Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
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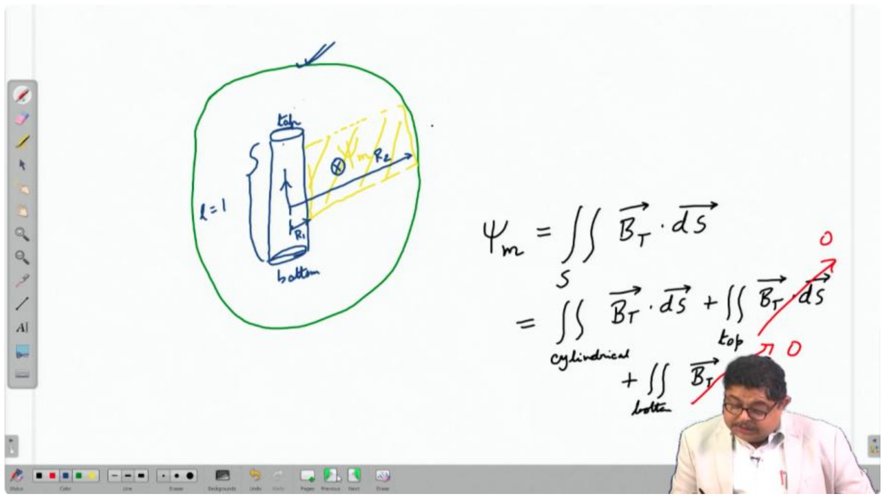
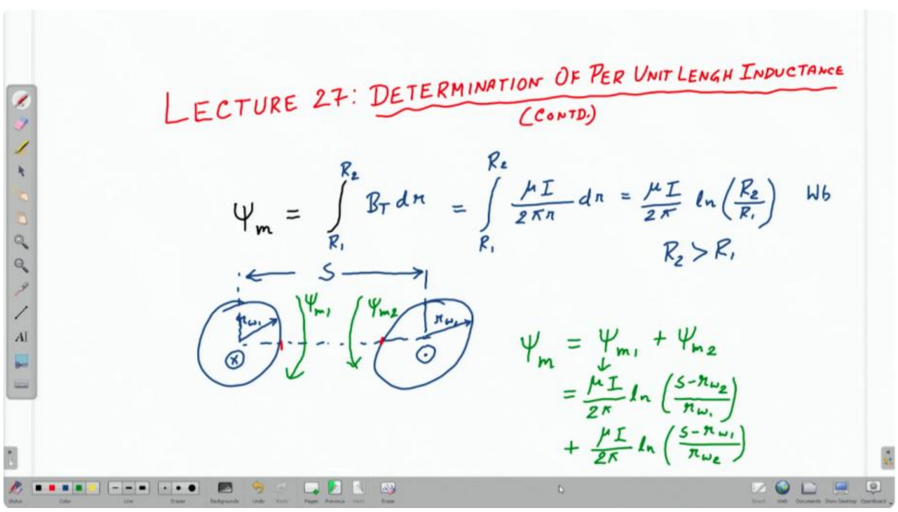
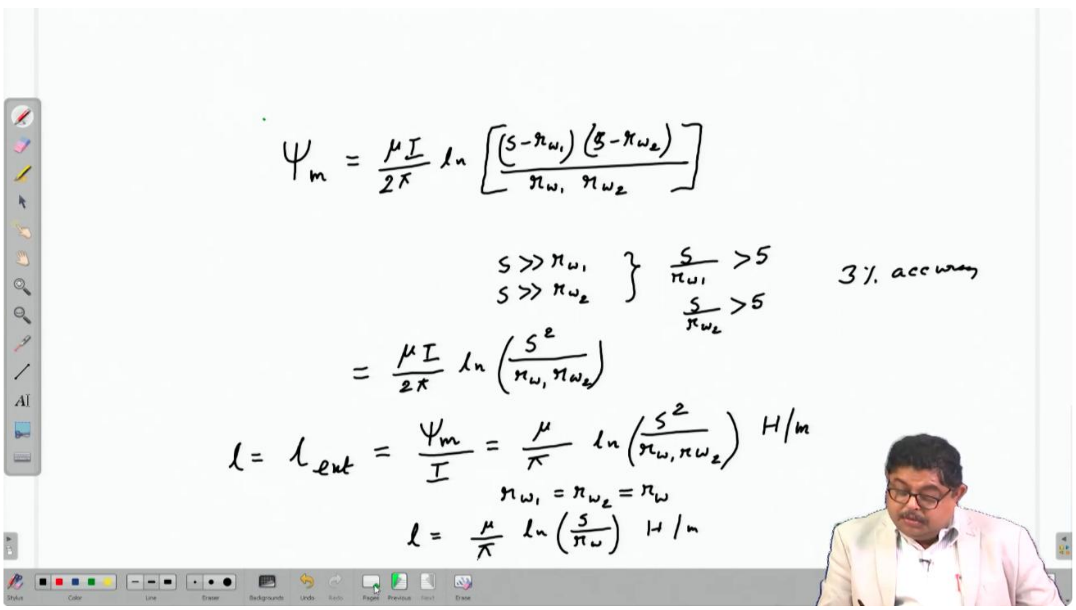
Week :06
Lecture 27: Determination of Per Unit Length Inductance (Continued)

Welcome to the 27th lecture of the course on EMIMC and Signal Integrity Principles, Techniques and Applications. We were discussing determination of per unit length inductance in our previous lecture that we are continuing. We have seen that the total flux linked by the single current, single isolated current that it contributed only by the circumference the surface that is cylindrical surface of the open transverse surface. So that we will now evaluate by integration because the field is varying with r, so we will have to be careful there, so that will be simply the for an unit length per unit length means the surface now the this surface I can say that this whatever surface I am seeing that is of length is 1, so only the radius matters, so that means I can write B t instead of ds we have already seen that it has become instead of a vector product it is a just simple multiplication. So B t ds will become B t dr and this will be from r1 the nearer part to the further part, so r1 to r2 and let us put the value of B t r1 r2 mu i i is i enclosed by 2 pi dr, so that will give us mu i by 2 pi ln r2 by r1 where for reminder we are saying that r2 is greater than r1 and the unit will be Weber because it is flux. So now instead of isolated where we consider a two-wire system carrying equal currents i in opposite direction, so our picture is somewhere two wires are there and let us say that they are carrying opposite directed current, so in one where if I assume this the other one I will be using this and this one let us say its radius is rw1 where that is why this one let us say that its radius is rw2 and the separation between their two centers that is s and let us join a hypothetical line here, so this current is flowing inside, so I will apply right hand rule to find the flux, so the flux will be in this direction I will say it is pi m1 now for this it is coming out, so this is also in the same direction that means the total flux will be added, so total flux of these two conductor system will be pi m1 plus pi m2 now already we have got expression. Now here I forgot to tell you that what whenever we are using two conductor system definitely there will be proximity effect the magnetic field will get distorted etcetera but so that means whatever we have derived for an isolated wire will not hold good here but we are neglecting the proximity effect we are considering in our model as if the two wires are isolated which is not the case but that is the flow of our model or assumption of our model if you want more realistic picture you will have to take it for a better model you will have to develop. So now we can put the values that this is mu y i is same in both mu y by 2 pi ln r2 what is r2 you see in this case r2 means up to this one so what is r2 here can I say that s minus rw2 is r2 so my expression will be s minus rw2 by what is r1, r1 is r1 means here so that I can write as rw1 this is for phi m1 and for phi m2 I will write mu y by 2 pi ln for it by similar way I can say that r2 in this case will be s minus rw1 by it will be rw2. Here we are assuming that the two wires are of different radius this is a general expression.

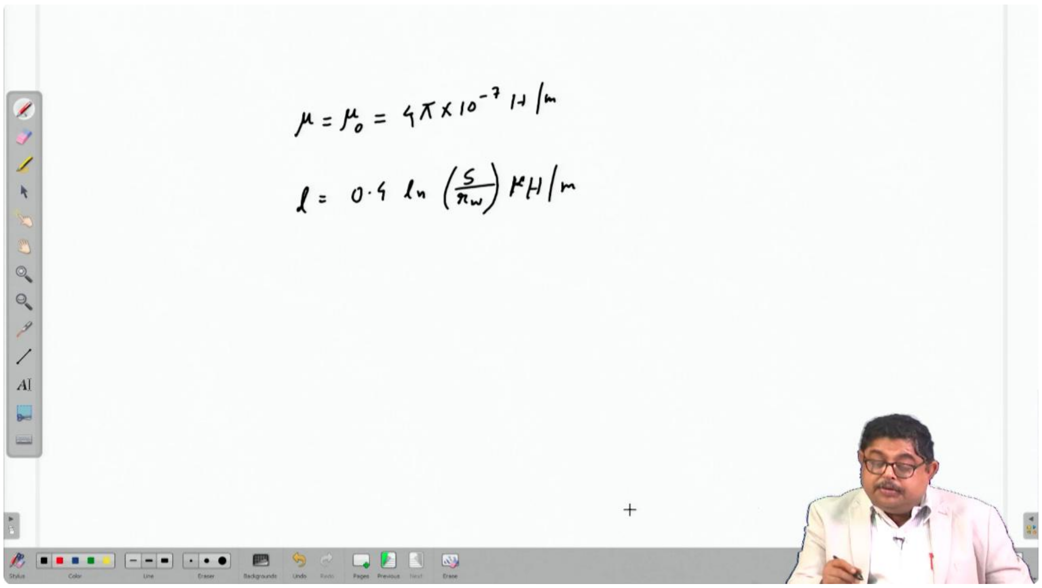




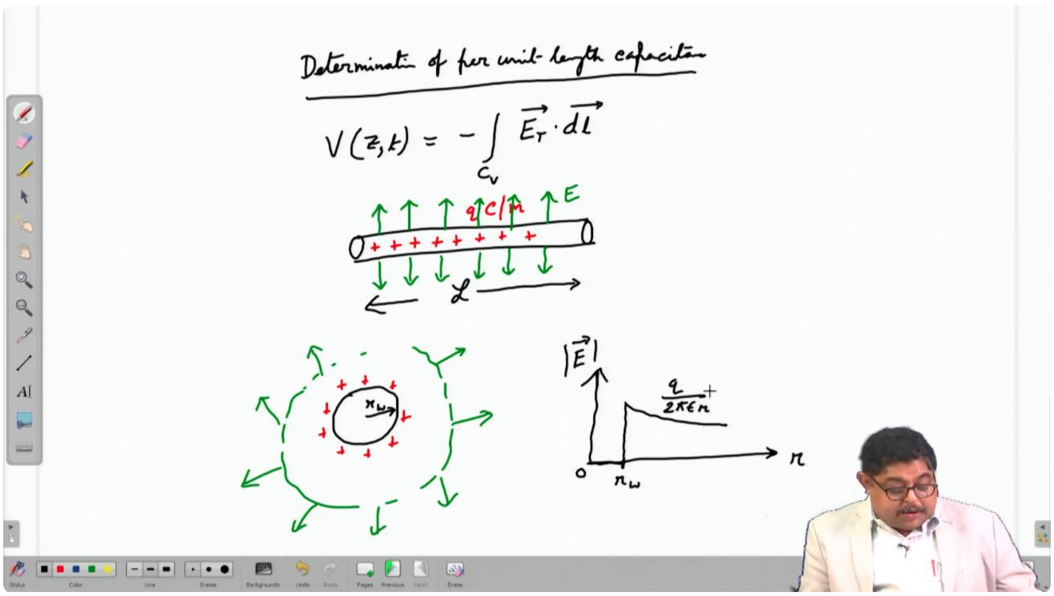
so phi m is mu y by 2 pi the total flux of the system ln s minus rw1 into rs minus rw2 by rw1 into rw2. Now here we can further simplify we can make some assumption that s is the separation of the two wires separation between their centers that in general is much larger than radius of either of the wire so our assumption is s is much larger than rw1 s is also much larger than rw2 so that means in the numerator we can write 2 pi ln s square by rw1, rw2.So if actually this assumption this is if s by rw1 is greater than 5 or s by rw2 is greater than 5 this assumption is justified or the result that will come up from this assumption that will be accurate up to 3 percent within 3 percent the error will be. If you want more realist more accurate result you should not do this, this equation you can put the values but we are proceeding with this assumption knowing that 3 percent accurate result will get. So now we have related flux with current so we can easily find what is l or l external to be specific but let us say that we will call this l so because internal inductance is not more that is simply flux of the system by current that is mu by pi ln s by rw1, rw2 and inductance is henry per meter per unit length inductance. Now if both wires are same then we put rw1 is equal to rw2 and that time l becomes mu here mu by pi this will be s square mu by i now s square by rw square so that can come out and I can write mu by pi ln is equal to let us say r, rw is s by henry per meter.



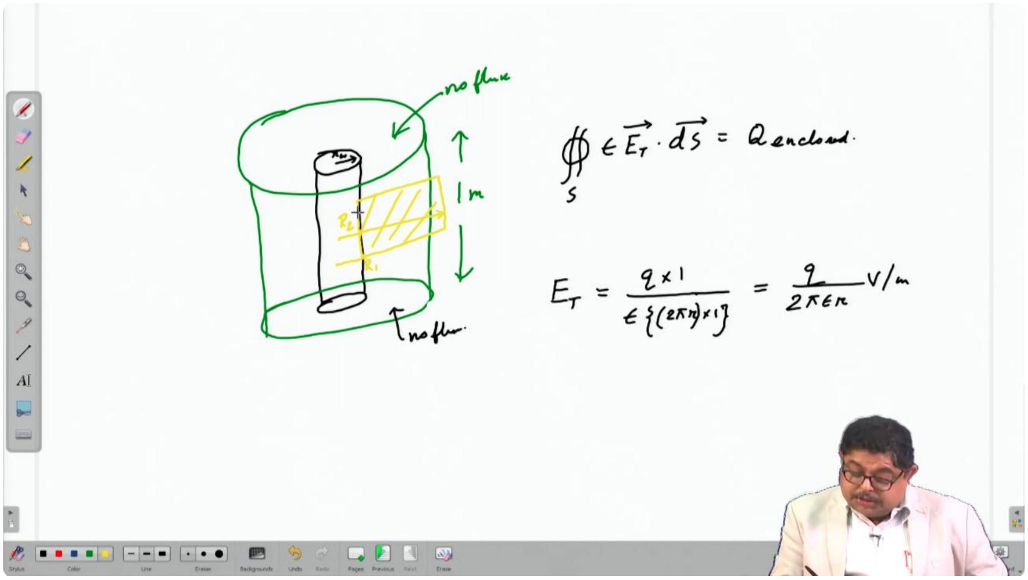
Now if we put the value of mu if mu is equal to mu naught that means it is in free space or the medium between the two transmission two lines are free space then it is mu naught and you know the value of mu naught 4 pi into 10 to the power minus 7 henry per meter. So you get l is equal to if you do the calculation 0.54 ln s by rw henry micro henry per meter I have absorbed that 10 to the power minus 6 there in micro henry per meter. Now so this is about determination of an inductance that means from the geometry and the medium you saw that if we know the separation distance if we know the wire radius and if we know the intervening medium then you can derive the per unit length capacitance per unit length inductance.



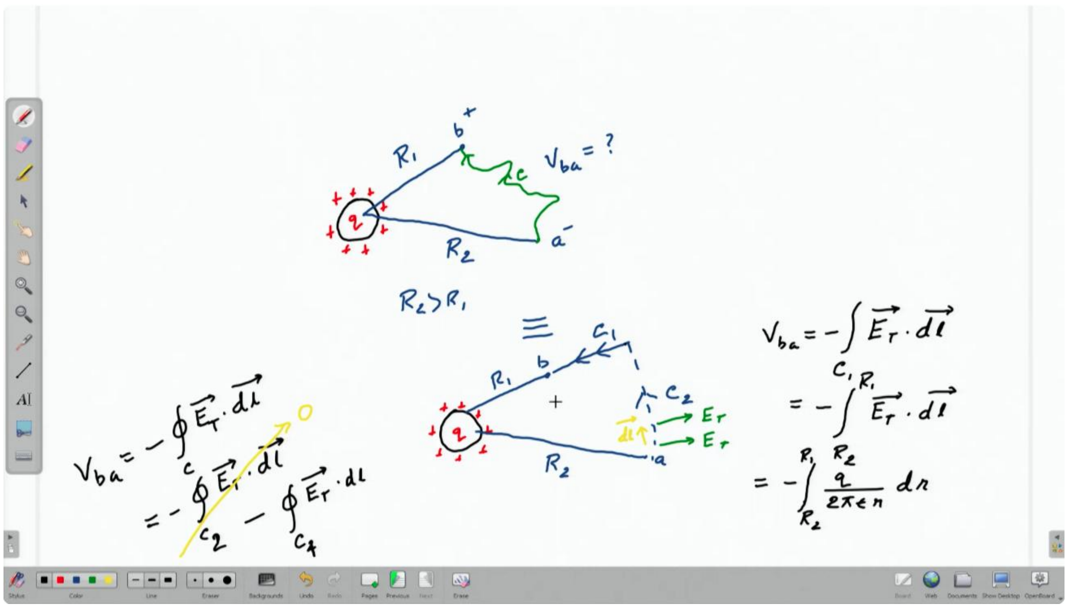
Now let us go to determination of per unit length capacitance. So, here again we have seen the transmission line can be analyzed in a static way. So we can write V Z t is equal to minus contour across any path E t dot d line integral path. So again consider a an isolated conducting wire carrying a per unit length charge distribution Q coulomb per meter. So let us say that there are charge in the line it is like static case so this distribution is Q coulomb per meter and this length first let us call l actually determination time will put l is equal to 1 and we know that if we have this type of charge what will happen to the electric field electric field will be like this again we are not considering the end points because at the end there will be some fringing etcetera all those we are neglecting. So there will be this is the green colored ones are the electric field that will come out from here E fields. So now we know that because of symmetry the that means basically if I now see the cross section of the wire so it is R w here and the charge is on the outside surface because in the conductor there cannot be any charge so this charge distribution is like this and let us consider a Gaussian surface and from that we know that this electric field will be like this symmetrical because the charge distribution is symmetrical and you know all of you will be able to plot the what is the E versus R distribution so this is your E magnitude that you know that up to R w there is not anything unlike the magnetic case where inside also there are magnetic fields but here this will be 0 sorry 0 this is R w and then we will get suddenly a jump in the electric field and then we have seen that it falls down with a 2 pi by epsilon simply by doing Gauss's law applying Gauss's law you can find this out that this will be the electric field distribution.



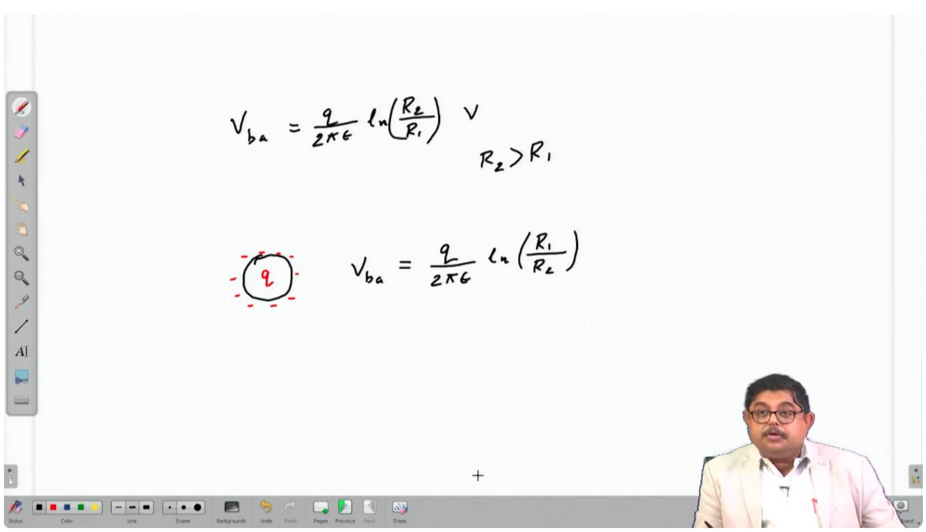
Now to determine this transverse electric field we consider a cylindrical Gaussian surface of radius R so let me take a cylinder and its radius is R w and let me take a Gaussian surface around it and now let me take that this cylinder is length is 1 meter and we know since the from our previous structure that the fluxes are all going away that means they are all like this so this top does not contribute any flux because the top in top there is the ds vector that is on top but E t is going away so now we can write the equation which governs this that is epsilon E t dot ds is equal to q enclosed so no flux from the top no flux from the bottom so only the cylindrical curved surface contributes to flux so and that flux you can easily calculate by your knowledge of Gauss's law that it will be q into 1 by epsilon 2 pi R into 1 is equal to q by 2 pi epsilon R volts per meter. Now again the same question that if we have this surface we will have to find the flux electric flux now inside there is no flux so in this cylindrical portion what is the flux so if I call this as R1 from the center and this distance is R2 from the center so in this cylindrical surface that means in the transverse plane what is the flux that is got.



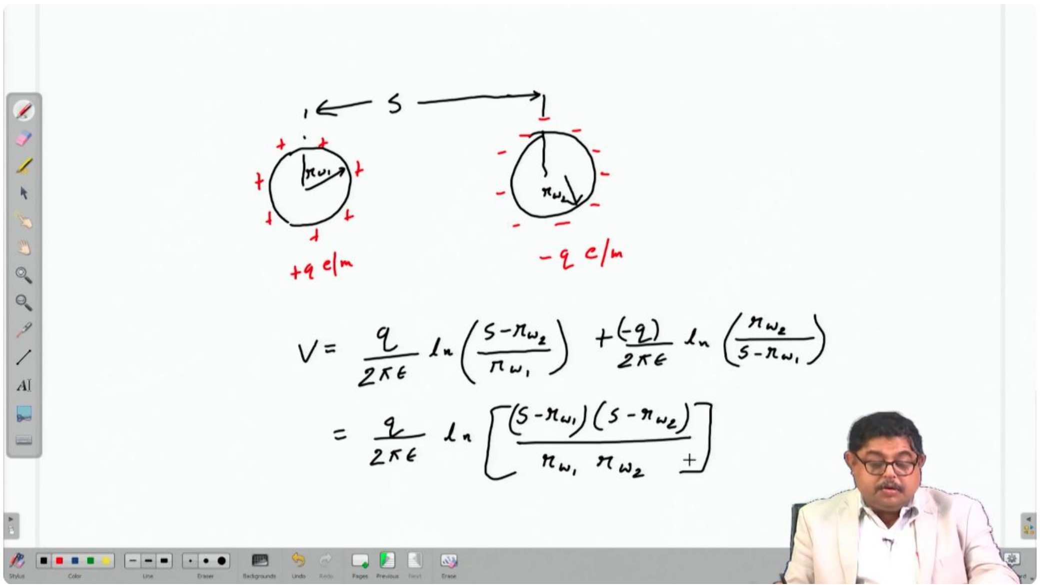
So, before that let us find the question that suppose pose the question like this suppose I have a charge here let the charge is like this plus q charge and what is the voltage that is there between two points suppose arbitrarily I am taking one point B that is at distance R1 from the charge this is a point charge I have shown it in bigger way but this is a point charge and another point let us say at B deliberately I have taken A further away from B and let its distance is called R2 so R2 is greater than R1 and what is the voltage between the point A and B where I am starting the integral from A I am going to be E dot dl I am finding so basically I am finding what is VBA so I can go line integral so I can go on any path like this to reach this let us call this path as C. So, since I am going from A to B so let me call that this is at A is at a lower potential and B is at a higher potential. So, instead of these an equivalent problem is I can write like this that same charge q and I am first going to A from here that is I am going to A and then from there I am taking a semicircular path or let me put it in dash this is my path of integration A and then from there this is my point B this is R1 so I am coming back in this path this will also do so let me call that this is my C2 this is my C1 so I am reaching here so that means first I am starting from here I am going by C2 this is a part of a circle with radius R2 I am reaching at this point and then coming back these are same because for line integral it is immaterial what path I take and I know that all the ETs are like this from the charge distribution you see ET will be like this. So, let us take the integral VBA that is our objective that is minus C ET dot dl I can break these originally it was path C I can breaking it into C1 ET dot dl minus C2 first I am going by C2 so let me write the C2 then C1 ET dot dl. So along C2 you see ET and dl vectors what will be dl vectors these are my dl vectors at various points so dl and C E2 ET they are orthogonal so that means this part will go out this will give rise to 0 only along this point the ET and dl they are in the same direction so I can write now VBA is equal to minus C1 ET dot dl is equal to minus C1 means I am starting from R2 because these distance is R2 and these distance is R1 ET dot dl I know the value of ET what is ET Q by 2 pi epsilon R and dl I am writing as dr because in this path they are same .



so I can see that VBA will become Q by 2 pi epsilon ln R2 by R1 volts where R2 is greater than R1. Just similarly you can do yourself a problem that if you have a negative charge distribution for that you will see that VBA you will have to redo just do not put Q is equal to minus because many things are different there but this will give rise to the result I am doing anyone can easily do it but you will have to do it ln R1 by R2 with this



now we are in a position to see what happens if we have two conductors one is positively charged another is negatively charged again we have brought we are not considering any proximity etc. so this is rw1 this is rw2 and this distribution is plus Q coulomb per meter this distribution is minus Q coulomb per meter so what is our V? V will be for this one this isolated one Q by 2 pi epsilon ln S minus rw2 again you see that what is the maximum oh I have not put the S part so S is the distance between the two centers so R2 is in this case S minus rw2 by rw1 plus Q by 2 pi epsilon ln rw2 by S minus rw1. Q charge this Q I assume actually this Q if positive and negative both are together this Q will become a minus Q this portion is so this will be again Q by 2 pi epsilon ln S minus rw1 into S minus rw2 by rw1 rw2.



Then assuming S is much larger than rw1 rw2 we get V is equal to Q by 2 pi epsilon ln S square by rw1 rw2 then taking rw1 is equal to rw2 is equal to rw we get V is equal to Q by pi epsilon ln S by rw and then per unit length capacitance C is equal to Q by V is equal to pi epsilon by ln S by rw. So, if epsilon is equal to epsilon naught C becomes you can put the value epsilon naught means 1 by 36 pi into 10 to the power 36 pi into 10 to the power minus 9 farad per meter if you put this value it will be 27.78 by ln S by rw pico farad per meter. So, you can cross check we have got L we have got C what is the L C this is you can put the value it will come as 1.11 into 10 to the power minus 17 and you can see that this is L C will be equal to mu into epsilon naught. So, you can mu naught into epsilon naught you can put that value. So, as I said that 1 by this will be 1 by root over L C should is coming to 2.999 into 10 to the power 8 meter per second which is the velocity of light in free space. So, we have determined both inductance and capacitance from first principles. Now, we will see some practical problems in the next class. Thank you.