Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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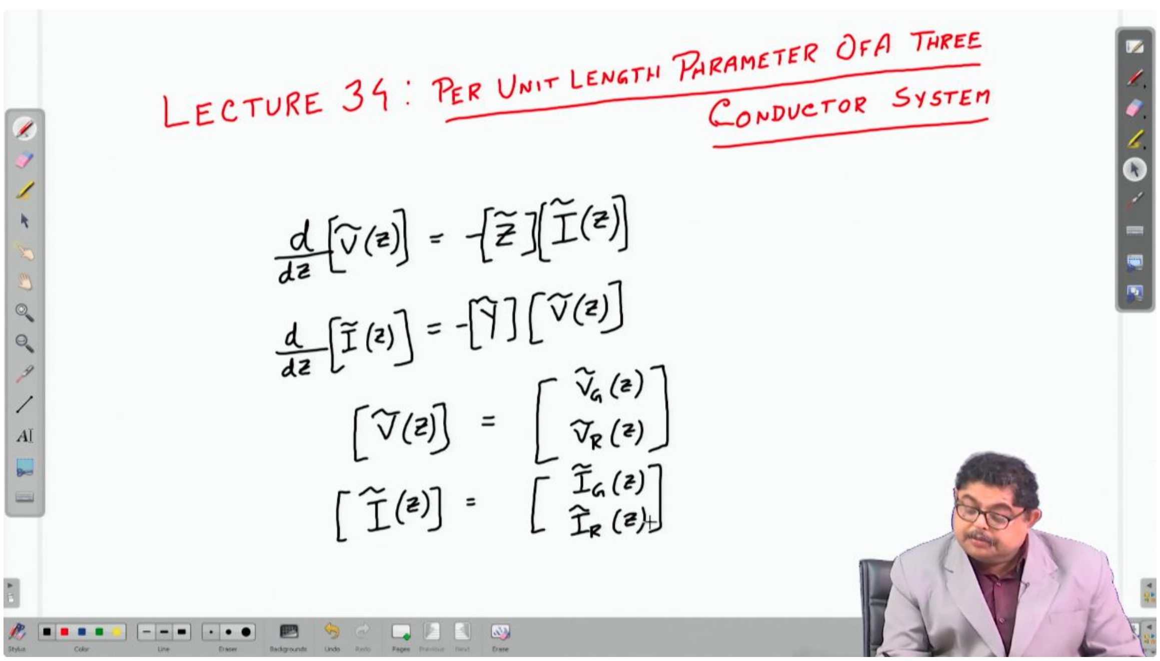
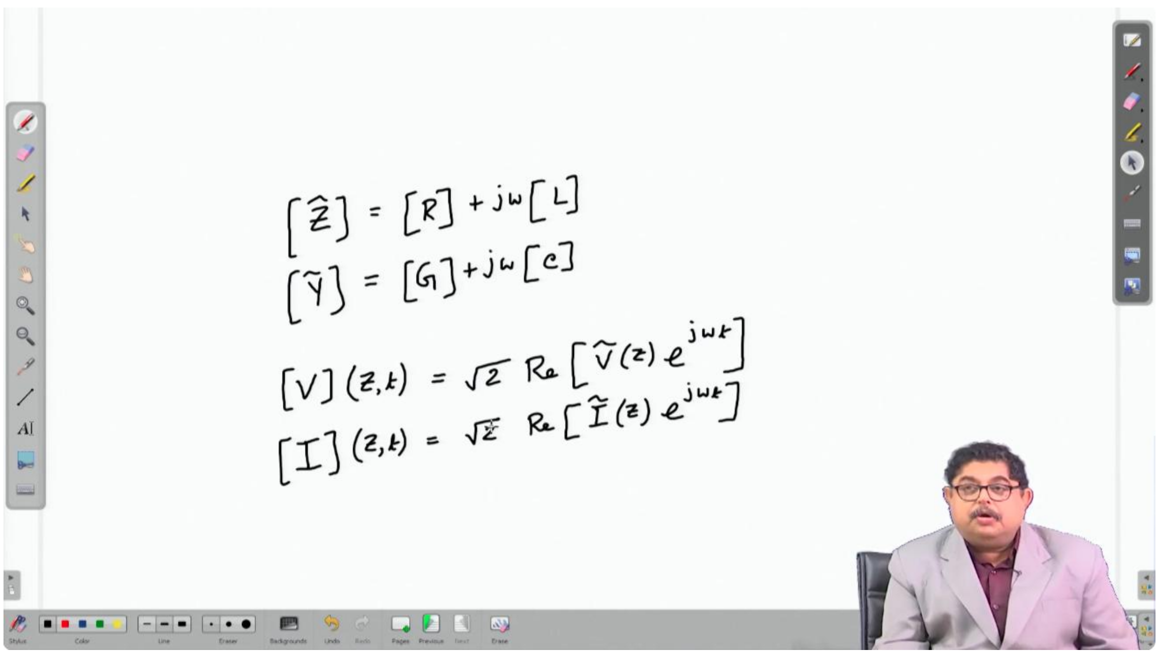
Week :07
Lecture 34: Per Unit Length Parameter of a Three Conductor System

Welcome to 34th lecture of the NPTEL course on EMIMC and Signal Integrity Principles, Techniques and Applications. Now, in the last class we have derived the multi conductor transmission line model MTL equations. So, today we will derive the per unit length parameter of a three conductor line system. So, in the while deriving multi conductor line we have seen the RLGC matrices and based on that we have written that in sinusoidal steady state and just recollecting that we have d dz the space variation of voltage along the line can be written as Z matrix into I Z matrix and similarly d dz this also will be matrix. All phasor, phasor means you have sinusoidal steady state. So, this V Z matrix it is actually vector consisting of the generator part and the receptor part.

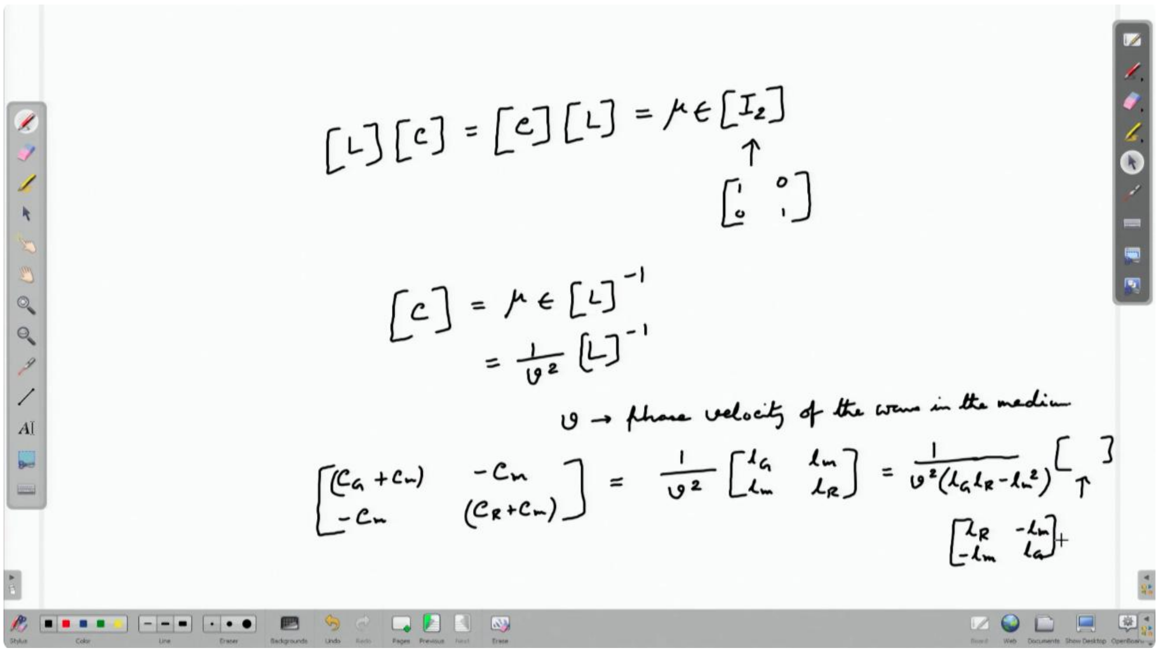


Similarly, the current vector that is also and also we can write the impedance matrix consisting of R matrix plus L matrix. Similarly, the impedance matrix a admittance matrix that is conductance matrix plus capacitance matrix. So, if we can solve these equations and we know these matrices we can write the time domain voltage and currents from this phasor currents by using the relations V no phasor that is why I am not giving any tilde. Now, here some books write this equal to phasor real part of V Z e to the power j omega t that is the definition of phasor, but I generally follow Harrington's book whose definition of phasor is root 2 real part of V Z e to the power j omega t.

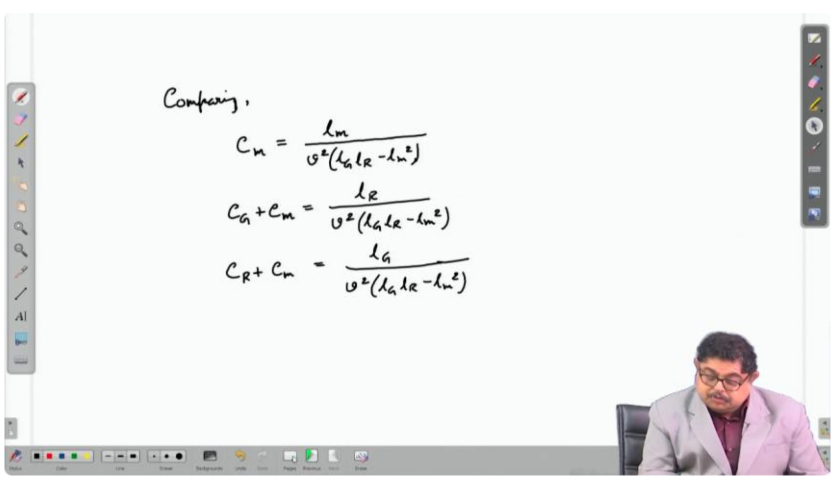
Similarly, I Z t is root 2 real I Z e to the power j omega t. Actually this root 2 if we place then in the power equation it is simply product of voltage and current no half terms come, but it is up to you how you define if this root 2 is not there then in the power you will have to take half into V into t. So, this is the notation we will use unless and until stated otherwise all AC voltages are in RMS values.



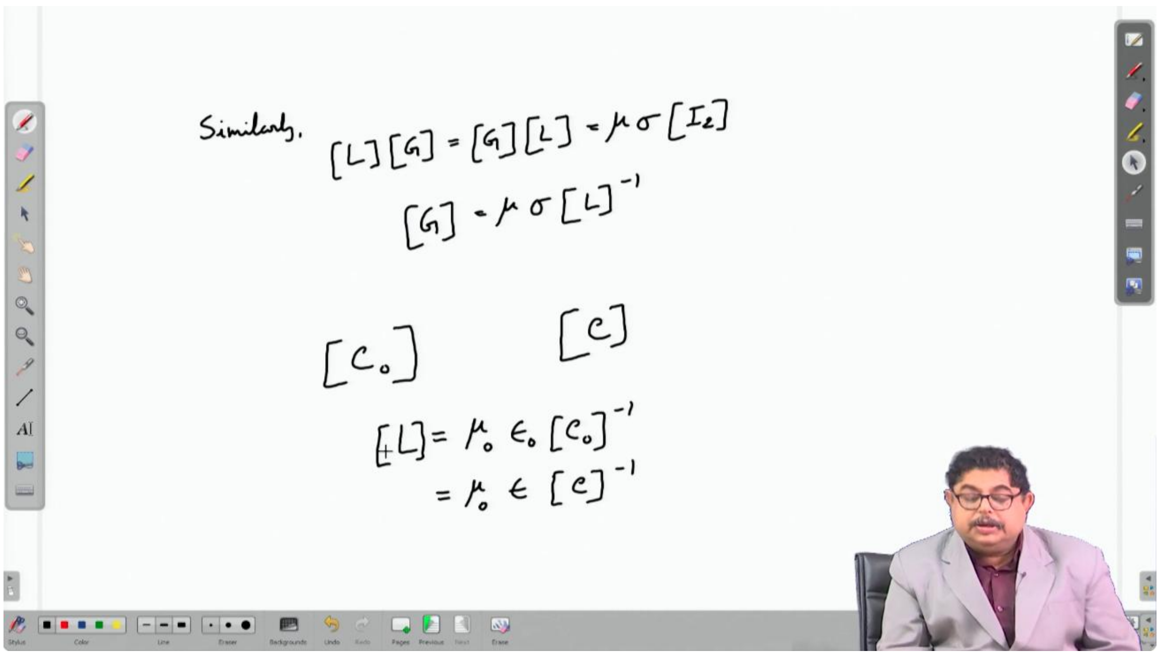
Now, if the surrounding medium is homogeneous we can use this that L matrix into C matrix is equal to C matrix into L matrix and that is equal to actually this product is already a diagonal matrix. So, mu epsilon into I 2 a 2 by 2 identity matrix where I am just saying this I 2 is nothing, but 1 1 0 0. So, this is our I 2. So, with the help of this we can determine if L is known we can found C by knowing the medium parameters similarly if C is known we can find. So, in some cases determining L is easier in some cases determining C is easier. So, according to the problem you can choose which one to use which one to determine first then from the other. So, we know that if L is known then C will be equal to mu epsilon L inverse and mu epsilon is 1 by v square L inverse where v is phase velocity phase velocity of the wave in the medium in case of our transmission line it is the voltage and current wave they move with this phase velocity. So, for a 3 conductor line we know that we have already seen C is C g plus C m is equal to 1 by v square L g L m L m L r that is 1 by v square into L g L r into 1 by v square minus L m square into we have no space. So, I am writing actually there will be a matrix. So, this matrix I am again writing here that this is inverse. So, transpose of that L r minus L m minus L m L g ok.



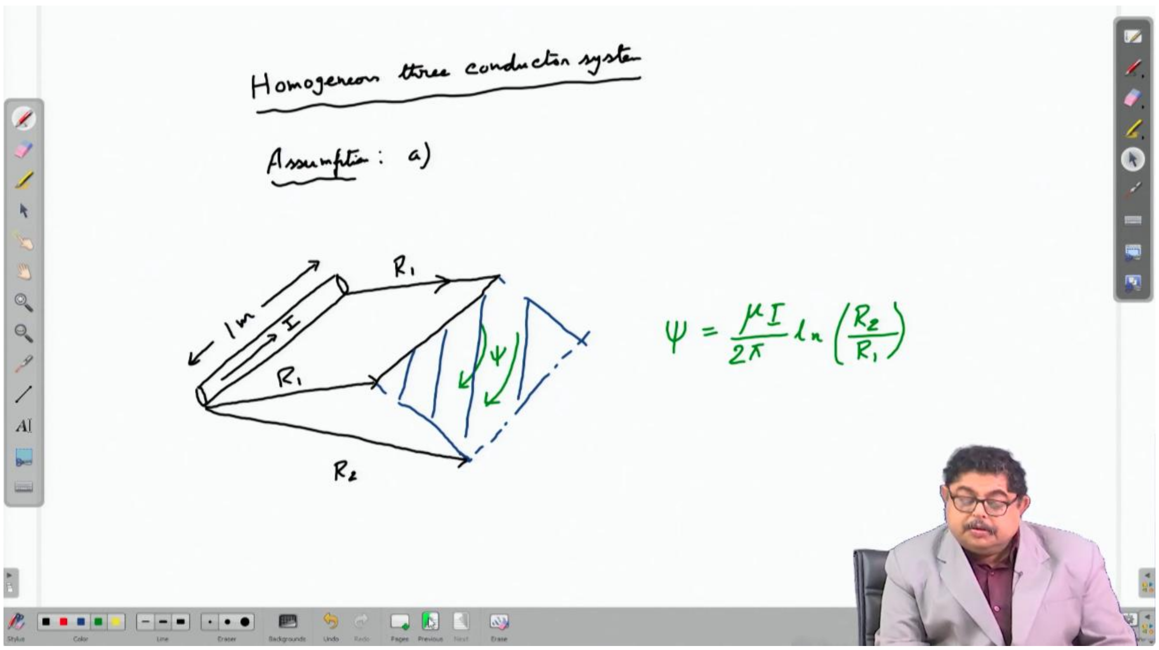
So, now, you can compare left hand side with right hand side that gives you that. So, I can write comparing you get C m is equal to L m by v square L g L r minus L m square C g plus C m is equal to L r by v square into L g L r minus L m square and C r plus C m is equal to L g by v square into L g L r minus L m ok. So, once you know L m you can find C m.



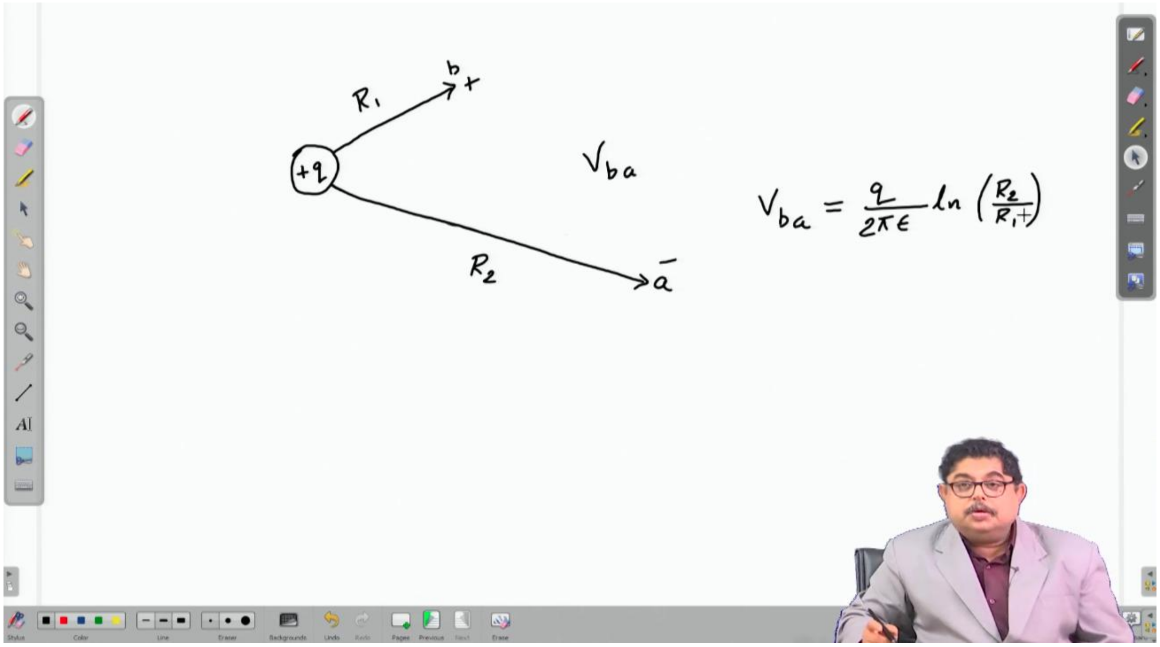
similarly L into G is equal to G into L is equal to mu conductivity into I 2. So, g is equal to mu L inverse. Now, for inhomogeneous medium L matrix is not affected because the inhomogeneous medium also generally we assume non ferromagnetic medium. So, the inhomogeneity is in the dielectric thing not in the permeability. So, L remains unchanged. So, if we designate. So, the trick is if we designate the per unit length capacitance matrix with dielectric removed then that means, replaced with free space let us call that per unit length matrix as C naught and with the dielectric inhomogeneity let us call that is C. So, in both the cases the you see L is will satisfy this L is equal to mu naught epsilon naught C naught inverse and also mu naught epsilon C inverse. So, this one you can easily find out. So, for inhomogeneous medium we need to determine the per unit length capacitance matrix with and without dielectric. So, twice you need to evaluate C L you can evaluate also this is L matrix ok.



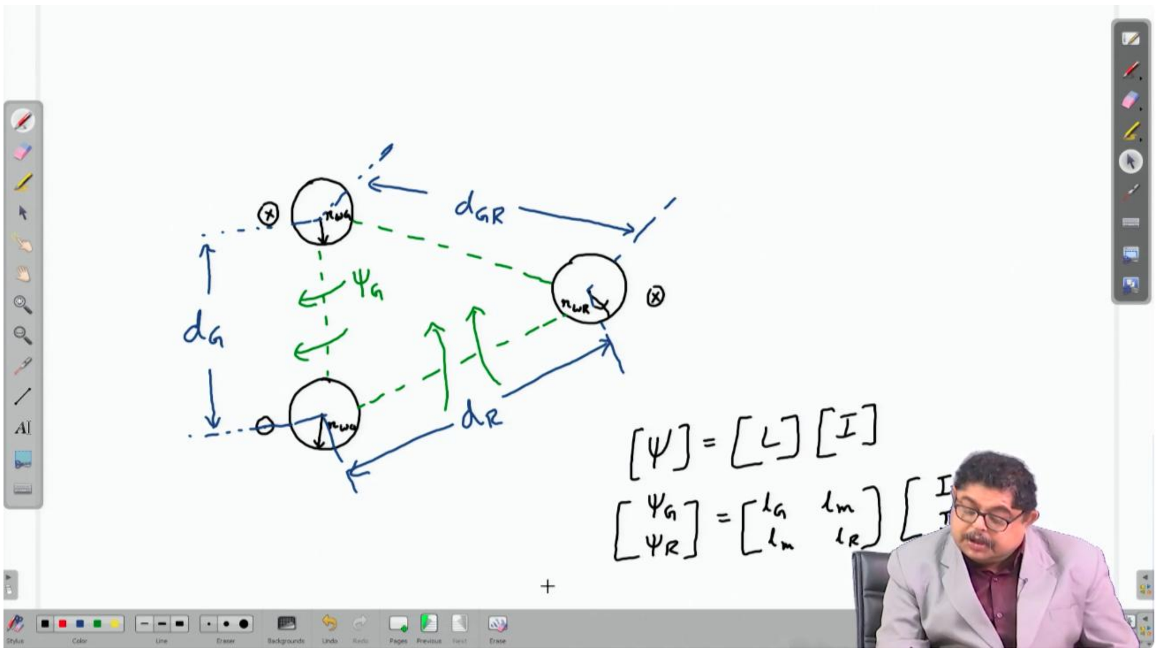
So, with this preliminaries let us move to our per unit length parameters of a three conductor line system homogeneous. So, homogeneous three conductor system. So, the assumption is wires are separated sufficiently. So, that the charge and current distributions around the peripheries of the wires are essentially uniform. Now, this you know that may not be practical, but this is the assumption of the model that we do not have any proximity effect. So, even if we have three ah conductors that they are sufficiently away we assume they are sufficiently away. So, that current distribution charge distribution is uniform throughout the conductors cross section. Now, so we will attack the problem, but before that I want you to recall that we have solved some fundamental things for deriving per unit length parameters of two conductors. So, one thing was that if I have a conductor of 1 meter length and through it a current I is flowing and I have a surface. So, through this surface so that means, I have a surface whose one end is r 1 distance away another end is r 2 distance away from one of the ends and so we can see that through this surface how much flux is passing away that we have already found that this phi is equal to mu by 2 pi ln ln r 2 by r 1 this we derived from the first principles. So, this one you should remember because we will be using this results and also so this will be required for determination of l if you do not want to determine c then this is sufficient,

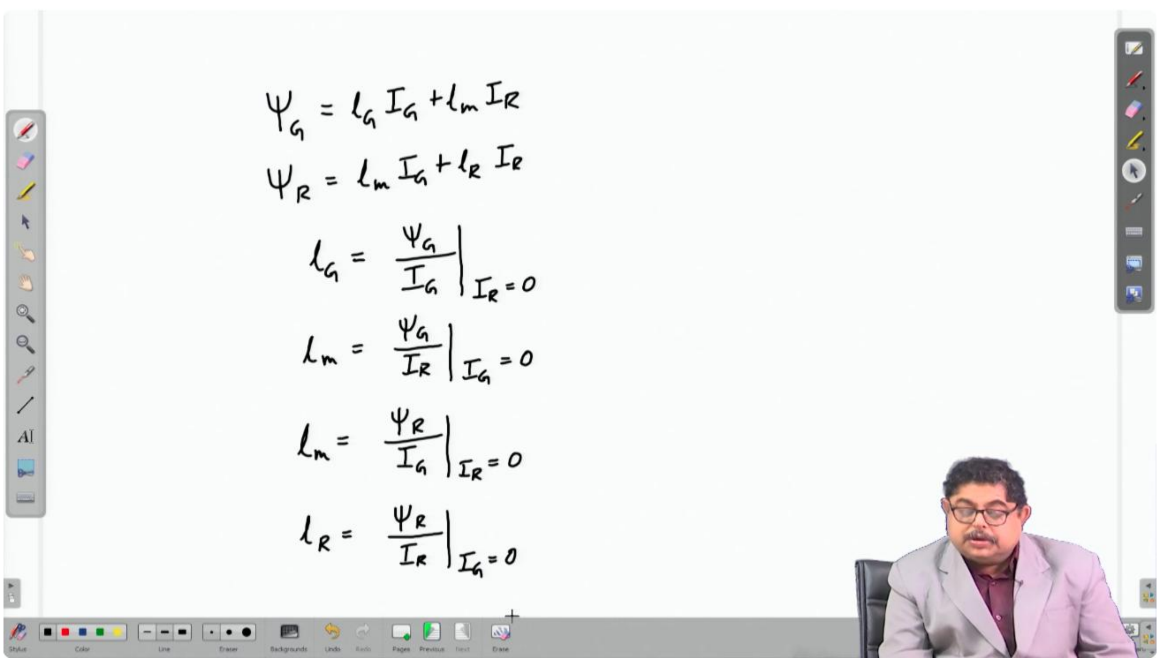


but in cases where you want to find out c then this one will be required that at a distance r 1 I have a point B at a distance sorry at a distance r 2 let me have a point A. So, this is my minus potential this is my assumed potential is e what is the potential of point B with respect to potential of point A that means, what is V B A that we have seen is equal to q by 2 pi epsilon epsilon I am writing if it is epsilon naught it will be that ln r 2 by r 1. So, these two fundamental result will be using this will be for per unit length capacitance determination will use this.

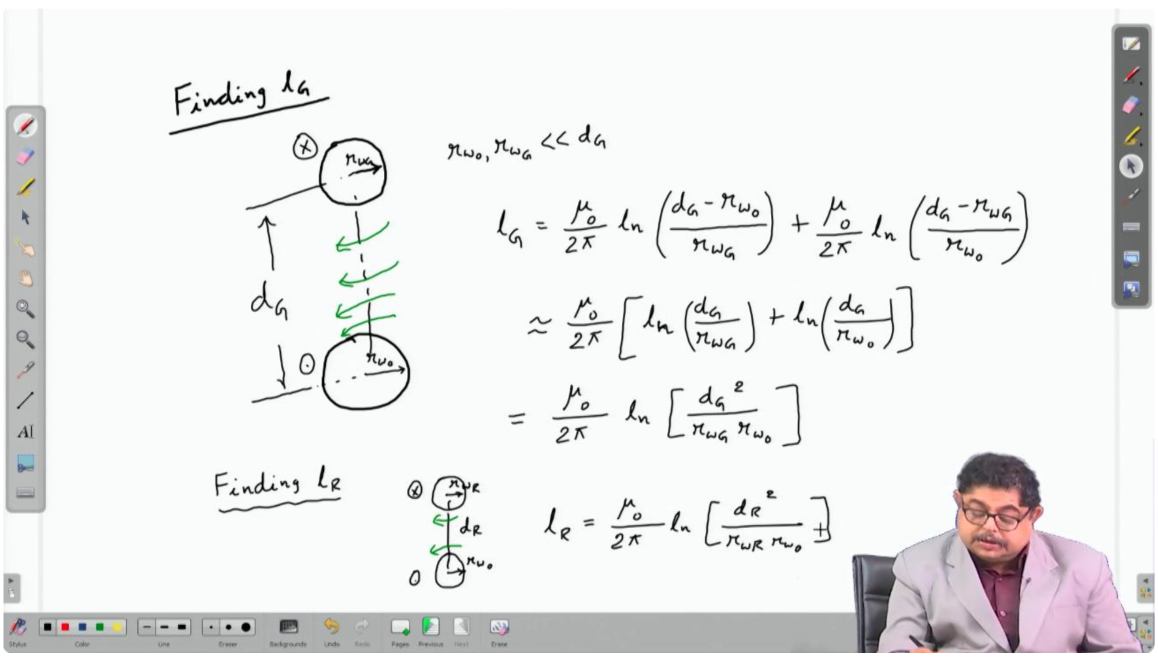


So, now, let us come to our problem that let us say we have three wires. So, this is the generator wire, this is the reference wire r w 0 and this is the generator wire this is the receptor wire ok. So, this one the let us say the current is going inside. So, through the reference it should come out and this since it is coming out. So, this one also will come back through here. So, this is also this is the current. Now, let us give the distances things. So, let us call this d G, let us call this distance d r. So, this distance d G r. So this on this green lines I am trying to find out what will be the flux directions. So, let us see for the generator the generator the current is going inside the board. So, I place my thumb inside the board. So, the direction of my fingers that shows that flux is in this direction. Similarly, for the reference one the current is coming out.So, I make my thumb come out of the board. So, the flux is also in this direction same direction. Now let us do this that for reference for receptor the again it is coming in. So, the flux will be in this direction for reference it is coming out. So, the flux is in the same direction and for this I am not showing now. So, here now we know that flux it is equal to inductance into current. So, psi G psi r is equal to you know L G L M L R I G I R or I can write that. L G I G is equal to L G I G plus L M I R psi r is equal to L M I G plus L R I R. So, this I can now find L G what will be L G? L G is psi G by I G when I R is equal to 0 what is L M? L M is psi G by I R when I G is equal to 0 also from the second equation L M is psi G by I G when I R is equal to 0 or L R is equal to psi R by I R. So now that means, what will be the procedure? So, any of these per unit length parameter inductance parameter can be found by applying current through one of the conductor and opening the other conductor. That means, if I pass the current through suppose generator conductor it will return through reference conductor and let me open the receptor conductor then I will be able to find L G. Similarly, I will be able to find L M by the third expression. On the other hand if I pass the current through receptor conductor that will be returning through the reference conductor and let me make the generator conductor open. So, that will give me that L M and L R. So, for L M there are two ways by which I can do that for L G and L R there are only one way by the how we can do that.





Now, let us say that finding L G. So, problem reduces to that this is my R W G this is my R W O and this is my R W G. This is D G there are no receptor here because to find L G I have made I R 0. So, now, oh let me see as before let us say it is coming like this this is the direction of current. So, what will be the flux direction? You can easily see that by applying right hand rule the fluxes are both additive these are the fluxes due to the upper one this is the flux due to the lower ones flux like this. So, I can easily find out that L G is mu 0 by 2 pi L n remember the formula R 2 by R 1 what is R 2 D G minus R W 0 D G minus R W 0 by R W G plus due to this one the due to the reference conductor mu 0 by 2 pi L n D G minus R W G by R W 0. So, it is approximately we are assuming that R W 0 or R W G is much less than D G. So, approximately we can say mu 0 by 2 pi L n D G by R W G plus L n D G by R W 0. So, that is mu 0 by 2 pi L n D G square by R W G R W 0 ok. So, I think from the it can be easily guess that if we put L R for finding L R we will have to make the L G is not there. So, the case will be similar that you have R W R you have R W O you have current like this and this distance is D R you can easily see that flux also will be same like this additive and that means, your L R will be also having a similar expression that mu 0 by 2 pi L n D R square by R W R R W 0 that is understandable.



What about L m? Let us see because L m will be a bit different. Now, as I said either generator can be excited you have the formula for that or receptor can be excited you have the formula for that. So, it is up to you anyone you can do results will be same let us say that we are exciting generator. So, what will be the picture? Now, in this case 3 will be there because this is mutual that is why the receptor is also there.We are generating the generator exciting the generator. So, current direction in a W the receptor we need not show. So, this is our D G this is our D G R and this is our D R. So, these are hypothesis hypothetical lines where I will check what is the flux directions. So, here you can see that this one the we are exciting the generator. So, flux direction is for this current the flux will be putting the right hand here. So, flux you see on this surface not on this surface. So, let me better erase it. On this surface we are interested that mutual means between this surface. So, what is the current is going here? So, the flux is coming like this here. Now, this one it is coming here flux is again going like here. So, this is phi R. So, now, I can write the formula that L m first write letters for the generator. So, mu 0 by 2 pi L n what is the maximum distance? It is from this start here we are assuming that D G is greater than D G R that means, receptor is within the generator and the reference. So, maximum distance is D G by D G R plus mu naught by 2 pi L n it is maximum distance is D R and minimum distance is R naught. So, it is mu 0 by 2 pi L n D G D R by D G R R omega naught. So, that completes the finding of the L matrix per unit length parameters L matrix. So, for homogeneous line from this C can be obtained. If not homogeneous I have told how to do it. So, we complete it here today. Thank you.