

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

Professor name: Prof. Amitabha Bhattacharya

Department name: Electronics and Electrical Communication Engineering

Institute name: IIT Kharagpur

Week :06

Lecture 28: Per Unit Length Parameters of Various Two Wire Lines

Welcome to the 28th lecture of the course on EMI-EMC and Signal Integrity Principles, Techniques and Applications. We were discussing per unit length parameters of two-wire line system. We will see various types of two-wire lines today, their per unit length parameters. First with our developed formulae that we derived for per unit length inductance and capacitance in the previous class. Let us take an example. These are two 5 gauge wires of a typical ribbon cable with center to center spacing of 50 meters. Let us find calculate the per unit length inductance and capacitance of these two-wire line system. So, we know that 5 gauge wires that means the diameter will be 0.5 millimeter. So, the radius of the wires are 0.25 millimeter. This is given s means 1 milli is 0.001 ohm inch. So, we know that it is 50 into 0.001 into 2.54 centimeter. So, what is inductance? I am writing L_e because it is basically external inductance as we know that these predominance. So, L_e is also equal to L_e . So, $0.4 \ln s$ by $\rho r w$. So, I can put $0.4 \ln$ this s 50 into 0.001 into 2.54 in centimeter. So, for $r w$ also if I put it in centimeter it is 0.025. So, the unit that we have seen that for this 0.4 means it is microhenry per meter. So, that will give us a 0.65 microhenry per meter is our inductance per unit length inductance and we can also calculate the per unit length capacitance that will be 27.78. We are obviously assuming free space because these values 27.78 is for that. So, \ln of 1.27 by 0.25 both are in millimeters here. So, \ln of 1.27 is 17.09 picofarad per meter. So, you can cross check that whether L_e and C comes as $\mu_0 \epsilon_0$ naught or 1 by C square. So, that you can cross check it will come.

LECTURE 28: PER UNIT LENGTH PARAMETERS OF VARIOUS TWO WIRE LINES

5 gauge wires
 $s = 50 \text{ mils}$

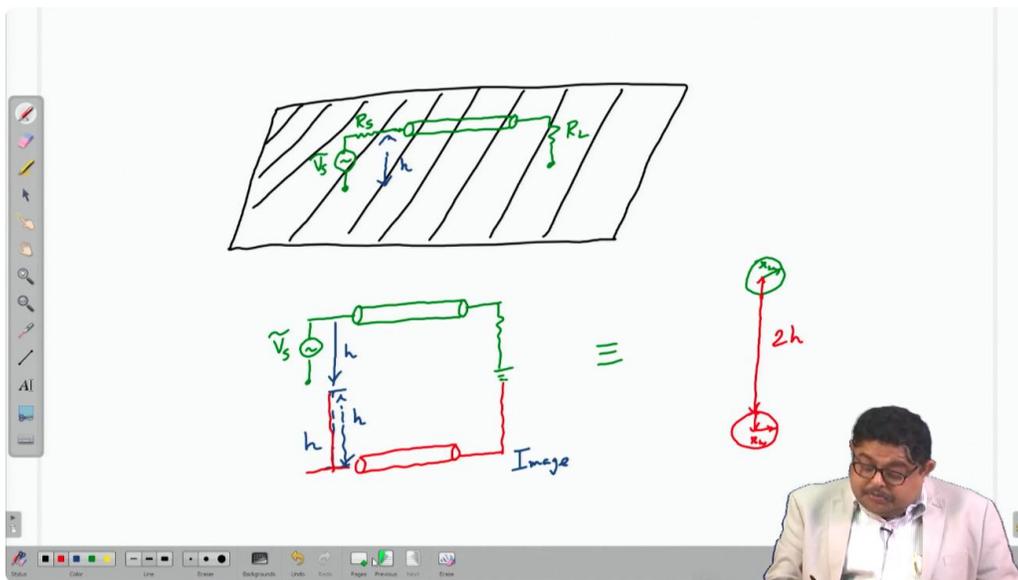
$r_w = 0.25 \text{ mm}$
 $s = 50 \times 0.001 \times 2.54 \text{ cm}$

$L_e = 0.4 \ln \left(\frac{s}{r_w} \right) = 0.4 \ln \left(\frac{50 \times 0.001 \times 2.54}{0.025} \right) \mu\text{H/m}$
 $= 0.65 \mu\text{H/m}$

$C = \frac{27.78}{\ln \left(\frac{1.27}{0.25} \right)} = 17.09 \text{ pF/m}$

$L_e C = \mu_0 \epsilon_0 = \frac{1}{c^2}$

Now let us see single wire above a ground plane. So, let us say that we have a infinite actually finite, but we are considering it to be infinite or very large you can say that there I have a transmission line a single wire transmission line it is terminated on this ground and it is having a voltage source V_s and it is connected to this line by a source and let us say that this height of this line above the ground plane is h . The ground plane now how to tackle this I think you know image theory by image theory we can remove this ground plane and we can think that we have basically one transmission line V_s and then this is at a distance of h at another distance h we know we can consider there is another one and they are connected with this. So, let me show again that this line is h . So by that we got actually two it is now in this is the image because of this infinite ground plane I can consider that there are image. So basically now I am left with this problem. So this problem is equivalent to that I have in cross section if I see this is a W then there is another one that also its radius r_w and the separation sorry between the two centers is now $2h$.



So let us find the per unit length parameters of these. So I can say that there will be a capacitance because there is a this transmission line is separated from the a thing so from the ground plane by a distance h so there will be a per unit length capacitance so that if I call c_1 then the equivalent to a line is this with a capacitance c_1 then there will be another c_1 with a capacitance this so this is c_1 the image of that is c_1 . So we can say that this one suppose we call that this is again equivalent to if I call this c_2 so easily you can see that what is c_2 c_2 is c_1 c_1 2 capacitors are in series c_1 series c_1 . So two capacitors if they are in series that is c_1 into c_1 by 2 c_1 so that becomes c_1 by 2 so we

can say what is C_2 it is C_1 by 2 so that C_2 value is our required thing so or you can say that this was our requirement .

The diagram shows three equivalent circuit representations of a single conductor. The first shows a single capacitor C_{one} connected to ground. The second shows two capacitors C_{one} connected in series between two terminals. The third shows a single capacitor C_{two} connected between the two terminals. Below the diagrams, the following equations are written:

$$C_{two} = C_{one} \text{ series } C_{one}$$

$$= \frac{C_{one}}{2}$$

so let us go that what is C_1 now C_1 is $2 C_2$ so from image theory we got this now what is this value of C_2 C_2 is already we developed this one that if there are two lines and we know that their formula so between them there is a capacitance C_2 that C_2 is given by $\pi \epsilon$ by $\ln r_2$ by r_1 so from that formula we can put 2 into C_2 will be $\pi \epsilon$ by $\ln r_2$ what is their maximum distance that is $2h$ and minimum distance is r_w so this is farad per meter where we have assumed that h is greater that is why we are calling it r_2 so h is greater than r_2 . So once we have determined capacitance we can find what is L_e of this structure L_e is $\mu \epsilon$ by C_1 is equal to $\mu \epsilon$ by $2 \pi \epsilon \ln 2h$ by r_w so that is $\mu \epsilon$ by $2 \pi \ln 2h$ by r_w henry per meter. So, you see by applying image theory and our already developed formulas for two conductor line we could also take tackle the single conductor line.

The diagram shows a single conductor circuit with two terminals and a capacitor C_{two} . The following equations are written:

$$C_{one} = 2 C_{two}$$

$$= 2 \frac{\pi \epsilon}{\ln \left(\frac{2h}{r_w} \right)} \text{ F/m}$$

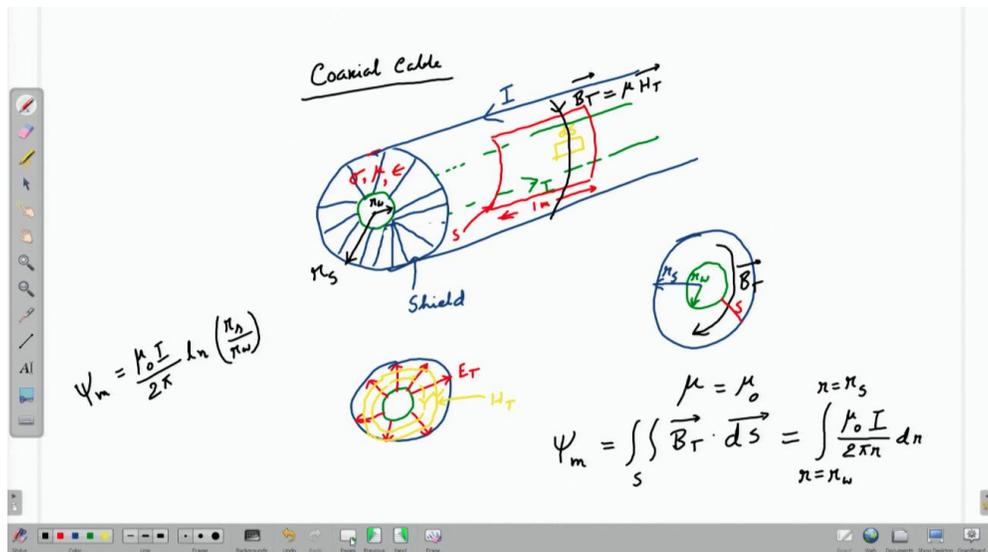
$h \gg r_w$

$$L_e = \frac{\mu \epsilon}{C_{one}} = \frac{\mu \epsilon}{2 \pi \epsilon} \ln \left(\frac{2h}{r_w} \right)$$

$$= \frac{\mu}{2 \pi} \ln \left(\frac{2h}{r_w} \right) \text{ H/m}$$

Now let us come to a very important two wire line that is a coaxial cable you have studied coaxial cable so we will now see a coaxial cable. Coaxial means axis is similar that means the two cables have different radius but their axis is similar that is why it is called coaxial cable. First let me draw the coaxial cable so this is the outer radius inner radius let me say this the center is same so here I can now say this is shield. So this is the I can say this is the shield the blue color one is the outer conductor the so it is let us say that it is radius let me call it r_s r shield and for the inner conductor its radius is r_w now that is called the central conductor and we can say that this parameters of the shield is μ σ ϵ and epsilon because it is made up some dielectric material. Now you have seen so this is a cable so I will have to draw the so this is the cable and this portion is not visible that is why we are drawing it inner conductor so this is the outer conductor visible at the cross section only you can see the inner conductor. So you have already seen this analysis before only we will find the per unit length inductance and capacitance that probably you have done in your undergraduate classes or may not have done so I will do that again you know the field structure of this one that if this is the geometry then we know that the electric field is like this no fields go out that is the beauty of this it does not have any outside radiation so this is the transverse electric field and the magnetic field is magnetic field or let me take magnetic field is concentric rings so these are its like this. So from symmetry of the structure you have already know these also you know the values of this field etcetera. Now if we place a per unit length positive charge distribution in the inner wire and a per unit length negative charge distribution in the inner surface of the shield the resulting charge distribution will be uniform around the periphery of both the inner and outer conductors regardless of the value of r_w or r_s so in this case no proximity effect takes place though the two conductors are here but the field structure is uniform so in this case there is no proximity effect. Similarly if a current i flows along the inner wire surface and returns so a current i is suppose flowing like this and returning here along the interior surface of the shield this is interior surface of the shield the resulting currents will be uniformly distributed around the periphery of the conductors so here also no proximity effect that is why coaxial cable has its field structure is quite good and we can now find out that so this ϵ_t and ϵ_{ht} already this type of thing we have seen so we have our formulas now only thing that we need to see is that if I draw a surface a cylindrical surface. So this is my cylindrical surface and length let us see since we will be determining per unit length conduct per unit length parameter so let me say that this length is 1 meter and let us take that there is a small area so what color is left let us consider a small area on that conductor that we are calling ds so and the current is flowing you see if you put current here or here so if I put my thumb like this the magnetic field or magnetic flux density will be like this to my fingers so I can see that we will have to catch a little bit of black so the flux is coming like this so you can say that there is a b_t like this and b_t we know is nothing but μ into h_t vectors. So to

your better thing I can show the cross section of if I take a cross section in this red colored surface it will be something like this and what is the bt color I have taken black so it will be bt is like this so this is your bt direction and so there if you want to see that s is on that cross section s will be something like this is the this open surface as a cross section it will look like this. Now we assume the dielectric shield to be non magnet non ferromagnetic so we can assume that its mu is mu naught and we can find what is the flux linked to these surfaces so that will be s this thing we have already done again I am just showing that because this is a that time two wires simply where this is also a two-wire system but this is coaxial so to remove any confusion I am again redoing that what is the bt dot ds so you see that same everything applies here that ds portion that will be simply bt and ds they are the direction of ds and bt they are everywhere on the cylindrical surface s is collinear so this bt dot ds will instead of a dot product it will be simply scalar multiplication so this will result in r is equal to what is your r r 1 is r w and what is r 2 r 2 is you see it is r s because beyond that that means here if I show that this is your r w and this is your r s so between them only the flux is there so what is the total flux of the system linked to a surface s that will be simply mu naught i by 2 pi r dr this thing we have derived earlier the h t so let me write here itself that I know that phi m will be psi m will be mu naught i by 2 pi 1 by r will give me after integration it will give me ln r s by r w .



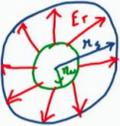
so now I am in a position to find l_e let us go to the next page so l_e is nothing but μ_m by i that is μ_0 by $2\pi \ln(r_s/r_w)$ is equal to μ_0 value we know if we put that it will become $0.2 \ln(r_s/r_w)$ micro Henry per meter so we have done again we have derived actually we could have used the result but just to show you that how to determine again I have shown how to determine the total magnetic flux penetrating a unit length surface between the wire and the shield interior surface so from that I got the inductance now how to find the capacitance of this line capacitance per unit length capacitance for that it will be even easier that you know these fields that it is are like this e_t and r_s/r_w m not let me show r_w and and this is r_s so we know what is the expression for e_t you can assume there is a positive charge distribution and a negative charge distribution so e_t will be equal to q by $2\pi\epsilon_0 r$ volt per meter this is because of this structure is symmetry just by putting an Gaussian surface between inner conductor and outer conductor you can find e_t so the voltage between the inner wire and the inner surface of the outer wire assuming inner wire is at higher potential so let me write that that is an important thing that we assuming inner wire potential is higher and then the outer ones one so what is the voltage that we now need to see v is equal to $-\int e_t \cdot dl$ and it is the voltage is from the lower potential to higher potential so lower potential is r_s and higher potential is r_w by your assumptions and so this again e_t and dl vector because if I go from here to here the dl vector is along dr and e_t is also along dr radial so instead of dot product it will be simply a scalar multiplication so that I know I can now put the value of e_t so q by $2\pi\epsilon_0$ into $\ln(r_s/r_w)$ due to this minus sign this \ln will be give us so from this we can get what is C C is q by v so it is $2\pi\epsilon_0$ by $\ln(r_s/r_w)$ equal to now we know this value if it is free space or you can assume that it is a dielectric with dielectric constant ϵ_r in that case you know that ϵ_0 is equal to $\epsilon_r \epsilon_0$ is equal to ϵ_0 naught ϵ_r ϵ_0 naught is permittivity of free space so that value yesterday I have told you that 136π etcetera so that if you put it will be $55.56 \epsilon_r$ by $\ln(r_s/r_w)$ you see always I am trying to get all the constant values factored in now in other theory classes generally this is not done but EMC is a practical subject and there an engineer should know at least this ballpark values so if as far as possible everything should be in some numbers so just by putting ϵ_r just by putting r_s/r_w you should have an idea that what the value will be near about so practical engineers they have this type of things in theory we do not do all these things we just leave it the constant value needs to be put but in practical cases this is important that unnecessarily do not carry the constants you factored in there so this 55.56 is very important to an EMC engineer may not be so important to a EMC theory practice where EMC theory theorist so this is like this and you can check whether LE and C their product if you take their product whether it is coming to ϵ_r by C square. So let us do an example or time is up so we will take that example next day then we will go to the actual this is just determination of L and C for various two type two wire conductors now we will develop the model for a radiated

susceptibility model for a two-wire line that we will take up in the next class including this example thank you.

$$L_e = \frac{\Psi_m}{I} = \frac{\mu_0}{2\pi} \ln\left(\frac{\pi_D}{\pi_w}\right)$$

$$= 0.2 \ln\left(\frac{\pi_D}{\pi_w}\right) \mu H/m$$

Capacitance



$$E_T = \frac{q}{2\pi\epsilon r} \text{ V/m}$$

any inner wire potential is higher

$$V = - \int_{\pi_w}^{\pi_D} \vec{E}_T \cdot d\vec{l} = \frac{q}{2\pi\epsilon} \ln\left(\frac{\pi_D}{\pi_w}\right)$$

$$C = \frac{2\pi\epsilon}{\ln\left(\frac{\pi_D}{\pi_w}\right)} = \frac{55.56 \epsilon_n}{\ln\left(\frac{\pi_D}{\pi_w}\right)} \text{ pF/m}$$

$\epsilon = \epsilon_0 \epsilon_r$