

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

Professor name: Prof. Amitabha Bhattacharya

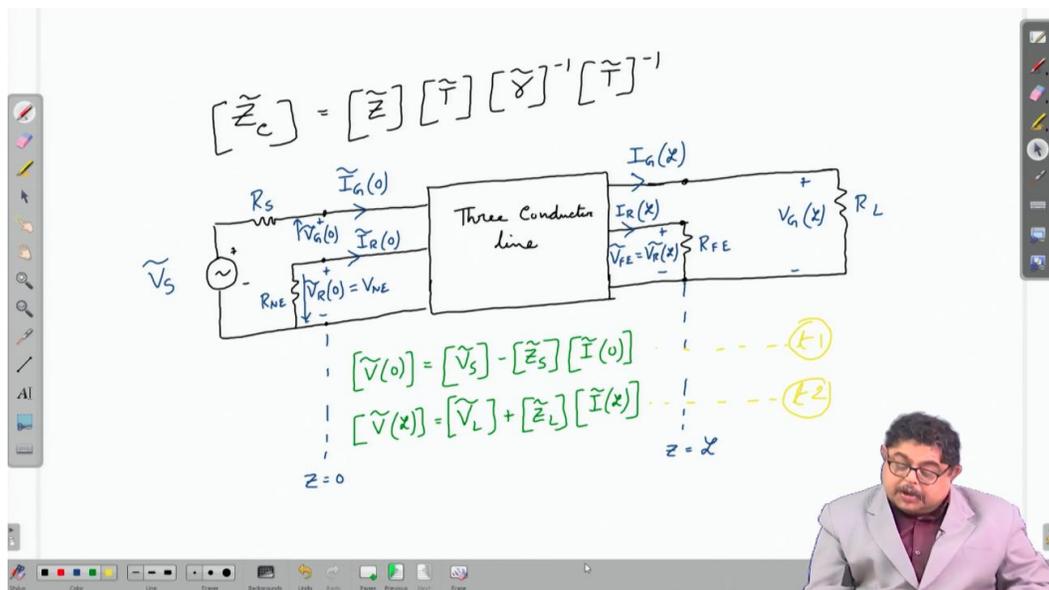
Department name: Electronics and Electrical Communication Engineering

Institute name: IIT Kharagpur

Week :08

Lecture 38: Determination of Terminal Currents of a three conductor system

Welcome to the 38th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. In the last class, we were discussing how to find the terminal currents of a three conductor line system. So, today we will do that. In the last class, I just introduced these that this is the structure and I have written two equations actually you can think them as boundary condition, but for lines we call them terminal conditions T 1, T 2 that and we also define there that you see that there were this \tilde{V} matrix, \tilde{V} s is the excitation we are only exciting the generator. So, that is why it is a \tilde{V} s is a vector 2 into 1 vector. So, \tilde{V} s for generator it is \tilde{V} s the voltage source and for receptor there is no excitation that is why it was 0. Similarly, in the right hand side also we have kept a provision for excitation, but so that that matrix is \tilde{V} I matrix you see here this \tilde{V} I matrix, but you have noticed that there is no excitation to the generator or receptor on the load side that is why you have put 0 0 to \tilde{V} I matrix and we have \tilde{Z} s etcetera. So, appropriately they were given.



us V_L is equal to characteristic impedance Z_c to the power minus γL . I_m plus and equation 3 it will give me I_L is equal to t_L . So, this is the equation number 5. So, this is equation number 5. So, this is equation number 6. I_m plus minus e to the power γL . So, these two equations I am calling equation number 7. So, again this one if I put into terminal condition at L that means this V_L is equal to this V_L has another expression. I have derived another expression. So, these two I will equate. So, this is equation number 7. So, now just manipulating these. So, we can write Z_c minus Z_L . So, we got another equation. This is our equation number 8. So, 6 and 8 you can see 6 that is also a similar type that was at the source end or Z is equal to 0 end and 8 is at Z is equal to L . So, they can be combined and in a compact form.

Load side, $Z = Z_L$

$$E_1(4) \rightarrow [V(x)] = [Z_c] [\tilde{T}] \left([e^{-\gamma x}] [\tilde{I}_m^+] + [e^{\gamma x}] [\tilde{I}_m^-] \right) \quad \dots (7)$$

$$E_1(3) \rightarrow [I(x)] = [\tilde{T}] \left([e^{-\gamma x}] [\tilde{I}_m^+] - [e^{\gamma x}] [\tilde{I}_m^-] \right)$$

$$[Z_c] [\tilde{T}] \left([e^{-\gamma x}] [\tilde{I}_m^+] + [e^{\gamma x}] [\tilde{I}_m^-] \right) = [Z_L] [\tilde{T}] \left([e^{-\gamma x}] [\tilde{I}_m^+] - [e^{\gamma x}] [\tilde{I}_m^-] \right)$$

$$\Rightarrow ([Z_c] - [Z_L]) [\tilde{T}] [e^{-\gamma x}] [\tilde{I}_m^+] + ([Z_c] + [Z_L]) [\tilde{T}] [e^{\gamma x}] [\tilde{I}_m^-] = [\tilde{V}_L] \quad \dots (8)$$

So, we can write. So, you can see that these actually comprises a set of four simultaneous equations. You see V is 2 into 1, V_L is 2 into 1. So, there are four equations I_m is I_m plus is 2 into 1, I_m minus is 2 into 1. So, that means, there are four set of simultaneous four equations in the four unknowns which is our I_m plus, I_m minus, I_m plus, I_m minus. Now, they can be solved to get voltage or current along any point on the line. However, we are not so interested to solve them generally, but we are interested to solve the terminal voltages $V_r 0$, $V_r L$ etcetera because $V_r 0$ is our $V_n e$, the near end voltage in the receptor. Similarly, $V_r L$ is our far end voltage on the receptor. So what we do, we just make a trick. Please look at equation 5 and 7. This is equation 7. You see it is $V_L I_L$, right hand side is in terms of I_m , I_m plus, I_m minus. Let us go to 5, where is 5 here, 5. Here also V and I are in terms of I_m plus and I_m minus.

$$\begin{bmatrix}
 ([\tilde{z}_c] + [\tilde{z}_s]) [\tilde{\tau}] \\
 ([\tilde{z}_c] - [\tilde{z}_l]) [\tilde{\tau}] [e^{-\tilde{\gamma}x}]
 \end{bmatrix}
 \begin{bmatrix}
 ([\tilde{z}_c] - [\tilde{z}_s]) [\tilde{\tau}] \\
 ([\tilde{z}_c] + [\tilde{z}_l]) [\tilde{\tau}] [e^{-\tilde{\gamma}x}]
 \end{bmatrix}
 = \begin{bmatrix}
 [\tilde{I}_m^+] \\
 [\tilde{I}_m^-]
 \end{bmatrix}$$

$$= \begin{bmatrix}
 [\tilde{V}_s] \\
 [\tilde{V}_l]
 \end{bmatrix}$$

So, that says us that we can write V of L, I of L, right hand side is V L I L as a matrix I L. That means, since equation 5 and 7, both are in terms of I m plus and I m minus, we know that there will be a relation between V L I L and V 0 I 0 and that we are representing by a matrix I L that is called chain parameter matrix. Before that, let me give a number to this one. This will be our equation 9. So how to get this chain parameter matrix? Obviously, from equation 5 and 7, if we eliminate I m plus and I m minus, we will get this type of equation. So, this one is called chain parameter matrix. So, what is psi L? It will be a 2 by 2 matrix. And the moment I write this, I can define what is psi L? Each element of this matrix, what is phi 11 L? From this easily I can find out that it is V L by V 0 given I 0 is equal to 0. What is the meaning of I 0 is equal to 0? Let us look at equation 5. So, I 0 is equal to 0 implies I m plus is equal to I m minus.

$$\begin{bmatrix}
 \tilde{V}(x) \\
 \tilde{I}(x)
 \end{bmatrix}
 = \begin{bmatrix}
 \tilde{\Phi}(x)
 \end{bmatrix}
 \begin{bmatrix}
 \tilde{V}(0) \\
 \tilde{I}(0)
 \end{bmatrix}
 \quad \dots \dots (9)$$

↑
Chain parameter matrix

$$\begin{bmatrix}
 \tilde{\Phi}_{11}(x) & \tilde{\Phi}_{12}(x) \\
 \tilde{\Phi}_{21}(x) & \tilde{\Phi}_{22}(x)
 \end{bmatrix}$$

$$\tilde{\Phi}_{11}(x) = \left. \frac{[\tilde{V}(x)]}{[\tilde{V}(0)]} \right|_{\tilde{I}(0)=0^+}$$

So, I can write that from equation 5, I_0 is equal to 0 implies I_m plus is equal to I_m minus. So, now I will have to evaluate with this knowledge, what is V_0 , what is V_L ? Then I will have to divide and I will get it. So, what is V_0 ? So, if I put V_0 , for V_0 you can what is the equation? V_0 is this. So, $Z_c t$ into $2 I_m$ plus I can write $Z_c t$ into $2 I_m$ plus and so I_m plus. $Z_c t$ into $2 I_m$ plus is equal to half t inverse Z_c inverse V_0 . You see the order while inverting the post multiplier here, it comes as pre multiplier that you should remember. So, now, this one you put into equation 7 that means for V_L . So, putting in equation 7, we get V_L is equal to equation 7 you can see $V_L Z_c t e$ to the power minus $\gamma L I_m$ plus e to the power $\gamma L I_m$ minus. So, that is what I will write $Z_c t$ then e to the power minus γL plus e to the power $\gamma L I_m$ plus. So, in place of I_m plus I will put half t inverse Z_c inverse V naught. So, now, I am in a position V_L by V_0 . So, I can write the first element of the chain parameter matrix e to the power minus γL plus e to the power $\gamma L t$ close it t inverse and Z_c inverse. So, this is the first element of the matrix. So, similarly, we will determine the other chain parameter matrix other elements of the chain parameter matrix and then we can relate the $V_f e$ and $V_n e$ with the source conditions source we know. So, from that we will be able to find out $V_f e$ and $V_n e$ that we will see in the next class. Thank you.

$$E_1(0) \rightarrow [\tilde{I}(0)] = 0 \Rightarrow [\tilde{I}_m^+] = [\tilde{I}_m^-]$$

$$[\tilde{V}(0)] = [\tilde{Z}_c] [\tilde{T}]^2 [\tilde{I}_m^+]$$

$$\therefore [\tilde{I}_m^+] = \frac{1}{2} [\tilde{T}]^{-1} [\tilde{Z}_c]^{-1} [\tilde{V}(0)]$$

Putting in Eq. 7,

$$[\tilde{V}(x)] = [\tilde{Z}_c] [\tilde{T}] \left([e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \right) \frac{1}{2} [\tilde{T}]^{-1} [\tilde{Z}_c]^{-1} [\tilde{V}(0)]$$

$$[\tilde{\Phi}_{11}(x)] = \frac{1}{2} [\tilde{Z}_c] [\tilde{T}] \left([e^{-\tilde{\gamma}x}] + [e^{\tilde{\gamma}x}] \right) [\tilde{T}]^{-1} [\tilde{Z}_c]^{-1}$$