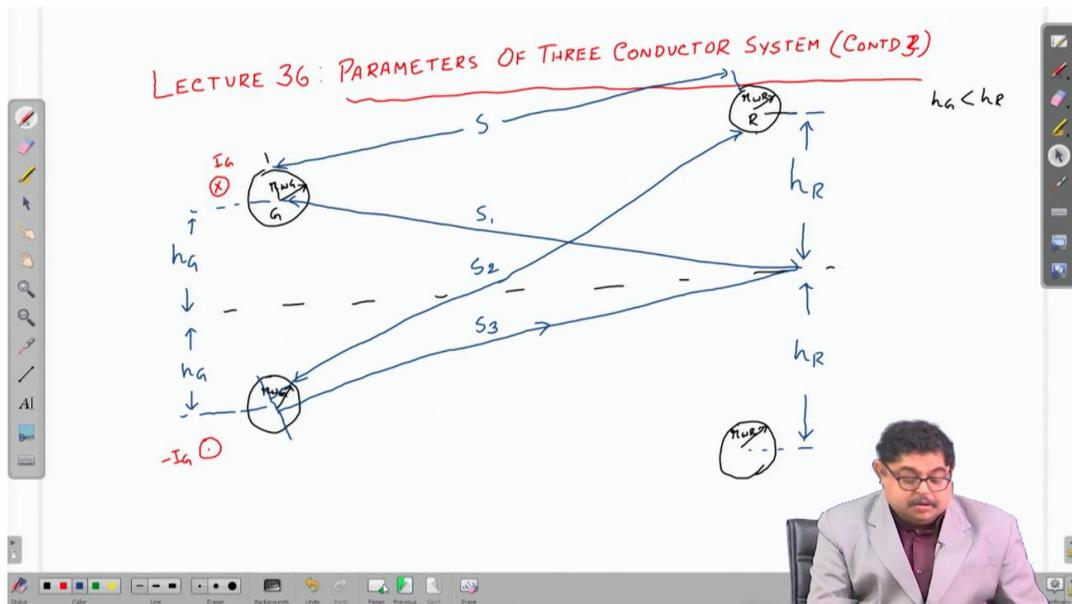


Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.  
 Professor name: Prof. Amitabha Bhattacharya  
 Department name: Electronics and Electrical Communication Engineering  
 Institute name: IIT Kharagpur  
 Week :08  
 Lecture 36: Parameters of Three Conductor System (Continued)

Welcome to the 36th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. In the previous lectures, we are seeing the per unit length parameter matrices of three conductor systems. We are continuing that discussion in this lecture. So, already I have given you this problem, introduced the problem in the last class. Now, the ground plane that will replace, so we will have the structure like this. Suppose this was the ground plane, now it is removed and we know from his image theory that there will be the images of both the generators and receptors. So, receptor was at a higher height,  $h_g$  is less than  $h_r$  that we have taken. So, let us give some current distribution. So, let us say that this is my  $I_g$ . So, now I have got through this image, the current is coming back returning, so it is minus  $I_g$ . Let me also give the radius as  $r_w g$ , this one  $r_w$  naught, this is an image, so this will be also  $r_w g$ , this is  $r_w r$ , this is  $r_w r$ . Now, I will have to give the distances as said that, this slanting distance is  $s$  between the two centers. This distance let me call  $s_1$ . This distance let me call  $s_2$  and this distance is  $s_3$ . This will be the circuit by applying image theory.



Now, let us determine  $L_g$ ,  $L_r$  and  $L_m$  one by one for finding  $L_g$ . Now, the receptor is not there,  $r_w g$ ,  $r_w g$ . So, this is  $h_g$ , this is also  $h_g$  when use theory,  $I_g$  is equal to  $L_g$ .  $I_g$  minus  $I_g$ . Now, what is the flux direction? Apply right hand rule. So, the current is going inside. So, I can say that flux will be here. Similarly, this one is coming out. So, the flux is additive, but remember that by image theory only in the upper portion of the ground plane, the flux is to be considered because in the bottom portion what is happening that if we try to find it will be wrong because in actually there is no lower portion, no flux can exist below the ground plane. So, only in the upper portion the flux to be considered. So, in this case already  $I_r$  is equal to 0 when finding  $L_g$ . So, what is  $L_g$ ? I am explicitly writing  $\mu_0$  by  $2\pi \ln$ , here you try to understand that is why let me write only magnetic flux on the upper portion of the ground plane. So, ground plane line to be considered. So, what is the maximum distance? Flux is coming only up to this, you see only up to this not coming behind. So,  $H_g$  by  $R_w g$  plus due to this  $\mu_0$  by  $2\pi \ln$  what is the maximum distance to which the flux is going that is  $2H_g$  and what is the minimum distance to which it is going?  $H_g$ . So,  $L_g$  is  $\mu_0$  by  $2\pi \ln 2H_g$  by  $R_w g$ . Similarly I am not considering separately  $L_r$ ,  $L_r$  will be similar. So,  $L_r$  will be  $\mu_0$  by  $2\pi \ln 2H_r$  by  $R_w r$ .

For finding  $L_g$

Only magnetic flux on the upper portion of the ground plane line to be considered

$I_R = 0$   
 $L_g = \frac{\Psi_g}{I_g} \Big|_{I_R=0}$   
 $= \frac{\mu_0}{2\pi} \ln \left( \frac{h_g}{r_w g} \right) + \frac{\mu_0}{2\pi} \ln \left( \frac{2h_g}{h_g} \right)$   
 $= \frac{\mu_0}{2\pi} \ln \left( \frac{2h_g}{r_w g} \right)$

Similarly,  $L_r = \frac{\mu_0}{2\pi} \ln \left( \frac{2h_r}{r_w r} \right)$

Now let us see  $l_m$ . There is an image that I need not consider. So, I have not written this that this is R w r. So, this distance is given as  $s$ . So, mutual means on this surface I will have to find. This is  $s_1$ . This is  $s_2$ . This is my assumed is  $s_3$ . Now I am exciting generator, but for mutual I am finding on the surface what is the flux passing. So, you see the flux directions again apply the right hand rule. So, the current is going inside. So, the flux is now the return one the flux is coming this. So, that also is like this right here. I can now write  $l_m$  is equal to  $\mu_0$  by  $2\pi l n$ . Now for the generator current going inside. So, what is the maximum it is going? It can go up to  $s_1$  because only the upper portion is to be considered. So,  $s_1$  and minimum it can go up to  $s$  plus  $\mu_0$  by  $2\pi$  due to this image  $l_n$ . How much it can go? It can go up to  $s_2$  maximum and minimum is  $s_3$ . So, it is  $\mu_0$  by  $2\pi l n s_1 s_2$  by  $s s_3$ . You see that  $s$  is only given in the problem, but  $s_1 s_2 s_3$  is my assumptions. So, from geometry I will have to find what is the relation of this term argument of  $l_n$ . So, let us see now it is obvious that  $s_2$  no this one that this is at the same height. So,  $s_1$  and  $s_3$  are same because this is the image the generator and its image. So, from there the same points distance.

For finding  $l_m$

$$l_m = \frac{\mu_0}{2\pi} \ln \left( \frac{s_1}{s} \right) + \frac{\mu_0}{2\pi} \ln \left( \frac{s_2}{s_3} \right)$$

$$= \frac{\mu_0}{2\pi} \ln \left( \frac{s_1 s_2}{s s_3} \right)$$

So, I can write that  $s_1$  is equal to  $s_3$ . So,  $\ln$  becomes  $\mu_0$  by  $2\pi \ln s_2$  by  $s$ . So,  $s_2$  needs to be determined. So, I need to find  $s_2$  from geometry. So, this is the generator, this is the receptor and this is you can see the image and this was my ground plane. So, this point is  $s$ , this point is  $s_2$ . Let me draw a perpendicular from here. So, I can say that this is  $h_g$  and this one is  $h_r$ . Also, let me draw another perpendicular here and call this length of this one as  $y$ . So, this one also will be  $y$ . From Pythagoras theorem first I can write about this one this is also a perpendicular. So I can write that  $s_2$  square this bigger one  $s_2$  square is equal to  $h_r$  plus  $h_g$  whole square plus  $y$  square and from the smaller one I can write  $s$  square is equal to  $h_r$  minus  $h_g$  whole square plus  $y$  square. So, eliminating  $y$  I can say that  $s_2$  square minus  $h_r$  plus  $h_g$  whole square is equal to  $s$  square minus  $h_r$  minus  $h_g$  whole square. So I can say that for  $s_2$  square is equal to  $s$  square plus  $h_r$  plus  $h_g$  whole square minus  $h_r$  minus  $h_g$  whole square is equal to  $s$  square plus  $4 h_r h_g$ . So putting this in  $\ln$ ,  $\ln$  is  $\mu_0$  by  $2\pi$  then this is square but I want  $s_2$  by  $s$ . So  $s_2$  by  $s$  a half will come  $\ln$   $1 + 4 h_r h_g$  by  $s^2$ . So this is now  $h_r h_g$  all are given  $s$  is also given. So  $\ln$  is obtained.

$$s_1 = s_3$$

$$l_m = \frac{\mu_0}{2\pi} \ln \left( \frac{s_2}{s} \right)$$

To find  $s_2$  from geometry

$$s_2^2 = (h_r + h_g)^2 + y^2$$

$$s^2 = (h_r - h_g)^2 + y^2$$

$$s_2^2 - (h_r + h_g)^2 = s^2 - (h_r - h_g)^2$$

$$\therefore s_2^2 = s^2 + \left\{ (h_r + h_g)^2 - (h_r - h_g)^2 \right\}$$

$$= s^2 + 4 h_r h_g$$

$$l_m = \frac{\mu_0}{2\pi} \frac{1}{2} \ln \left( 1 + 4 \frac{h_r h_g}{s^2} \right) +$$

So see a problem consider two solid wires at a height of 2 centimeter above a ground plane and separated among them by 2 centimeter. Then the per unit length parameters of the system. Two solid wires that means one wire here in this case they are at same height. So two wires this is their  $r_w$ , same  $r_w$  and  $r_w$  is given as 16 mils at a height of

2 centimeter. And separated among them by 2 centimeter. This is also 2 centimeter. So find the per unit length parameter. So what will be  $l_g$ ?  $l_g$  is  $\mu_0$  by  $2\pi$   $\ln$   $2h_g$  by  $r_w$ . So we can put the values 2 into 10 to the power minus 7 into  $\ln$   $2h_g$  is 2 centimeter everything is in centimeter  $r_w$  only we will have to bring to centimeter 16 mil multiplied by this comes to inch multiplied this by 2.54. And answer will be  $h_g$  per meter. So this will be 0.918 micro henry per meter. Similarly  $l_r$  also this case will be same. So  $l_r$  is 0.918 micro henry per meter  $l_m$  will be you have that formula. So if you put it 10 to the power minus 7 into  $\ln$   $1 + \frac{4}{s}$  into  $2$  into  $2$  by  $4$  you can see the formula  $4h_r$  by  $s$  square. So that 4 so that will give you 0.161 micro henry per meter.

#

Diagram showing two parallel wires separated by 2 cm, with a height of 2 cm from a dashed ground line.

$r_w = 16 \text{ mil}$

$$l_g = \frac{\mu_0}{2\pi} \ln \left( \frac{2h_g}{r_w} \right) = 2 \times 10^{-7} \times \ln \left( \frac{2 \times 2}{16 \times 0.001 \times 2.54} \right) \text{ H/m}$$

$$= 0.918 \mu\text{H/m}$$

$$l_r = 0.918 \mu\text{H/m}$$

$$l_m = 10^{-7} \times \ln \left( 1 + \frac{4 \times 2 \times 2}{s} \right) = 0.161 \mu\text{H/m}$$

$z_c$  of each isolated circuit is  $l_g$  into  $v$ . So  $l_g$  is 0.918 into 10 to the power minus 6 into  $v$  is 3 into 10 to the power 8. So that gives you 275.4 ohm. Now determine  $c_m$ ,  $c_m$  is  $1/m$  by  $v$  square  $l_g$   $l_r$  minus  $l_m$  square all these values you have. So if you put it will be 2.19 pico farad per meter. Now come to  $c_g$  plus  $c_m$  which requires the knowledge of  $l_g$  by this  $v$  square  $l_g$   $l_r$  minus  $l_m$  square. This denominator already we have used. So  $c_g$  plus  $c_m$  will be 12.487 pico farad per meter. So  $c_g$  becomes 10.297 pico farad per meter. So  $z_c$  of one circuit in presence of other is equal to root over  $l_g$  by  $c_g$ . So that if you do it will be 298.58 ohms. So you see that from 275 ohm it increased to almost 299 ohm. So that means the characteristic impedance is increasing in this case.

$Z_c$  of each isolated circuit =  $0.918 \times 10^{-6} \times 3 \times 10^8 = 275.4 \Omega$   
 $C_m = \frac{l_m}{v^2(l_1 l_2 - l_m^2)} = 2.19 \text{ pF/m}$   
 $C_G + C_m = \frac{l_G}{v^2(l_1 l_2 - l_m^2)} = 12.487 \text{ pF/m}$   
 $C_G = 10.297 \text{ pF/m}$   
 $Z_c$  of one ckt. in presence of other =  $\sqrt{\frac{l_G}{C_G}} = 298.58 \Omega$

So after seeing this we have seen the per unit length parameters again but this time for three conductor system. Now you are ready to go to the mathematical model for crosstalk. So first we will evaluate crosstalk in frequency domain then we will see in the time domain. Going to time domain is not much problem first we will see the frequency domain. So this is the actual crosstalk model. So now you are ready with all the necessities or prerequisites for going to modeling crosstalk. So we already have seen this equation  $\frac{dV}{dz}$  in frequency domain  $V(z)$  is equal to minus  $V(z)$ . So let me call this equation equation 1 this equation equation 2. So you know these are coupled first order differential equation because  $V(z)$  if I try to solve  $V(z)$  I am getting  $I(z)$  for  $I(z)$  solution again  $V(z)$  is knowledge of  $V(z)$  is required. So I need to uncouple them so that either  $V$  or either  $I$  is present in equation. So to do that we differentiate this each equation with respect to  $z$  and substitute the other. So we will continue this discussion in the next class is crosstalk model development. Thank you.

Crosstalk Model

$$\frac{d}{dz} [\tilde{V}(z)] = -[\tilde{Z}] [\tilde{I}(z)] \quad \dots \text{---} \textcircled{1}$$

$$\frac{d}{dz} [\tilde{I}(z)] = -[\tilde{Y}] [\tilde{V}(z)] \quad \dots \text{---} \textcircled{2}$$