Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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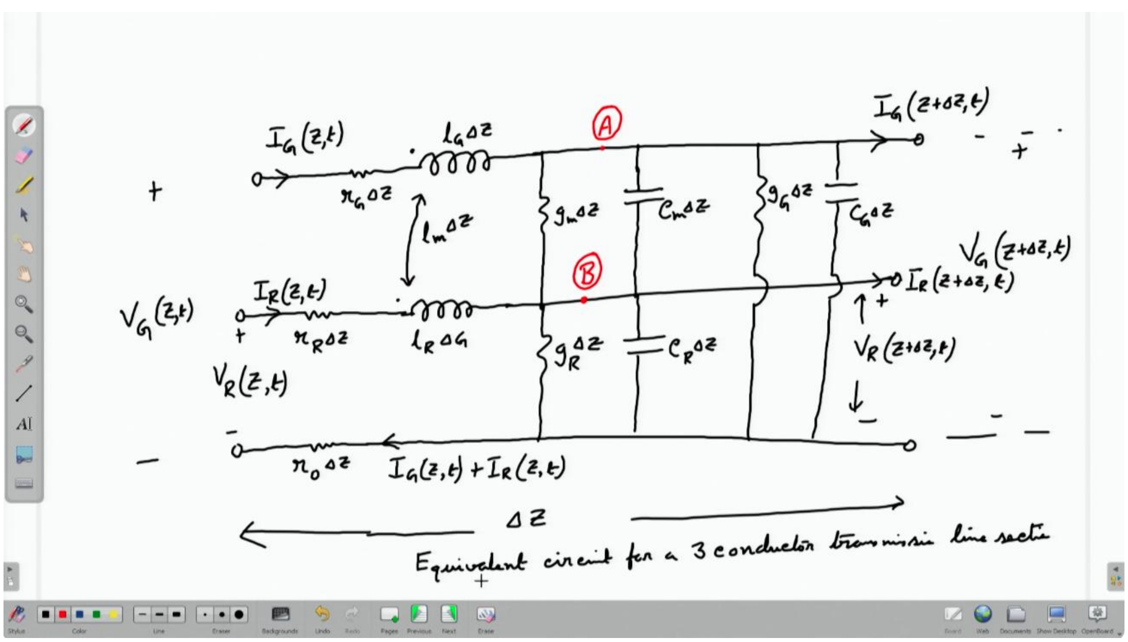
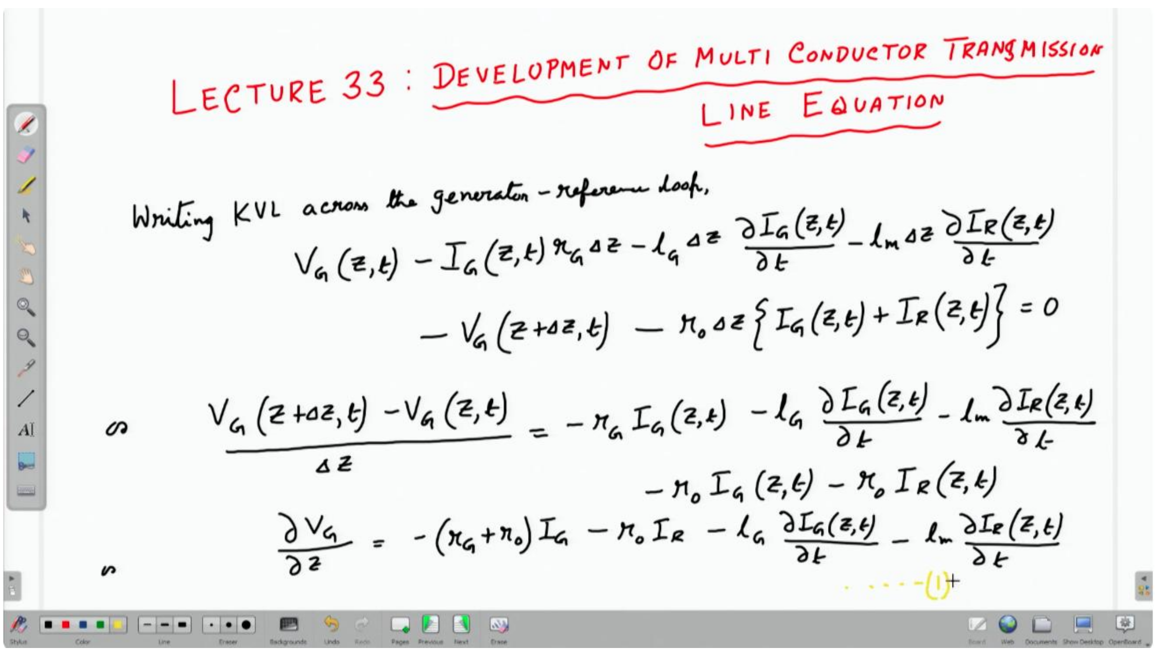
Week :07
Lecture 33: Development of Multi Conductor Transmission Line Equation

Welcome to the 33rd lecture on of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. We were discussing three conductor line, we have found its equivalent circuit for a section of the line in the last class. Now, today we will derive or develop its transmission line equation. So, we can go to the previous class that this is the line. So, here let me do one more thing that let me put the two nodes, one is this node let me call A and this node let me call B. Now, here what I will do simply that I will find between the two conductor at a time we will write the KVL and KCL equations. So, you see that if first I will write across the generator reference loop. So, that means the topmost and the bottom most conductor along that if I take KVL equation then you see that I can take it suppose I am starting from here then Vg Zt minus Rg delta z I z minus Lg delta z del I g del t minus Lm delta z del Ir del t minus Vg g plus delta z t this is the voltage Vg del g plus delta z t minus R naught into I z into I g z t plus Ir z t that should be equal to 0.

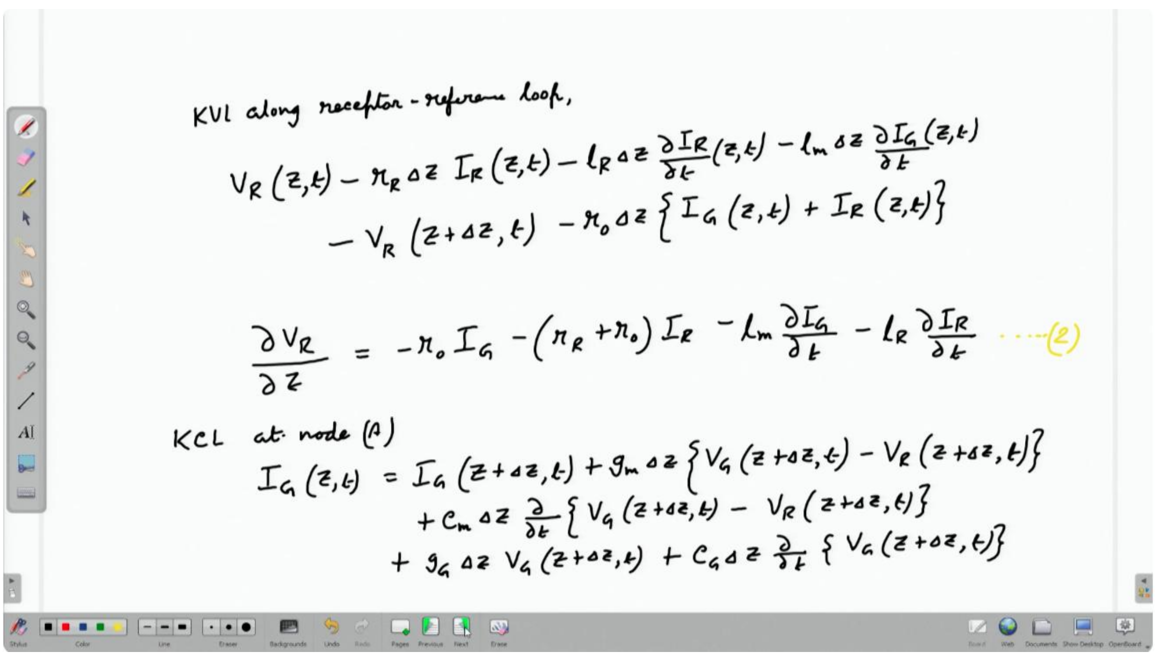


So, let me write that. So, writing KVL across the generator reference loop. So, what I said now I will write. You can see it Vg Zt minus I g Zt Rg delta z minus Lg delta z del Ir del Ir del Ir del z del I g del t minus Lm delta z this current will be the receptor current. Then the loop Vg g plus delta z t minus the in the reference path the resistance it has. So, R naught delta z then I g Zt plus Ir Zt. So, that is equal to 0. Now before writing other equations let me make it you know that I will have to bring the two voltages together. So, I can write for Vg g plus delta z t minus Vg Zt divided by delta z that will be minus Rg I g Zt then minus Lg del I g del t minus Ir z t. So, I can write for Vg g minus Lm del Ir del t minus R 0 I g Zt minus 0. Now, if I take limit delta z goes to 0. So, I can write del Vg del z is equal to now I will club the I g and other terms together. So, minus Rg plus R naught I g minus R naught I r minus Lg del I g del t minus Lm del Ir del t. So let me call this equation, equation number 1.

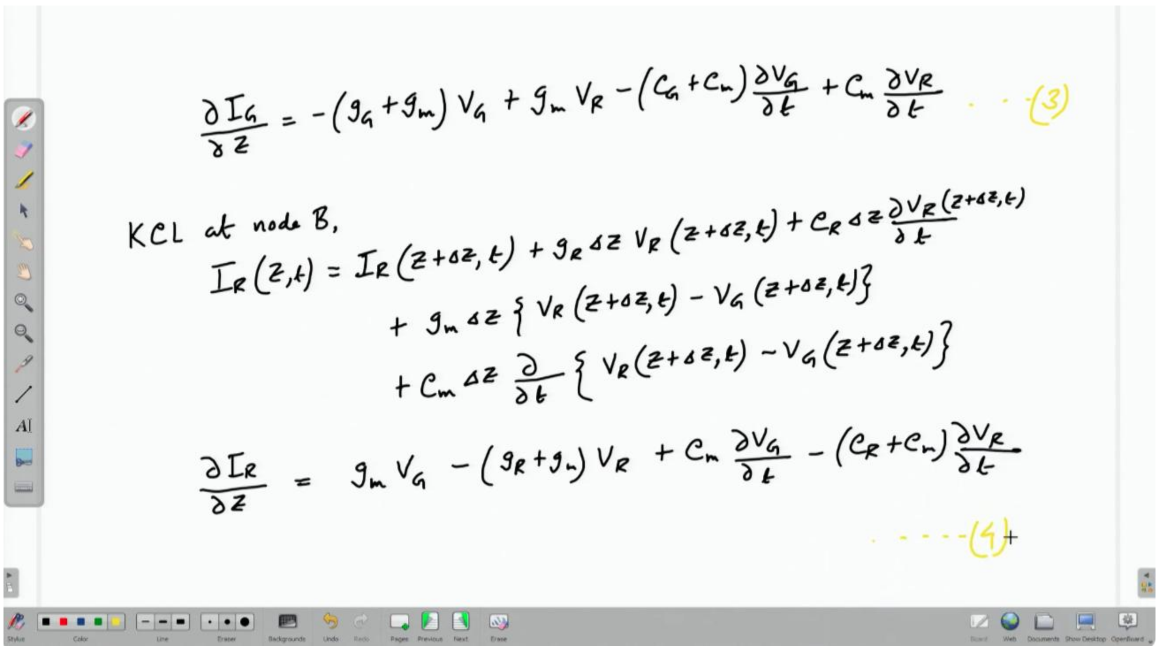


Now how many such equations will be there you see this one loop I have taken along the generator reference another loop is there the receptor reference. So, there also one KVL will come then there are two nodes as you have seen A B in node A I can write a KCL equation in node B I will write a KCL equation. So, there will be four such equations let me write them one by one in the same fashion. So, this KVL along receptor reference loop. So, that will give me V r just exactly similar to the previous one. You keep the equivalent circuit in front of you till I am doing that that makes it easier instead of every time going there and looking at it. This is now I g because I am writing in difference of the mutual one that will give me that will be based on I g. So, again I can I will do that V r.

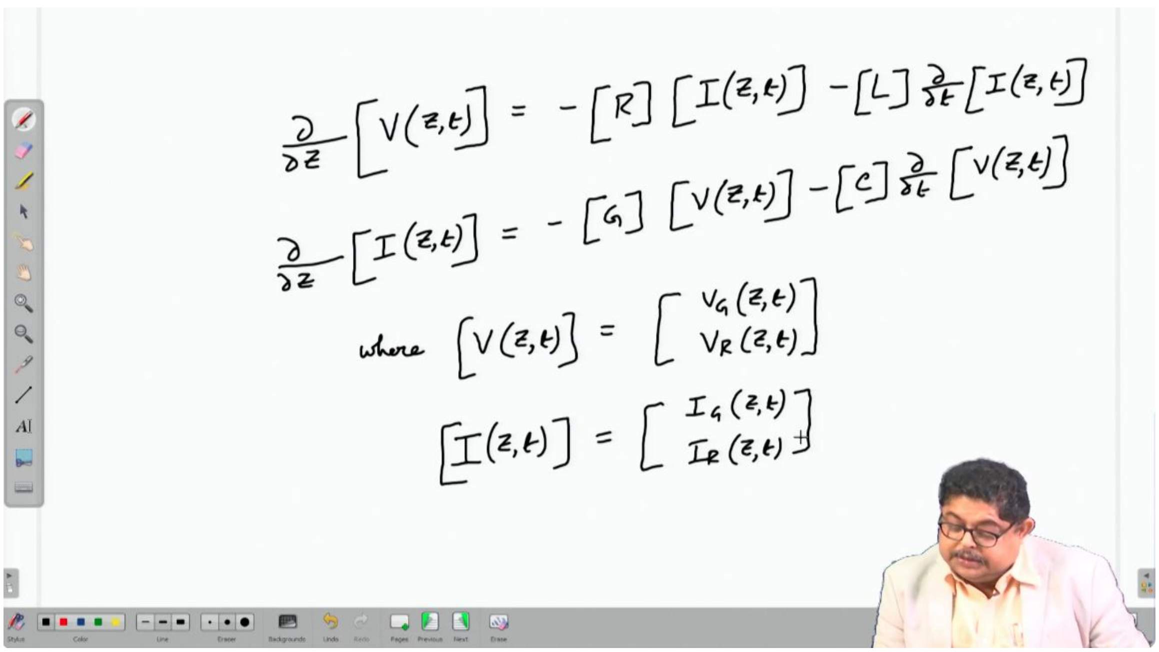
V r g plus delta z t minus V r z t and I will divide by delta z. So, that will be outside separate then I will take the delta z goes to 0 because this will be as online. So, I will get another equation that will be del V r del z is equal to minus R naught I g minus R r plus R naught I r minus L m. L m delta I g delta t minus L r delta z. This will be my equation number 2. Now, I will write KCL along at node a. So, you see at node a the incoming currents are I g z t then the outgoing current is I g. Now, in the there is a current g m I am assuming that this is positive. So, there will be the through g m delta z there will be a current that will depend on the difference of voltages between V g and V r. Then through c m also there will be a outgoing current that will be del del t V g minus V r then through g g there will be a current delta z that will depend on the voltage here V g j plus delta z t and then through c g again there will be a voltage that is del del t of V g g plus delta z. So, I can write that that there is one in going incoming current to node a all others are outgoing current. So, I can write as I g z t is equal to I g z plus delta z t plus g m delta z V g z plus delta z t minus V r z plus delta z t plus c m delta z t plus c m delta z del del t of V g g plus delta z t minus V r z plus delta z t plus g g plus g g delta z V g t plus c g delta z del del t V g g plus delta z t.



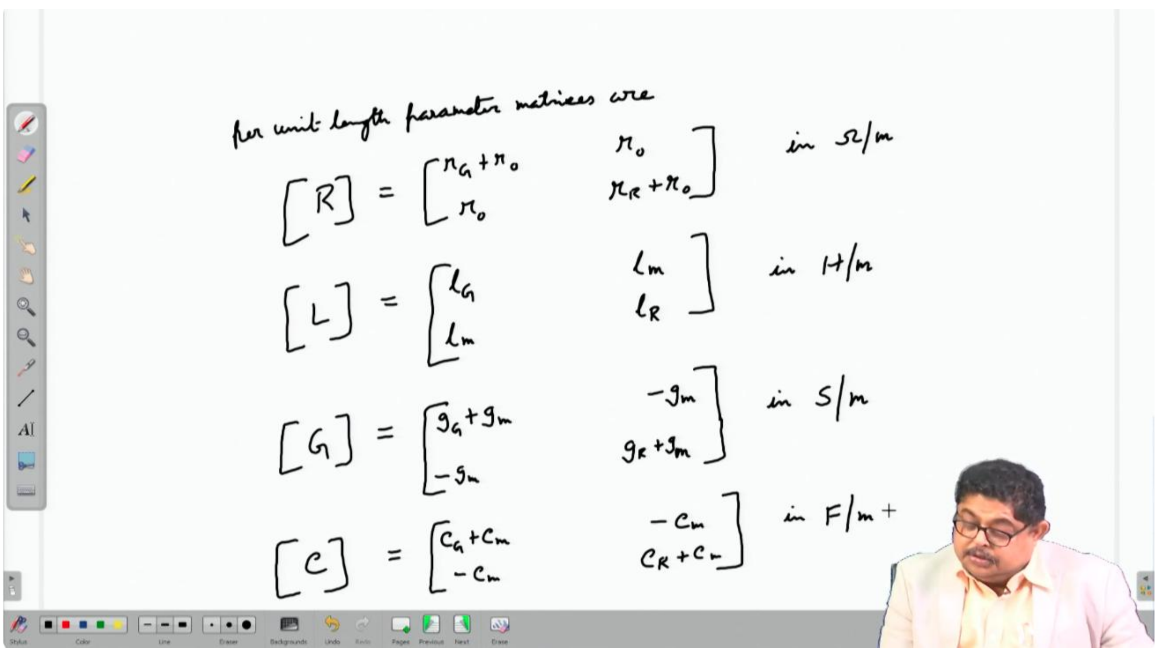
No more current now again the same technique that I g g plus delta z t minus I g z t keep it one side all others another side divide by delta z then take limit of delta z to 0 that will give you a delta I g delta z is equal to minus g g plus g m V g plus g m V r minus c g plus c m del V g del t plus c m del V r del t. So, this will be your equation number 3. Similarly, the last one K c l at node b in similar fashion you can write that I r z t will be I r z plus delta z t plus g r delta z V r z plus delta z t plus c r delta z del V r del t z plus delta z t plus g m delta z V r z plus delta z t plus c r delta z t minus V g z plus delta z t plus c m delta z sorry c m means there will be V r z plus delta z t minus V g z plus delta z t. So, again taking the two currents to one side taking limit you will get the fourth equation del I r del z is equal to g m V g minus g r plus g m V r plus c m del V g del t minus c r plus c m del V r del t. So, this will be your equation number 4.

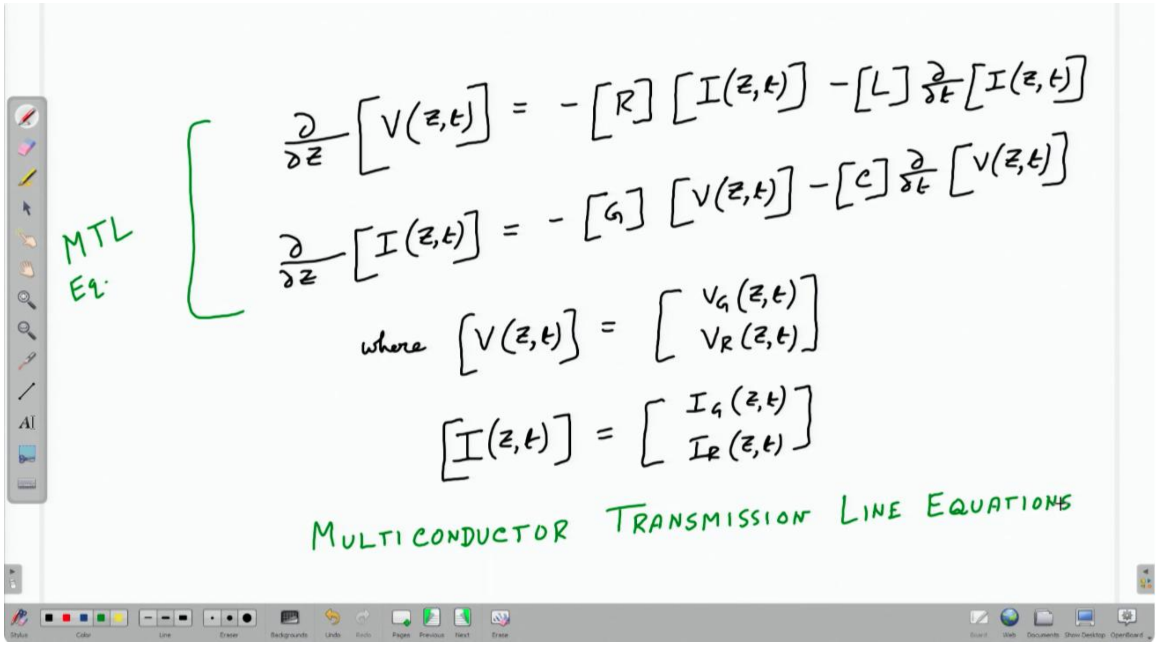


So, four equations you have got and this partial differential equations can be it is written by showing explicitly all the terms we can write it in more compact form and for that we can just write it as in matrix form oh sorry del del z V j t plus g m del V r is equal to minus r I z t minus del del t I z t. The first two KVL equations can be written like this and second two KCL equations can be just written like this. Just see the four equations we can easily write them in this matrix form. You see such elegant where what is V j t? V j t is nothing, but a vector V g z t V r z t. Similarly, what is I vector I matrix that is also a vector it is I g and I r. So, in matrix form the four equations are just written as two equations like our two wire transmission line system and what is this four matrix R L G C you can just look at those equation 1 2 3 4 and that will tell you the per unit length length parameters.



So, the per unit length parameter matrix matrices are first is what is our R matrix? This is your where is equation 1. So, you see what is R matrix? Can I say that R matrix the first one the it will be a 2 by 2 matrix the r 11 will be R g plus r naught r 12 will be r naught then go to the second equation r 21 will be r naught r 22 will be r r plus r naught. So, write that. So, r is R g plus r naught r naught this is r naught r r plus r naught and its unit is in ohms per meter. Similarly, there will be a 2 by 2 L matrix. So, let us again go to equation 1. So, what is L matrix? The L 11 will be L g L 12 will be L m L 21 will be L m L 22 will be L r. So, I will write that that it is L g L m L r and what is its unit in Henry per meter then there will be g matrix conductance matrix. So, they will come from the third and fourth equation. So, third equation where is this is third equation. So, what will be g 11 g g plus g m g 12 g m g 21 is g m g 22 g r plus g m. Now, here it will be g 11 is g g plus g m, but g 12 will be minus g m because in general we have taken the equation as minus g into V g t. You see that our equation is minus g into s o those which are positive terms they will become negative in the per unit length parameters. So, I can easily write that g is equal to g g plus g m this is minus g m this is also minus g m g r plus g m. And what is its unit it will be Siemens per meter conductance unit is Siemens. So, per unit length parameter is Siemens per meter and finally, there will be per unit length capacitance matrix. So, let us see it will also come from equation third and fourth also it is minus c. So, remember so where is 3 yes. So, it is c 11 will be c g plus c m c 12 will be minus c m c 21 will be minus c m c 22 will be c r plus c m. So, it is c g plus c m minus c m it is minus c m it is c r plus c m mean farad per meter. So, this sorry this equation is actually called MLT equation or multi conductor transmission line equation. For our case 3 conductor case it is becoming these if it is 4 conductor it will be a these vectors will be 3 I Z T will also be 3 and the per unit length parameters they will be 3 by 3 matrices. So, there will be 9 terms in that etcetera. So, sorry multi conductor transmission line. So, MTL M T L equations multi conductor. So, this is the equation for multi conductor transmission line equation. So, we have derived it as I said that from the same approach as we take for 2 conductor line these things were derived.





And now in the next class we will move from here we will have to find various things finally, also we will this is the transmission line equations then we will have to find the for 3 conductor again we will have to derive the per unit length parameters of various 3 conductor lines and then from that we will move to a model of a crosstalk. So, with that I finish today's lecture we have done a great job today that multi conductor transmission line is completely new to you. So, that we have derived from basic principles of KVL and KCL. Thank you.