

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

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Lecture 40: Determination of Crosstalk in a Lossless Line Immersed in Homogeneous Medium

Welcome to the 40th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. We have seen how to determine the terminal currents in a three conductor line. Now, we will see the determination of crosstalk in a lossless line immersed in homogeneous medium. So, our assumption is first assumption is perfect conductor, lossless means that ideal conductor which is not possible, but we are assuming. Then medium also surrounding medium is lossless and homogeneous. So, for a lossless line we know that R will be 0, the G matrix will be also 0, the Z matrix will be simply $j\omega L$ and the Y matrix will be $j\omega C$. Since the line is immersed in a homogeneous medium, the per unit length inductance and capacitance matrix satisfy this equation L into C is equal to $\mu\epsilon$ I_2 a identity matrix. So, already this product is diagonal. So, this is given.

LECTURE 40: Determination of Crosstalk in a lossless line immersed in homogeneous medium

- Perfect conductor
- Medium is lossless and homogeneous

$$[R] = 0 = [G]$$
$$[\tilde{Z}] = j\omega [L]$$
$$[\tilde{Y}] = j\omega [C]$$
$$[L][C] = \mu\epsilon [I_2]$$

So, let us see what is \tilde{Y} it is simply minus omega square mu epsilon i 2, we know this is nothing, but beta square i 2 where beta is the phase constant of the line sorry of the medium beta is phase constant of the medium. So, you see the product \tilde{Y} into \tilde{Z} is already diagonal. We use previously \tilde{T} matrix for diagonalizing the this matrix. In lossy case it was not diagonal, lossy in homogeneous case, but now it is already diagonal. So, please remember how we define \tilde{T} matrix that $\tilde{T}^{-1} \tilde{Y} \tilde{Z} \tilde{T}$ is equal to gamma square which is a diagonal one, but already $\tilde{Y} \tilde{Z}$ is diagonal can I say that \tilde{T} matrix is a diagonal matrix. So, \tilde{T} is i 2 and what is this gamma square matrix? It is now $\tilde{J} \tilde{Y}$ is now $\tilde{J} \omega$ C, $\tilde{J} D$ is now $\tilde{J} \omega$ L. So, can I say that gamma is $\tilde{J} \omega$ C into L, but we know this product C into L is mu epsilon i 2. So, from electromagnetic theory we can say 1 by root over mu epsilon is v, v is the phase velocity in the medium $\tilde{J} \omega$ by v i 2. So, or we can also write $\tilde{J} \beta$ i 2 ok. So, we got that gamma the eigen function or the propagation constant that is also in this case diagonal.

$$\begin{aligned} [\tilde{Y}] [\tilde{Z}] &= -\omega^2 \mu \epsilon [I_2] \\ &= -\beta^2 [I_2] \end{aligned}$$

$\beta \rightarrow$ phase constant of the medium

$$[\tilde{T}]^{-1} [\tilde{Y}] [\tilde{Z}] [\tilde{T}] = [\tilde{\gamma}^2]$$

$$[\tilde{T}] = [I_2]$$

$$[\tilde{\gamma}^2] = j\omega [C] j\omega [L]$$

So, $[\tilde{\gamma}] = j\omega [C] [L]$

$$\begin{aligned} &= j\omega \mu \epsilon [I_2] \\ &= j\frac{\omega}{v} [I_2] \\ &= j\beta [I_2] \end{aligned}$$

Then let us see what is $Z C$? Let us recall what was the definition of $Z C$? $Z C$ was Z you will just see your previous notes that will show you $Z T$ gamma inverse T inverse. So, Z is $\tilde{J} \omega$ L matrix, T is a identity matrix, i t i 2 is 1 by i 2 gamma inverse. So, 1 by $\tilde{J} v$ by omega then i 2 because this and then i 2 inverse is also i 2. So, we can easily see that this is nothing, but v into L. So, $Z C$ is v into L. What is $Y C$? This is the characteristic impedance matrix. So, that is v into L. What is $Y C$? It is $Z C$ inverse. So, it is 1 by v L inverse. So, but we know that L into C where is that L into C is mu epsilon i 2. So, utilizing that I can put that 1 by v 1 by mu epsilon 1 by v is root over mu epsilon by mu epsilon into C that is 1 by root over mu epsilon that is again v into C.

So, Y_C is also Z_C is v into L, Y_C is v into C ok. So, this is as far as the various constants of the multi conductor line.

$$\begin{aligned}
 [\tilde{Z}_c] &= [\tilde{Z}][\tilde{T}][\tilde{Y}]^{-1}[\tilde{T}]^{-1} \\
 &= j\omega[L][I_1] \frac{1}{j} \frac{v}{\omega} [I_2][I_2] \\
 &= v[L] \\
 [\tilde{Y}_c] &= [\tilde{Z}_c]^{-1} = \frac{1}{v} [L]^{-1} \\
 &= \frac{1}{v} \frac{1}{\mu\epsilon} [c] \\
 &= \frac{\sqrt{\mu\epsilon}}{\mu\epsilon} [c] \\
 &= v[c]
 \end{aligned}$$

Now, let us see what happens to chain parameter matrix. So, what was our $\frac{1}{2} \ln \frac{Z_C + Z_0}{Z_C - Z_0}$ that was half Z_C to the power minus gamma L. This was the definition. So, Z_C we have just now seen it is v into L t is i 2, then e to the power minus gamma L e to the power gamma L t inverse will be also i 2 and Z_C inverse is 1 by v L inverse. So, you can see that these two. So, e to the power minus gamma L plus e to the power plus gamma L by 2 that is cos hyperbolic gamma L. So, can I write that cos hyperbolic gamma L and since it is a matrix I will have to write i 2 all others are getting cancelled. Now, for lossless line gamma we know alpha is 0. So, gamma is equal to j beta. So, this becomes cos beta L i 2 L. What about $\frac{1}{2} \ln \frac{Z_C + Z_0}{Z_C - Z_0}$ it was given as half Z_C to the power minus gamma L minus e to the power gamma L t inverse. So, this becomes half v into L t is i 2 this is as it is e to the power minus gamma L this is also i 2. So, if I want to make a sin hyperbolic term there is a minus. So, that minus I can take and also there is a requirement of that will come later. So, that minus it becomes v these two goes here v into L into sin hyperbolic gamma L already L is there. So, no need of i 2. Now, for lossless case again gamma is equal to j beta. So, we know that it will be j v L sin beta L or for our later use I will write it as minus j omega L sorry let me is equal to minus j omega L sin beta L by beta L into L because since beta is equal to omega by v. So, this is phi 1 2.

$$\begin{aligned}
\tilde{\Phi}_{11}(x) &= \frac{1}{2} [\tilde{Z}_c] [\tilde{\Gamma}] \{ [e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \} [\tilde{\Gamma}]^{-1} [\tilde{Z}_c]^{-1} \\
&= \frac{1}{2} v [L] [I_2] \{ [e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \} [I_2] \frac{1}{v} [L]^{-1} \\
&= \cosh(\tilde{\gamma}x) [I_2] \\
&\quad \tilde{\gamma} = j\beta \\
&= \cos(\beta L) [I_2] \\
\tilde{\Phi}_{12}(x) &= \frac{1}{2} [\tilde{Z}_c] [\tilde{\Gamma}] \{ [e^{-\tilde{\gamma}x}] - [e^{\tilde{\gamma}x}] \} [\tilde{\Gamma}]^{-1} \\
&= \frac{1}{2} v [L] [I_2] \{ [e^{-\tilde{\gamma}x}] - [e^{\tilde{\gamma}x}] \} [I_2] \\
&= -v [L] \sinh(\tilde{\gamma}x) \\
&= -j v [L] \sin(\beta x) = -j \omega x \frac{\sin(\beta x)}{(\beta x)} [L] \quad \text{since } \beta = \frac{\omega}{v} +
\end{aligned}$$

then we come to phi 2 1 L that is half t. So, e to the power minus gamma L minus e to the power gamma L t inverse. So, it is half i 2 e to the power minus gamma L. 1 by v L inverse. So, this is clearly a if I take a minus out this is a sin hyperbolic term. So, I can write minus sin hyperbolic gamma L by v L inverse. Now, gamma for lossless line will be j beta. So, I can write minus j sin beta L by v 1 by mu epsilon into c is equal to minus j v these are manipulation sin beta L c is equal to again in that form j omega L sin beta sorry sorry these are all capital c. This is phi 2 1.

$$\begin{aligned}
\tilde{\Phi}_{21}(x) &= \frac{1}{2} [\tilde{\Gamma}] \{ [e^{-\tilde{\gamma}x}] - [e^{\tilde{\gamma}x}] \} [\tilde{\Gamma}]^{-1} [\tilde{Z}_c]^{-1} \\
&= \frac{1}{2} [I_2] \{ [e^{-\tilde{\gamma}x}] - [e^{\tilde{\gamma}x}] \} [I_2] \frac{1}{v} \\
&= -\frac{\sinh(\tilde{\gamma}x)}{v} [L]^{-1} \\
&= -j \frac{\sin(\beta x)}{v} \frac{1}{\mu \epsilon} [c] \\
&= -j v \sin(\beta x) [c] \\
&= -j \omega x \frac{\sin(\beta x)}{(\beta x)} [c]
\end{aligned}$$

The last one is half the to the power minus gamma L plus e to the power gamma L t inverse i 2 e to the power minus gamma L is equal to cos hyperbolic for lossless line I can write cos beta L into m i 2. So, the chain parameters got determined. Now, our job is to putting this all these values into equation 10 which was our i 0. So, that will give us the solution for i 0. So, we get minus j omega L sin beta L by beta L L. So, so this now just we can make that some manipulation here.

$$\begin{aligned}\tilde{\Phi}_{22}(x) &= \frac{1}{2} [\tilde{T}] \{ [e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \} [\tilde{T}]^{-1} \\ &= \frac{1}{2} [I_2] 2 \cosh(\tilde{\gamma}x) [I_2] \\ &= \cos(\beta x) [I_2]\end{aligned}$$

Putting all these in $[\tilde{I}(0)]$ eqn. (10),

$$\begin{aligned}(-j\omega x \frac{\sin(\beta x)}{(\beta x)} [L] - \cos(\beta x) [\tilde{Z}_S] - \cos(\beta x) [\tilde{Z}_L] + [\tilde{Z}_L] (-j\omega x) \frac{\sin \beta L}{\beta L} [e] [\tilde{Z}_S]) \\ [\tilde{I}(0)] \\ = ([\tilde{Z}_L] \{ -j\omega x \} \frac{\sin \beta x}{\beta x} [e] - \cos \beta x) [\tilde{V}_S] + [\tilde{V}_L]\end{aligned}$$

So, already we have got the result, so I can write over cos beta L j dash. So, so so this is your equation number 12 ok. So, this will give us a solution for i naught. Now, putting this in equation 11 we can find an equation for i L, but we would not do that we will use another trick and then we will solve for the i naught and i L and from that our cross talk values V n e and V f e will determine.

$$\begin{aligned}\Rightarrow [\cos \beta L ([\tilde{Z}_S] + [\tilde{Z}_L]) + \frac{\sin \beta L}{\beta L} (j\omega x [L] + [\tilde{Z}_L] j\omega x [e] [\tilde{Z}_S])] [\tilde{I}(0)] \\ = (\cos \beta L [I_2] + \frac{\sin \beta L}{\beta L} [\tilde{Z}_L] j\omega x [e]) [\tilde{V}_S] - [\tilde{V}_L]\end{aligned}$$

..... (12) +

In equation 12 replace i_0 by $-i_L$ as you know that i_0 goes into the circuit and i_L goes out of the outside the circuit. So, that is why this minus sign comes Z_L by Z_S Z_S by Z_L V_S by V_L and finally, V_L by V_S . So, using reciprocity we will do this. So, for a reciprocal circuit we can do this. So, from equation 12 we will be able to generate equation 13 which will be giving us the solution for i_L ok. So, that we will see in the next class. Thank you.

In Eqn (12) →
replace

$[\tilde{I}(0)]$	by	$-\tilde{I}(x)$
$[\tilde{Z}_L]$	by	$[\tilde{Z}_S]$
$[\tilde{Z}_S]$	by	$[\tilde{Z}_L]$
$[\tilde{V}_S]$	by	$[\tilde{V}_L]$
$[\tilde{V}_L]$	by	$[\tilde{V}_S]^+$

and

