Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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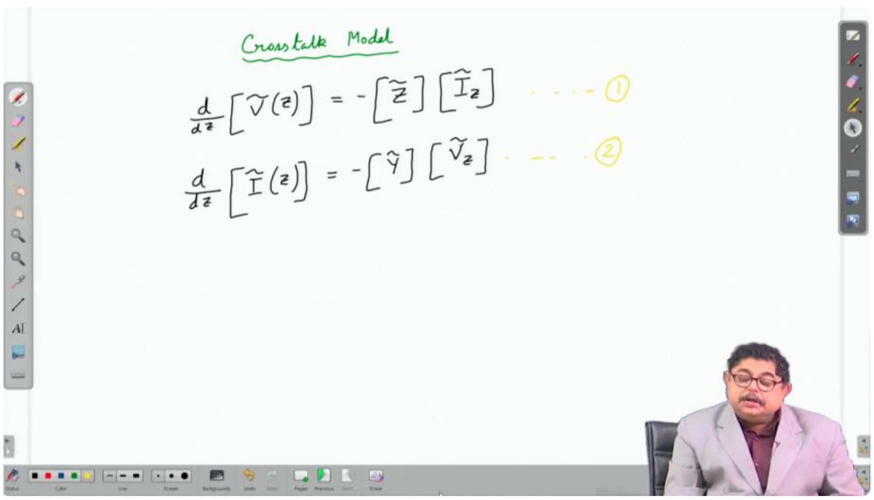
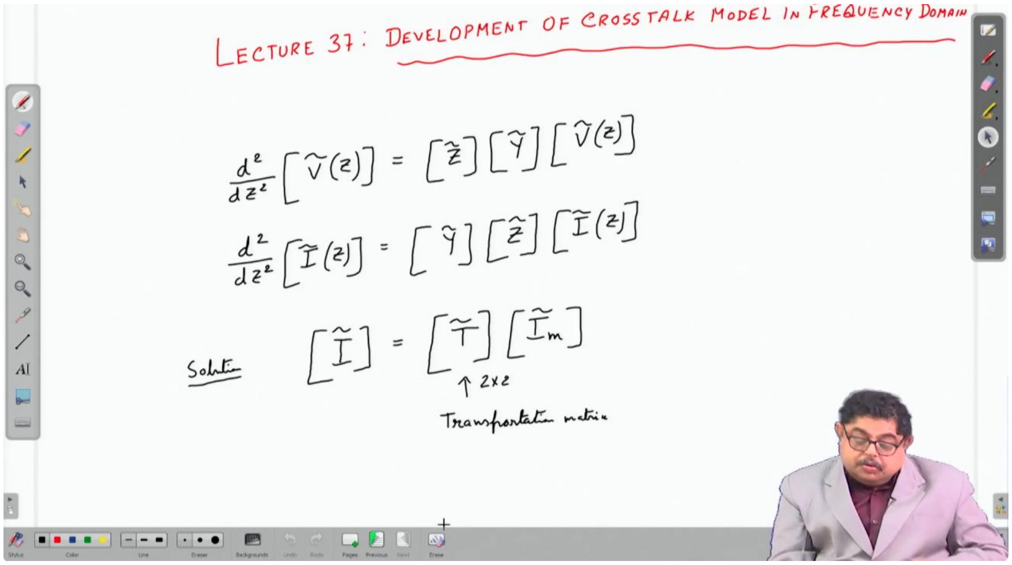
Week :08
Lecture 37: Development of crosstalk model infrequency domain

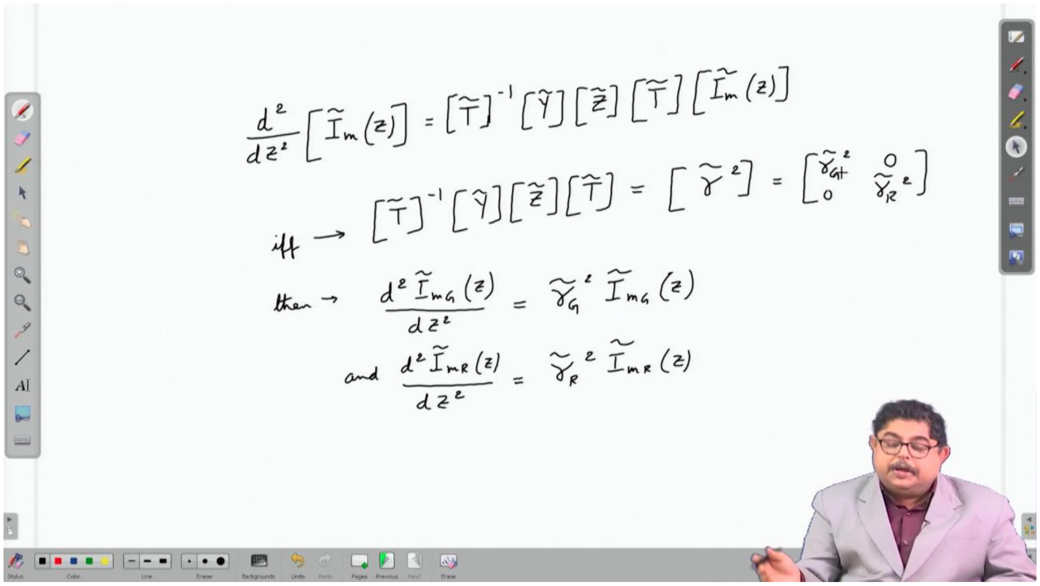
Welcome to 37th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. In the previous class, we have started developing cross talk model. So, in this class, we will go further with that model. We have written up to these.



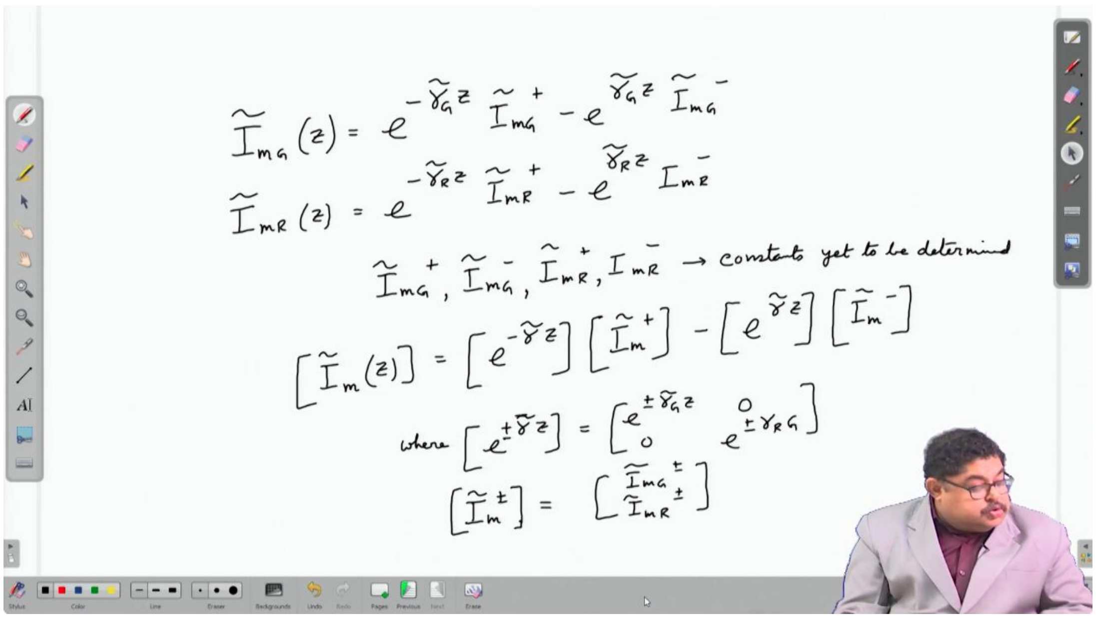
So, if we differentiate it with respect to z and substitute the other that you know in transmission line, you have done that type of things. So, we get d 2. See now the equations are uncoupled. You see in the first equation only V z is there, I z is not there. In the second equation I z is there, V z is not there. Also note the order of multiplication for z and y. In the first one, z is the pre multiplier. In the second one, z is the post multiplier. They do not commute. So, please do not say that z and z into y is equal to y into z because the matrices are not diagonal. In general, in lossy transmission line, they do not commute. We will see later that for lossless case, they will commute. Now, either of these equation can be solved. If we either can find I z along the whole line, current along the whole line or voltage along the whole line, we can determine from the other what is the relation by that previous 1 and 2. So, let us solve the second equation. The general solution we know can be obtained in terms of I is equal to some transportation matrix T. So, this IM are the modal currents. Basically for waves, there will be a forward going wave, there is a backward going current into F. So, that we are calling modal currents and T is a transportation matrix. I and IM both are vectors of dimension 2 into 1. T is transportation matrix, its order is 2 into 2. So, if we substitute this solution, this is the solution we are assuming.



We substitute this solution in the second uncoupled equation that means in this equation number 2. So, what we get? We get sorry in this second uncoupled equation, I have shown it wrongly. This one, this equation we are putting that solution. So, we are getting d 2 dz 2 IM z is equal to T inverse. The second equation y is the pre multiplier. So, if we can found find a transportation matrix T so that y into z is equal to T inverse into z gets diagonalized, then the modal equation also becomes uncoupled. That means, what I am meaning is if T inverse y becomes and this one is diagonal that means it will be gamma g square 0 0 gamma square. If this is a diagonal matrix, so if we can find a T matrix such that T inverse into y into z into T is diagonal, then this IM they gets uncoupled because in IM there are IM g and IM r. So, with this diagonalization IM g and IM r will be separated and we will be able to solve them. So, that time you see with this we can write this equation. So, if this then we get d 2 IM g z dz 2 is equal to gamma g square 0 0 gamma g square IM g z and d 2 IM r dz 2. So, this gamma g square it is now obvious that gamma g square and gamma r square are the eigenvalues of y into g matrix. So, gamma g square and gamma r square are eigenvalues, you know eigenvalues that you have done in matrix. So, sorry and they are referred to propagation constants of the multi conductor transmission line. So, now once this is done we can write the solutions we know now they will be separately we can treat them.

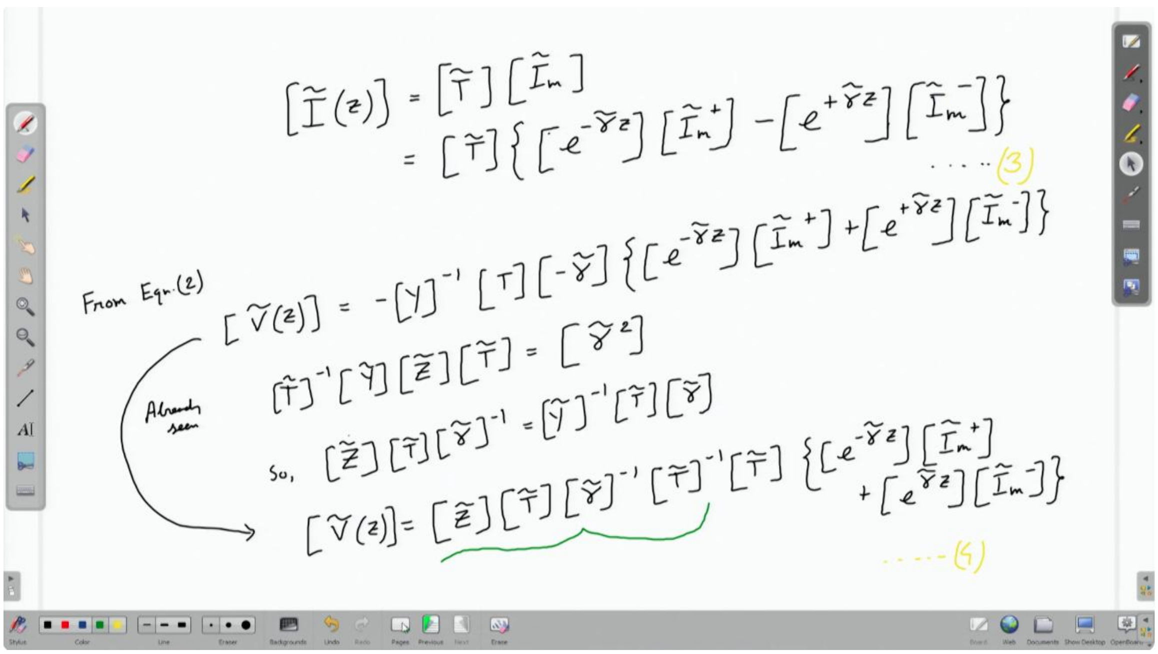


So, we can write as IM g z will be e to the power minus gamma g z IM g plus minus e to the power gamma g z IM g minus IM r z is equal to. So, this is the solution now because of the uncoupled nature due to that T matrix. So, now several things are to be noted why I am saying uncoupled you see in IM g is assumed solution there is no IM r term. In IM g solution there is the IM g plus there is an IM g minus. Similarly in IM r there is only IM r plus and IM r minus. Now, also note the minus sign we know that current waves the forward wave and backward wave there is a minus between them to keep the direction of propagation same. In voltage waves both of them adds in current waves they oppose. And what are these IM j IM g plus IM g minus IM r plus IM r plus IM g m r minus are constants yet to be determined. So, you see how I got this type of form you see this equation this is a well known equation that from simple harmonic motion we know it into in complex notation e to the power minus gamma z is a solution ok. The cos sign they change the form because if you twice differentiate cos there will be a minus sign come. Similarly if you twice differentiate sign there will be a minus sign come but for exponential terms e to the power minus gamma z or e to the power plus gamma z there is no minus sign comes that is why it is better and there is a relation by Euler's identity always it can be related to appropriate sinusoidal thing. So, that is why I have written this that it will be sum of this. Now, let us write everything in a compact matrix form. So, in compact matrix form I can now again go back I have once shown it but we will write it like this that IM z is equal to e to the power minus gamma z IM plus minus e to the power gamma z IM plus minus ok. And where e to the power minus gamma z is equal to or I can say plus minus in one way I can solve it e to the power e to the power plus minus gamma g z 0 0 e to the power plus minus gamma r z IM. These are all for later references because we will manipulate these matrices that time this compactness will help us IM g IM g plus minus this also then we write this IM r plus minus.

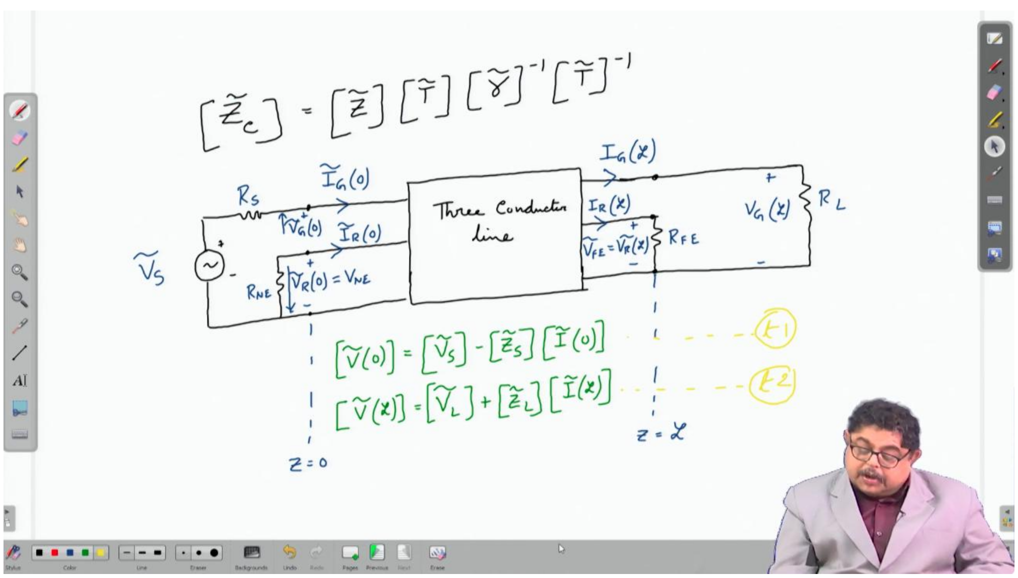


So, now actual currents we have seen the solution we have taken as this that I z is equal to t matrix into IM matrix. So, now we will put the assumed solution. So, e to the power minus gamma z, this is IM plus minus e to the power plus gamma z IM minus. So, let me call this equation number 3. Now, from equation 2 we have to write this equation 2. Can we say that what was equation 2 let us see, this was equation 2. So, what is V z? It is minus Y inverse d d z of I z. So, from equation 2 I can write V z is equal to minus Y inverse then d d z of I z. So, I z I know. So, it will be t minus gamma e to the power minus gamma z. So, this is equation number 4. Already we have seen this relation that t inverse Y j t is equal to gamma square. So, we have Y inverse t into gamma. You see this minus gamma, this minus sign and this minus sign will go.So, now, already we have seen that I am again writing already seen t inverse Y all phasors that is why tilde over each matrix is equal to gamma square. So, we can write that we can write z t. So, gamma inverse is equal to Y inverse t gamma. So, instead of this Y inverse t gamma this minus will go. I will write z t gamma inverse here.

So, this equation will come here that V z is equal to z t gamma inverse. So, this is the equation number 5. Now, here I will do a trick that is because I know so I am doing I can always do this then I will go on writing that second bracket thing e to the power minus gamma z I m plus plus e to the power gamma z I m minus. So, let me call this equation, equation number 4. So, equation number 3 was expression of I j I z equation 4 is expression of V z. Now, you see what was I z? I z is t into second bracket that bracket has not been closed there. Here also I have t into the second bracket that means, this term if it is I z. So, what is this term V z is equal to something into current voltage is equal to something into current. So, can I say that this one is something like impedance. So, this is actually the characteristic impedance of the line.



So, this says that I can define a characteristic impedance as z t gamma inverse. So, that means, we can write V j is equal to z c into I z. So, in a compact notation I know this, but I have not done one thing that we have not determined the 4 unknown constants I m g plus I m g minus I m r plus I m r minus that we have not done. So, that we will do now, how to do it? We will find the terminal conditions. So, it is a 3 conductor line. So, we will have 2 input ports, we have 2 output ports, one port is between generator and reference, another port is between receptor and reference. So, in input side 2 ports, output side 2 ports. So, we will find 4 terminal conditions from which 4 unknown constants we will define. So, for that So, this is the source side, the source side I have a voltage source and the source impedance z s is there, I will in actual writing I will take it as d s s, but while showing I am calling it r s, then the current that is entering through port of the generator is I g, it is at z is equal to 0. So, I am writing I g 0, R n e is the lumped circuitry that is in the near side of the source for receptor, I am calling that full circuitry as R n e, you can again call it Z n e etcetera, but for simplicity we are calling it R n e.So, voltage developed between the receptor port is V r 0 that is V n e. Similarly, the current that is coming to the coming out of the generator port, I am calling it I g l, the current that is carried there should be a port that is I r l, then this one I am calling R f e, the this is this voltage plus minus assumed V f e, this is basically V r l. And this voltage is V g l plus minus, this is R l again load, R f e is the lumped effect of all the circuitry associated, there is another voltage to be shown here at V g 0 plus and minus, so it is from here to here. And I can write some terminal conditions that V naught is equal to V s minus, you say I am writing Z s, but actually I have shown R s that I explained. So, so V 0 is the matrix, so V g 0 and V r 0 are two elements of this, this is a vector is equal to V s minus Z s into I naught and V l, again V l comprises of V g l and V r l is equal to V l is the port voltage plus Z l into I l. So, these equations are my terminal equation, I am calling this T 1, I am calling this T 2,



where what is V s, V s from the circuit we can see, V s is the source voltage for receptor, there is no source voltage, what is V l, you see for V l there are V l, there are no voltages, all the voltages are in this lump part. Then what is Z s, Z s is in our case R s 0 0 R n e near end and Z l will be R l 0 R f e. So, I have written terminal conditions and this is the terminal circuit model. So, from this we will have to find the four unknown constants, once we find that with this terminal conditions we will be able to evaluate V f e and V n e that is our data m from the start I have said that, that from all other knowledge I will have to finally find V f e V n e. Today time is up, in the next class we will start developing that. Thank you.