

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.  
 Professor name: Prof. Amitabha Bhattacharya  
 Department name: Electronics and Electrical Communication Engineering  
 Institute name: IIT Kharagpur  
 Week :08  
 Lecture 39: Derivation of Chain Parameter Matrix

Welcome to the 39th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. In the last class, we were deriving the elements of chain parameter matrix, we will continue that discussion in the present lecture. So, we have already discussed  $\phi_{11}$ . Now, let us find the  $\phi_{21}$ . From the definition, you can see what is  $\phi_{21}$ . It will be when  $I_0$  is equal to 0. So, the same condition that  $I_m$  plus that holds. So, already we derived in the previous one the value of  $V_0$ . So, I am not deriving it here. Now, I can derive the  $I_L$ , I think from where is  $I_L$ , this is equation number 7. So, from equation number 7, it will be  $T$  into  $I_m$  into  $e$  to the power minus  $\gamma L$  minus  $e$  to the power plus  $\gamma L$  whole thing multiplied by  $I_m$  plus. So,  $I_L$  is equal to  $T$ . So, in  $V_0$  expression, you can see there is a  $I_m$  plus. So, I can write now what is  $\phi_{21}$  that will be simply half  $T$ .

LECTURE 38: DERIVATION OF CHAIN PARAMETER MATRIX

$$\left[ \tilde{\Phi}_{21}(x) \right] = \frac{\left[ \tilde{I}(x) \right]}{\left[ \tilde{V}(0) \right]} \left[ \tilde{I}(0) = 0 \right]$$

$$\left[ \tilde{I}(0) = 0 \right] \Rightarrow \left[ \tilde{I}_m^+ \right] = \left[ \tilde{I}_m^- \right]$$

$$\left[ \tilde{I}(x) \right] = \left[ \tilde{T} \right] \left\{ \left[ e^{-\tilde{\gamma}x} \right] - \left[ e^{\tilde{\gamma}x} \right] \right\} \left[ \tilde{I}_m^+ \right]$$

$$\left[ \tilde{\Phi}_{21}(x) \right] = \frac{1}{2} \left[ \tilde{T} \right] \left\{ \left[ e^{-\tilde{\gamma}x} \right] - \left[ e^{\tilde{\gamma}x} \right] \right\} \left[ \tilde{T} \right]^{-1} \left[ \tilde{Z}_c \right]^{-1}$$

Now, next we determine. This time is equal to  $V_0$  is 0. So, if you look at  $V_0$  expression,  $V_0$ , if it is 0,  $I_m$  plus is equal to minus  $I_m$  minus. So, that is the only change. So,  $V_0$  is equal to 0 implies  $I_m$  plus is equal to minus  $I_m$  minus. So, we know again from the source side equation  $I_0$  will be  $T$  to the power minus  $I_m$  plus and again

from the load side equation I L will be equal to T into e to the power minus gamma L plus e to the power gamma L I m plus. So, this becomes phi 2 L.

$$[\tilde{\phi}_{22}(x)] = \frac{[\tilde{I}(x)]}{[\tilde{I}(0)]} \Big|_{[\tilde{v}(0)] = 0}$$

$$[\tilde{v}(0)] = 0 \Rightarrow [\tilde{I}_m^+] = -[\tilde{I}_m^-]$$

$$[\tilde{I}(0)] = [\tilde{T}] z [\tilde{I}_m^+]$$

$$[\tilde{I}(x)] = [\tilde{T}] \{ [e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \} [\tilde{I}_m^+]$$

$$\text{So, } [\tilde{\phi}_{22}(x)] = \frac{1}{2} [\tilde{T}] \{ [e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \} [\tilde{T}]^{-1}$$

Similarly, I will have to find phi 1 2 L. So, from the defining a matrix, I think equation 9 probably as per as I remember, it is also V 0 is equal to 0. So, I m plus is equal to minus I m minus. So, V L will be equal to Z C. C and I 0 is equal to 2 T I m plus. So, the last element phi 1 2 L is half Z C. D will be a T inverse. So, you have derived all the chain parameter elements. Now, the terminal conditions T 1 and T 2 can be substituted in 9 to give a form of solution for the terminal currents that may be used to evaluate crosstalk.

$$[\tilde{\phi}_{12}(x)] = \frac{[\tilde{v}(x)]}{[\tilde{I}(0)]} \Big|_{[\tilde{v}(0)] = 0}$$

$$[\tilde{I}_m^+] = -[\tilde{I}_m^-]$$

$$[\tilde{v}(x)] = [\tilde{Z}_c] [\tilde{T}] \{ [e^{-\tilde{\gamma}x}] - [e^{\tilde{\gamma}x}] \} [\tilde{I}_m^+]$$

$$[\tilde{I}(0)] = 2 [\tilde{T}] [\tilde{I}_m^+]$$

$$\text{So, } [\tilde{\phi}_{12}(x)] = \frac{1}{2} [\tilde{Z}_c] [\tilde{T}] \{ [e^{-\tilde{\gamma}x}] - [e^{\tilde{\gamma}x}] \} [\tilde{T}]^{-1}$$

So, let us see that, that means, what we are doing we are now knowing the chain parameter matrix, this T 1 and T 2. Now, we will put we know chain parameter matrix. So, we will now find the current I 0. Actually our now aim is to find I 0 from I 0 we will find I 0. This is the definition. Now, here we can put the values of V L and V 0. We own D star by I naught I 0. So, we will write I 0 because that is our goal. So, we will put the values of V L and V 0 from T 1 to T 2. So, in place of V L we will write V L plus Z L I L plus I L plus Z L plus I L. And Z is into I 0 plus. I have written, but here again I L has come. Now, my interest is not I L I will I will determine later. So, again from the equation 9 that is chain parameter matrix definition I will put the value of I L. So, I will write V L plus Z L for I L I am writing 5 2 1 2 1 L. The right hand side is everything in I naught. So, I am not changing anything in RHS. The equations are a bit tedious, but if you understand how we are proceeding it is not difficult to write. So, have we reached our goal? No, because still this V naught has come. So, this V naught value I will put from T 1 the terminal condition 1. So, from T 1 V L plus Z L. For V naught I am writing V S minus Z S. So, I naught plus nothing else I will do. So, I will write V S minus Z S. So, you see that we have arrived at an equation where there is apart from R S, there is a V S. Apart from I naught all are source term all are either V S or V L. Chain parameter matrix is the circuits thing. So, we know how to determine it. So, now, just a bit modification and rearrangement so that all the source term goes to the right and I naught comes to the left.

From Eq. (9)  $\rightarrow [\tilde{V}(x)] = [\tilde{\Phi}_{11}(x)] [\tilde{V}(0)] + [\tilde{\Phi}_{12}(x)] [\tilde{I}(0)]$   
 $\infty [\tilde{V}_L] + [\tilde{Z}_L] [\tilde{I}(x)] = [\tilde{\Phi}_{11}(x)] ([\tilde{V}_S] - [\tilde{Z}_S] [\tilde{I}(0)]) + [\tilde{\Phi}_{12}(x)] [\tilde{I}(0)]$   
 $\infty [\tilde{V}_L] + [\tilde{Z}_L] ([\tilde{\Phi}_{21}(x)] [\tilde{V}(0)] + [\tilde{\Phi}_{22}(x)] [\tilde{I}(0)])$   
 $= [\tilde{\Phi}_{11}(x)] ([\tilde{V}_S] - [\tilde{Z}_S] [\tilde{I}(0)]) + [\tilde{\Phi}_{12}(x)] [\tilde{I}(0)]$   
From (1)  $\rightarrow [\tilde{V}_L] + [\tilde{Z}_L] ([\tilde{\Phi}_{21}(x)] ([\tilde{V}_S] - [\tilde{Z}_S] [\tilde{I}(0)]) + [\tilde{\Phi}_{22}(x)] [\tilde{I}(0)])$   
 $= [\tilde{\Phi}_{11}(x)] ([\tilde{V}_S] - [\tilde{Z}_S] [\tilde{I}(0)]) + [\tilde{\Phi}_{12}(x)] [\tilde{I}(0)]$

So, rearranging we can write all. So, now, this is our desired I naught equation and I call it as a equation number 10. So, the terminal current at Z is equal to 0 that means, I 0

can be solved from this equation 10. Once  $I_0$  is known we can solve for  $I_L$  how?  $I_0$  is solution for  $I_L$  because we from the start said that we will find the terminal currents  $I_0$  and  $I_L$ . So,  $I_{naught}$  is determined. Now, equation 9 gives you  $I_L$  is equal to 2. Here  $I_0$  is here, but  $V_{naught}$  has come we know how to eliminate  $V_S$ . So, using T 1 terminal condition 1 we can eliminate it  $I_L$  is equal to. So, or we can write just this is rearrangement already we have got our things. Now, I should not close this bracket  $\phi_{21}$  into  $J$  dash then I should close the first bracket this is my equation number 11 it gives me  $I_L$ . So,  $I_L$  gets determined from this 11. So, already we have seen that from equation 10  $I_{naught}$  gets determined putting that value in equation 11 for  $I_{naught}$  I get  $I_L$ . Now, do it we will do it for a simple or some assumption we will make that a homogeneous lossless line immerse in homogeneous media and we will check whether we are getting the consistent result also that result will be useful because most of the time at microwave frequencies we will be dealing with lossless homogeneous medium that we will do in the next lecture. Thank you.

$$\begin{aligned} \text{or } & \left( [\tilde{\phi}_{12}(x)] - [\tilde{\phi}_{11}(x)] [\tilde{z}_s] - [\tilde{z}_L] [\tilde{\phi}_{22}(x)] + [\tilde{z}_L] [\tilde{\phi}_{21}(x)] [\tilde{z}_s] \right) [\tilde{I}(0)] \\ & = \left( [\tilde{z}_L] [\tilde{\phi}_{21}(x)] - [\tilde{\phi}_{11}(x)] \right) [\tilde{V}_s] + [\tilde{V}_L] \quad \dots\dots\dots (10) \\ \text{Solution for } & [\tilde{I}(x)] \\ \text{Eq. (9)} & \rightarrow [\tilde{I}(x)] = [\tilde{\phi}_{21}(x)] [\tilde{V}(0)] + [\tilde{\phi}_{22}(x)] [\tilde{I}(0)] \\ \text{Using (10)} & \rightarrow [\tilde{I}(x)] = [\tilde{\phi}_{21}(x)] \left( [\tilde{V}_s] - [\tilde{z}_s] [\tilde{I}(0)] \right) + [\tilde{\phi}_{22}(x)] [\tilde{I}(0)] \\ \text{or } & [\tilde{I}(x)] = [\tilde{\phi}_{21}(x)] [\tilde{V}_s] + \left( [\tilde{\phi}_{22}(x)] - [\tilde{\phi}_{21}(x)] [\tilde{z}_s] \right) [\tilde{I}(0)] \quad \dots\dots\dots (11) \end{aligned}$$