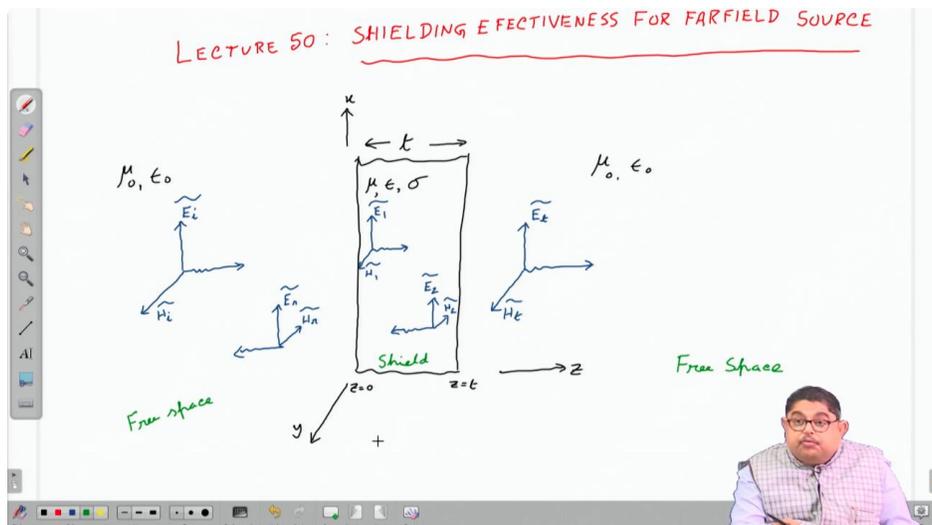


Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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 Week :10
 Lecture 50: Shielding Effectiveness for Farfield Source

Welcome to the lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. We were seeing we have seen the definition of shielding effectiveness in the last class. We have seen that there can be two types of sources near field or far field. So, first we have take up the far field case. So, we draw the a far field source picture. This is a shield. So, let me write that this is your, this is the shield with its electrical parameters μ ϵ σ . This is a free space. This is also a free space this side extending up to infinity that is why no reflected transmitted wave is coming. So, this is the picture. Now, we need to find what is the ratio of E_i by E_t . Now, to do that E_i is known we are assuming that whatever the source has send. So, that electrical field strength is known at the shield. So, we now need to find E_t . We have two interfaces of discontinuous materials. One is free space to shield material that is at z is equal to 0. Similarly, we have another discontinuity at z is equal to 2 inter plane that is from shield to free space. So, there we have some knowledge of the boundary condition of the electrical magnetic fields that we will now apply to get that ratio. So, we have already seen discussed that a far field source always at its far field that the wave becomes like plane waves. So, we can assume that all these waves incident wave reflected wave transmitted wave then the wave that is starting from the z is equal to 0 interface inside the shield. Similarly, the wave that is E_2 that is the wave that is starting from the z is equal to t interface inside the shield all of them are plane waves.



So, we can easily write their expressions that E_i is I can say $E_i e^{-j\beta z}$ to the power minus $j\beta z$ in our picture we have assumed it to be x directed. And this β is the phase constant of the free space because this is happening at free space. So, that is $\omega \mu_0$ into ϵ_0 naught similarly, H_i will be E_i by $\eta_0 e^{-j\beta z}$ to the power minus $j\beta z$ this is for the plane wave it is y directed and what is η_0 ? η_0 is the free space impedance that is $\sqrt{\mu_0/\epsilon_0}$. So, this is the incident wave for reflected wave I will write the E_r and H_r . So, E_r will be $E_i e^{+j\beta z}$ because the wave is going in the negative z direction a y sorry a x and H_r will be minus E_r by $\eta_0 e^{+j\beta z}$ a y then we have E_1 . So, E_1 will be $E_i e^{-j\beta z} + E_r e^{+j\beta z}$ in it is σ . So, it has the complex part that means, the α is also present. So, it is $e^{-\alpha z} e^{-j\beta z}$ where α is the propagation constant of the shield material. So, α is $\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$ and this is a complex number. So, that is generally say that $\alpha + j\beta$. So, that is E_1 . So, H_1 will be E_1 by $\eta e^{-\alpha z} e^{-j\beta z}$. So, what is η ? The impedance is of the shield medium. So, that is $j\omega\mu / (\sigma + j\omega\epsilon)$ and that is generally written as $\eta_0 \tan \theta$.

So, E_1 then there will be E_2 , E_2 is E_2 vector is E_2 the magnitude part into $e^{-\alpha z} e^{-j\beta z}$ to the power plus αz a x H_2 minus H_2 by $\eta e^{-\alpha z} e^{-j\beta z}$ because it is going opposite to E_1 or opposite to the incident wave direction then I have E_t and H_t . So, E_t is magnitude $e^{-\alpha z} e^{-j\beta z}$ again β has written because this is free space and H_t is $\eta_0 e^{-\alpha z} e^{-j\beta z}$ to the power minus $j\beta z$ a y. So, all the waves which we have drawn there we have written their mathematical expression and these are the extra variables that we use throughout. So, that means now you see we have the we will have to determine basically we will have to determine E_t , but in the process we have also got the μ another three variables E_r , E_1 and E_2 . So, we have total unknown four variables E_r , E_1 , E_2 and E_t . So, we need four equations to solve for these four unknowns and that we will get from the boundary conditions at the two interfaces. For electric field there will be one boundary condition for magnetic field there will be another boundary condition. So, there are two interfaces. So, there will be total four equations boundary conditions concerning all these four variables and from that we will be able to solve for our desired one that E_t by E_1 .

$$\vec{E}_i = \tilde{E}_i e^{-j\beta_0 z} \hat{a}_x$$

$$\vec{H}_i = \frac{\tilde{E}_i}{\eta_0} e^{-j\beta_0 z} \hat{a}_y$$

$$\vec{E}_r = \tilde{E}_r e^{+j\beta_0 z} \hat{a}_x$$

$$\vec{H}_r = -\frac{\tilde{E}_r}{\eta_0} e^{+j\beta_0 z} \hat{a}_y$$

$$\vec{E}_t = \tilde{E}_t e^{-\tilde{\gamma} z} \hat{a}_x$$

$$\vec{H}_t = \frac{\tilde{E}_t}{\tilde{\eta}} e^{-\tilde{\gamma} z} \hat{a}_y$$

where $\beta_0 = \omega \sqrt{\mu_0 \epsilon_0}$ $\tilde{\gamma} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$
 $\tilde{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \tilde{\eta} \angle \alpha_n$

So, we can say that the boundary condition at z is equal to 0 for electric field is that electric field is continuous. So, tangential electric field is continuous. So, I can say that at z is equal to 0 enforcing the continuity of tangential electric field. So, magnetic field is continuous we know that at a conductor boundary and the boundary there are no current on the z is equal to 0 surface. So, magnetic field tangential component will also be continuous. Similarly, at z is equal to t also. So, we will get the electric field tangential electric field continuity and tangential magnetic field continuity again because of the reason that we are not putting any source current source at z is equal to t interface. So, we will have E_I at z is equal to 0 plus E_r you see your diagram. Let me go there that at z is equal to 0 I can say that E_I plus E_0 will be equal to E_1 plus E_2 at z is equal to 0. Similarly, at z is equal to t what will happen to the electric field components E_1 plus E_2 is equal to E_t for magnetic fields similarly we will be able to write. So, let us write that is equal to E_1 z is equal to 0 plus E_2 z is equal to 0 E_1 z is equal to t plus E_2 z is equal to t is equal to E_1 z is equal to t is equal to E_t z is equal to t H_I z is equal to 0 plus H_r z is equal to 0 plus H_1 z is equal to 0 plus H_2 z is equal to 0 and H_3 z is equal to 0 plus H_4 z is equal to 0 and H_1 z is equal to t plus H_2 z is equal to t is equal to 0. Now, we have our mathematical forms of plane wave. So, there we can put this. So, E_I is this. So, E_I at z is equal to 0 will be $E_I A_x E_r$ at z is equal to 0 will be $E_r A_x$ similarly E_1 at z is equal to 0 will be $E_1 A_x H_1$ will be E_1 by η to the power that will go because it is at z is equal to 0. So, like that we will be able to write.

At $z=0$
 Enforcing the continuity of tangential electric and magnetic fields:

$$\vec{E}_i \Big|_{z=0} + \vec{E}_r \Big|_{z=0} = \vec{E}_1 \Big|_{z=0} + \vec{E}_2 \Big|_{z=0}$$

$$\vec{E}_i \Big|_{z=t} + \vec{E}_r \Big|_{z=t} = \vec{E}_t \Big|_{z=t}$$

$$\vec{H}_i \Big|_{z=0} + \vec{H}_r \Big|_{z=0} = \vec{H}_1 \Big|_{z=0} + \vec{H}_2 \Big|_{z=0}$$

$$\vec{H}_i \Big|_{z=t} + \vec{H}_r \Big|_{z=t} = \vec{H}_t \Big|_{z=t}$$

So, let us write that. So, E_i plus E_r is equal to E_1 plus E_2 let me call this equation equation 1. So, this is equation number 1. Then the next one will be E_i to the power minus γt plus E_r to the power plus γt is equal to E_t to the power minus $j\beta_0 t$. Let us call this equation number 2. Then the H's. So, that we will write as E_i by η_0 minus E_r by η_0 is equal to E_1 by η minus E_2 by η . So, let me call this as equation number 3. And the last one will be E_i by η_0 to the power minus γt minus E_r by η_0 to the power plus γt is equal to E_t by η to the power minus $j\beta_0 t$. So, let me call it equation 4. So, I have got 4 equations. Now, just we will have to do algebraic manipulation.

$$\vec{E}_i + \vec{E}_r = \vec{E}_1 + \vec{E}_2 \quad \dots (i)$$

$$\vec{E}_i e^{-\gamma t} + \vec{E}_r e^{+\gamma t} = \vec{E}_t e^{-j\beta_0 t} \quad \dots (ii)$$

$$\frac{\vec{E}_i}{\eta_0} - \frac{\vec{E}_r}{\eta_0} = \frac{\vec{E}_1}{\eta} - \frac{\vec{E}_2}{\eta} \quad \dots (iii)$$

$$\frac{\vec{E}_i}{\eta_0} e^{-\gamma t} - \frac{\vec{E}_r}{\eta_0} e^{+\gamma t} = \frac{\vec{E}_t}{\eta} e^{-j\beta_0 t} \quad \dots (iv)$$

So, that we will do now. E_i plus E_r is equal to E_1 plus E_2 . E_i plus E_r is equal to E_1 plus E_2 plus E_2 and 3 gives me that magnetic field 1 that E_i by η naught minus E_r by η naught is equal to E_1 by η naught plus E_2 by η naught minus. That one will be minus because it is in the opposite direction. That is all. So, now we can add these two equations. So, adding what we get is E_i is equal to half E_1 plus η naught by η naught plus half E_2 1 minus η naught by η naught. There is no η naught. So, let me call this one as equation 5. So, adding 1 and 3, I have got E_i in terms of E_1 and E_2 . Similarly, if I add adding 2 and 4. You can easily see. So, I will add this one and this one. So, again you see. So, I will get an expression of E_1 . So, let us do that. 2 means $E_1 e$ to the power minus γt plus $E_2 e$ to the power γt is equal to $E_2 e$ to the power minus $j \beta$ naught t and $E_1 e$ to the power minus γt minus $E_2 e$ to the power γt is equal to $E_1 e$ to the power minus η naught t minus η naught t e to the power minus $j \beta$ naught t . So, if I add, I get E_2 is equal to half 1 minus η by η naught $E_1 e$ to the power minus γt e to the power minus $j \beta$ naught t . I hope I am correct. So, this equation 4, this is not 2 and 4. This was actually subtracting 2 and 4. So, adding 2 and 4, I have not let me do that. So, adding 2 and 4, this is 2. Let me check again. 2 is E_1 plus E_2 is that and from 4, I can write $E_1 e$ to the power minus γt minus $E_2 e$ to the power plus γt is equal to η by η naught e to the power this. So, this is minus. So, I will get E_2 , E_1 . So, this is wrong. This is not E_1 . I will get because if I add E_1 , E_1 is half 1 plus e to the power this term also will come. So, I will get e to the power minus $j \beta$ naught t . So, that will be my equation number 6.

(i) $\rightarrow \tilde{E}_i + \tilde{E}_r = \tilde{E}_1 + \tilde{E}_2$

(ii) $\rightarrow \frac{\tilde{E}_i}{\eta_0} - \frac{\tilde{E}_r}{\eta_0} = \frac{\tilde{E}_1}{\eta_1} - \frac{\tilde{E}_2}{\eta_2}$

Adding. $\tilde{E}_i = \frac{1}{2} \tilde{E}_1 \left(1 + \frac{\eta_0}{\eta_1}\right) + \frac{1}{2} \tilde{E}_2 \left(1 - \frac{\eta_0}{\eta_2}\right)$ (v)

Adding (i) & (ii)

$$\begin{aligned} \tilde{E}_1 e^{-\tilde{\gamma}t} + \tilde{E}_2 e^{\tilde{\gamma}t} &= \tilde{E}_1 e^{-j\beta_0 t} \\ \tilde{E}_1 e^{-\tilde{\gamma}t} - \tilde{E}_2 e^{\tilde{\gamma}t} &= \frac{\eta_0}{\eta_1} \tilde{E}_1 e^{-j\beta_0 t} \end{aligned}$$

$\tilde{E}_1 = \frac{1}{2} \left(1 + \frac{\eta_0}{\eta_1}\right) \tilde{E}_2 e^{-j\beta_0 t}$ (vi)

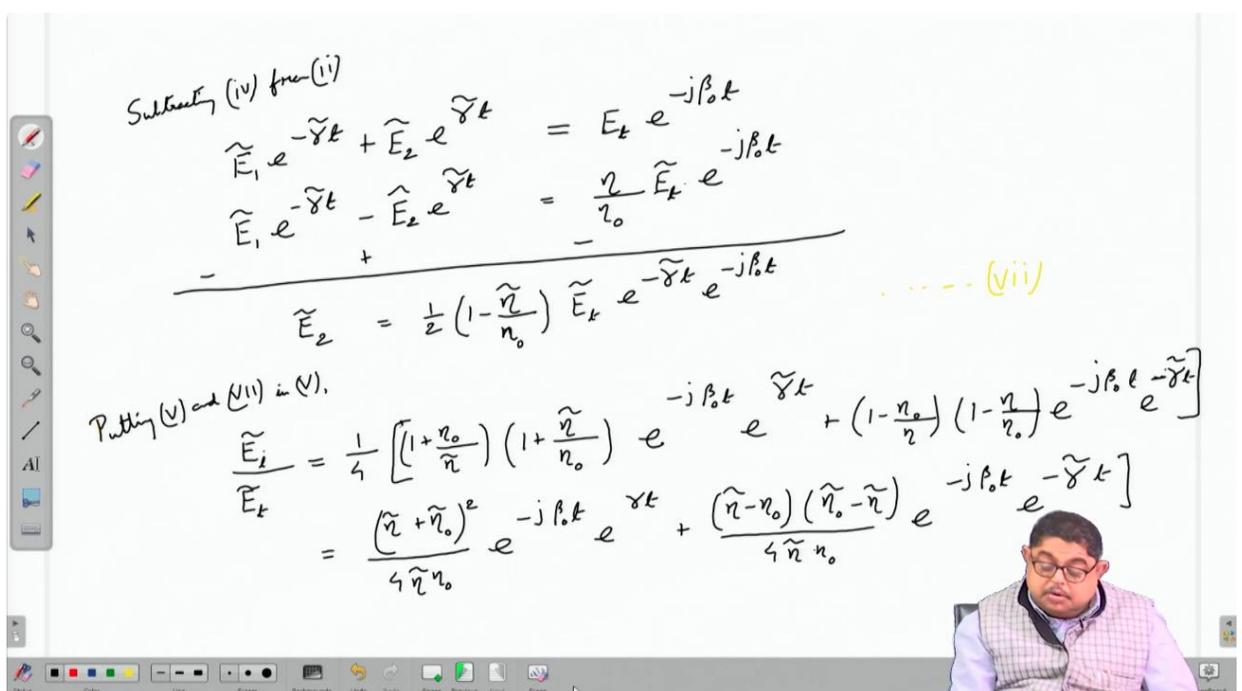
And now, I will subtract. Subtracting, I have already got E 1. I will have to get E 2 in terms of E t. Subtracting 4 from 2, what we get was what I have done that E 1 e to the power minus gamma t plus E 2 e to the power gamma t is equal to E t e to the power minus gamma t eta naught t. And here, I will get E 1 e to the power minus gamma t minus e to the power gamma t is equal to eta by eta naught e t e to the power minus gamma t. So, beta naught t I will now subtract. So, E 1 will go out. I will get E t E 2 is equal to half 1 minus eta 0 E t e to the power minus gamma t e to the power minus j beta naught t. So, this one I will call equation number 7 fine. And then, I will put this. So, 6 give me an expression of E 1 in terms of E t. The 7 give me an expression of E 2 in terms of E t. So, if I put this into 5, I get everything in terms of E i and E t. So, look at your 5. What was 5? So, E i is equal to half here if you put E 1 in. So, E t will come here if you put E 2 from equation 7, E t will come. So, you are getting E i is equal to something into E t. So, you will get shielding effectiveness. So, we get E i by E t is equal to eta naught plus 1 by 4 into 1 plus eta naught by eta 1 plus eta naught by eta. So, that gives you eta plus eta naught square by 4 eta eta naught e to the power minus j beta naught t e to the power gamma t plus eta minus eta naught into eta naught minus eta by 4 eta eta naught e to the power minus j beta naught t e to the power minus j beta naught. Now, note that only there because already I have taken it common. So, there will be 2 equations. Now, this one I can finally, write as eta naught eta naught plus eta square by 4 eta eta naught into 1 minus eta naught minus eta whole square by eta naught plus eta whole square e to the power minus 2 alpha t e to the power minus j 2 beta naught t into e to the power alpha t e to the power j beta t e to the power minus j beta naught t. Just writing like this because I will interpret this result in this way. So, that we will see in the next class. Thank you.

Subtracting (iv) from (i)

$$\begin{aligned} \tilde{E}_1 e^{-\gamma t} + \tilde{E}_2 e^{\gamma t} &= E_t e^{-j\beta_0 t} \\ \tilde{E}_1 e^{-\gamma t} - \tilde{E}_2 e^{\gamma t} &= \frac{\eta}{\eta_0} \tilde{E}_t e^{-j\beta_0 t} \end{aligned}$$

$$\tilde{E}_2 = \frac{1}{2} \left(1 - \frac{\tilde{\eta}}{\eta_0} \right) \tilde{E}_t e^{-\gamma t} e^{-j\beta_0 t} \quad \dots \dots (vii)$$

Putting (v) and (vii) in (v).

$$\begin{aligned} \frac{\tilde{E}_i}{\tilde{E}_t} &= \frac{1}{4} \left[\left(1 + \frac{\eta_0}{\tilde{\eta}} \right) \left(1 + \frac{\tilde{\eta}}{\eta_0} \right) e^{-j\beta_0 t} e^{\gamma t} + \left(1 - \frac{\eta_0}{\tilde{\eta}} \right) \left(1 - \frac{\tilde{\eta}}{\eta_0} \right) e^{-j\beta_0 t} e^{-\gamma t} \right] \\ &= \frac{(\tilde{\eta} + \eta_0)^2}{4 \tilde{\eta} \eta_0} e^{-j\beta_0 t} e^{\gamma t} + \frac{(\tilde{\eta} - \eta_0)(\tilde{\eta}_0 - \tilde{\eta})}{4 \tilde{\eta} \eta_0} e^{-j\beta_0 t} e^{-\gamma t} \end{aligned}$$


$$\begin{aligned}
 \frac{\tilde{E}_i}{\tilde{E}_t} &= \frac{(\tilde{n} + n_0)^2}{4\tilde{n}n_0} e^{-j\beta_0 t} e^{\gamma t} - \frac{(\tilde{n} - n_0)^2}{4\tilde{n}n_0} e^{-j\beta_0 t} e^{-\gamma t} \\
 &= \frac{(\tilde{n} + n_0)^2}{4\tilde{n}n_0} e^{-j\beta_0 t} e^{\gamma t} \left[1 - \frac{(\tilde{n} - n_0)^2}{4\tilde{n}n_0} e^{-2\gamma t} \right] \\
 &= \frac{(n_0 + \tilde{n})^2}{4\tilde{n}n_0} \left[1 - \frac{(n_0 - \tilde{n})^2}{(n_0 + \tilde{n})^2} e^{-2\gamma t} e^{-j2\beta_0 t} \right] e^{\gamma t} e^{j\beta_0 t} e^{-j\beta_0 t}
 \end{aligned}$$

