Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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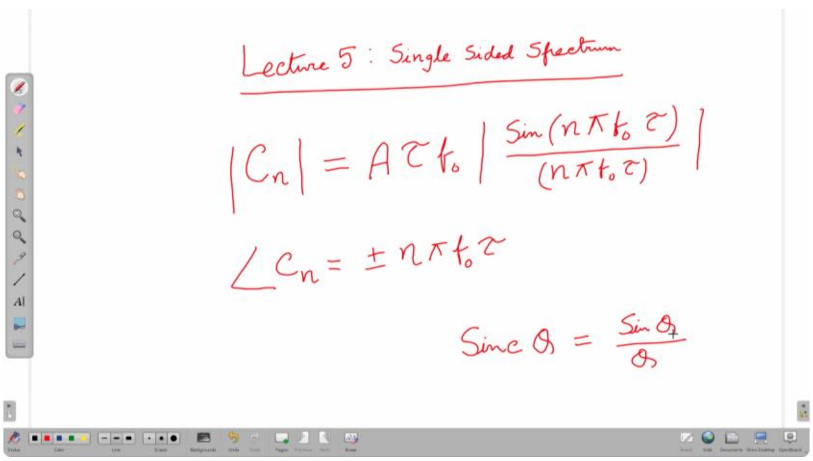
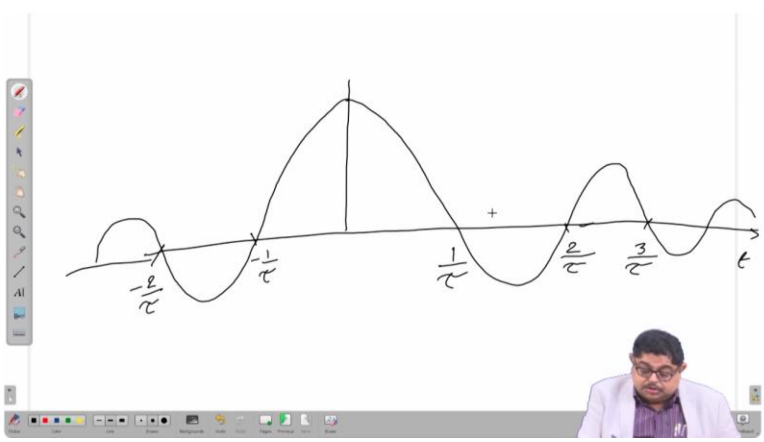
Week :01
Lecture 1: Introduction to Electromagnetic Environment
Lecture 5: Single Sided Spectrum

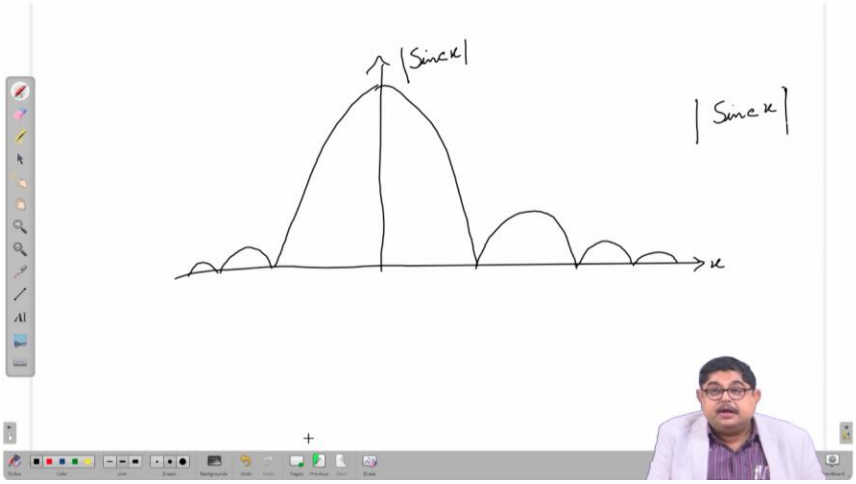
Welcome to the fifth lecture of the course on EMI, EMC and Signal Integrity Principles, techniques and applications. Now, in our last lecture we were discussing the discrete spectrum of a rectangular pulse train. Now, there we have seen our magnitude of the spectral components, and phase of the spectral components were seen.



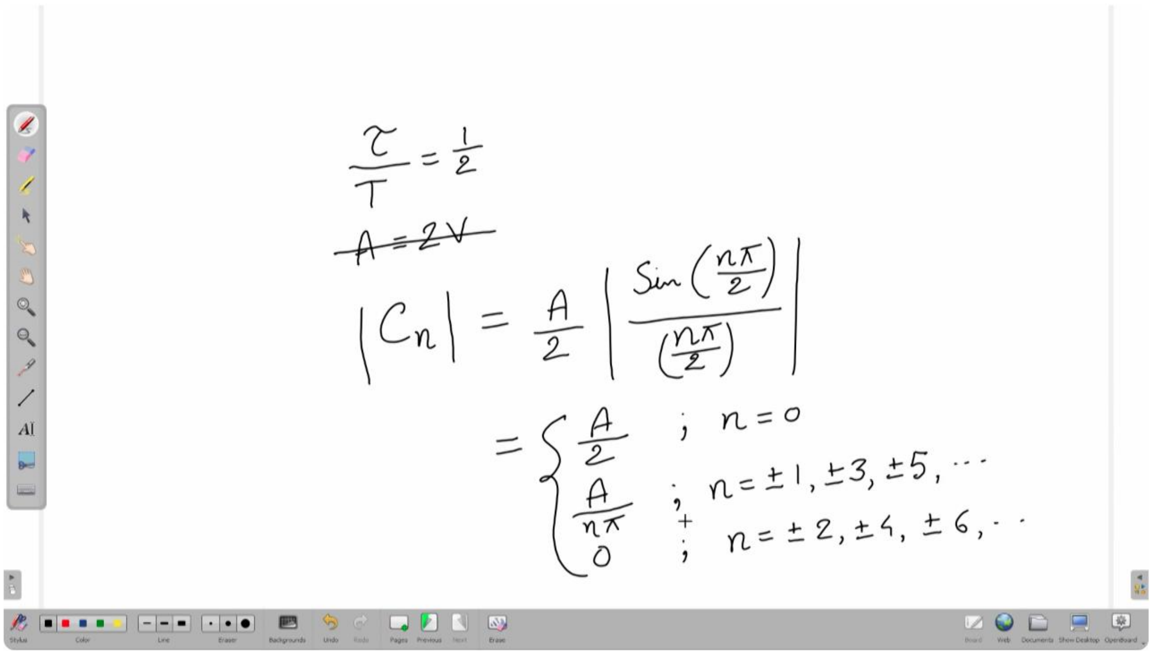

So, that will be generally calling a sinc function. where this thetas are in radians. Now, this in some books particularly in communication books also this sinc function comes and in some books this definition is a bit different there it is sinθ/Πθ, but we would not use that definition we will be using throughout this EMC course this sinθ/θ definition. And you know the behavior of this function that this function if I plot actually it is like this.



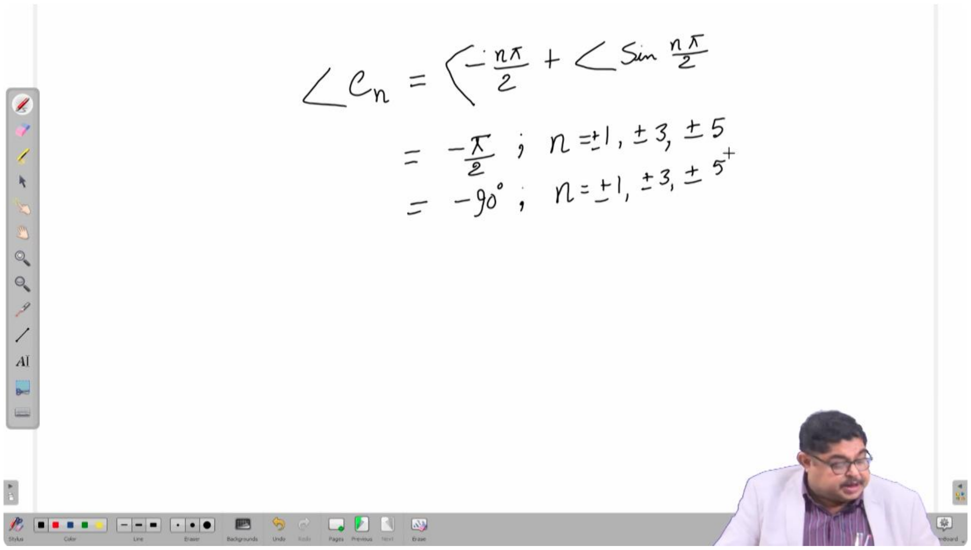
So, gradually you see it is likes something like cosine function because at n is equal to 0 the value of the function is maximum and that value is the peak and there are for every n by tau this function has a 0 or null and you see that at gradually the peaks are going down, but this function has negative values also in some portion that means, over from 1 pi by 1 by tau to 2 pi this is in time domain you are saying and here also you have minus 1 by tau minus 2 by tau etcetera. So, the peaks are gradually going down. So, when we take the magnitude of this this negative portion will come here. So, that time we can draw it like this like this this is the magnitude of sinc sinc by x if I plot it over x.



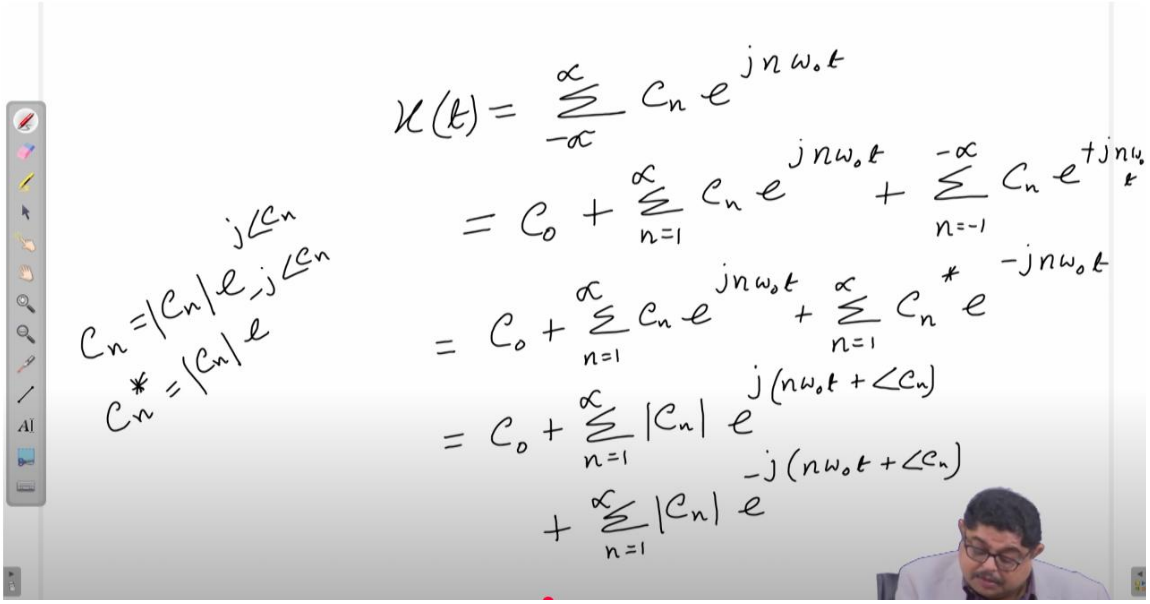

So, this is my magnitude of sinc x ok. So, you have seen our magnitude spectrum was something like this the envelope of that actually we have discrete spectrum. So, basically we have all only at the spectral components we have the values in other places it was not there etcetera. Now, in the previous expression you have the expression of the thing now let us take some particular values. So, that it will be meaningful suppose one of the common practice in industry is to take the duty cycle of the rectangular pulse strain to be 50 percent. So, if we take duty cycle that means, tau by t is 50 percent that is half and also for convenience let us take a some value let us take a is the amplitude. So, 2 volt. So, what will be the value of C n magnitude C n magnitude will be a or a I need not take anything in terms of a let us write. So, A/2 sin pi by 2 then sin n pi by 2 by n pi by 2. So, you see that when n is an even function even number then this whole sink thing will go to 0. So, at n is equal to so, I can write that this function first let us see what is n is equal to 0. So, at n is equal to 0 this thing will be 1 always sink sin 0 by 0 you know that it is indeterminate. So, we will have to take the limit and then put the value and by that it gets 1. So, it is a by 2 when n is equal to 0 then for it will be 0 when n is equal to plus minus 2 plus minus 4 plus minus 6 etcetera and for odd values of n we have this sin n pi by 2 will be 1. So, we can write it will be a by n pi for n is equal to plus minus 1 plus minus 3 plus minus 5 and what will happen in this case to the phase spectrum. So, phase we can say it will be minus angle of minus n pi by 2 plus angle of phase of sin n pi by 2.



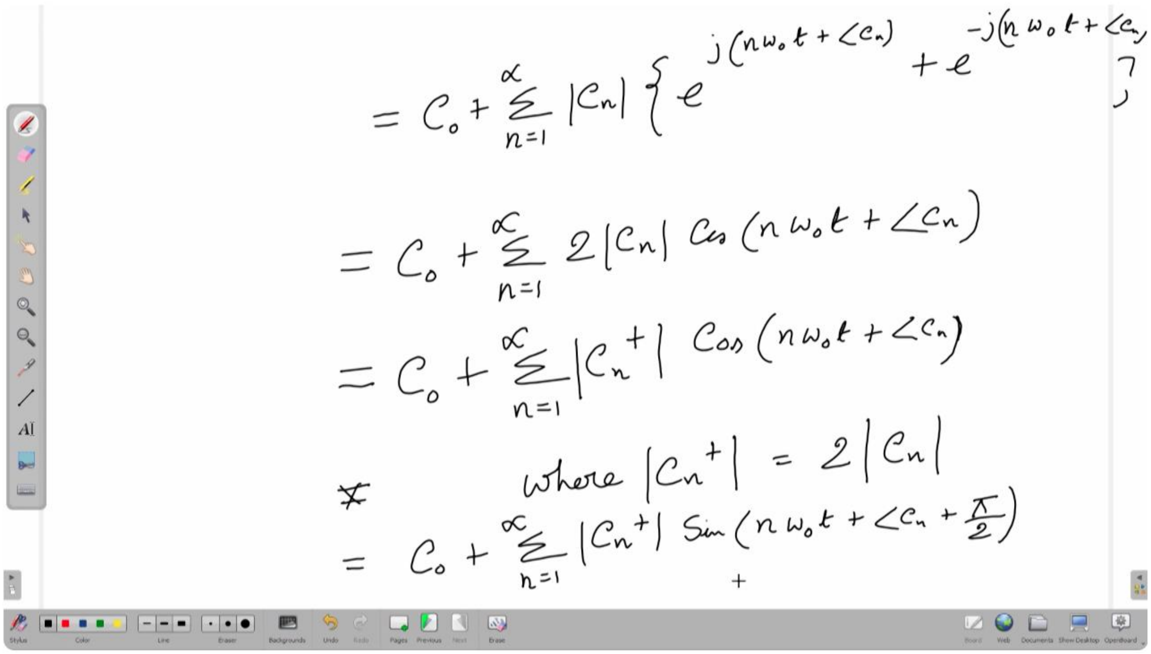

So, what is angle of sin n pi by 2 you see for n odd values the whole thing the spectral components does not exist only it exist for the odd values of n. So, for odd values of n this value is 1 that means, the phase is 0. So, the phase that time will be minus pi by 2 that means, is equal for n is equal to plus minus 1 plus minus 3 plus minus 5 etcetera. So, we can say that minus 90 degree n is equal to plus minus 1 plus minus 3 plus minus 5. So, now here one thing is there that you see that the you know that Fourier coefficient that means, the spectrum for is a double sided spectrum that means, you have positive frequencies you have negative frequencies that is why you are saying that n is equal to plus minus etcetera. Now you know that actually physically negative frequency does not have any meaning, but since spectrum is a mathematical operation. So, to have the mathematical really a physical reality intact this mathematical necessity was there that the frequency should be negative also. So, physically it does not have any meaning. So, EMC engineers they are basically engineers they are not so bothered about all those mathematical things. So, they thought that can we have a representation where the negative spectrum need not be shown.



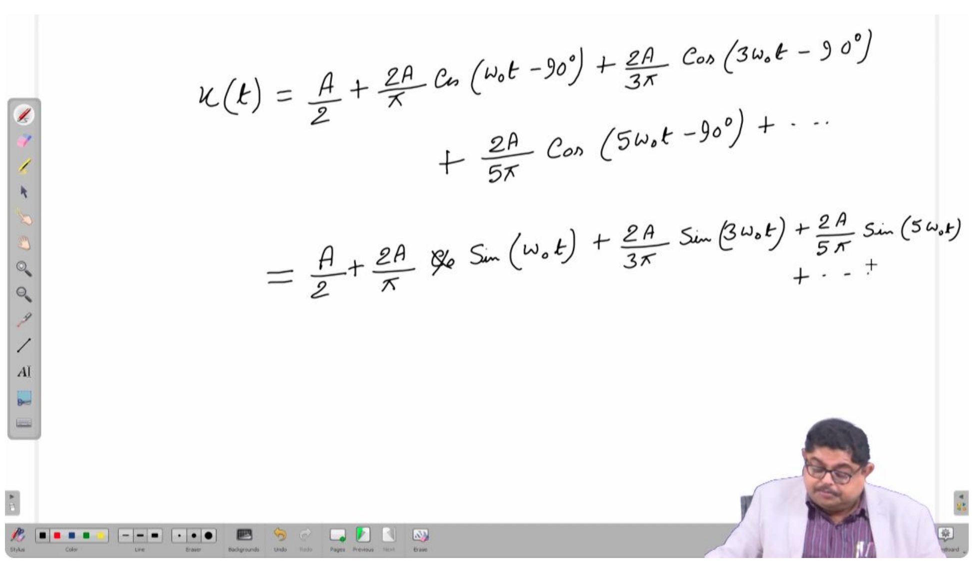

So, that is called single sided spectrum that we will discuss now. So, what they did? So, for single sided spectrum they saw that what are the spectral components or what is the value of C n in case of negative n. So, for negative n we know that what will be C of minus n C of minus n will be again by the 1 by T T 1 to T 1 plus T x T e to the power minus j minus n omega naught T d T. So, that if we do 1 by T many of you know this because these are taught in signals and system course, but just if anyone is not conversant I am saying this that x T e to the power j n omega naught T d T. So, that we know is we can say nothing, but it is C n complex conjugate. So, now if we attempt to write x T in terms of this negative spectral components then we know that x T can be written in terms of its spectral coefficient as C n e to the power j n omega naught T. Now I can break it into two parts one is minus infinity to 0 C n e to the power or let me break it like a something different one let me write it again x T is equal to minus infinity to infinity C n e to the power j n omega naught T is equal to obviously, C 0 it is immaterial whether it is positive or negative, but first let me write the positive part n is equal to 1 to infinity C n e to the power j n omega naught T plus I know it can be n is equal to minus 1 to minus infinity C n e to the power e to the power plus j n omega naught T. So, now I can write C 0 plus this one positive part as before C n e to the power j n omega naught T plus this one I can write as for this is negative n. So, I can write C n star e to the power minus j n omega naught T this is negative n. So, this now let us say that our C n is let us break it into magnitude and phase part C n is e to the power j C n. So, what will be C n star that will be C n e to the power minus j C n. So, putting it here I can write C 0 plus n is equal to 1 to infinity C n e to the power j n omega naught T plus C n plus n is equal to 1 to infinity C n e to the power j minus j n omega naught T plus C n.



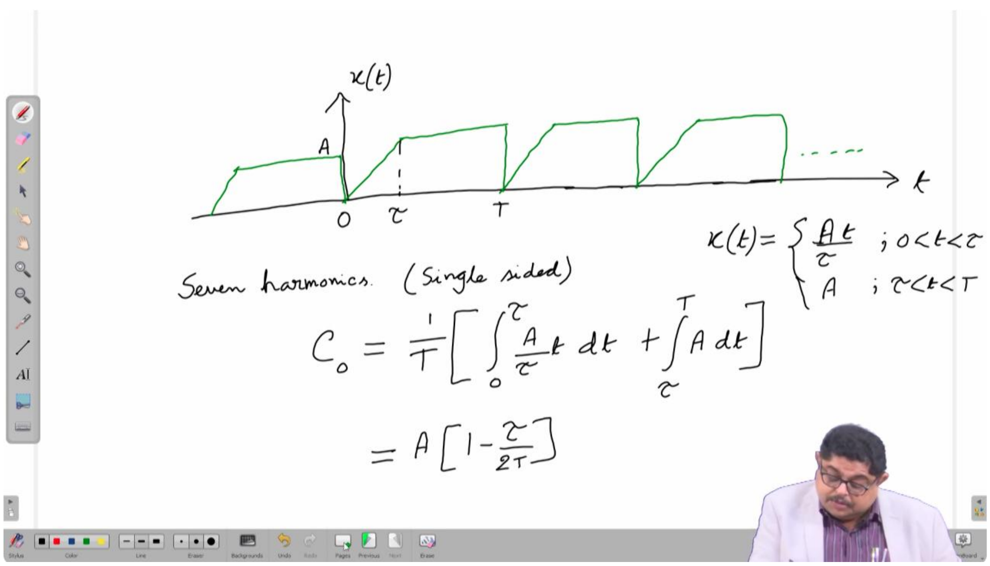

So, this I can now write as C 0 plus now everything is one sided n is equal to 1 to infinity C n e to the power j n omega naught T plus C n plus e to the power minus j n omega naught T plus C n. So, C 0 plus you know e to the power j alpha plus e to the power minus j alpha means 2 cos. So, I can write n is equal to 1 to infinity 2 C n cos n omega naught T plus C n ok. So, you see that instead of double sided spectrum now I can write it in terms of the spectrum is only from n is equal to 1 to infinity that means, only plus values will be there, but what I have to be careful that the spectral components have been multiplied by 2. So, I can now write that in terms of one sided spectrum C n plus. So, one sided spectrum spectral components I am saying C n plus. So, C n plus cos n omega naught T the phase part has not changed, but what I have lost is instead of e to the power j omega t a complex exponential I have got a cosine term. So, if I go to one sided spectrum I will have to have a cosine term not e to the power you know that e to the power j omega t that has a benefit actually that is an eigen value type of thing. So, if that is eigen value. So, if you have a signal expressed as e to the power j omega t or e to the power minus 2 j omega t etcetera then the output will be a scalar multiplication of that same form, but in this cos case or sin case that would come also I can write it. So, before that you let me know what is C n plus this is new term C n plus is equal to 2 C n also I can express this whole thing in terms of sin any cos can be expressed in terms of sin. So, then I will have sin n omega t plus C n sin pi plus theta is what sin pi by 2 plus theta is equal to this same thing. So, I can add plus pi by 2 if I want to write it in terms of sin.



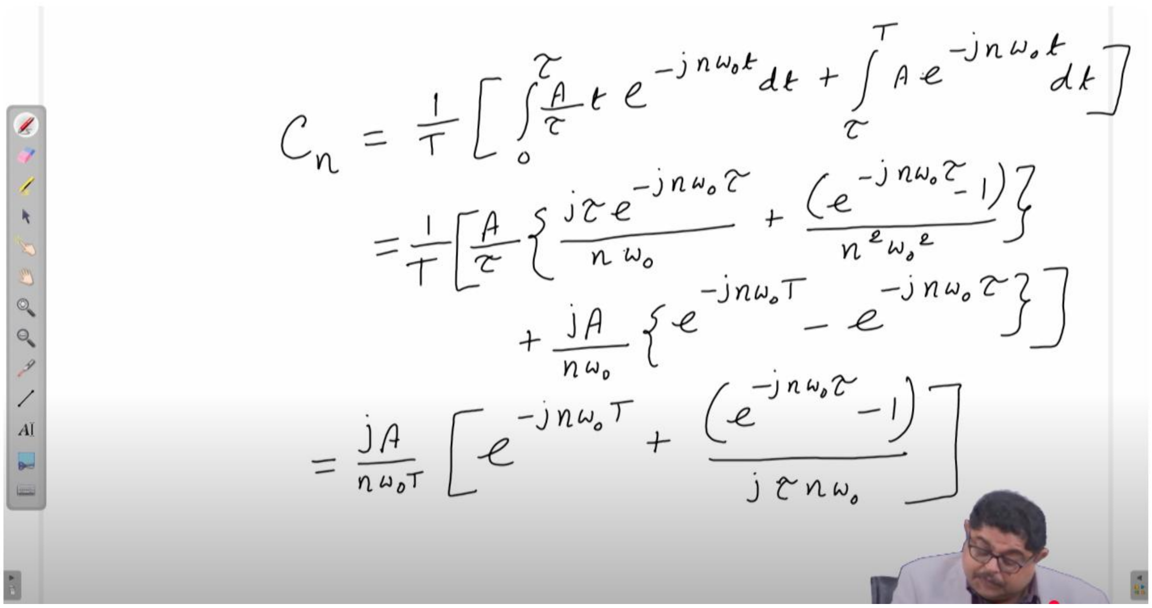

Now, EMC engineers generally prefer this double sided spectrum. So, we have seen the rectangular pulse strength spectrum if we write it in terms of the single sided spectrum I can write it that X t will be equal to a by 2 remember that the DC value does not change for one sided or double sided, but all other spectral components change. So, it will be 2 a by pi cos omega naught t minus 90 degree plus 2 a by 3 pi because we have values only for n is equal to 1 3 5 etcetera. So, it will be cos 3 omega naught t and for rectangular pulse strain our C n. So, phase is minus 90 degree plus 2 a by 5 pi cos 5 omega naught t minus 90 degree etcetera or if I want to write it in the same thing in terms of sin I will have to write it as 2 a by pi cos ah sorry sin. So, everything I will have to add 90 degree. So, it will be sin omega naught t plus 2 a by 3 pi sin 3 omega naught t plus 2 a by 5 pi sin 5 omega naught t etcetera ok. So, pretty simple probably you knew.



So, now let us go one step advance and let us see that what are the let us say harmonics first 7 harmonics of a waveform which looks like this. So basically you see in industry you cannot produce a sharp rectangular pulse ideal rectangular pulse. So, there will be a rise time also there will be a fall time I am not taking it now later we will take it. So, basically an industry ah pulse strain is always actually a trapezoidal sort of thing. So, instead of trapezoid we are first let us try this then we will go to trapezoid. So, what will be the Fourier coefficients for this? So, let us see because EMI will come that if I have a clock and if I have a sharp rise time there will be some high frequency components. So, let us estimate that. So, let us take what are the first 7 harmonics or spectral coefficients of these let us do it 7 harmonics per single sided. So, for doing that our algorithm will be first we will find the double sided spectrum we will multiply the magnitude part by 2. So, and the DC value will be kept unchanged. So, what will be our C 0? C 0 you know it is 1 by T you see actually this waveform is made of 2 parts one is there is a rising line then there is a flat portion. So, that flat portion is a and this one is from 0 to a it is going in a time tau. So, I will have to while integrating do that that means, X t I can say X t is having a tau by T sorry a by tau T for tau. So, for 0 less than t less than tau and is equal to a for tau less than t less than capital T. So, T is the time period. So, I will have to do this integration I will have to break it into 2 parts. So, a tau by T dt plus tau 2 T a dt. So, this if I do it is integration is integration it will give you 1 minus tau by 2 T .




and what will be C n the double sided C n it is 1 by n. So, you can do this the second integration is easier a is will just come out. So, you can make that the first integration is T integration of T e to the power minus j n omega naught T. So, you will have to use integration by parts. So, if you do that it is easy just to do integration by parts it will give you a by tau j tau e to the power minus j n omega naught tau by n omega naught plus…….So, this is for the first integration and the second integration is easier it will give you e to the power minus j n omega naught tcapital T  minus e to the power minus j n omega naught tau ok.  So, if you open up these one of the terms will get cancelled this tau tau will cancel.  So, this term and I think this term they will cancel. So, you will be left with j ah  this j will come. So, j a by n omega naught t e to the power minus j n omega naught t  plus e to the power minus j n omega naught tau minus 1 by j tau n omega naught .




So, in the second part you will have to you see that you can usually the practice is whenever you have this type of terms you take e to the power minus j n omega naught tau by T common. So, then you get e to the power minus j n omega naught tau by 2 minus e to the power  plus j n omega naught tau by 2. So, that will give you a sign type of term. So, if you do  that you will get that j a n omega naught t e to the power minus j n omega naught t  minus e to the power minus j n this common I am taking tau by 2. So, I will get a sinc term here sin n omega naught tau by 2 I can write it sink . Now here if we remember that e to the power minus j n omega naught t omega naught t is  2 pi. So, e to the power minus j n 2 pi. So, that will give us 1 and also omega naught  t is 2 pi. So, if we put that we get that c n is equal to j a by 2 pi n 1 minus e to  the power minus j n pi tau by T into that sinc n omega naught tau by 2 this is the c  n expression. So, we will proceed today the time is up the in the next class we will start  from here we will have to go to the single sided spectrum.  Thank you.

$$= \frac{jA}{n\omega_0 T}\left[e^{-jn\omega_0 T} - e^{-jn\omega_0\frac{\tau}{2}}\left\{\text{Sinc}\left(n\omega_0\frac{\tau}{2}\right)\right\}\right]$$

$$e^{-jn\omega_0 T} = 1$$

$$\omega_0 T = 2\pi$$

$$C_n = \frac{jA}{2\pi n}\left[1 - e^{-jn\pi\left(\frac{\tau}{T}\right)}\,\text{Sinc}\left(n\omega_0\frac{\tau}{2}\right)\right]$$