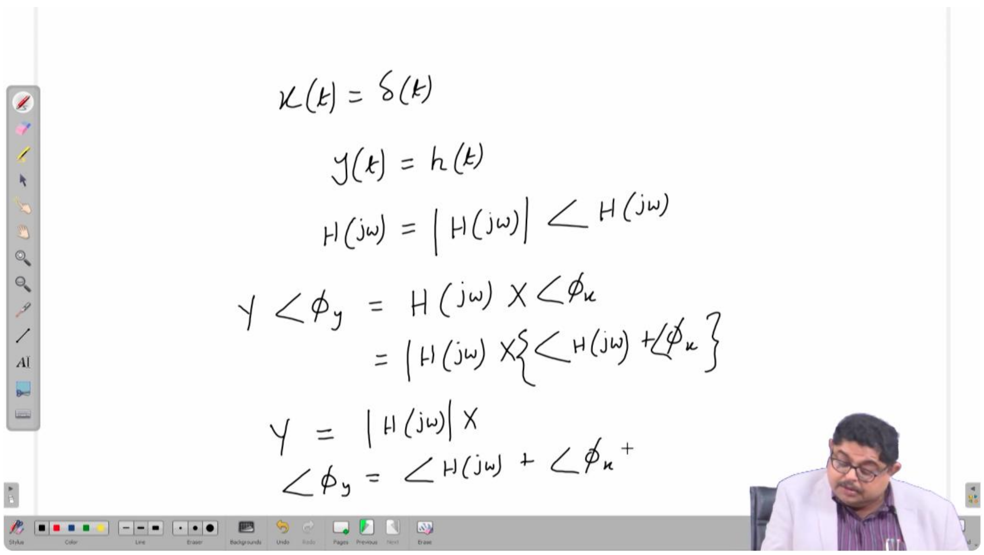
So, in terms of them how we can write x t x t will be 1.5 it is same it is not changed, but this is 0.7547 cos pi t by 2 plus 122.48 degree plus 0.3184 cos pi t plus 90 degree sorry plus 0.3184 cos pi t plus 90 degree sorry plus 0.217 cos 3 pi t by 2 plus 101.98 degree etcetera up to 7 component. If you want to write it in terms of sin it will be 1.5 plus 0.7547 sin then pi t by 2 plus 122.48 degree plus 90 degree plus 0.3184 sin pi t plus 180 degree plus 0.217 sin 3 pi t by 2 plus 0.217 sin 3 pi t by 2 plus 191.98 degree plus etcetera. So, by this way you can do the spectrum.



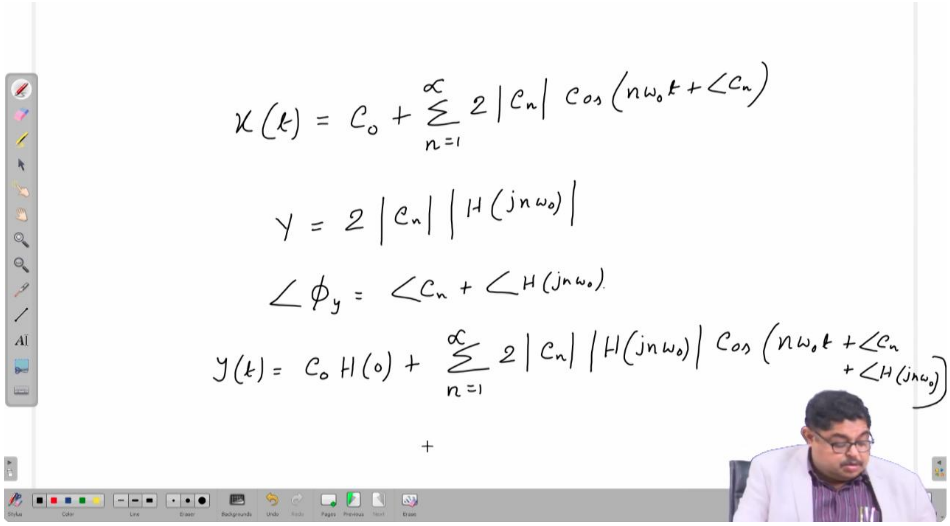
Now let us come to this lectures topic. So, we have seen one sided spectrum. Now let us see that generally the our input signal will pass through a system and we will get an output. So, suppose our input to the system is this we assume a linear system for non-linear system the analysis is different I think you know what is a linear system which obeys the principle of superposition and principle of scaling. So, this things I think you taught. So, in electronics this is the meaning of linear system not the mathematical meaning that y is equal to m x plus c that is not electronics way of saying linear system in electronic systems linear system means it should obey superposition also it should obey scaling. So, in that sense we have a linear system. So, this is a single input single output linear system and let us say that x t is a sinusoidal signal and let us say that x t just assume that it is x cos omega t plus some phase phi x. Now we know that in steady state the output will also be sinusoidal, but it is not like e to the power j omega t is thing in that case it would have been just a scalar multiplication, but here there will be a phase term added.So, output we know it will be cosine. So, let us, but and also multiplied by something because of the this systems performance. So, it will be cos frequency will be same since it is linear system frequency cannot change for non-linear system that may also change. So, it will be phi y.



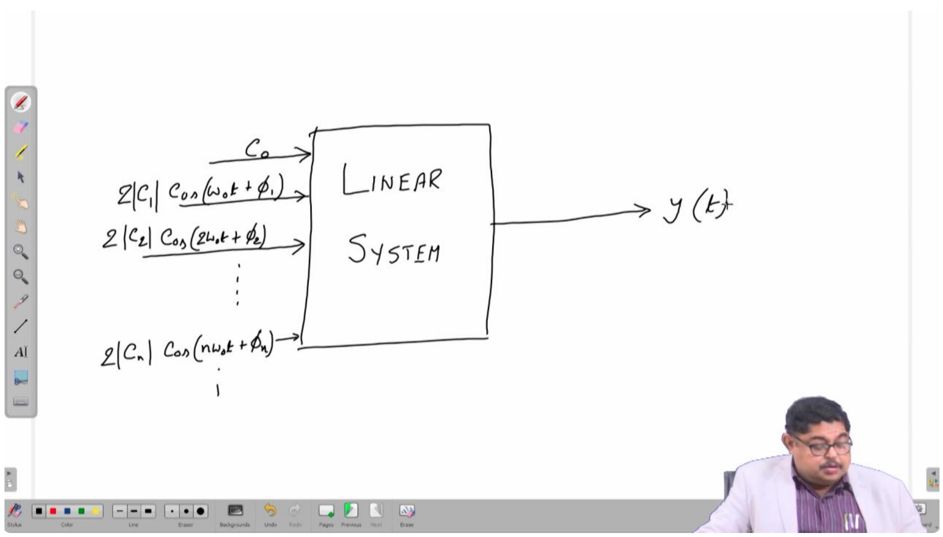
Now if this x t is a delta function I think all of you know direct delta function that it is a necessary because to represent that there is a function existing only at a point we have direct delta. So, if we have 0 initial condition for the system then we know that y t will be something let us call that h t and this is called the impulse response of the system. Now the in frequency domain or Fourier transform of this impulse response is called h j omega. We go to Fourier domain because the actually y t is nothing, but convolution of x t with h t, but y omega will be simply multiplication of x omega into h omega that is the simple multiplication that is why we go to frequency domain. So, h j omega is also a complex function. So, we can break it into a magnitude and phase form. So, we can write that it is h j omega. So, this one I think you know that this is called transfer function because it transfers input to output. So, we can write in frequency domain that y you see previously I have written y. So, y phase of y is equal to h j omega into x this was what I was saying that the y of omega is equal to h of omega into x of omega. So, that is written here. So, this one we can replace as h j omega into x into h j omega plus phi x angle phi x. So, we can write y is the magnitude part that will be h j omega into the magnitude part of the input x and phi y is equal to angle h of j omega plus angle of phi a this is very well known.



Now, suppose x t is periodic suppose our x t is periodic because in E M I E M C class we are concerned with periodic actually we are concerned with clock trend because you have seen that that definition of our digital device is which has so much clock frequency more than 9 kilohertz clock. clock is always periodic signal. So, let us say our x t is periodic and we have seen its Fourier series. So, let us say that our x t is in this form c 0 plus n is equal to one sided spectrum 2 c n cos of n omega naught t plus angle c n. We may pass each of these components through the system through the our linear system determine the each one is a sinusoid. So, determine the sinusoidal steady state response to each that means, suppose I am passing cos omega naught t to the system I am giving cos omega naught t what will be y j n omega naught. So, I can say the magnitude of the response. So, one by one I am giving. So, y is nothing, but 2 c n h j n omega naught this is the input part this is the system part this is the output part and phase part will be c n plus h j n omega naught since the system is linear superposition applies. So, can I say that y t is equal to c 0 h 0 sorry h of 0 plus a m e is equal to 1 to infinity 2 c n h j n omega naught cos n omega naught t plus c n plus angle h j n omega naught.

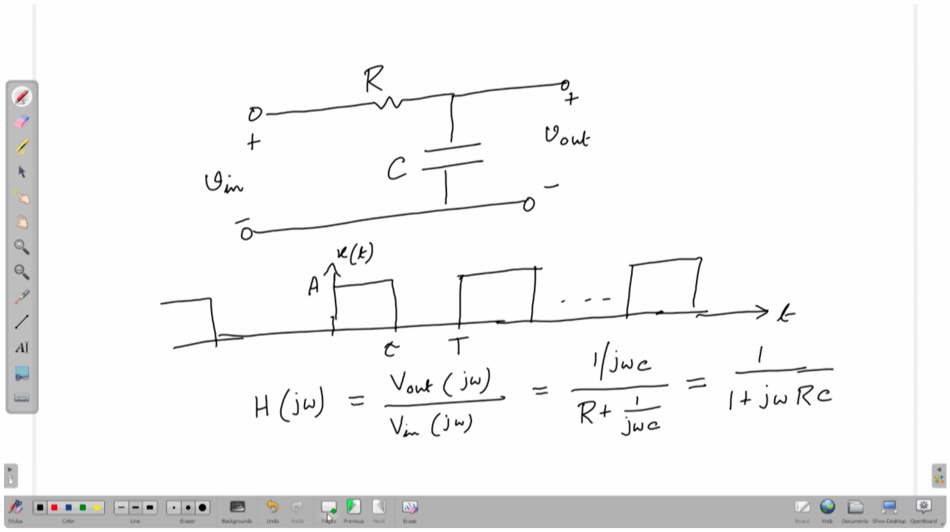


Basically what I am doing I can write pictorially like this that I am passing a c naught I am passing a 2 c 1 cos omega naught t plus phi 1 I am passing 2 c 2 cos 2 omega naught t plus phi 2 like this I am passing 2 c n cos n omega naught t plus phi n etcetera etcetera. So, all of them I am passing and this is a linear system. So, what I am getting is nothing, but y t.

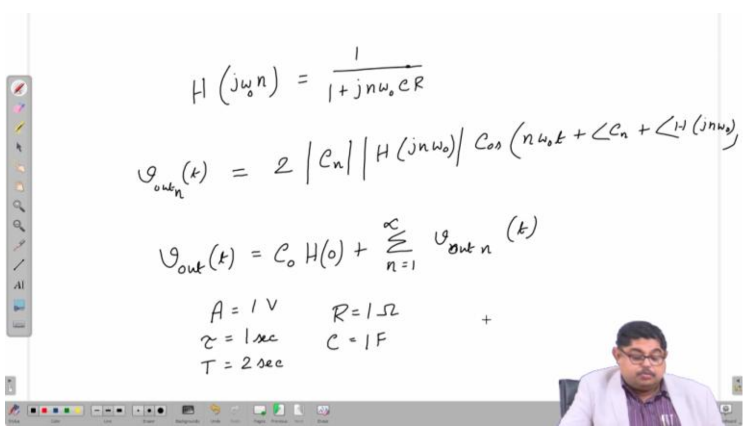


So, to exemplify let us see that suppose I have a simple R C circuit it is a linear system and so I am giving the input here and in R C this is C R R C means output is taken across C if it was C R output was taken across R, but it is R C.

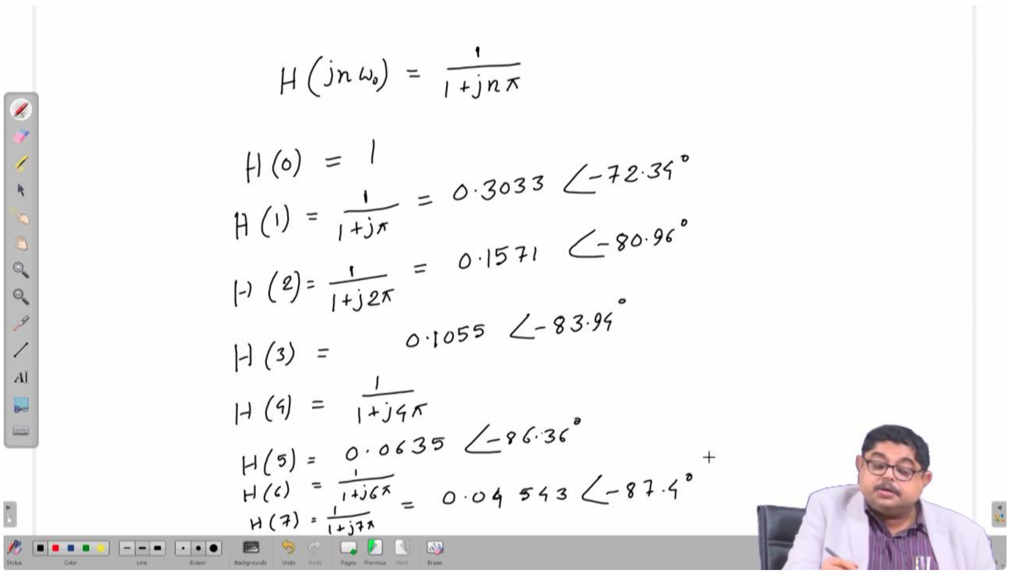
So, this you know is a low pass filter and I am giving there that our rectangular pulse A tau. So, we know this spectrum. So, we can easily find out what is h j omega h j omega is V out j omega by V in j omega. So, you know that what is that that is 1 by j omega C at steady state that is why I am not using S because that is also involving transient, but at steady state I know that this will be the thing. So, 1 by R plus j omega C that is 1 by 1 plus j omega R C.



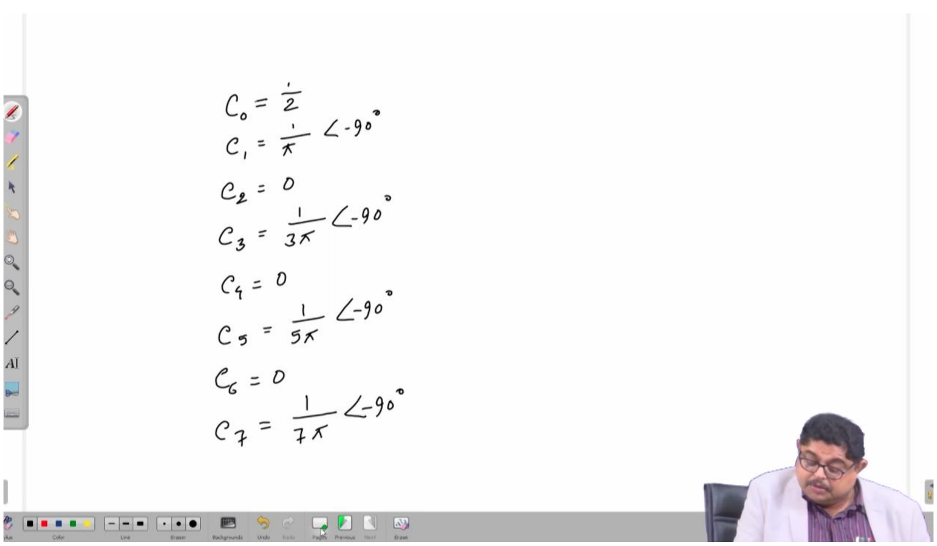
So, we can say that the at each harmonic frequencies that means at instead of omega I am not writing j omega naught n is equal to 1 by 1 plus j n omega naught C by R because my input signal is a discrete frequency points. So, I have I can evaluate h j omega n like this. So, what will be my V output T it will be 2 V output let me call C n this is separate. So, 2 C n h j n omega naught cos of n omega naught t plus C n plus h j n omega naught. So, my complete response this was at n values. So, but V out t will be C naught h naught plus this limit or some n is equal to 1 to infinity V out n t. Now, let us take some typical values that A is equal to 1 volt tau is 1 second T is 2 second that means, again we are taking duty cycle to be 50 percent and also take some R and A value R let us take 1 ohm and C let us take 1 farad. So, time constant R C is 1 second.



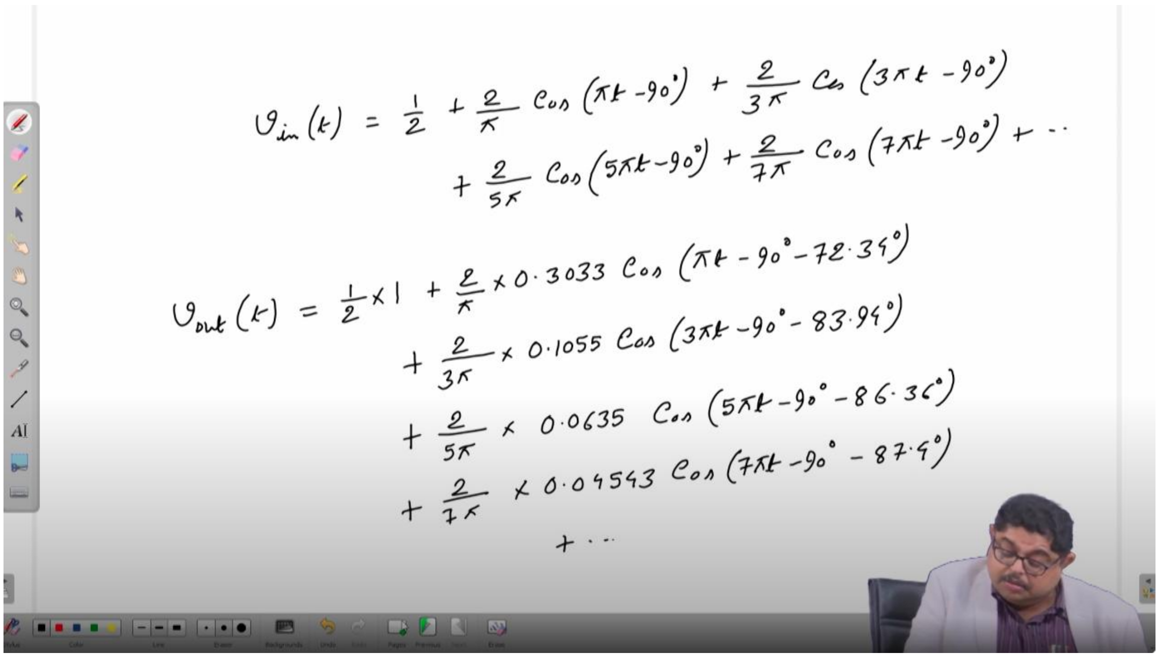
So, now what will be our h j n omega naught that will be 1 by 1 plus j n pi with those values. So, if we evaluate up to 7 thermonic because we have seen up to 7 thermonic those things. So, h naught h if I put n is equal to 0 h naught is 1 h 1 will be 1 by 1 plus j pi that is 0.3033 minus 72.34 degree h 2 is 1 by 1 plus j 2 pi that is 0.1571 minus 80.96 h 3 is similarly 0.1055 minus 83.94 degree h 4 is 1 by 1 plus j 4 pi h 5 is 0.0635 minus 86.36 degree h 6 is 1 by 1 plus j 6 pi h 7 is 1 by 1 plus j 7 pi and that is equal to 0.04543 minus 87.40degree. So, I have all h values.



So, for input signal we know that h 0 is half for those frequencies c 1 is 1 by pi minus 90 degree c 2 is 0 c 3 is 1 by 3 pi minus 90 degree c 4 is 0 c 5 is 1 by 5 pi minus 90 degree c 6 is 0 c 7 is 1 by 7 pi minus 90 degree.



So, now, I can say the if I double the magnitude. So, v in t is half plus 2 by pi cos pi t minus 90 degree plus 2 by 3 pi cos 3 pi t minus 90 degree plus 2 by 5 pi cos 5 pi t minus 90 degree plus 2 by 7 pi cos 7 pi t minus 90 degree and now since I have all the things available I can find each frequency components. So, half into 1 because h 0 is 1 plus 2 by pi into 0.3033 cos pi t minus 90 degree minus 72.34 degree plus 2 by 3 pi into 0.1055 cos 3 pi t minus 90 degree minus 83.94 degree plus 2 by 5 pi into 0.0635 cos 5 pi t minus 90 degree minus 86.36 degree plus 2 plus 7 pi into 0.04543 cos 7 pi t minus 90 degree minus 87.4 degree.



So, I now know what is the each spectral component. So, this is 0.5 the first harmonic its value is point magnitude is 0.1931 there is no second harmonic the third harmonic is third harmonic value is 0.0224 the fifth harmonic value is the seventh harmonic is. So, you see that EMC engineers can calculate by the technique shown that what are the each components and up to what component he is getting which is not satisfying the standard etcetera. So, he can elegantly use, but still you see that we had to give considerable mathematical labor to derive or come here because to have the spectrum you see we had to do lot of e to the power plus j sign etcetera then we had to multiply each etcetera. So, that was a laborious task, but in field EMC engineers cannot have so much time and so much may not have so much mathematical expertise to do all this. So, we will see is there any shortcut for all this that will be the lecture topic of our next class that shortcut. Thank you.

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\begin{aligned}
v_{out}(t) = 0.5 &+ 0.1931 \cos(\pi t - 162.34^\circ) \\
&+ 0.0224 \cos(3\pi t - 173.94^\circ) \\
&+ 0.0081 \cos(5\pi t - 176.36^\circ) \\
&+ 0.0041 \cos(7\pi t - 177.40^\circ) \\
&+ \cdots
\end{aligned}
$$