

Course name: EMI /EMC and Signal Integrity: Principles, Techniques and Applications.

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Lecture 19: Farfield Characteristics of Current Element: Some Discussion

Welcome to the course on EMI, EMC and Signal Integrity Principles, Techniques and Applications. Today is the 19th lecture. Yesterday, we were discussing the far field of current element or Hertzian Dipole. From there we derived several characteristic of the far field of a of an current element and from that we have seen that the far field is inversely proportional to the distance of the observation point from the current element and we have derived that extrapolation relation. So, let us take an example on that. Suppose I have a current element, it has a length of 1 centimeter, it carries a current of 1 ampere at 100 megahertz. Now, let us determine first whether it can be used as a current element or not. So, because that depends on what is the electrical length of this so of the antenna. So, at 100 megahertz let us find the lambda naught, lambda naught will be 3 meter. So, the dipole length is 1 centimeter that means, lambda naught so, we can say that dl by lambda naught that will be the electrical length that will come to be 1 by 300.

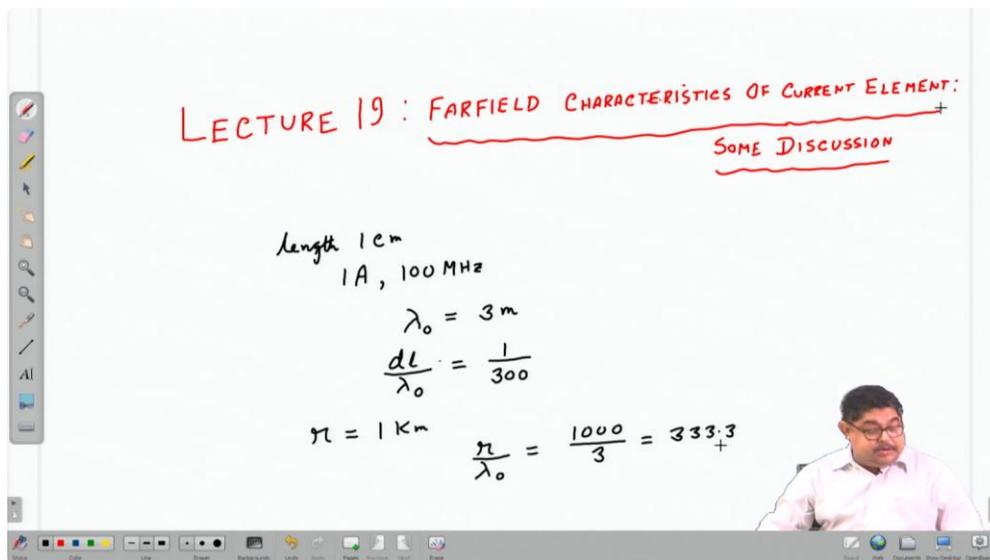
So, 1 by 300 now generally if it is less than 1 by 10, we call it a current element you can consider it as a current element. So, this is pretty less than 1 by 10. So, this is electrically a short antenna. Now, suppose we know or we want to calculate the far field at a distance of 1 kilometer, far field at a distance of 1 kilometer away and broadside to the antenna. I explained broadside yesterday so that means, theta is equal to 90 degree in our case so, broadside. Now, first check whether r is equal to 1 kilometer is at far field or not. So, let us do that the observation point. So, r by lambda naught is what r is 1 kilometer lambda naught is 3 meter. So, we can say that this ratio is 1000 by sorry 1000 by 3 that is 333.3 the ratio usually here we say that r by lambda naught I have given you the formula. So, this is an wire antenna. So, if it is more than 3 lambda naught we can consider it to be far field. So, this is much much greater. So, 1 kilometer away observation point is definitely at the far field. So, let us find the E far and H far.

LECTURE 19 : FARFIELD CHARACTERISTICS OF CURRENT ELEMENT:
SOME DISCUSSION

length 1cm
1A, 100 MHz

$$\lambda_0 = 3m$$

$$\frac{dl}{\lambda_0} = \frac{1}{300}$$

$$r = 1km \quad \frac{r}{\lambda_0} = \frac{1000}{3} = 333.3$$


So, we have derived those formulas E far is you can see the formula. So, I can write J then f mu f. So, mu is 4 pi into 10 to the power minus 7 mu f by 2 f is 100 megahertz mu f by 2 then e to the power minus j beta naught r by r. So, e to the power minus j beta naught that is 2 pi by 3 and this is 1000 by r is 10 to the power 3. So, the unit of electric field will be volts per meter. So, if you calculate this becomes that 6.2831 into 10 to the power minus 4 this j you can absorb in the phase part. So, if you do that phase part it will come as minus 2000 pi by 3 and j for j you can add a pi by 2. So, this is 6.2831 into 10 to the power minus 4 and this one if you do it will be minus 30 degree volts per meter.

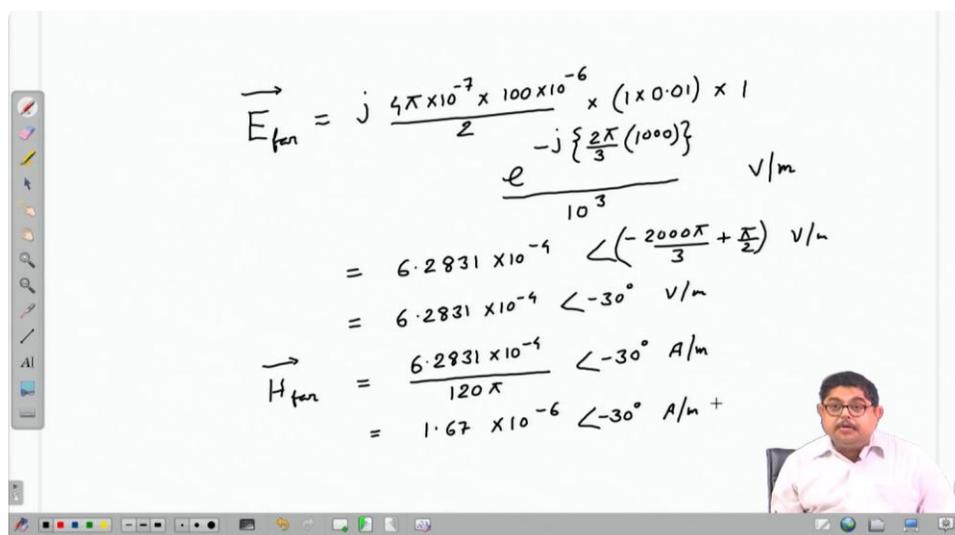
So, you can see that it is roughly 62.8 milli volt and it is phase is minus 30 degree. Now, H far is simple if you know E far you can easily find H far. So, H far is this same thing only you will have to divide by eta naught. So, 6.2831 into 10 to the power minus 4 by eta naught is 120 Pi or 377 whatever and this will be minus 30 degree then it is unit is ampere per meter. So, this one will be 1.67 into 10 to the power minus 6 minus 30 degree ampere per meter. So, these are the far fields.

$$\vec{E}_{far} = j \frac{4\pi \times 10^{-7} \times 100 \times 10^{-6} \times (1 \times 0.01) \times 1}{2} \frac{e^{-j\left\{\frac{2\pi}{3}(1000)\right\}}}{10^3} \text{ V/m}$$

$$= 6.2831 \times 10^{-4} \angle \left(-\frac{2000\pi}{3} + \frac{\pi}{2} \right) \text{ V/m}$$

$$= 6.2831 \times 10^{-4} \angle -30^\circ \text{ V/m}$$

$$\vec{H}_{far} = \frac{6.2831 \times 10^{-4}}{120\pi} \angle -30^\circ \text{ A/m}$$

$$= 1.67 \times 10^{-6} \angle -30^\circ \text{ A/m}$$


Now suppose now do the extrapolation problem that the magnitude of the far electric field of a hertzian dipole is measured at a distance of 100 meter let us say that 1 hertzian dipole its measurement is at at r is equal to 100 meter which is at far field that is given that the E far magnitude is 1 milli volt per meter let us say. So, let us extrapolate these two what is the E far at 1 kilometer. So, we can again check that whether both these 100 meter and 1 kilometer are in the far field. So, 100 meter means 100 meter is 100 by $3\lambda_0$ and that is 33.33. So, it is still greater than $3\lambda_0$. So, it is the measured field is indeed a far field and obviously, 1 kilometer will be 333.33 which is also at far field. So, we can apply our formula. So, from our formula E_{D2} this formula I gave in the last class. So, E_{D2} will be D_1 by D_2 E_{D1} magnitude. So, if you put this that 100 by 1000 into 1 milli volt per meter 1 milli volt per meter. So, it will be you can say 100 micro volt per meter.

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$$r = 100 \text{ m}$$

$$|\vec{E}_{far}| = 1 \text{ mV/m}$$

$$|\vec{E}_{far}| \text{ at } 1 \text{ km?}$$

$$100 \text{ m} = \frac{100}{3} \lambda_0 = 33.33 \lambda_0$$

$$|\vec{E}_{D2}| = \left(\frac{D_1}{D_2}\right) |\vec{E}_{D1}|$$

$$= \frac{100}{1000} \times 1 \text{ mV/m}$$

$$= 100 \mu\text{V/m}_+$$

So, you see in the far field we can extrapolate the things. Now, let us find in the far field what is the power that is being carried out. So, we can we know that at any point the pointing vector in phasor form is half real $\vec{E} \times \vec{H}^*$ this is the average power flow density vector. So, in our case we can come that it will be we have only E_θ and H_ϕ components. So, they can give me what if we take their cos product we get $E_\theta H_\phi$ U_r minus E_r let me write phasor $E_r H_\phi$ if they have all the components I am writing the general expression U_θ , but in our case you see E_r is not there. So, this U_θ component would not be there. So, our power ah pointing vector is pointing towards U_r

direction that means, away from the antenna there is an average power flow density that means, power is flowing radially outwards from the antenna and we know what is this half you see our expressions $E_\theta H_\phi^*$. So, real part of that if you take it will come as $I dl$ square you have all these expressions E_θ and H_ϕ just take H_ϕ^* that is the complex conjugate of that and you will get it. I am not going into this simple mathematical details $E_\theta H_\phi^* \sin^2 \theta$ by r^2 then $U r$. So, if you simplify that it will come to be because there is 4π you can put the value β_0 you can express in terms of λ_0 etcetera. So, this gives you $15 \pi dl$ by λ_0^2 whole square I am writing like this because this is the electrical length then there will be the current that is flowing through the current element then $\sin^2 \theta$ by r^2 $U r$ and its unit will be watts per meter square. So, this is the power flow density. So, what is the power flow? We can integrate this over a sphere of radius r enclosing the antenna that will give us the total power radiated P_{rad} radiated power P_{rad} is that $\vec{S}_{avg} \cdot d\vec{s}$ vector and that we know that the radius. So, that it is a surface. So, over a sphere the surface is a spherical surface. So, that will have the integrals 0 to 2π 0 to 2π generally we take θ from 0 to 2π or you can take any other choice of θ , but it should be 180 degree and the for ϕ generally we take full 0 to 2π . So, this $\vec{S}_{avg} \cdot d\vec{s}$, \vec{S}_{avg} you can put $15 \pi dl$ by λ_0^2 square $\sin^2 \theta$ by r^2 $U r$ dot. What is the surface elemental surface area that is $r^2 \sin \theta d\theta d\phi$ and its direction is \hat{u}_r and its unit will be watts.

The image shows a handwritten derivation on a whiteboard. The first part calculates the average power flow density \vec{S}_{avg} as the real part of $\vec{E} \times \vec{H}^*$. It then simplifies this to $\frac{1}{2} \frac{(\tilde{I} dl)^2}{(4\pi)^2} \eta_0 \beta_0^2 \sin^2 \theta \frac{1}{r^2} \hat{u}_r$, which is further simplified to $15 \pi \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}|^2 \frac{\sin^2 \theta}{r^2} \hat{u}_r$ W/m². The second part calculates the total radiated power P_{rad} by integrating $\vec{S}_{avg} \cdot d\vec{s}$ over a spherical surface, resulting in $\int_0^{2\pi} \int_0^\pi 15 \pi \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}|^2 \frac{\sin^2 \theta}{r^2} \hat{u}_r \cdot \pi r^2 \sin \theta d\theta d\phi \hat{u}_r$ Watts.

$$\vec{S}_{avg} = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*]$$

$$= \frac{1}{2} \text{Re} [\tilde{E}_\theta \tilde{H}_\phi^* \hat{u}_r - \tilde{E}_r \tilde{H}_\theta^* \hat{u}_\theta]$$

$$= \frac{1}{2} \frac{(\tilde{I} dl)^2}{(4\pi)^2} \eta_0 \beta_0^2 \sin^2 \theta \frac{1}{r^2} \hat{u}_r$$

$$= 15 \pi \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}|^2 \frac{\sin^2 \theta}{r^2} \hat{u}_r \text{ W/m}^2$$

$$P_{rad} = \oint \vec{S}_{avg} \cdot d\vec{s}$$

$$= \int_0^{2\pi} \int_0^\pi 15 \pi \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}|^2 \frac{\sin^2 \theta}{r^2} \hat{u}_r \cdot \pi r^2 \sin \theta d\theta d\phi \hat{u}_r$$

Watts

So, if I carry out this integration it is you will get it is basically a you see $\sin^3 \theta$ integration there are no ϕ variation. So, 0 to 2π $d\phi$ will give you a 2π and $\sin^3 \theta d\theta$ that is integration from 0 to π that will give you $4/3$. So, ultimately you will get $80 \pi^2 dl^2$ by λ_0^2 whole square then $I^2 r$ I square sorry 15

into 2 that will give you ah 15 into 2 30 30. So, it will be 40 not it will be 40 pi square this watts. Now this I can write as an RMS value for current because generally that is specified.

So, all others are same pi square dl by lambda naught square then I can write it as I by root 2 ok. So, then this one will become 80 pi square into root 1. So, this is the your radiated power. So, we can consider conceptually that as if it is this power is useful power, but we can consider everything in terms of circuit theory that this is getting ah dissipated into a resistance which is useful resistance that is called radiation resistance. So, and we can denote I by root 2 as I rms. So, I can say that p rad is 80 pi square dl by lambda naught square into I rms square and that is getting dissipated across a I square r type of thing. So, I can say this is equal to I rms square half I rms square into r rad. Similarly we say that when r is dissipating r should be small, but for an antenna we want that r rad should be higher because it is the useful thing actually there is no ready resistance in the universe, but in free space in the whole universe, but we consider that whole power where it is going actually it is getting radiated, but we consider it as if it is dissipating across r rad. So, from this I can say that r rad is r rad is that is I square r that is not half y half will come I square r. So, r rad is p rad by I rms square and that is a very well known formula 80 pi square dl by lambda naught square and r rads unit is ohm. So, let us see that what is the radiation resistance of a current element. So, suppose I have a 1 centimeter long current element at 300 megahertz that means, the current is a sinusoidal current at 300 megahertz. So, what will be it is r rad if you calculate it is radiation resistance will come out to be if you put this formula pi square is roughly 10. So, 800 into this so, that will be 79 milliohm. So, in order to radiate 1 watt of power we require a rms current. So, I want to radiate 1 watt power through this antenna. So, what will be the rms current? I rms will be you see that I want to radiate 1 watt. So, I rms will be 1 by root over 1 by that 79 milliohm ampere and that will require quite a huge current 3.56 ampere rms current will be required just to radiate 1 watt of power. So, a 1 centimeter current element actually will be burnt if you give this.

Handwritten derivation for radiation resistance of a current element:

$$\begin{aligned}
 &= 40 \pi^2 \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}|^2 \text{ W} \\
 &= 80 \pi^2 \left(\frac{dl}{\lambda_0}\right)^2 \left|\frac{\tilde{I}}{\sqrt{2}}\right|^2 \text{ W} \\
 P_{\text{rad}} &= 80 \pi^2 \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}_{\text{rms}}|^2 \text{ W} \\
 &= |\tilde{I}_{\text{rms}}|^2 R_{\text{rad}} \\
 R_{\text{rad}} &= \frac{P_{\text{rad}}}{|\tilde{I}_{\text{rms}}|^2} = 80 \pi^2 \left(\frac{dl}{\lambda_0}\right)^2 \Omega
 \end{aligned}$$

1 cm current element @ 300 MHz

$$R_{\text{rad}} = 79 \text{ m}\Omega$$

1W power $\rightarrow I_{\text{rms}} = \sqrt{\frac{1}{79 \times 10^{-3}}} \text{ A} = 3.56 \text{ A}$

Now if I want to do it at 3 mega let us say I lower the frequency suppose at 3 megahertz then the radiation resistance becomes 7.9 micro ohm and so, to radiate again 1 watt of power we require I_{rms} required is 356 ampere. So, you see the what is the conclusion from this that current element is a very inefficient radiator. So, we need to have in our EMC experiment etcetera we cannot use this current element we want to go for some other better antenna, but it is an elemental antenna because you see current element any arbitrary antenna is made of you can say made from this current element. If I have an antenna like this so, there I can make that this is a current element, this is a current element, this is a current element like that if I do then and I know from each what is the field. So, by linearity I can find out the field. So, this is a basic structure. So, and also people have antenna researcher have researchers have found that the far field structure that it gives whatever the characteristic of the far field that is same for all these arbitrary antennas because it is made of them. So, the characteristic what we have seen for current element that is written for any type of wire antenna ok, for antenna or even aperture antennas also because they can be also by applying some principles can be thought of as made from current elements not directly some duality etcetera type of things are required to understand how they are made, but the structure you can take that for all practical antennas they are the they gives the structure. So, in the next class we will find a better antenna which is a dipole antenna. Thank you.

@ 3 MHz
 $R_{rad} \rightarrow 7.9 \mu\Omega$
 $I_{rms} \rightarrow 356 A !!!$

The diagram shows a black oval representing a current element. Inside the oval, there is a red dashed line. A small cross is drawn on the left side of the oval.