

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

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Lecture 35: Parameters of Three Conductor Systems (Continued)

Welcome to 35th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. We have seen in the previous class the determination of inductance matrix for a three conductor transmission line. Now, today we could have ended it there, but just to show that capacitance matrix also can be determined by recalling the second basic sub problem that we have developed for two conductor lines. I will show you that. So, the starting point there will be Q is equal to C into V that means, Q_G, Q_R is equal to C_G plus C_M minus C_M minus C_M C_R plus C_M . Now, this form as you can see is different from L matrix because here direct evaluation of C_G, C_R is not possible always with them a C_G is always with plus C_M, C_R also is with C_M . So, we do a thing we invert the equation. So, write it as V is equal to P into Q that means, where what is P matrix that is we inverted C inverse into Q . So, now we can write V_G, V_R is equal to $P_G, P_M, P_M, P_R, Q_G, Q_R, Q_R, Q_R, Q_G, Q_R$. So, we will first determine P matrix from P matrix we will go to C matrix. So, you can easily write that what is P_G ? P_G is V_G by Q_G when Q_R is 0.

LECTURE 35: PARAMETERS OF THREE CONDUCTOR SYSTEMS (CONTD.)

$$[q] = [c][v]$$
$$\begin{bmatrix} q_G \\ q_R \end{bmatrix} = \begin{bmatrix} C_G + C_M & -C_M \\ -C_M & C_R + C_M \end{bmatrix} \begin{bmatrix} V_G \\ V_R \end{bmatrix}$$
$$[v] = [p][q] = [c]^{-1}[q]$$
$$\begin{bmatrix} V_G \\ V_R \end{bmatrix} = \begin{bmatrix} p_G & p_m \\ p_m & p_R \end{bmatrix} \begin{bmatrix} q_G \\ q_R \end{bmatrix}$$
$$p_G = \frac{V_G}{q_G} \Big|_{q_R = 0}$$

Let me take a new page P M is equal to V R by Q G when Q R is equal to 0. What is P M is V G by Q R when Q G is equal to 0 and V R is equal to V R by Q R when Q G is equal to 0.

The whiteboard contains the following equations:

$$P_m = \frac{V_R}{Q_G} \Big|_{Q_R=0}$$

$$P_m = \frac{V_G}{Q_R} \Big|_{Q_G=0}$$

$$P_R = \frac{V_R}{Q_R} \Big|_{Q_G=0}$$

In the bottom right corner, there is a video feed of a man with glasses and a mustache, wearing a light-colored jacket over a dark shirt.

So, let us find one by one this P parameters. So, finding P G. So, in this case the receptor is not there, R W G, R W naught this is my D G. So, this is my D G and I have a V G plus minus assumed then Q G let us put plus. So, I have a Q G here similarly I have a minus Q G here. So, I am writing it minus. So, what is P G? From our basic sub problem we can write $\frac{1}{2\pi\epsilon_0} \ln \frac{D_G}{R_W G}$ for the first one $\frac{D_G}{R_W G}$ plus $\frac{1}{2\pi\epsilon_0} \ln \frac{D_G}{R_W 0}$ is equal to $\frac{1}{2\pi\epsilon_0} \ln \frac{D_G^2}{R_W G R_W 0}$. So, the receptor P R that will also be similar. So, I can directly write $\ln \frac{D_R^2}{R_W 0}$ only finding P R .

Finding p_a

Finding p_a

$$p_a = \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_g}{r_{wg}}\right) + \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_g}{r_{w0}}\right)$$

$$= \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_g^2}{r_{wg} r_{w0}}\right)$$

Finding p_R

$$p_R = \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_R^2}{r_{wR} r_{w0}}\right)$$

finding P m is only critical finding P m here also the charge you can put either on the generator circuit or on the receptor circuit we are as before doing it for generator, but receptor is present here. So, this is my R W G, this is my R W 0, this is my R W R receptor ok. So, this is my D G, this is my R W R receptor this is my D R and this is my D G R. So, I will write Q G here a charge Q naught or same charge Q G it is a negative charge here and I will have to find what is V R. So, easily you can find that P m is equal to 1 by 2 pi epsilon naught ln again we are assuming that D G is greater than D G R. So, D G you see we are finding it here. So, from here the charge the maximum is R here and the minimum is here. So, D G by D G R plus 1 by 2 pi epsilon naught ln again this charge. So, the maximum is here D R and minimum is R W naught. So, that gives you 1 by 2 pi epsilon naught ln D G D R by D G R R W naught. Now, you can once you know P matrix you can easily find you know that P is equal to C inverse. So, you can determine C ok.

Finding μ_m

$d_G > d_{GR}$

The diagram shows three wires with charges $q_G \oplus$, q_R , and $q_G \ominus$. Distances d_G , d_{GR} , and d_R are marked. A magnetic field B_m and voltage V_R are shown.

$$\mu_m = \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_G}{d_{GR}}\right) + \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_R}{r_w}\right)$$

$$= \frac{1}{2\pi\epsilon_0} \ln\left(\frac{d_G d_R}{d_{GR} r_w}\right)$$

$[P] = [C]^{-1}$

Now, let us find a problem for which this assumption is not true. So, take a problem consider a 3 wire cable and this is the generator, this is the reference, this is the receptor. So, you see that the receptor is not in between generator and reference. So, consider a 3 wire ribbon cable composed of 3 wires whose radius. So, all the radius are same R W this is also R W this is also R W given. So, this is a problem R W is 7.5 mils and adjacent separations adjacent separations are all 50 mils. That means, this is the separation is given as 50 mils finding l_g . So, you see that we will use the easier one that is inductance method. So, when I am finding l_g the this is R W this is also R W . So, separation is D let us take the current is. So, this is our I_g this is also our I_g or you can call it minus I_g and on a hypothetical line I am trying to find that if the what is the flux direction whether they add or not. So, in this case putting applying the right hand rule the flux is in this direction similarly here the flux is in this direction. So, I can write l_g is equal to μ_0 by 2π l n . So, what is the maximum distance D minimum distance is R W plus μ_0 by 2π l n D by R W is equal to μ_0 by π l n D by R W . Also we know that l r will be similar. So, l r will be also μ_0 by π l n D by R W .

 $n_w = 7.5 \text{ mls}$
 adjacent
 separate $\rightarrow 50 \text{ mls}$

Finding L_G

$$L_G = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{r_w}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{d}{r_w}\right)$$

$$= \frac{\mu_0}{\pi} \ln\left(\frac{d}{r_w}\right)$$

Finding L_R

$$L_R = \frac{\mu_0}{\pi} \ln\left(\frac{d}{r_w}\right)$$

Let us see 1 m. So, this is g this is 0 reference this is receptor. Now let us excite the generator. So, I have a current I_g and that will return. So, minus I_g there is no current no excitation on it, but this part will determine what is the flux thing. So, you see that if I apply for this current plus I_g it is going here. So, the this will be due to plus I_g whereas, if I take this current that return current minus I_g apply the right hand rule the flux will be in this direction, but who is nearby nearby one is the reference conductor. So, net flux will be this this I call net flux that means, this is a case where the two fluxes are opposing each other. So, I can write 1 m is equal to μ_0 . So, since this is a μ_0 by 2π due to the plus I_g current \ln maximum distance is $2D$ and minimum distance is D and another one will be μ_0 by 2π \ln D by RW . Now, since net is in this side that I know so, I put like this so that means, the final answer is μ_0 by 2π \ln D by $2RW$.

Now, the values are given D and RW . So, if I put those values I_g will be equal to I_g is μ_0 by π \ln D by RW will be 50 by 7.5 you can put that μ_0 value you can put 4π into 10 to the power minus 7 . So, 4 into 10 to the power minus 7 4 into 10 to the power minus 7 \ln D by RW that is 50 by 7.5 50 by 7.5 . If you do this you will get 0.759 microhenry per meter. L_R L_R is also same expression. So, L_R also is equal to 0.759 microhenry per meter and L_M is equal to μ_0 by 2π that means, 2 into 10 to the power minus 7 \ln 50 by 15 . If you do it it will be 0.24 microhenry per meter. So, we have got the per unit length parameters.

Finding L_m

$$L_m = -\frac{\mu_0}{2\pi} \ln\left(\frac{2d}{d}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{d}{\pi\omega}\right)$$

$$= \frac{\mu_0}{2\pi} \ln\left(\frac{d}{2\pi\omega}\right)$$

$$L_G = 4 \times 10^{-7} \ln\left(\frac{50}{7.5}\right) = 0.759 \mu\text{H/m}$$

$$L_R = 0.759 \mu\text{H/m}$$

$$L_m = 2 \times 10^{-7} \ln\left(\frac{50}{15}\right) = 0.24 \mu\text{H/m}$$

Now, let us see an interesting thing. Suppose we know that what is characteristic impedance of an circuit, two wire circuit or in this case we can say isolated circuit. So, we know that characteristic impedance of an isolated circuit or isolated two wire circuit. Where line that is we know it is L_g into V , V is the phase velocity L_g is the per unit length inductance. So, Z_c in this case will be how much L_g is 0.759 into 10 to the power minus 6 and if I assume free space then V will be 3 into 10 to the power 10 this much ohm. So, this will be 227.7 ohm ok. Also you see it could have been found from the receptor and reference pair they also will give you the same thing. Now, what is per unit length capacitance? So, we know per unit length for capacitance matrix we need to first determine C_m always do that first find C_m . So, C_m is related to per unit length inductances and that we have seen that C_m is L_m by V square L_g L_r minus L_m square this we have seen in the previous class. So, if you put the value L_m value you know 0.24 into 10 to the power minus 6 by V square means 3 into 10 to the power 8 square into 0 both this L_g and L_r are same 0.759 into 10 to the power minus 6 whole square minus 0.24 into 10 to the power minus 6 whole square that will be your C_m . And I have done the calculation you can do it in your calculator. So, it is 5.14 picofarad per meter. Now, we know now you can find what is C_g plus C_m C_g plus C_m is L_g by V square the denominator is same L_g L_r minus L_m square is equal to. So, just by looking at L_g value because this denominator you have evaluated keep it once. So, 16.265 picofarad per meter. So, from this C_m you know so, C_g comes out C_g is 11.125 picofarad per meter also in this case C_r plus C_m also give you the same value. So, C_r will be 11.125 picofarad per meter. Now, this is for a 3 receptor line. Now, you see what

will be the characteristic impedance of it any 2 wires line system in presence of third that means, when the receptor is present what is the characteristic impedance.

Characteristic impedance of an isolated two wire line = $l_g \sqrt{C_g}$

$$Z_c = 0.759 \times 10^{-6} \times 3 \times 10^{10} \Omega = 227.7 \Omega$$

$$C_m = \frac{l_m}{v^2(l_a l_R - l_m^2)} = \frac{0.24 \times 10^{-6}}{(3 \times 10^8)^2 \{ (0.759 \times 10^{-6})^2 - (0.24 \times 10^{-6})^2 \}}$$

$$= 5.19 \text{ pF/m}$$

$$C_a + C_m = \frac{l_g}{v^2(l_a l_R - l_m^2)} = 16.265 \text{ pF/m}$$

$$C_a = 11.125 \text{ pF/m}$$

$$C_R = 11.125 \text{ pF/m}^+$$

So, characters or let me write Z_c . So, Z_c of a 2 wire line in presence of a receptor that I can see is equal to root over L_g by C_g and you calculate now you know both. So, this is 11.159 into 10 to the power minus 6 by 11.125 into 10 to the power minus 12. If you do this you will get 261.2 ohm. So, isolated circuit has 228 ohm and this is 260 ohm. Why? The presence of receptor has changed the characteristic impedance. Actually characteristic impedance embodies everything not only the material things also the geometry. So, now, the geometry has changed you have another conductor here by that is why the current distribution or etcetera have changed. So, that is reflected in the characteristic impedance value. So, this you should remember that characteristic impedance that is why is fundamental that it tells you everything about the circuit.

Z_c of a two wire line in presence of a receptor

$$= \sqrt{\frac{l_g}{C_a}} = \sqrt{\frac{0.759 \times 10^{-6}}{11.125 \times 10^{-12}}} = 261.2 \Omega$$

Now, there is another interesting problem I will start this problem in the next class, but just some time is there. So, I want to introduce the problem that suppose I have a 2 conductor line not a 3 conductor line. So, this is also another problem that. So, let us see. But I have a ground plane nearby infinite or semi infinite ground plane ok. So, let me call this is the ground plane and this one is at a height of h_g from the ground plane, this one is at a height of h_r from the ground plane the 2 heights are not same and arbitrarily we have taken that the receptor is at a higher height. So, this is receptor this is the generator and you see the current is not returning any other there should be the separation between the 2 lines is S . So, find the per unit length parameters of this system 2 conductor system above an infinite ground plane. So, you see that since a ground plane is there you cannot call it a pure 2 conductor line. So, you will have to do something we will have to take the help of some theory to replace this system because you actually what is happening the current oh the current let me see that I have only one current let us see that the generator is given the current and the current is just going that means what is happening actually it is radiating the current if it goes basically it returns by displacement current. So, current is going you see there is no return path. So, basically it is a radiation problem, but we are not dealing with radiation here we will try to make it through some conductor problem that transmission problem that it is going actually the due to that current there is a current distribution created here. Obviously, a charge distribution is created due to that a current distribution is created on the ground plane by that it is returning by the displacement current between these it is returning. So, this is the problem in the next class we will start from here. Thank you.

