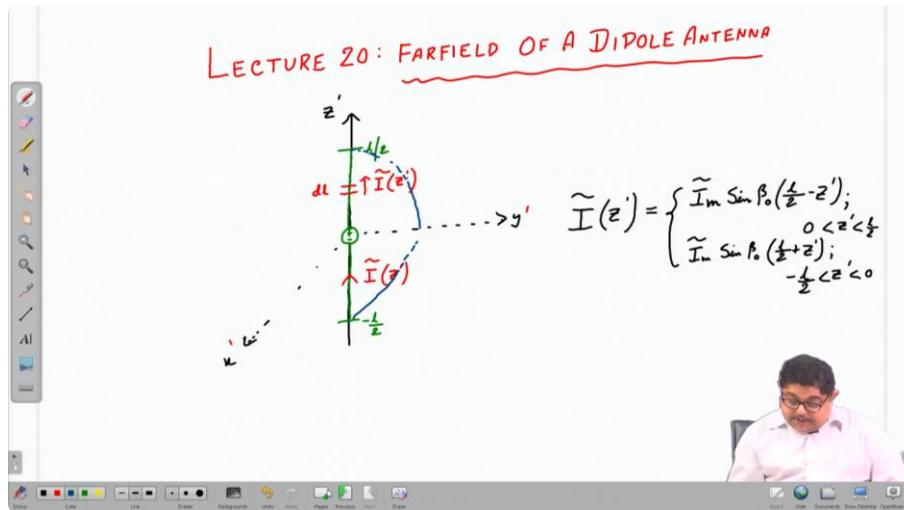


Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.
Professor name: Prof. Amitabha Bhattacharya
Department name: Electronics and Electrical Communication Engineering
Institute name: IIT Kharagpur
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Lecture 20: Farfield of Dipole Antenna

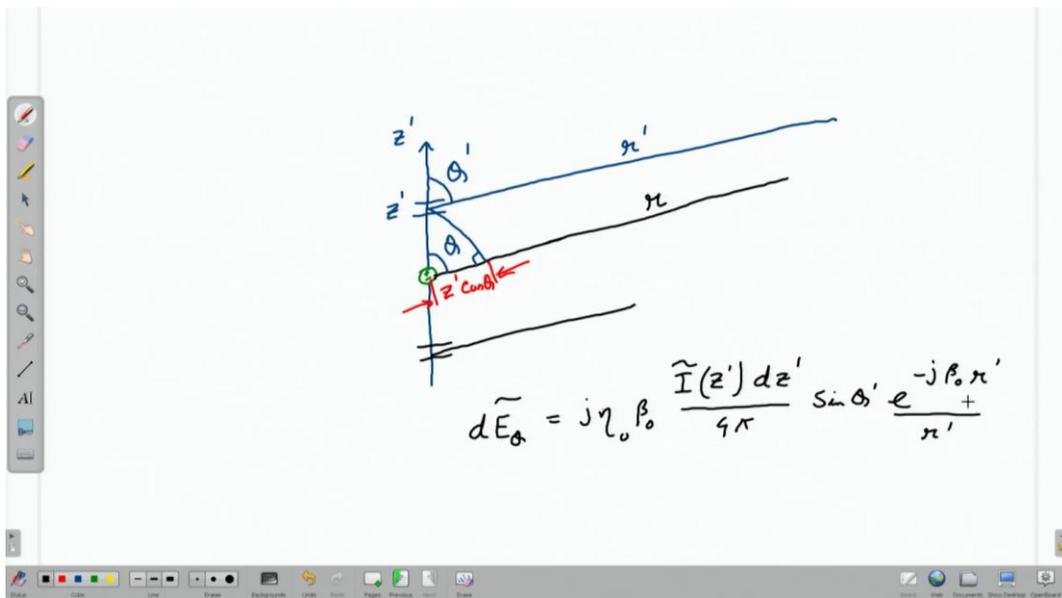
Welcome to the 20th lecture of the course on EMIMC and Signal Integrity Principles Techniques and Applications. In previous class we have discussed the necessity of current element, but we also saw that it is a very inefficient radiator, its radiation resistance is very low. So, we need to find a better radiator fortunately a better radiator is available that is a dipole antenna. Also you see that the current along the current element is assumed to be constant. So, at the end points current is non-zero. Now, that is not a practical case because any antenna it is at the end points of the antenna the current should go to 0. Obviously, if they are from the displacement current also is goes, but the conduction current should go to 0 at the end otherwise there will be charge accumulation etcetera at the end of the antenna. So, what is a dipole antenna? It is a thin wire and driven by a voltage source at the midpoint voltage source or current source any source at the midpoint. So, first let me draw the current element in sorry dipole antenna. So, let me draw the current element in the is a dipole antenna its length is L . Now, I have chosen my coordinate axis to be collinear or at the same point with the voltage source. We are assuming the voltage source does not occupy any current any space it is just a point and there is our coordinate. Now, the dipole antennas length is L and it is aligned along our coordinate systems Z axis, but somehow I am calling it Z dash axis why because actually let us take a convention that generally is taken in antenna analysis that source things are given as primed quantities like Z dash. Since our antenna currents etcetera antenna's current etcetera along the Z axis we are calling it Z dash to distinguish it that in observation point that is at unprimed coordinate. So, actually I should have called this also Y dash and this is X dash. So, our source coordinates are prime coordinates and observation coordinates are unprimed coordinates. Now, a reasonable guess for this as I said that antennas any finite antennas current should go to 0 at the end point. So, you see a guess also people have found by measurements that some more or less this is the case that what is the current distribution on this antenna that means what is I Z dash it is basically obviously it is maximum at the source point and goes to 0 at end point. So, people have found that this is a reasonable guess that the maximum also it is we have seen that it should depend on antenna's wavelength through beta naught. So, $\sin \beta \text{naught } L \text{ by } 2 \text{ minus } Z \text{ dash}$ for $0 \text{ less than } Z \text{ dash less than } L \text{ by } 2$ and $I \text{ m } I \text{ m}$ is a constant $\sin \beta \text{naught } L \text{ by } 2 \text{ plus } Z \text{ dash}$. So, for $\text{minus } L \text{ by } 2 \text{ minus less than } 0$.

So, you see here if I put first check whether at Z dashed the current is maximum or not you see that at Z dashed is equal to 0 the current is $\sin \beta naught L$ by 2 also the current is continuous at Z dashed is equal to 0 and at Z dashed is equal to L by 2 it goes to 0. So, you can say that roughly the current distribution is sinusoidal over the now let us consider that DL I have shown let us consider it as a current element. So, I can think that there are different current elements one as I have shown on above another one. So, there are many such current elements along the Z dashed line and I know the field far field radiated by this current element. So, if I can properly take the vector addition of all the fields radiated by all the current elements I will get the far field of this dipole antenna actually that is what I said in the previous lecture that if you know the current element well you can construct the far fields by simply taking the help of linearity and properly taking the vector addition of all the radiated field.



So, now let us see that picture that far field means I am far from the antenna. So, let us consider that again that current element thing let me consider and suppose the current element is here. So, the and observation point now I it is very away. So, let us say that and also this is the centre. So, if there is another current element here now I can say that the r from this one and r from this current element they are parallel rays they are all meeting at the far field at point P point P is far away, but from here when I look they are all parallel. So, this is my r dashed because this one I am taking as my thing this is my z dashed and I consider another one that is or the voltage source here. So, here is the voltage source and there is the coordinate. So, that one I consider as another. So, from here also there is a going to the observation point actually I should not show the P because P is not there it cannot be seen all are parallel rays and I can say that this length is r and this angle this angle is θ dashed and this angle I should then call θ and now what is the difference in path length of the two because you know that e to the

power minus $j\beta_0 r$. So, this ray is also traversing this ray is also traversing, but due to the position of this one with respect to this one there should be a difference in path length. So, what is that for that I will have to take a perpendicular on this line and see that what is the distance. So, obviously, this with respect to this this is having a smaller distance. So, that I need to calculate that what is this length that means what is this length. So, from the geometry you can easily see that this is $Z \cos \theta$. Now, again concentrate on this current element at $Z \cos \theta$. So, what is the field at point P due to this current element? So, that now call me dE_θ because I will be having many such E field I will have to some name. So, dE_θ is equal to look at your expressions sorry it is $\eta_0 \beta_0 I Z \cos \theta dZ$ by $4\pi r^2 \sin \theta$ to the power minus $j\beta_0 r$ dashed by r dashed ok. You see likewise others for others also I can write and let us then write them all in terms of r . Now, actually this $Z \cos \theta$ that is much much less than r . So, in the amplitude terms they may be assumed to be same as r that means this r dashed term this is actually not the part of this phase term. So, r dashed for all r dashed can be considered as equal to r in amplitude part, but you know that phase is very sensitive because the small change in phase actually that gives antenna its property its different current elements on it they their fields with a small change of those r dashed the field may be widely separated.



So, suppose to emphasize this point let us say I will again come back here that suppose 2 far field points are one is at r is equal to 1 kilometer and another is 1001 meter that means both are almost same in the amplitude suppose f is equal to 300 megahertz. So, what is $\beta_0 r$ dashed let us call ah this this is r dashed is 1001. So, $\beta_0 r$ not r dashed

you can calculate lambda naught value what will be lambda naught? Lambda naught is 3 into 10 to the power 8 by 3 into 10 to the power 8. So, it is 1 meter. So, so beta naught 0 is ah it will be if you calculate 2 pi 2 pi by 1 into r dashed that is 1001 or you can call this one even smaller let us take it even smaller that 1000.5 m. So, it will be 1000.5 that will be 360180 degree and let us calculate beta naught r it is 2 pi by 1 into 1000 that is 360000. So, you see e to the power beta naught r dashed and e to the power beta naught r their difference is in phase 180 degree that means if one value is positive another is negative. So, even though the separation is only 0.5 meter the phase of the field at two points are 180 degree out of phase ok. So, that is why we cannot neglect or we cannot take that all r dashed r is equal to r in phase term, but in amplitude term they does not matter. So, we can take that.

$r = 1 \text{ km}$
 $r' = 1000.5 \text{ m}$
 $f = 300 \text{ MHz}$
 $\lambda_0 = \frac{3 \times 10^8}{3 \times 10^8} = 1 \text{ m}$
 $\beta_0 r' = \frac{2\pi}{1} \times 1000.5 = 3,60,180^\circ$
 $\beta_0 r = \frac{2\pi}{1} \times 1000 = 3,60,000$

So, now if I go to the previous page what is the r dashed? So, r dashed is can I write r minus j dashed cos theta ok. So, now, d e theta I can write here that ah let me since or here I write d e theta is j eta naught beta naught I z dashed d z dashed by 4 pi sin theta naught e to the power minus j beta r minus j dashed cos theta by r ok. So, d so, d e theta we have found.

$$d\tilde{E}_\theta = j\eta_0\beta_0 \frac{\tilde{I}(z') dz'}{4\pi} \sin\theta' \frac{e^{-j\beta_0 r'}}{r'}$$

$$r' = r - z' \cos\theta$$

$$d\tilde{E}_\theta = j\eta_0\beta_0 \frac{\tilde{I}(z') dz'}{4\pi} \sin\theta' \frac{e^{-j\beta_0 (r - z' \cos\theta)}}{r}$$

So, now, we can integrate. So, the total things actually now our integration because became easier because it is an integration over z axis only everything we converted in terms of z axis. So, I can write the far field of the dipole that will be j eta naught beta naught for all theta naughts we can also take a theta dash we can also take theta because this is the far field they are all parallel. So, sin theta is there now I can take out that r outside and e to the power minus j beta naught r this part also I have taken and there will be an I z dashed where what is I z dashed that is the now it is an integration I z dashed is equal to minus L by 2 to L by 2 I z dashed e to the power j beta naught z dashed cos theta d z dashed. So, this one will be this let us see whether in the previous one yes. So, this so, now, we can put the value of I m, but I z dashed if we look where this I z dashed if you see the expression. So, and if you say this integral this integral is symmetric about z dashed is equal to 0. So, I can write is equal to 2 integration from 0 to L by 2 I by putting the value of I z dashed sin beta naught L by 2 minus z dashed e to the power j beta naught z dashed cos theta d z dashed. So, this is z dashed I will have to integrate. So, you see here is a z dashed. So, the way is to convert this sin term into by Euler's identity you can write it as e to the power j beta naught L by 2 minus z dashed minus e to the power minus j beta naught L by 2 minus z dashed by 2 j and proceed. So, that thing will give you and then you will get these and these two together. So, you do that integration that is a pretty simple integration that will give you minus 2 I m by beta naught cos of beta naught L by 2 cos theta minus cos of beta naught L by 2 ok this will be the outcome of that integration. So, we got I j is this. So, we will have to put I z dashed here.

$$\tilde{E}_{\theta} = j \eta_0 \beta_0 \frac{\sin \theta}{4 \pi r} e^{-j \beta_0 r} \tilde{I}(z')$$

where,

$$\tilde{I}(z') = \int_{-L/2}^{L/2} \tilde{I}(z') e^{j \beta_0 z' \cos \theta} dz'$$

$$= 2 \int_0^{L/2} \tilde{I}_m \sin \beta_0 \left(\frac{L}{2} - z' \right) e^{j \beta_0 z' \cos \theta} dz'$$

$$= \frac{-2 \tilde{I}_m}{\beta_0} \left[\cos \left(\frac{\beta_0 L}{2} \cos \theta \right) - \cos \left(\beta_0 \frac{L}{2} \right) \right]$$

So, e theta will become j eta naught beta naught sin theta by 4 pi r e to the power minus j beta naught r minus 2 I m by beta naught into cos of beta naught L by 2 cos theta

whether this space no minus cos of beta naught L by 2 then sin theta. Let me check previous word ah minus 2 I m by beta naught this minus 2 I m by beta naught is ok.

So, I can write it as minus j eta naught I m e to the power minus j beta naught r by 2 pi r into a function of theta a function of theta where what is that function f theta is cos beta naught L by 2 cos theta minus cos beta naught L by 2 multiplied by the sin theta from here. So, there is a theta dependent term and there is an remaining portion here. So, this is the dipoles field and this one we can further put the value of eta naught that is 120 pi. So, it will go. So, then I can write that finally, e theta is so, e theta is minus j 60 I m e to the power minus j beta naught r by r f theta. So, we have derived this later we will see actually that was our aim you can write I think the magnetic field also from here just you divide by 120 pi that will be your magnetic field and so, from there we will try to arrive at our desired thing we have seen two antennas one is current element using that current element we have found the far field of a dipole antenna. So, from there we will go in the next class we will discuss some more things and then we will go to our radiated emission model actually that is our aim. So, we are modeling the antenna part which is the heart of the radiated emission problem. Thank you.

$$\begin{aligned} \tilde{E}_{\theta} &= j \eta_0 \beta_0 \frac{\sin \theta}{4\pi r} e^{-j\beta_0 r} \left(\frac{-2\tilde{I}_m}{\beta_0} \right) \left[\cos\left(\beta_0 \frac{L}{2} \cos \theta\right) - \cos\left(\beta_0 \frac{L}{2}\right) \right] \\ &= -j \eta_0 \tilde{I}_m \frac{e^{-j\beta_0 r}}{2\pi r} F(\theta) \\ \text{where } F(\theta) &= \left[\cos\left(\beta_0 \frac{L}{2} \cos \theta\right) - \cos\left(\beta_0 \frac{L}{2}\right) \right] \sin \theta \\ \tilde{E}_{\theta} &= -j \frac{60 \tilde{I}_m e^{-j\beta_0 r}}{r} F(\theta) \end{aligned}$$