

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

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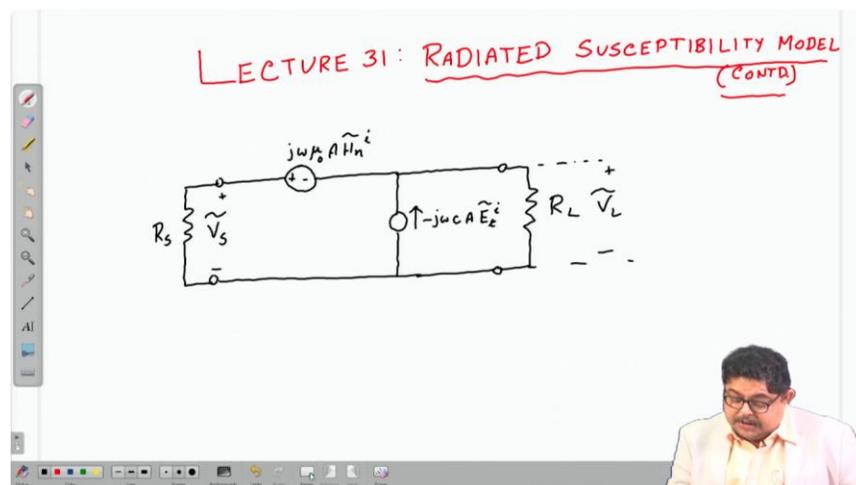
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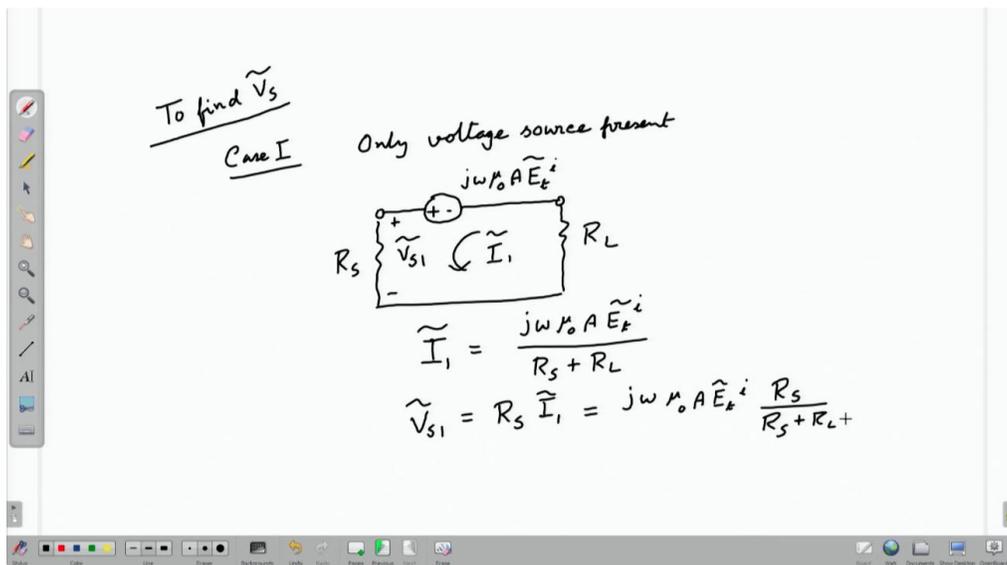
Week :07

Lecture 31: Radiated Susceptibility Model (Continued)

Welcome to the 31st lecture on the course on EMIMC and Signal Integrity Principles, Techniques and Applications. We were considering our discussion on Radiated Susceptibility Model, we have not finished that so today also in this lecture we will continue that. Up to this we have seen that by simplifying by making some simplifying assumption our model ultimately becomes these there are only two induced sources or dependent sources due to the plane wave that has been incident on the two way transmission line, one is a voltage source dependent on the normal component of the incident electromagnetic field, normal component of the magnetic field of the incident electromagnetic field and there is a current source which is proportional to the tangent transverse component of the electric field of the incident electromagnetic field. So, due to them at the RS end that means at the termination which we have lumped as an VS on that there will be a source voltage or a voltage developed that we are calling VS and on the far side that is on the load side there will be a voltage developed at VL we will have to determine what is this VS and VL that is our job. If we do that the model will be finished, model development will be finished. So, now you see that there are two sources so we can apply superposition in one case we will apply that only one source is we will take one source at a time the other one will be not present and then we will find the two voltages VS and VL due to that then again we will put the other source and the previous source we will put it off and find the VS and VL then when the two sources are present superposition says that you can add up those voltages so that will do.



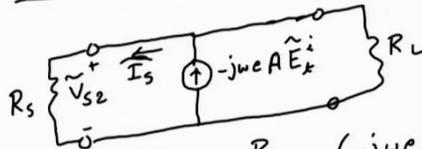
So, let us say that first we want to find  $\tilde{V}_s$  to find  $\tilde{V}_s$ . So, the first case is only voltage source is present, voltage source is present so that means the current source to make it off we will have to open it so the circuit will become now that there is the voltage source. So, the voltage that will be developed at this end let us call that  $\tilde{V}_{s1}$  because this is the case 1 so  $\tilde{V}_{s1}$  is this is our assumed voltage direction so you know that there will be a current this is plus minus so a loop current  $\tilde{I}_1$  will be developed we can easily find what is the value of  $\tilde{I}_1$ ,  $\tilde{I}_1$  will be  $j\omega\mu_0 A \tilde{E}_z^i$  divided by  $R_S + R_L$  so  $\tilde{V}_{s1}$  we can write that it is  $R_S \tilde{I}_1$  sorry I will have to write  $\tilde{I}_1$  these are all phasors. So,  $\tilde{V}_{s1}$  is nothing but  $j\omega\mu_0 A \tilde{E}_z^i$  into  $R_S$  by  $R_S + R_L$ .



So, let us come to case 2 that only current source present so that means there is only a current source a voltage source if it is not present we make it short it does not have any impedance so we are not putting any impedance here. So, this side is our  $R_L$  this is our  $R_S$  this one we know the value was minus  $j\omega\mu_0 A \tilde{E}_z^i$  this one I have written wrongly you know that this depends on  $HNI$ . So,  $j\omega\mu_0 A \tilde{E}_z^i$  must be  $\tilde{I}_1$  so this one also change will become  $HNI$ . So, this is  $\tilde{E}_z^i$  let me write clearly  $A \tilde{E}_z^i$  this is  $R_S$  this is  $R_L$  now let me call it  $\tilde{V}_{s2}$  because this is case 2 and let me call that this current at this node it will one portion will come here. So, we know actually you can see that  $R_S$  and  $R_L$  are in parallel so we can easily write what is the value of  $\tilde{I}_S$ ,  $\tilde{I}_S$  is equal to  $R_L$  by  $R_S + R_L$  into minus  $j\omega\mu_0 A \tilde{E}_z^i$ . So,  $\tilde{V}_{s2}$  is equal to  $\tilde{I}_S$  into  $R_S$  so it is  $R_S R_L$  by  $R_S + R_L$  minus  $j\omega\mu_0 A \tilde{E}_z^i$ . So, now applying superposition we have got  $\tilde{V}_s$  which is  $\tilde{V}_{s1}$  plus  $\tilde{V}_{s2}$  and that is  $R_S$  by  $\tilde{V}_{s2}$  plus  $R_S$  plus  $R_L$   $j\omega\mu_0 A \tilde{E}_z^i$ .

Case II

Only current source present



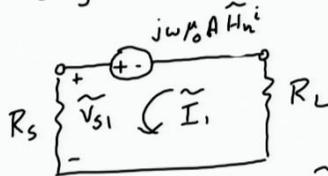
$$\tilde{I}_s = \frac{R_L}{R_s + R_L} (-j\omega_c A \tilde{E}_k^i)$$

$$\tilde{V}_{s2} = R_s \tilde{I}_s = \frac{R_s R_L}{R_s + R_L} (-j\omega_c A \tilde{E}_k^i)$$

$$\tilde{V}_s = \tilde{V}_{s1} + \tilde{V}_{s2} = \frac{R_s}{R_s + R_L} (j\omega_{p0} A \tilde{H}_n^i) + \frac{R_s R_L}{R_s + R_L} (-j\omega_c A \tilde{E}_k^i)$$

To find  $\tilde{V}_s$   
Case I

Only voltage source present



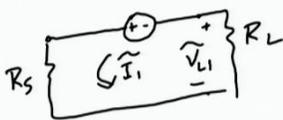
$$\tilde{I}_1 = \frac{j\omega_{p0} A \tilde{H}_n^i}{R_s + R_L}$$

$$\tilde{V}_{s1} = R_s \tilde{I}_1 = j\omega_{p0} A \tilde{H}_n^i \frac{R_s}{R_s + R_L}$$

So, VS is complete similarly we can find VL to find VL so again case 1 same as before only voltage source present and now I am readily writing that so there is only the voltage source RS RL this is that one. So, now we are interested to find this VL1 so VL1 already we have seen the I1, I1 is like this so VL1 will be RL into I1 so it will be, but you see that the current is oppositely directed to our assumed voltage so I will have to put a minus so that will be minus RL by RS plus RL into J omega mu naught A HNI. Then case 2 is this one is shorted now RL this is RS and there is a current source here so that will give rise to a VL2. So, here we can find that there is a current coming here IL so we can say that IL will be RS by RS plus RL into minus J omega CA ETI. So, we can see that VL2 that is in the same direction as IL so VL2 will be minus RS RL by RS plus RL into J omega CA ETI phasor. So, now we know that VL is equal to VL1 plus VL2 is equal to minus RL by RS plus RL into J omega mu naught A HNI minus RS minus RL by RS plus RL into J omega CA ETI. I hope it has visible .

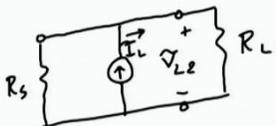
To find  $\tilde{V}_L$

Case I



$$\tilde{V}_{L1} = -R_L \tilde{I}_1 = -\frac{R_L}{R_S + R_L} (j\omega \mu_0 A H_n^i)$$

Case II



$$\tilde{I}_L = \frac{R_S}{R_S + R_L} (-j\omega C A \tilde{E}_t^i)$$

$$\tilde{V}_{L2} = -\frac{R_S R_L}{R_S + R_L} (j\omega C A \tilde{E}_t^i)$$

$$\tilde{V}_L = \tilde{V}_{L1} + \tilde{V}_{L2} = -\frac{R_L}{R_S + R_L} (j\omega \mu_0 A H_n^i) - \frac{R_S R_L}{R_S + R_L} (j\omega C A \tilde{E}_t^i)$$

so we have completed the model let us see an example. Consider a ribbon cable of RS 50 ohm and RL 150 ohm. The wire radius is 7.5 mils and the two cables separate two cables are separated cable separation is 50 mils these are standard values you will see that separation 50 mils is coming in times. Find the unit induced terminal voltages in the cable that means find VS and VL they are the terminal induced voltages in the cable when a 100 megahertz 100 megahertz is incident broadside to this line. The electric field magnitude is given to be 10 volt per meter and polarized in X direction. So, you see

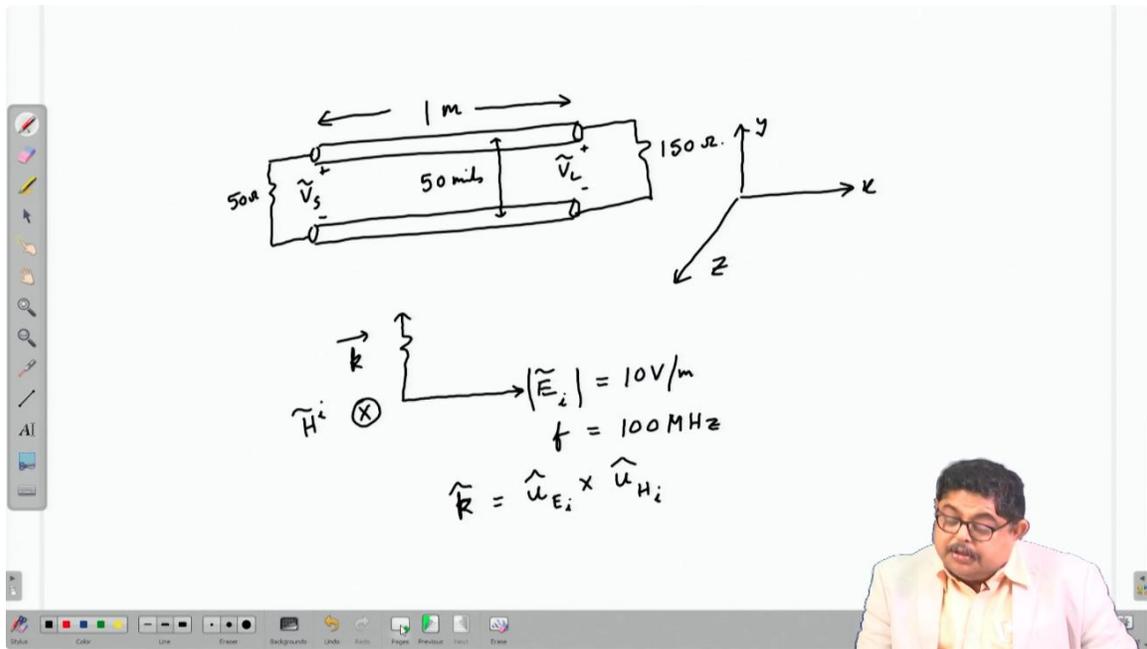
I have written it explicitly that you will have to apply your knowledge of electromagnetic theory that a uniform plane wave it is propagating in Y direction and the electric field is polarized in X direction. So, with that you will have to find what is the induced fields and then finally, what will be the radiated susceptibility that means what is the voltages they will induce to a passive wire which is not energized, but this wave is energizing it.

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1 m ribbon cable  
 $R_S = 50 \Omega$   
 $R_L = 150 \Omega$   
 Wire radius = 7.5 mils  
 Cables separated  $\rightarrow$  50 mils  
 $\tilde{V}_S, \tilde{V}_L = ?$   
 100 MHz plane wave travelling in the X-Y plane in Y direction  
 is incident broadside to this line. The electric field mag<sup>th</sup> is  
 10 V/m and polarized in X direction

So now, from our model you can easily solve it that this is the wire it is 1 meter ribbon cable. So, this is 1 meter to wire cable. So, this is also 1 meter it is said that their separation is 50 mils that means here to here you have 50 mils and this side it is terminated by 150 ohm this side it is terminated by 50 ohm. So, we can call this is our  $V_S$  this is our  $V_L$  and regarding the wave it has said that it is going in the Y direction in our case we have taken X as this direction Y as this direction and positive Z is this direction. So, it is said that wave is going in Y direction that means this is the direction in which the wave is going the wave is E field is polarized like this. So, this is its E field  $E_i$  is given as  $E_i$  magnitude is 10 volt per meter frequency given is 100 megahertz and so if the wave goes in Y direction E field is in X direction what is the direction of the magnetic field we know that this is called propagation direction k vector. So, k vector always is unit k vector I am not k. So, let me write like this unit vector k is always u E i vector into actually this three form a couplet u vector of H i this i always can say that unit vector along the electric field unit vector along the magnetic field their cross product will give you the propagation direction of the uniform plane wave for uniform plane wave this is

always true. So, here this is your Y u E i is your X. So, what will be your H i direction that you can easily see that X cross what will give you Y will it X Z if it is X Z it will give you minus Y, but if it is X this direction is minus Z that will give you plus Y. That means, you know that the magnetic vector that will be we can easily say that this will be the direction of H i that it is going inside whereas, our positive direction is this. So, this much information we got. Now, actually first thing to be checked is whether our model is applicable here.



So, you can see that what is the  $f$ ?  $f$  is 100 megahertz. So, what is  $\lambda$  for 100 megahertz it is  $3 \times 10^8$  by 100 megahertz means  $10^8$  into  $10^6$  to the power 6. So, that is 3 meter and our length is 1 meter. So, can I say that  $L$  is equal to  $\lambda$  by 3. So, it is not sufficiently electrically short. So, our model is not applicable here, but so, first you will check that and you can say that it is not applicable. So, you will have to do the exact analysis, but let us assume that in this case it is less than  $10\ \Omega$ . So, that we can apply the model actually it cannot be applied. Now, let us proceed, but this should be always the first check and then you can proceed. Now, what is your  $E_i$ ?  $E_i$  is in  $x$  direction is it in the transverse direction because we have said that radiated susceptibility or those voltages will be induced only if you have a transverse component of induced electric field, incident electric field and normal component of incident magnetic field, but you see in this case  $E_i$  is not transverse, incident electric field is not transverse. So, no current source will be generated that means there will not be any current source is the incident magnetic field normal? Yes, it is normal. So, a

voltage source will be generated. So, we can now draw that this will be the circuit that I have a voltage source then I have a 150 ohm load I have a 50 ohm load here. So, what is the voltage source? Let us calculate that let me call this  $V_s$  into  $L$  because whatever we calculated that was per unit length voltage source. So, we will have to multiply it with the length. So,  $V_s$  into  $L$  that is  $j\omega\mu_0 A H_n^i$  into  $h_n^i$  into already  $L$  has been put in  $s$  into  $L$ . So, that is there. So, this if you put the value  $j$  into  $2\pi$  into  $10$  to the power 8 directly I am writing then  $\mu_0$  is  $4\pi$  into  $10$  to the power minus 7 into area, area is 1 meter into 50 ohm 50 mil. So, 50 into I am continuing here 50 into 0.001 that has brought me to inch that has multiply with 2.54 that has brought me to centimeter multiply with 10 to the power minus 2 that has brought me to meter. Then what is  $h_n^i$ ?  $h_n^i$  I can say that it is  $E_i$  because  $E_i$  value is only given  $h_n^i$  directly not given, but for plane wave we know that  $E_i$  divided by  $\eta_0$  the free space impedance that is  $120\pi$  that will give me the value of  $h_n^i$  it is only the magnitude part. So, I can say this. So, this if you put the values it will come to  $j26.6$  mV actually I am putting  $j$ . So, let me remove this thing. So,  $V_s$  into  $L$   $j\omega\mu_0 A H_n^i$  or it was there  $h_n^i$ . So, with that I can easily that means, this  $V_s L$  is nothing, but  $j$  right. So,  $V_s$  is 50 by 50 plus 150 into  $j26.6$  milli volt that will give you  $j6.65$  milli volt and  $V_L$  will be minus 150 by 50 plus 150 into  $j26.6$  milli volt that will give you minus  $j19.95$  milli volt. So, you have solved it.

Handwritten calculations and circuit diagram:

$$\lambda_0 = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

$$\alpha = \frac{\lambda_0}{3}$$

Circuit diagram showing a voltage source  $\tilde{V}_s$  in series with a  $50 \Omega$  resistor and a  $150 \Omega$  resistor.

$$\tilde{V}_s \alpha = j\omega\mu_0 A H_n^i$$

$$= j2\pi \times 10^8 \times 4\pi \times 10^{-7} \times (1 \times 50 \times 0.001 \times 2.54 \times 10^{-2}) \times \frac{E_i}{\eta_0}$$

$$= -j 26.6 \text{ mV}$$

$$\tilde{V}_s = \frac{50}{50+150} \times -j 26.6 \text{ mV} = -j 6.65 \text{ mV}$$

$$\tilde{V}_L = \frac{-150}{50+150} \times -j 26.6 \text{ mV} = j 19.95 \text{ mV}$$

Now, let me change the problem everything remaining same let me say that let the picture is like this the line is same all the things are same terminal impedances are same I will just change the polarization of the field 50 ohm. So, what I will do I will now say that the wave is moving in this direction and the E field is polarized in this the value of electric field is same. So, the moment I do that you see E field has a transverse component and you can check what is the H field this is your k vector. So, you see that you require u j should be equal to u or u j u x you require u y cross something. So, H N i in which direction it will come y into if it is in positive z direction that means, then it is ok that means, in this case the H vector will be H i will be in positive z direction. So, this now E field has transverse component. So, u i is 10 volt per meter and H field has normal component H z and its value will be 10 by 120 pi ampere per meter. So, in this case both the voltage source and current source will be present already we have found that what is V s into L that will be j 26.6 milli volt, but this I time you see the H z is in the opposite direction than the previous case in previous case it was going inside in this case it is coming out. So, in equivalent circuit I will have to remember that that means, the voltage source polarity will be reverse also let me calculate what will be I SL I SL will be minus j omega c E t i E t i A. Now, here we will have to calculate C. So, we know for two well line how to find C. So, what will be C? So, that we formula we need to put that j omega c means pi you can see your formula per unit length capacitance up to conductor line epsilon naught then l n s by r w then E y into A A you know how to find that 1 into 1.27 into 10 to the power minus 3. So, I SL its unit will be ampere and if you do this calculation it will come as minus j 0.1168 ampere.

Diagram of a two-wire transmission line with terminal impedances of  $50 \Omega$  and  $150 \Omega$ .

Handwritten equations:

$$\hat{u}_k = \hat{u}_y \times \hat{u}_z$$

$$\tilde{E}_y = 10 \text{ V/m}$$

$$\tilde{H}_z = \frac{10}{120\pi} \text{ A/m}$$

$$\tilde{V}_s = j 26.6 \text{ mV}$$

$$\tilde{I}_s = -j \omega c \hat{E}_t \cdot \hat{A} = -j \omega \frac{\pi \epsilon_0 \epsilon_n}{\ln(\frac{s}{rw})} \tilde{E}_y (1 \times 1.27 \times 10^{-3}) \text{ A}$$

$$= -j 0.1168 \text{ A}$$

So, now my model is this it is a 50 ohm here I will have to put it the voltage source is reverse it is j 26.6 milli volt then there will be a current source and it is minus j 0.1168 milli ampere. If you have shown have I said milli ampere no minus j this should be milli ampere. So, this then there will be a 150 ohm resistance and you have V L here plus minus you have V s here plus minus by putting the model that V s will be minus previously you have determined. So, minus j 6.65 milli volt minus j 4.38 milli volt that will be V L will be 150 by 50 plus 150 j 26.6 milli volt minus 50 into 150 by 50 plus 150 j 0.1168 milli ampere. So, if you do you will get. So, note that the contribution from the two equivalent sources get subtracted in V L, but get added in V s this is an important thing that in the source side the contributions from both the sources are added and here it is subtracted. So, with that we complete the radiated susceptibility model in the next class we will start this radiated susceptibility, but that we do not call radiated susceptibility cross talk which is a near field effect. This is a far field effect that is why we assume plane wave, but if a near field thing that means, one near field source one source affects a two way to a two line conductor in its near field how to tackle that that is the domain of cross talk that we will discuss in the next class. Thank you.

$$\tilde{V}_s = j6.65 \text{ mV} - j4.38 \text{ mV} = j2.27 \text{ mV}$$

$$\tilde{V}_L = -\frac{150}{50+150} (j26.6 \text{ mV}) - \frac{50 \times 150}{50+150} (j0.1168 \text{ mA})$$

$$= -j24.33 \text{ mV}$$

+